Problem #2A
Date: 10/25/93
Problem Originator: [redacted]
Reviewer: [redacted]

Student Problem

Title: Electric Field and Electric Potential analysis in a spherical system

Abstract:

The electric field and electric potential were calculated and plotted in a spherical system. The system consists of two volume charges and a surface charge that was surrounded by an insulator. The relative permitivity of the insulator was varied.

As the relative permittivity of the insulator was varied, the electric field and the electric potential showed an insignificant change .

[redacted]

Ken Kaiser

Stating known values:

$r_1 := 0, 0.0001 .. 0.001$ m

$r_2 := 0.001, 0.0011 .. 0.003$ m

$r_3 := 0.003, 0.0031 .. 0.006$ m

$\varepsilon_o := \frac{1}{36 \cdot \pi} \cdot 10^{-9}$ F/m

$\varepsilon_1 := \varepsilon_o$ F/m

$\varepsilon_2 := 5 \cdot \varepsilon_o$ F/m

$\rho_s := 40 \cdot 10^{-6}$ C/m^2

$\rho_{v1} := 30 \cdot 10^{-6}$ C/m^3

$\rho_{v2} := 50 \cdot 10^{-6}$ C/m^3

I. Solution of the general Poisson's equation

Poisson's equation is;

$$Del^2 \cdot \Phi := -\frac{\rho_v}{\varepsilon}$$

... this yields;

$$\frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 \cdot \frac{d}{dr}\Phi\right) := -\frac{\rho_v}{\varepsilon}$$

Integrating this equation twice gives;

$$\Phi := \frac{-\rho_v \cdot r^2}{6 \cdot \varepsilon} - \frac{A}{r} + B$$

For region 3, where there is no volume charge, Poisson's equation is;

$$Del^2 \cdot \Phi := 0$$

$$\Rightarrow \frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 \cdot \frac{d}{dr}\Phi\right) := 0$$

Integrating this equation twice gives;

$$\Phi := \frac{-A}{r} + B$$

II. Specifying the boundary conditions

Since there are three separate materials in the spherical system, three different versions of the generic Poisson's equation solution are needed to describe the system. Two equations for the regions containing volume charges and another for the region with no volume charge. Since there are two unknowns for each equation, six separate boundary conditions will be needed to solve for the four unknowns. The boundary conditions are as follows;

$$\Phi_1(0) := \text{Finite}$$

$$\Phi_1(0.001) := \Phi_2(0.001)$$

$$\Phi_2(0.003) := \Phi_3(0.003)$$

$$\Phi_3(0.006) := 0$$

$$D_{n2} - D_{n1} := \rho_s$$

$$D_{n3} - D_{n2} := 0$$

... at the interfaces

III. Solving for the unknowns with the boundary conditions

Since the electrical potential at r=0 is finite, A must equal zero in the equation for the potential in the first region resulting in the equation;

$$\Phi_1 := \frac{-\rho_v \cdot r_1^{\,2}}{6\cdot\varepsilon_1} + B$$

Stepping through values of relative permittivity of 8, 10, and 12 for region #3

Case #1: $\varepsilon_3 := 8\cdot\varepsilon_o$ F/m

Initial guesses;

$B := 500$ $C := 500$ $D := 500$ $E := 500$ $F := 500$

Given

$$\frac{-\rho_{v1}\cdot 0.001^2}{6\cdot\varepsilon_1} + B = \frac{-\rho_{v2}\cdot 0.001^2}{6\cdot\varepsilon_2} - \frac{C}{0.001} + D$$

$$\frac{-\rho_{v2}\cdot 0.003^2}{6\cdot\varepsilon_2} - \frac{C}{0.003} + D = -\frac{E}{0.003} + F$$

$$-\frac{E}{0.006} + F = 0$$

$$\varepsilon_1\cdot\left(\frac{-\rho_{v1}\cdot 0.001}{3\cdot\varepsilon_1}\right) - \varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.001}{3\cdot\varepsilon_2} + \frac{C}{0.001^2}\right) = \rho_s$$

$$\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.003}{3\cdot\varepsilon_2} + \frac{C}{0.003^2}\right) - \varepsilon_3\cdot\left(\frac{F}{0.003^2}\right) = 0$$

$$\begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} := \text{Find}(B, C, D, E, F) \qquad \begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 605.73 \\ -0.905 \\ -299.274 \\ -0.003 \\ -0.572 \end{bmatrix}$$

Verifying the solutions;

$$\frac{-\rho_{v1}\cdot 0.001^2}{6\cdot\varepsilon_1} + B = 605.165$$

$$\frac{-\rho_{v2}\cdot 0.001^2}{6\cdot\varepsilon_2} - \frac{C}{0.001} + D = 605.165$$

$$\frac{-\rho_{v2}\cdot 0.003^2}{6\cdot\varepsilon_2} - \frac{C}{0.003} + D = 0.572$$

$$-\frac{E}{0.003} + F = 0.572$$

$$-\frac{E}{0.006} + F = 0$$

$$\varepsilon_1\cdot\left(\frac{-\rho_{v1}\cdot 0.001}{3\cdot\varepsilon_1}\right) - \varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.001}{3\cdot\varepsilon_2} + \frac{C}{0.001^2}\right) = 4\cdot 10^{-5}$$

$$\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.003}{3\cdot\varepsilon_2} + \frac{C}{0.003^2}\right) - \varepsilon_3\cdot\left(\frac{F}{0.003^2}\right) = 0$$

Using the constants obtained from the boundary conditions to solve for the equations;

$$\Phi 1_1(r_1) := \frac{-\rho_{v1}\cdot r_1^{\,2}}{6\cdot\varepsilon_1} + B \quad \text{V}$$

$$\Phi 1_2(r_2) := \frac{-\rho_{v2}\cdot r_2^{\,2}}{6\cdot\varepsilon_2} - \frac{C}{r_2} + D \quad \text{V}$$

$$E1_1(r_1) := -\frac{d}{dr_1}\Phi 1_1(r_1) \quad \text{V/m}$$

$$E1_2(r_2) := -\frac{d}{dr_2}\Phi 1_2(r_2) \quad \text{V/m}$$

$$\Phi 1_3(r_3) := -\frac{E}{r_3} + F \quad \text{V}$$

$$E1_3(r_3) := -\frac{d}{dr_3}\Phi 1_3(r_3) \quad \text{V/m}$$

Case #2: $\varepsilon_3 := 10\cdot\varepsilon_o$ F/m

Initial guesses;

$B := 500$ $C := 500$ $D := 500$ $E := 500$ $F := 500$

Given

$$\frac{-\rho_{v1}\cdot 0.001^2}{6\cdot\varepsilon_1} + B = \frac{-\rho_{v2}\cdot 0.001^2}{6\cdot\varepsilon_2} - \frac{C}{0.001} + D$$

$$\frac{-\rho_{v2}\cdot 0.003^2}{6\cdot\varepsilon_2} - \frac{C}{0.003} + D = -\frac{E}{0.003} + F$$

$$-\frac{E}{0.006} + F = 0$$

$$\varepsilon_1\cdot\left(\frac{-\rho_{v1}\cdot 0.001}{3\cdot\varepsilon_1}\right) - \varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.001}{3\cdot\varepsilon_2} + \frac{C}{0.001^2}\right) = \rho_s$$

$$\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.003}{3\cdot\varepsilon_2} + \frac{C}{0.003^2}\right) - \varepsilon_3\cdot\left(\frac{F}{0.003^2}\right) = 0$$

$$\begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} := \text{Find}(B, C, D, E, F) \qquad \begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 605.616 \\ -0.905 \\ -299.389 \\ -0.003 \\ -0.457 \end{bmatrix}$$

Verifying the solutions;

$$\frac{-\rho_{v1}\cdot 0.001^2}{6\cdot\varepsilon_1}+B=605.051$$

$$\frac{-\rho_{v2}\cdot 0.001^2}{6\cdot\varepsilon_2}-\frac{C}{0.001}+D=605.051$$

$$\frac{-\rho_{v2}\cdot 0.003^2}{6\cdot\varepsilon_2}-\frac{C}{0.003}+D=0.457$$

$$-\frac{E}{0.003}+F=0.457$$

$$-\frac{E}{0.006}+F=0$$

$$\varepsilon_1\cdot\left(\frac{-\rho_{v1}\cdot 0.001}{3\cdot\varepsilon_1}\right)-\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.001}{3\cdot\varepsilon_2}+\frac{C}{0.001^2}\right)=4\cdot 10^{-5}$$

$$\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.003}{3\cdot\varepsilon_2}+\frac{C}{0.003^2}\right)-\varepsilon_3\cdot\left(\frac{F}{0.003^2}\right)=0$$

Using the constants obtained from the boundary conditions to solve for the equations;

$$\Phi2_1(r_1):=\frac{-\rho_{v1}\cdot r_1^{\,2}}{6\cdot\varepsilon_1}+B \quad \text{V}$$

$$\Phi2_2(r_2):=\frac{-\rho_{v2}\cdot r_2^{\,2}}{6\cdot\varepsilon_2}-\frac{C}{r_2}+D \quad \text{V}$$

$$E2_1(r_1):=-\frac{d}{dr_1}\Phi1_1(r_1) \quad \text{V/m}$$

$$E2_2(r_2):=-\frac{d}{dr_2}\Phi1_2(r_2) \quad \text{V/m}$$

$$\Phi2_3(r_3):=-\frac{E}{r_3}+F \quad \text{V}$$

$$E2_3(r_3):=-\frac{d}{dr_3}\Phi1_3(r_3) \quad \text{V/m}$$

Case #3: $\varepsilon_3 := 12 \cdot \varepsilon_o$ F/m

Initial guesses;

B := 500 C := 500 D := 500 E := 500 F := 500

Given

$$\frac{-\rho_{v1} \cdot 0.001^2}{6 \cdot \varepsilon_1} + B = \frac{-\rho_{v2} \cdot 0.001^2}{6 \cdot \varepsilon_2} - \frac{C}{0.001} + D$$

$$\frac{-\rho_{v2} \cdot 0.003^2}{6 \cdot \varepsilon_2} - \frac{C}{0.003} + D = -\frac{E}{0.003} + F$$

$$-\frac{E}{0.006} + F = 0$$

$$\varepsilon_1 \cdot \left(\frac{-\rho_{v1} \cdot 0.001}{3 \cdot \varepsilon_1} \right) - \varepsilon_2 \cdot \left(\frac{-\rho_{v2} \cdot 0.001}{3 \cdot \varepsilon_2} + \frac{C}{0.001^2} \right) = \rho_s$$

$$\varepsilon_2 \cdot \left(\frac{-\rho_{v2} \cdot 0.003}{3 \cdot \varepsilon_2} + \frac{C}{0.003^2} \right) - \varepsilon_3 \cdot \left(\frac{F}{0.003^2} \right) = 0$$

$$\begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} := \mathrm{Find}(B, C, D, E, F) \qquad \begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 605.54 \\ -0.905 \\ -299.465 \\ -0.002 \\ -0.381 \end{bmatrix}$$

Verifying the solutions;

$$\frac{-\rho_{v1}\cdot 0.001^2}{6\cdot\varepsilon_1}+B=604.974$$

$$\frac{-\rho_{v2}\cdot 0.001^2}{6\cdot\varepsilon_2}-\frac{C}{0.001}+D=604.974$$

$$\frac{-\rho_{v2}\cdot 0.003^2}{6\cdot\varepsilon_2}-\frac{C}{0.003}+D=0.381$$

$$-\frac{E}{0.003}+F=0.381$$

$$-\frac{E}{0.006}+F=0$$

$$\varepsilon_1\cdot\left(\frac{-\rho_{v1}\cdot 0.001}{3\cdot\varepsilon_1}\right)-\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.001}{3\cdot\varepsilon_2}+\frac{C}{0.001^2}\right)=4\cdot 10^{-5}$$

$$\varepsilon_2\cdot\left(\frac{-\rho_{v2}\cdot 0.003}{3\cdot\varepsilon_2}+\frac{C}{0.003^2}\right)-\varepsilon_3\cdot\left(\frac{F}{0.003^2}\right)=0$$

Using the constants obtained from the boundary conditions to solve for the equations;

$$\Phi3_1(r_1):=\frac{-\rho_{v1}\cdot r_1^2}{6\cdot\varepsilon_1}+B \quad \text{V}$$

$$\Phi3_2(r_2):=\frac{-\rho_{v2}\cdot r_2^2}{6\cdot\varepsilon_2}-\frac{C}{r_2}+D \quad \text{V}$$

$$E3_1(r_1):=-\frac{d}{dr_1}\Phi1_1(r_1) \quad \text{V/m}$$

$$E3_2(r_2):=-\frac{d}{dr_2}\Phi1_2(r_2) \quad \text{V/m}$$

$$\Phi3_3(r_3):=-\frac{E}{r_3}+F \quad \text{V}$$

$$E3_3(r_3):=-\frac{d}{dr_3}\Phi1_3(r_3) \quad \text{V/m}$$

IV. Consolidating all the plot sections

$r := 0, 0.0001 .. 0.006$ m

Electric Potentials;

$\Phi1(r) := \text{if}\left(r<0.001, \Phi1_1(r), \text{if}\left(r<0.003, \Phi2_2(r), \Phi1_3(r)\right)\right)$ V

$\Phi2(r) := \text{if}\left(r<0.001, \Phi2_1(r), \text{if}\left(r<0.003, \Phi2_2(r), \Phi2_3(r)\right)\right)$ V

$\Phi3(r) := \text{if}\left(r<0.001, \Phi3_1(r), \text{if}\left(r<0.003, \Phi3_2(r), \Phi3_3(r)\right)\right)$ V

Electric Fields;

$E1(r) := \text{if}\left(r<0.001, E1_1(r), \text{if}\left(r<0.003, E2_2(r), E1_3(r)\right)\right)$ V/m

$E2(r) := \text{if}\left(r<0.001, E2_1(r), \text{if}\left(r<0.003, E2_2(r), E2_3(r)\right)\right)$ V/m

$E3(r) := \text{if}\left(r<0.001, E3_1(r), \text{if}\left(r<0.003, E3_2(r), E3_3(r)\right)\right)$ V/m

V. Consolidating all the plot sections to obtain a more readable graph of region 3

$r_a := 0.003, 0.0031 .. 0.006$ m

Electric Potentials for region 3;

$\Phi1_a(r_a) := \Phi1_3(r_a)$ V

$\Phi2_a(r_a) := \Phi2_3(r_a)$ V

$\Phi3_a(r_a) := \Phi3_3(r_a)$ V

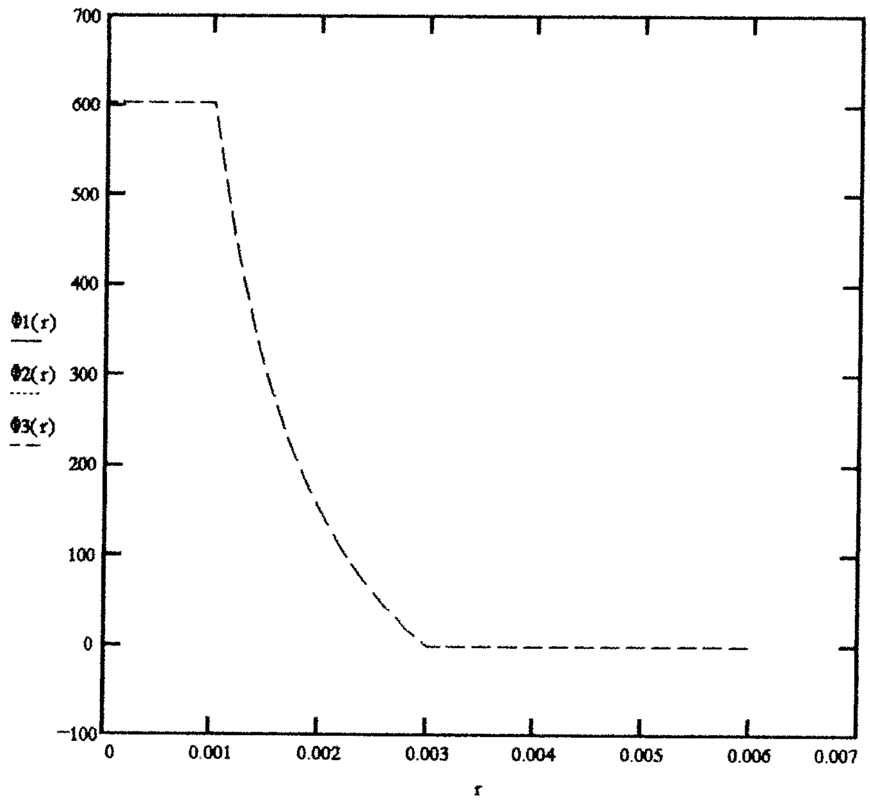
Electric Fields for region 3;

$E1_a(r_a) := -\frac{d}{dr_a}\Phi1_a(r_a)$

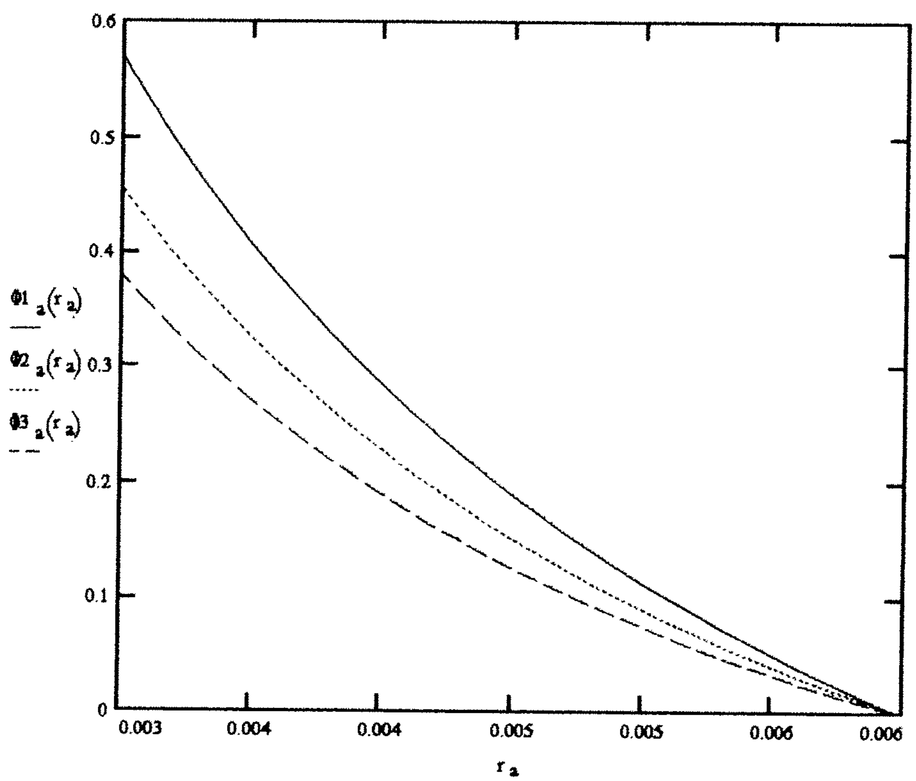
$E2_a(r_a) := -\frac{d}{dr_a}\Phi2_a(r_a)$

$E3_a(r_a) := -\frac{d}{dr_a}\Phi3_a(r_a)$

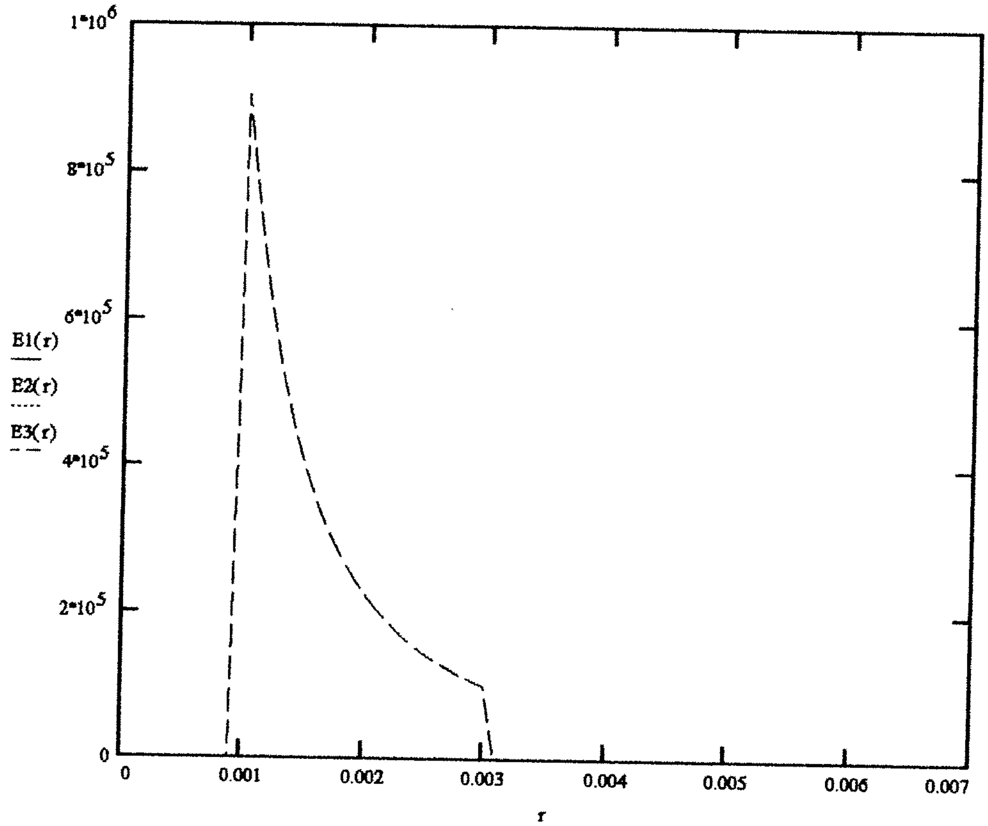
VI. The Electric Potential as a function of the radius



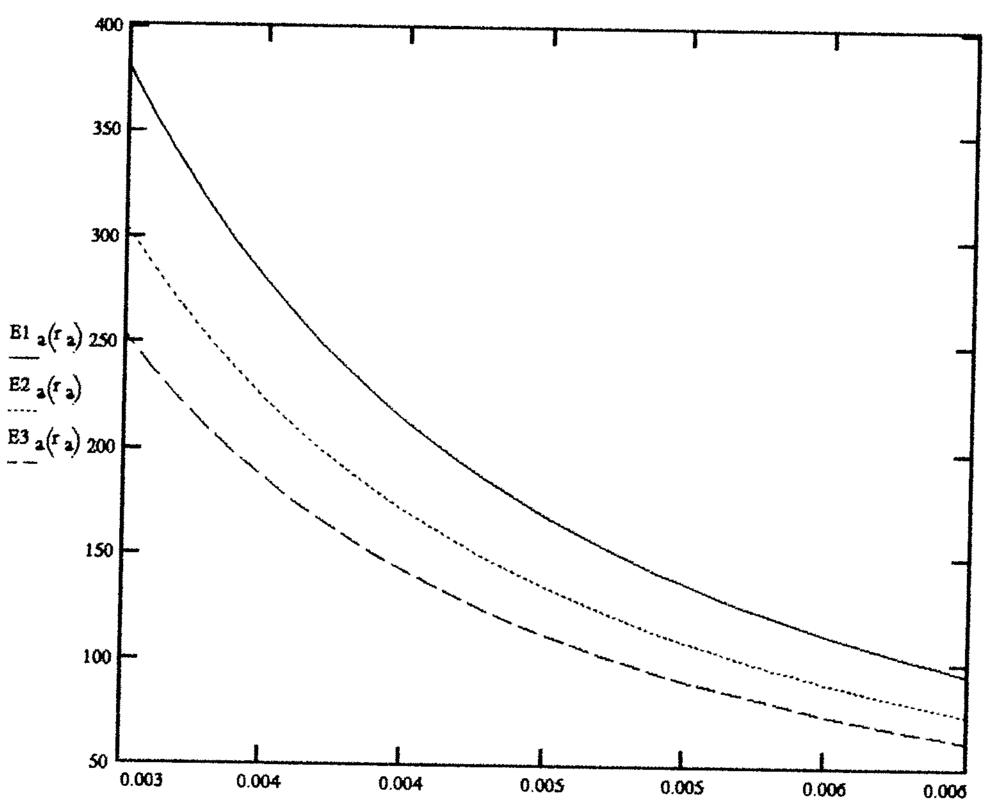
VII. The Electric Potential as a function of the radius for region 3



VIII. The Electric Field as a function of the radius



IX. Plotting the Electric Field as a function of the radius in region 3



The electric potential and electric field plots differ slightly for the three different relative permittivity of 8, 10, and 12 for region 3. It can be concluded that varying the relative permitivy of the outer conductor from 8 to 12 has little effect on the electric potential and the electric field.

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[redacted]

Problem Number #2A
Date: 10/18/93
Problem Originator: [redacted] A
Reviewer: [redacted]

Student Problem

Project Title: **Analysis of Three Industrial Grade Cable Prototypes**

Abstract:

Three prototypes of an industrial grade cable are presented in this paper. The analysis involves using polyfoam, polyvinyl and neoprene rubber for the outermost insulator and determining the electric potential and electric field functions for each design. The permittivity of the inner insulator remains constant. The cable is modeled as being infinitely long with copper as the center conductor. The general form of the electric potential function is derived using Poisson's equation. In addition, details concerning the boundary conditions for the cable are provided in the paper. The electric potential and electric field functions are plotted for each of the prototypes.

The electric potential and electric field display significant variation for each of the prototype cable designs as a result of the different relative permittivities associated with the three materials.

PROBLEM SET-UP:

Given:

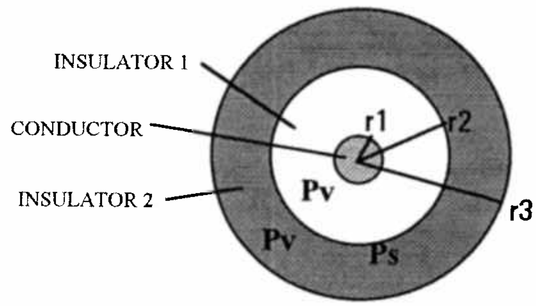
$\varepsilon_o = \dfrac{10^{-9}}{36\cdot\pi}$ **F/m** $\qquad \rho_s := 45\cdot 10^{-6}$ **C/m^2** $\qquad \rho_v := \left(20 - 12\cdot r^2\right)\cdot 10^{-6}$ **C/m^3**

$r_1 := 0.002$ **m**

$r_2 := 0.015$ **m**

$r_3 := 0.030$ **m**

CABLE CROSS SECTION



Outer Insulator Materials:

polyfoam: $\varepsilon_{pf} := 1.05\cdot\varepsilon_o$

polyvinyl: $\varepsilon_{pv} := 3.2\cdot\varepsilon_o$

neoprene: (rubber) $\varepsilon_{nr} := 5\cdot\varepsilon_o$

Inner Insulator: $\varepsilon_{in} := 4\cdot\varepsilon_o$

Radius limits defined for Plotting:

$r_1 := 0.0005, 0.0006 .. 0.002$

$r_2 := 0.002, 0.003 .. 0.015$

$r_3 := 0.015, 0.016 .. 0.030$

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I. SOLUTION OF POISSON'S EQUATION

$$Del^2 \cdot \Phi := -\left(\frac{\rho_v}{\varepsilon}\right)$$

$$Del^2 \cdot \Phi := \frac{-\left(20 - 12 \cdot r^2\right)}{\varepsilon}$$

For Cylindrical coordinate systems: $Del^2 \cdot \Phi := \frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{d}{dr}\Phi\right)$

... only r varies in this problem (phi and z are excluded from the Laplacian formula)

Therefore: $\frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{d}{dr}\Phi\right) := \left(\frac{12 \cdot r^2 - 20}{\varepsilon}\right) \cdot 10^{-6}$

By Integration, the general form of the the potential function is as follows:

$$\Phi_g := \left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon}\right) + C_1 \cdot \ln(r) + C_2$$

II. BOUNDARY CONDITIONS

For the cable design, there are two materials used as insulators. The copper conductor is treated as a perfect conductor; therefore, the volume charge distribution in the conductor is equal to zero. Four boundary conditions are required since there are two materials. The volume charge distribution varies within the two insulators according to the equation stated above. For this region (0.002 < r < 0.030 m), the electric potential function is solved for each insulator. Hence, the four boundary conditions determine the equations necessary to solve for the four unknowns in the problem. The electric and potential functions are combined in piece-wise fashion to describe the entire cable.

$\Phi1(0.002) := 600$ **... for r = 0.002 m**

$\Phi2(0.030) := 0$ **... for r = 0.030 m**

$\Phi1(0.015) := \Phi2(0.015)$ **... for r = 0.015 m**

$D_{n2} - D_{n1} := \rho_s$ **... for r = 0.015 m**

==>where D_{n2} refers to the outer material and D_{n1} refers to the inner material. ρ_s is the surface charge at the interface of the inner and outer insulators.

III. DERIVATION OF THE FOURTH BOUNDARY CONDITION:

$$D_{n2} - D_{n1} = \rho_s$$

$$\varepsilon_2 \cdot E_{n2} - \varepsilon_1 \cdot E_{n1} = \rho_s$$

$$\varepsilon_2 \cdot (-Del \cdot \Phi 2) - \varepsilon_1 \cdot (-Del \cdot \Phi 1) = \rho_s$$

$$\Phi_g := \left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon}\right) \cdot 10^{-6} + C_1 \cdot \ln(r) + C_2$$ **... general form of the potential function**

$$\Phi 1 := \left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon_1}\right) \cdot 10^{-6} + W \cdot \ln(r) + X \qquad \Phi 2 := \left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon_2}\right) \cdot 10^{-6} + Y \cdot \ln(r) + Z$$

NOTE:

e_1 corresponds to the permittivity of the inner insulator and e_2 corresponds to the permittivity of the outer insulator--actual values for e_1 and e_2 are stated in the PROBLEM SET-UP.

By differentiating each potential function:

$$\rho_s := \varepsilon_2 \left[\left(\frac{10 \cdot r - 3 \cdot r^3}{\varepsilon_2}\right) \cdot 10^{-6} - \frac{Y}{r}\right] - \varepsilon_1 \cdot \left[\left(\frac{10 \cdot r - 3 \cdot r^3}{\varepsilon_1}\right) \cdot 10^{-6} - \frac{W}{r}\right]$$

$$\rho_s := \frac{W \cdot \varepsilon_1}{r} - \frac{Y \cdot \varepsilon_2}{r}$$ **... for r = 0.015 m** **...the fourth boundary condition.**

IV. DERIVATION OF THE THIRD BOUNDARY CONDITION:

$$\Phi 1(0.015) := \Phi 2(0.015)$$

$$\left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon_1}\right) \cdot 10^{-6} + W \cdot \ln(r) + X = \left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon_2}\right) \cdot 10^{-6} + Y \cdot \ln(r) + Z$$

V. THE FOUR BOUNDARY CONDITIONS:

1) $\Phi 1(0.002) = 600$

2) $\Phi 2(0.030) = 0$

3) $\left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon_1}\right) \cdot 10^{-6} + W \cdot \ln(r) + X = \left(\frac{0.75 \cdot r^4 - 5 \cdot r^2}{\varepsilon_2}\right) \cdot 10^{-6} + Y \cdot \ln(r) + Z$ **... for r = 0.015 m**

4) $\rho_s := \frac{W \cdot \varepsilon_1}{r} - \frac{Y \cdot \varepsilon_2}{r}$ **... for r = 0.015 m**

VI. DESIGN #1: POLYFOAM

$W := 500 \qquad X := 500 \qquad Y := 500 \qquad Z := 500$

Given

$$\left(\frac{0.75\cdot 0.002^4 - 5\cdot 0.002^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.002) + X = 600$$

$$\left(\frac{0.75\cdot 0.030^4 - 5\cdot 0.030^2}{\varepsilon_{pf}}\right)\cdot 10^{-6} + Y\cdot \ln(0.030) + Z = 0$$

$$\left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.015) + X = \left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{pf}}\right)\cdot 10^{-6} + Y\cdot \ln(0.015) + Z$$

$$\frac{W\cdot \varepsilon_{in}}{0.015} - \frac{Y\cdot \varepsilon_{pf}}{0.015} = \rho_s$$

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} := \text{Find}(W, X, Y, Z) \qquad \begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.078094\cdot 10^4 \\ 6.759988\cdot 10^4 \\ -3.163519\cdot 10^4 \\ -1.10446\cdot 10^5 \end{bmatrix}$$

...solutions of unknown constants for polyfoam.

****Refer to next page for verification of these solutions**

VII. ELECTRIC POTENTIAL AND ELECTRIC FIELD (POLYFOAM):

	Electric Potential	***Electric Field***
(r<0.002): **(copper is treated as a perfect conductor)**	$\Phi1_{pf}(r_1) := 600$	$E1_{pf}(r_1) := 0$
(0.002<r<0.015):	$\Phi2_{pf}(r_2) := \left(\frac{0.75\cdot r_2^{\,4}}{\varepsilon_{in}} - \frac{5\cdot r_2^{\,2}}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(r_2) + X$	$E2_{pf}(r_2) := -\frac{d}{dr_2}\Phi2_{pf}(r_2)$
(0.015<r<0.030):	$\Phi3_{pf}(r_3) := \left(\frac{0.75\cdot r_3^{\,4}}{\varepsilon_{pf}} - \frac{5\cdot r_3^{\,2}}{\varepsilon_{pf}}\right)\cdot 10^{-6} + Y\cdot \ln(r_3) + Z$	$E3_{pf}(r_3) := -\frac{d}{dr_3}\Phi3_{pf}(r_3)$

Verification of Solutions:

$$\left(\frac{0.75\cdot 0.002^4 - 5\cdot 0.002^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.002) + X = 600$$

$$\left(\frac{0.75\cdot 0.030^4 - 5\cdot 0.030^2}{\varepsilon_{pf}}\right)\cdot 10^{-6} + Y\cdot \ln(0.030) + Z = -1.455192\cdot 10^{-11}$$

$$\left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.015) + X = 2.229131\cdot 10^{4}$$

$$\left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{pf}}\right)\cdot 10^{-6} + Y\cdot \ln(0.015) + Z = 2.229131\cdot 10^{4}$$

$$\frac{W\cdot \varepsilon_{in}}{0.015} - \frac{Y\cdot \varepsilon_{pf}}{0.015} = 4.5\cdot 10^{-5}$$

VIII. DESIGN #2: POLYVINYL

$W := 500$ $X := 500$ $Y := 500$ $Z := 500$

Given

$$\left(\frac{0.75\cdot 0.002^4 - 5\cdot 0.002^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.002) + X = 600$$

$$\left(\frac{0.75\cdot 0.030^4 - 5\cdot 0.030^2}{\varepsilon_{pv}}\right)\cdot 10^{-6} + Y\cdot \ln(0.030) + Z = 0$$

$$\left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.015) + X = \left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{pv}}\right)\cdot 10^{-6} + Y\cdot \ln(0.015) + Z$$

$$\frac{W\cdot \varepsilon_{in}}{0.015} - \frac{Y\cdot \varepsilon_{pv}}{0.015} = \rho_s$$

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} := \mathrm{Find}(W, X, Y, Z)$$

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 5.583\cdot 10^{3} \\ 3.5297\cdot 10^{4} \\ -1.6878\cdot 10^{4} \\ -5.9024\cdot 10^{4} \end{bmatrix}$$

...constants for the polyvinyl analysis.

$$\frac{W\cdot \varepsilon_{in}}{0.015} - \frac{Y\cdot \varepsilon_{pv}}{0.015} = 4.5\cdot 10^{-5}$$

...verification of the solutions

IX. ELECTRIC POTENTIAL AND ELECTRIC FIELD (POLYVINYL):

Electric Potential / ***Electric Field***

(r<0.002): $\Phi 1_{pv}(r_1) := 600$ $E1_{pv}(r_2) := 0$

(0.002<r<0.015): $\Phi 2_{pv}(r_2) := \left(\frac{0.75\cdot r_2^{\,4}}{\varepsilon_{in}} - \frac{5\cdot r_2^{\,2}}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(r_2) + X$ $E2_{pv}(r_2) := -\frac{d}{dr_2}\Phi 2_{pv}(r_2)$

(0.015<r<0.030): $\Phi 3_{pv}(r_3) := \left(\frac{0.75\cdot r_3^{\,4}}{\varepsilon_{pv}} - \frac{5\cdot r_3^{\,2}}{\varepsilon_{pv}}\right)\cdot 10^{-6} + Y\cdot \ln(r_3) + Z$ $E3_{pv}(r_3) := -\frac{d}{dr_3}\Phi 3_{pv}(r_3)$

X. DESIGN #3: NEOPRENE RUBBER

$W := 500 \qquad X := 500 \qquad Y := 500 \qquad Z := 500$

Given

$$\left(\frac{0.75\cdot 0.002^4 - 5\cdot 0.002^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.002) + X = 600$$

$$\left(\frac{0.75\cdot 0.030^4 - 5\cdot 0.030^2}{\varepsilon_{nr}}\right)\cdot 10^{-6} + Y\cdot \ln(0.030) + Z = 0$$

$$\left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(0.015) + X = \left(\frac{0.75\cdot 0.015^4 - 5\cdot 0.015^2}{\varepsilon_{nr}}\right)\cdot 10^{-6} + Y\cdot \ln(0.015) + Z$$

$$\frac{W\cdot \varepsilon_{in}}{0.015} - \frac{Y\cdot \varepsilon_{nr}}{0.015} = \rho_s$$

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} := \mathrm{Find}(W, X, Y, Z) \qquad \begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3.9272\cdot 10^3 \\ 2.5007\cdot 10^4 \\ -1.2126\cdot 10^4 \\ -4.242\cdot 10^4 \end{bmatrix}$$

... constants for neoprene rubber analysis.

$$\frac{W\cdot \varepsilon_{in}}{0.015} - \frac{Y\cdot \varepsilon_{nr}}{0.015} = 4.5\cdot 10^{-5}$$

...verification of solutions

XI.ELECTRIC POTENTIAL AND ELECTRIC FIELD (NEOPRENE RUBBER):

	Electric Potential	***Electric Field***
(r<0.002):	$\Phi1_{nr}(r_1) := 600$	$E1_{nr}(r_2) := 0$
(0.002<r<0.015):	$\Phi2_{nr}(r_2) := \left(\frac{0.75\cdot r_2^{\,4}}{\varepsilon_{in}} - \frac{5\cdot r_2^{\,2}}{\varepsilon_{in}}\right)\cdot 10^{-6} + W\cdot \ln(r_2) + X$	$E2_{nr}(r_2) := -\frac{d}{dr_2}\Phi2_{nr}(r_2)$
(0.015<r<0.030):	$\Phi3_{nr}(r_3) := \left(\frac{0.75\cdot r_3^{\,4}}{\varepsilon_{nr}} - \frac{5\cdot r_3^{\,2}}{\varepsilon_{nr}}\right)\cdot 10^{-6} + Y\cdot \ln(r_3) + Z$	$E3_{nr}(r_3) := -\frac{d}{dr_3}\Phi3_{nr}(r_3)$

XII. PIECE-WISE COMBINATION OF THE FUNCTIONS

Electric Potential:

NOTE:
This Mcad version does not support "nested if-conditional" functions.

$r := 0, 0.0005 .. 0.030$

$\Phi a_{pf}(r) := \text{if}(r<0.015, \Phi 2_{pf}(r), \Phi 3_{pf}(r))$

$\Phi_{pf}(r) := \text{if}(r<0.002, \Phi 1_{pf}(r), \Phi a_{pf}(r))$ **...polyfoam outer insulator**

$\Phi a_{pv}(r) := \text{if}(r<0.015, \Phi 2_{pv}(r), \Phi 3_{pv}(r))$

$\Phi_{pv}(r) := \text{if}(r<0.002, \Phi 1_{pv}(r), \Phi a_{pv}(r))$ **...polyvinyl outer insulator**

$\Phi a_{nr}(r) := \text{if}(r<0.015, \Phi 2_{nr}(r), \Phi 3_{nr}(r))$

$\Phi_{nr}(r) := \text{if}(r<0.002, \Phi 1_{nr}(r), \Phi a_{nr}(r))$ **...neoprene rubber outer insulator**

Electric Field:

$Ea_{pf}(r) := \text{if}(r<0.015, E2_{pf}(r), E3_{pf}(r))$

$E_{pf}(r) := \text{if}(r<0.002, E1_{pf}(r), Ea_{pf}(r))$

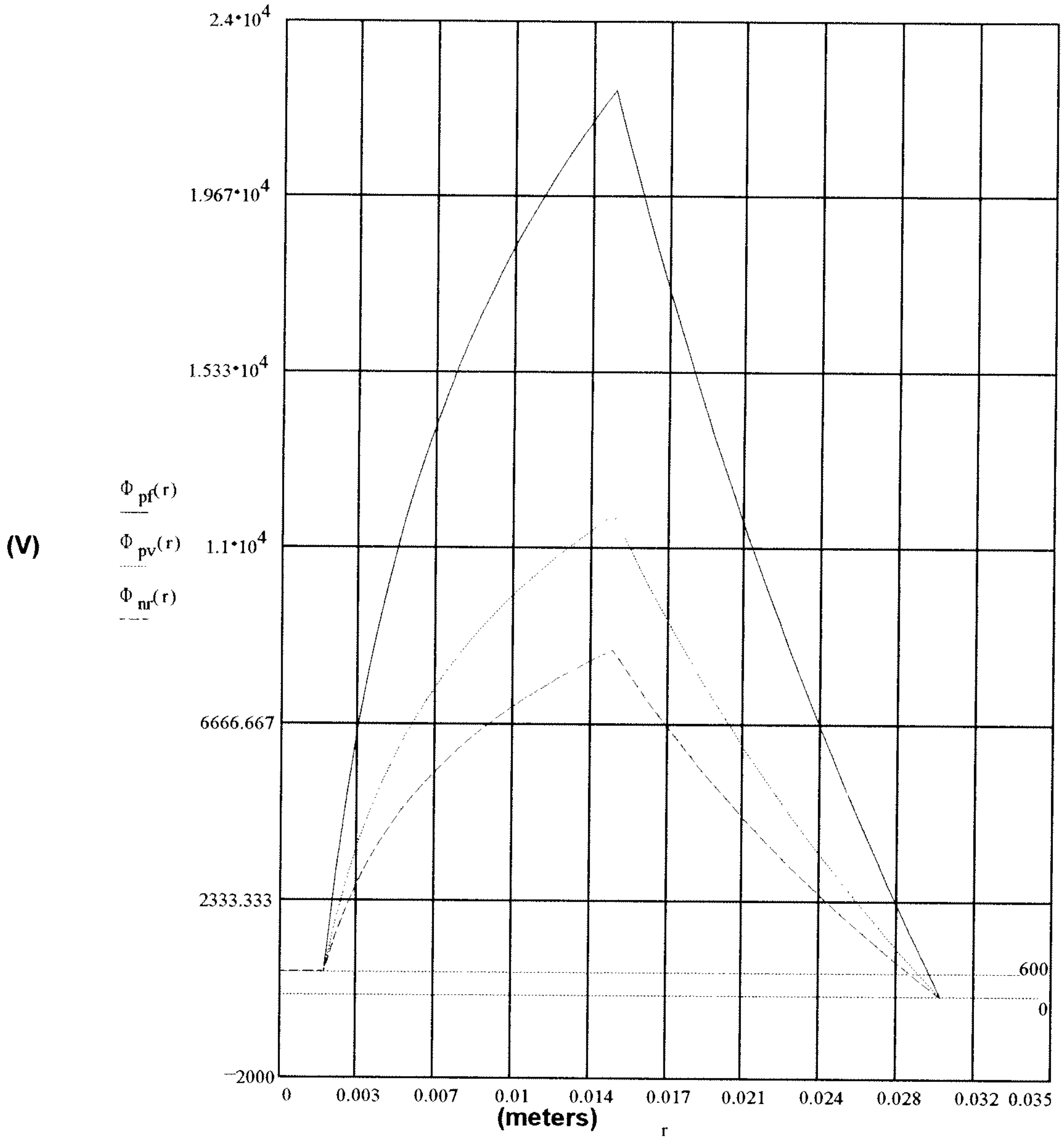
$Ea_{pv}(r) := \text{if}(r<0.015, E2_{pv}(r), E3_{pv}(r))$

$E_{pv}(r) := \text{if}(r<0.002, E1_{pv}(r), Ea_{pv}(r))$

$Ea_{nr}(r) := \text{if}(r<0.015, E2_{nr}(r), E3_{nr}(r))$

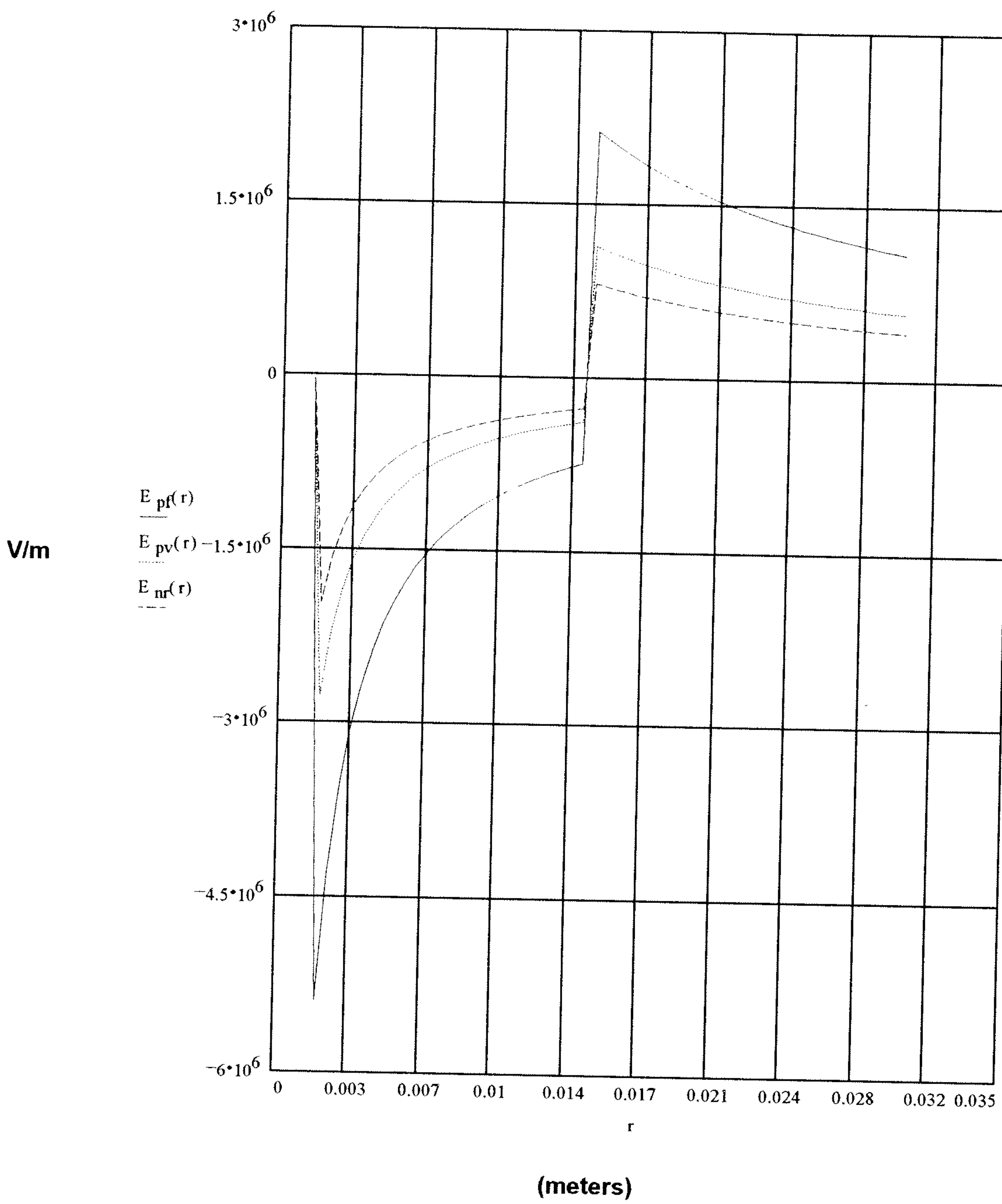
$E_{nr}(r) := \text{if}(r<0.002, E1_{nr}(r), Ea_{nr}(r))$

XIII. ELECTRIC POTENTIAL PLOT:



XIV. ELECTRIC FIELD PLOT:

Ken Kaiser



Summary of Results:

The electric potential and electric field display significant variation for each of the prototype cable designs as a result of the different relative permittivities associated with the three materials.

[redacted]

Problem Number #2
Date: September 15, 1993
Team Members: [redacted]

Student Problem

The Amusement Park Ride

A ride has an inner sphere of radius 5 meters and an outer sphere of radius 10 meters. The inner sphere, which has a volume charge density of 5 $\mu C/m^3$, also has a cushion material surrounding it. The relative permittivity of the cushion material is varied from 5 to 20 in steps of 5. This method is used in order to determine which material should be used to reduce the electric field inside the sphere. The outer surface of the cushion also has a surface charge density of 3 $\mu C/m^2$. The permittivities for this ride are $\varepsilon = 2\varepsilon_0$ for the inner sphere and ε_0 for the region between the inner and outer spheres. The outer sphere is at ground potential.

By evaluating Poisson's equation in the three separate regions, the potential and electric field distributions were determined. With this information, graphs were constructed to aid in the analysis of the problem. It was determined that as the permittivity is increased, in the region from 5 to 5.1 m, the electric field and the slope of the potential increase.

Using Poisson's equations in the spherical coordinate system, the constants were determined by integrating. Working from the inside of the sphere to the grounded outer sphere, boundary conditions were used to determine the constants of integration. At the interface between the cushion and the inner sphere (r = 5 meters), the normal electric flux densities are equal and the potentials V1 and V2 are also equal. At the interface between the cushion and the outer sphere (r = 5.1 meters), the difference between the electric flux densities is equal to the surface charge on the cushion. At this same interface the potentials V2 and V3 are equal. The last interface (r = 10 meters) is grounded, which implies V3 is equal to zero volts. The various properties outlined may be used in solving for the constants of integration at the various interfaces.

$\rho_v := 5\cdot 10^{-6}$ C/m^3 $\quad \rho_s := 3\cdot 10^{-6}$ C/m^2 $\quad \varepsilon_o := 8.854\cdot 10^{-12}$ F/m $\quad \varepsilon_1 := 2\cdot\varepsilon_o \quad \varepsilon_3 := \varepsilon_o$

$A_2 := 1\cdot 10^6 \quad A_3 := 1\cdot 10^6 \quad A_4 := 1\cdot 10^6 \quad A_5 := 1\cdot 10^6 \quad A_6 := 1\cdot 10^6$

$B_2 := 1\cdot 10^6 \quad B_3 := 1\cdot 10^6 \quad B_4 := 1\cdot 10^6 \quad B_5 := 1\cdot 10^6 \quad B_6 := 1\cdot 10^6$

$C_2 := 1\cdot 10^6 \quad C_3 := 1\cdot 10^6 \quad C_4 := 1\cdot 10^6 \quad C_5 := 1\cdot 10^6 \quad C_6 := 1\cdot 10^6$

$D_2 := 1\cdot 10^6 \quad D_3 := 1\cdot 10^6 \quad D_4 := 1\cdot 10^6 \quad D_5 := 1\cdot 10^6 \quad D_6 := 1\cdot 10^6$

Poisson's Equation for potential in the Spherical Coordinate System is solved below.

$$\frac{1}{r^2}\cdot\left[\frac{d}{dr}\left(r^2\cdot\frac{d}{dr}V(r)\right)\right] := \frac{-\rho_v}{\varepsilon}$$

$$\left[\frac{d}{dr}\left(r^2\cdot\frac{d}{dr}V(r)\right)\right] := \frac{-\rho_v}{\varepsilon}\cdot r^2$$

$$\left(r^2\cdot\frac{d}{dr}V(r)\right) := \frac{-1}{3}\cdot\frac{\rho_v}{\varepsilon}\cdot r^3 + C$$

$$\left(\frac{d}{dr}V(r)\right) := \frac{-1}{3}\cdot\frac{\rho_v}{\varepsilon}\cdot r + \frac{C}{r^2}$$

$$V(r) := \frac{-1}{6}\cdot\frac{\rho_v}{\varepsilon}\cdot r^2 - \frac{C}{r} + D$$

$$E := -\mathrm{grad}(V)$$

$$E_n := -\frac{d}{dr}V$$

$$E_n := \frac{1}{3}\cdot\frac{\rho_v}{\varepsilon}\cdot r + \frac{C}{r^2}$$

$$D_n := \varepsilon\cdot E_n$$

$$D_n := \varepsilon\cdot\left(\frac{1}{3}\cdot\frac{\rho_v}{\varepsilon}\cdot r + \frac{C}{r^2}\right)$$

Since the second term is undefined for r = 0, it is left out of the equation for V1(r).

$$V_1(r) := \frac{-1}{6}\cdot\frac{\rho_v}{\varepsilon}\cdot r^2 + A_2 \qquad V_2(r) := -\frac{A_3}{r} + A_4 \qquad V_3(r) := -\frac{A_5}{r} + C_6$$

$$D_{n1} := \varepsilon_1\cdot\left(\frac{1}{3}\cdot\frac{\rho_v}{\varepsilon_1}\cdot r\right) \qquad D_{n2} := -\varepsilon_2\cdot\left(\frac{A_3}{r^2}\right) \qquad D_{n3} := -\varepsilon_3\cdot\left(\frac{A_5}{r^2}\right)$$

$\varepsilon_2 := 5 \cdot \varepsilon_o$

Given

$$\varepsilon_1 \cdot \left(\frac{1}{3} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5 \right) = \varepsilon_2 \cdot \frac{A_3}{5^2} \qquad -\varepsilon_3 \cdot \left(\frac{A_5}{5.1^2} \right) + \varepsilon_2 \cdot \frac{A_3}{5.1^2} = \rho_s$$

$$-\frac{A_5}{10} + A_6 = 0 \qquad -\frac{A_5}{5.1} + A_6 = -\frac{A_3}{5.1} + A_4 \qquad -\frac{A_3}{5} + A_4 = \frac{-1}{6} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5^2 + A_2$$

$$\begin{bmatrix} A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} := \text{Find}\left(A_2, A_3, A_4, A_5, A_6\right) \qquad \begin{bmatrix} A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} -2.559 \cdot 10^5 \\ 4.706 \cdot 10^6 \\ -4.912 \cdot 10^5 \\ 1.472 \cdot 10^7 \\ 1.472 \cdot 10^6 \end{bmatrix}$$

$$\varepsilon_1 \cdot \left(\frac{1}{3} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5 \right) - \varepsilon_2 \cdot \frac{A_3}{5^2} = 0 \qquad \left[-\varepsilon_3 \cdot \left(\frac{A_5}{5.1^2} \right) + \varepsilon_2 \cdot \frac{A_3}{5.1^2} \right] = 3 \cdot 10^{-6} \qquad -\frac{A_5}{10} + A_6 = 0$$

$$\left(-\frac{A_5}{5.1} + A_6 \right) - \left(-\frac{A_3}{5.1} + A_4 \right) = 0 \qquad \left(-\frac{A_3}{5} + A_4 \right) - \left(\frac{-1}{6} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5^2 + A_2 \right) = 2.328 \cdot 10^{-10}$$

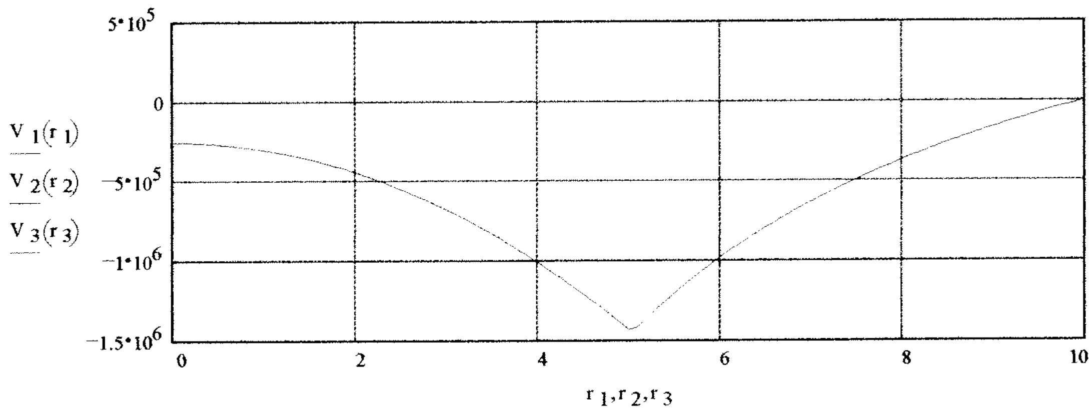
Ken Kaiser

$r_1 := 0, .05 .. 5 \quad r_2 := 5.0, 5.05 .. 5.1 \quad r_3 := 5.1, 5.15 .. 10$

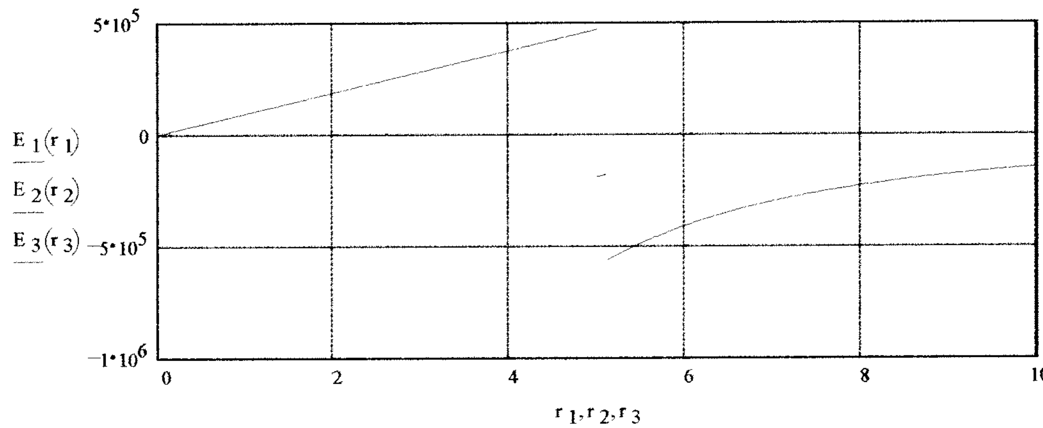
$V_1(r_1) := \frac{-\rho_v \cdot {r_1}^2}{6 \cdot \varepsilon_1} + A_2$ Volts $\qquad E_1(r_1) := -\frac{d}{dr_1} V_1(r_1)$ Volts/meter

$V_2(r_2) := \frac{-A_3}{r_2} + A_4$ Volts $\qquad E_2(r_2) := -\frac{d}{dr_2} V_2(r_2)$ Volts/meter

$V_3(r_3) := -\frac{A_5}{r_3} + A_6$ Volts $\qquad E_3(r_3) := -\frac{d}{dr_3} V_3(r_3)$ Volts/meter



The above graph shows that the potential is continuous in the region 0 < r < 10 meters.



The above graph shows that the electric field is not continuous in the region 0 < r < 10 meters.

$\varepsilon_2 := 10\cdot\varepsilon_o$

Ken Kaiser

Given

$$\varepsilon_1\cdot\left(\frac{1}{3}\cdot\frac{\rho_v}{\varepsilon_1}\cdot 5\right)=\varepsilon_2\cdot\frac{B_3}{5^2} \qquad -\varepsilon_3\cdot\left(\frac{B_5}{5.1^2}\right)+\varepsilon_2\cdot\frac{B_3}{5.1^2}=\rho_s$$

$$-\frac{B_5}{10}+B_6=0 \qquad -\frac{B_5}{5.1}+B_6=-\frac{B_3}{5.1}+B_4 \qquad -\frac{B_3}{5}+B_4=\frac{-1}{6}\cdot\frac{\rho_v}{\varepsilon_1}\cdot 5^2+B_2$$

$$\begin{bmatrix} B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} := \text{Find}(B_2, B_3, B_4, B_5, B_6) \qquad \begin{bmatrix} B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} -2.467\cdot 10^5 \\ 2.353\cdot 10^6 \\ -9.526\cdot 10^5 \\ 1.472\cdot 10^7 \\ 1.472\cdot 10^6 \end{bmatrix}$$

$$\varepsilon_1\cdot\left(\frac{1}{3}\cdot\frac{\rho_v}{\varepsilon_1}\cdot 5\right)-\varepsilon_2\cdot\frac{B_3}{5^2}=0 \qquad \left[-\varepsilon_3\cdot\left(\frac{B_5}{5.1^2}\right)+\varepsilon_2\cdot\frac{B_3}{5.1^2}\right]=3\cdot 10^{-6} \qquad -\frac{B_5}{10}+B_6=0$$

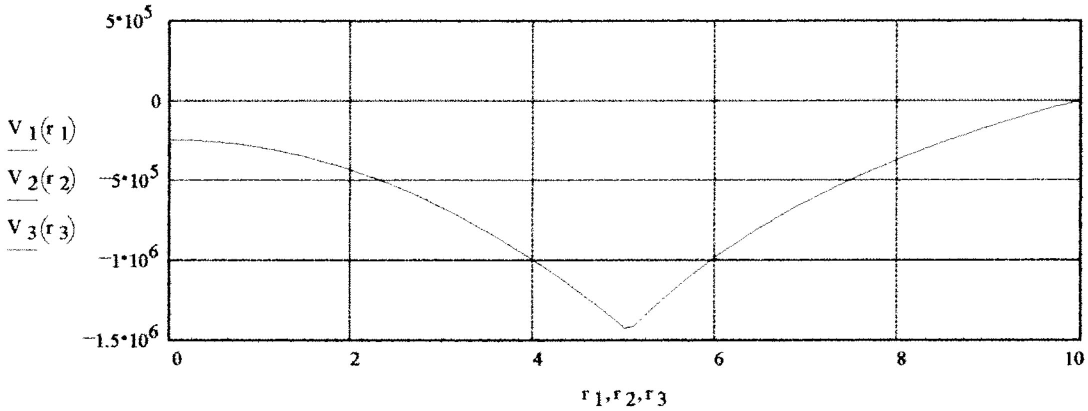
$$\left(-\frac{B_5}{5.1}+B_6\right)-\left(-\frac{B_3}{5.1}+B_4\right)=0 \qquad \left(-\frac{B_3}{5}+B_4\right)-\left(\frac{-1}{6}\cdot\frac{\rho_v}{\varepsilon_1}\cdot 5^2+B_2\right)=0$$

$r_1 := 0, .05 .. 5 \quad r_2 := 5.0, 5.05 .. 5.1 \quad r_3 := 5.1, 5.15 .. 10$

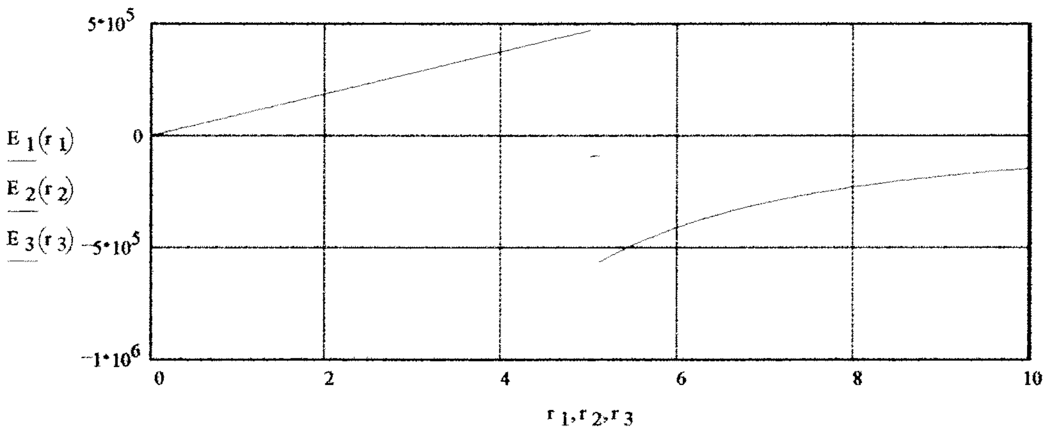
$V_1(r_1) := \frac{-\rho_v \cdot r_1^2}{6 \cdot \varepsilon_1} + B_2$ **Volts** $\quad E_1(r_1) := -\frac{d}{dr_1} V_1(r_1)$ **Volts/meter**

$V_2(r_2) := \frac{-B_3}{r_2} + B_4$ **Volts** $\quad E_2(r_2) := -\frac{d}{dr_2} V_2(r_2)$ **Volts/meter**

$V_3(r_3) := -\frac{B_5}{r_3} + B_6$ **Volts** $\quad E_3(r_3) := -\frac{d}{dr_3} V_3(r_3)$ **Volts/meter**



The above graph shows that the potential is continuous in the region 0 < r < 10 meters.



The above graph shows that the electric field is not continuous in the region 0 < r < 10 meters.

$\varepsilon_2 := 15 \cdot \varepsilon_o$

Given

$$\varepsilon_1 \cdot \left(\frac{1}{3} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5\right) = \varepsilon_2 \cdot \frac{C_3}{5^2} \qquad -\varepsilon_3 \cdot \left(\frac{C_5}{5.1^2}\right) + \varepsilon_2 \cdot \frac{C_3}{5.1^2} = \rho_s$$

$$-\frac{C_5}{10} + C_6 = 0 \qquad -\frac{C_5}{5.1} + C_6 = -\frac{C_3}{5.1} + C_4 \qquad -\frac{C_3}{5} + C_4 = \frac{-1}{6} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5^2 + C_2$$

$$\begin{bmatrix} C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} := \text{Find}(C_2, C_3, C_4, C_5, C_6) \qquad \begin{bmatrix} C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} -2.436 \cdot 10^5 \\ 1.569 \cdot 10^6 \\ -1.106 \cdot 10^6 \\ 1.472 \cdot 10^7 \\ 1.472 \cdot 10^6 \end{bmatrix}$$

$$\varepsilon_1 \cdot \left(\frac{1}{3} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5\right) - \varepsilon_2 \cdot \frac{C_3}{5^2} = 0 \qquad \left[-\varepsilon_3 \cdot \left(\frac{C_5}{5.1^2}\right) + \varepsilon_2 \cdot \frac{C_3}{5.1^2}\right] = 3 \cdot 10^{-6} \qquad -\frac{C_5}{10} + C_6 = 0$$

$$\left(-\frac{C_5}{5.1} + C_6\right) - \left(-\frac{C_3}{5.1} + C_4\right) = -2.328 \cdot 10^{-10} \qquad \left(-\frac{C_3}{5} + C_4\right) - \left(\frac{-1}{6} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5^2 + C_2\right) = 0$$

$r_1 := 0, .05 .. 5 \qquad r_2 := 5.0, 5.05 .. 5.1 \qquad r_3 := 5.1, 5.15 .. 10$

$$V_1(r_1) := \frac{-\rho_v \cdot r_1^{2}}{6 \cdot \varepsilon_1} + C_2$$ **Volts**

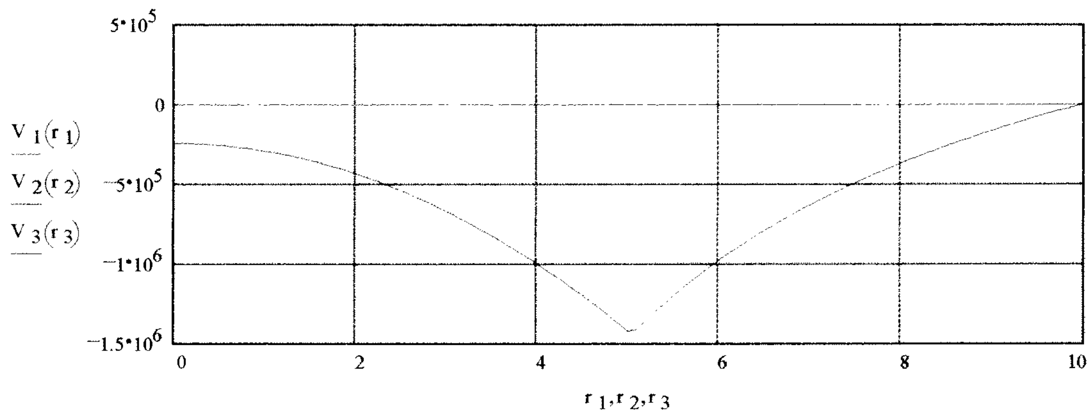
$$E_1(r_1) := -\frac{d}{dr_1} V_1(r_1)$$ **Volts/meter**

$$V_2(r_2) := \frac{-C_3}{r_2} + C_4$$ **Volts**

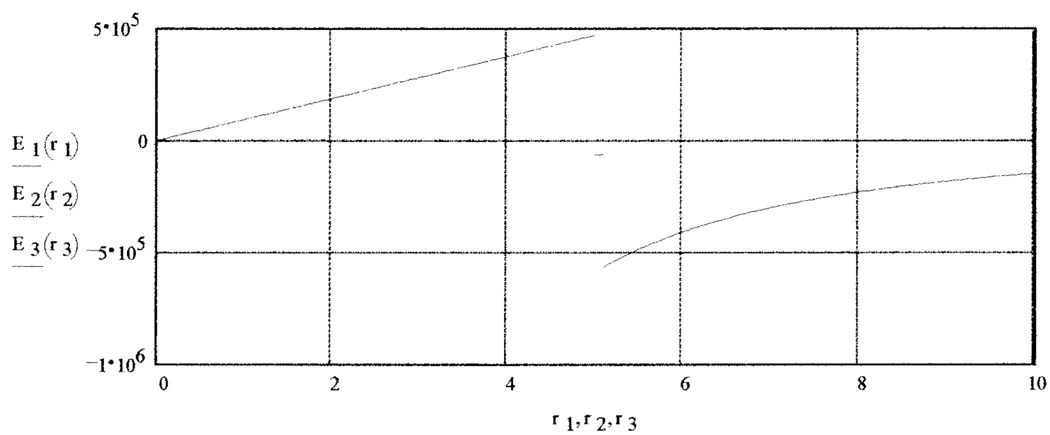
$$E_2(r_2) := -\frac{d}{dr_2} V_2(r_2)$$ **Volts/meter**

$$V_3(r_3) := -\frac{C_5}{r_3} + C_6$$ **Volts**

$$E_3(r_3) := -\frac{d}{dr_3} V_3(r_3)$$ **Volts/meter**



The above graph shows that the potential is continuous in the region 0 < r < 10 meters.



The above graph shows that the electric field is not continuous in the region 0 < r < 10 meters.

$\varepsilon_2 := 20 \cdot \varepsilon_o$

Given

$$\varepsilon_1 \cdot \left(\frac{1}{3} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5\right) = \varepsilon_2 \cdot \frac{D_3}{5^2} \qquad -\varepsilon_3 \cdot \left(\frac{D_5}{5.1^2}\right) + \varepsilon_2 \cdot \frac{D_3}{5.1^2} = \rho_s$$

$$-\frac{D_5}{10} + D_6 = 0 \qquad -\frac{D_5}{5.1} + D_6 = -\frac{D_3}{5.1} + D_4 \qquad -\frac{D_3}{5} + D_4 = \frac{-1}{6} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5^2 + D_2$$

$$\begin{bmatrix} D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} := \mathrm{Find}(D_2, D_3, D_4, D_5, D_6) \qquad \begin{bmatrix} D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = \begin{bmatrix} -2.421 \cdot 10^5 \\ 1.176 \cdot 10^6 \\ -1.183 \cdot 10^6 \\ 1.472 \cdot 10^7 \\ 1.472 \cdot 10^6 \end{bmatrix}$$

$$\varepsilon_1 \cdot \left(\frac{1}{3} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5\right) - \varepsilon_2 \cdot \frac{D_3}{5^2} = 0 \qquad \left[-\varepsilon_3 \cdot \left(\frac{D_5}{5.1^2}\right) + \varepsilon_2 \cdot \frac{D_3}{5.1^2}\right] = 3 \cdot 10^{-6} \qquad -\frac{D_5}{10} + D_6 = 0$$

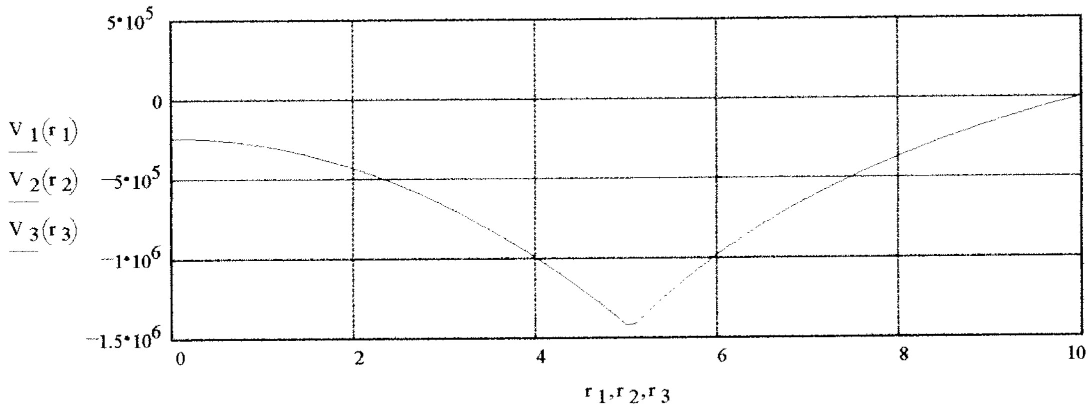
$$\left(-\frac{D_5}{5.1} + D_6\right) - \left(-\frac{D_3}{5.1} + D_4\right) = 0 \qquad \left(-\frac{D_3}{5} + D_4\right) - \left(\frac{-1}{6} \cdot \frac{\rho_v}{\varepsilon_1} \cdot 5^2 + D_2\right) = 2.328 \cdot 10^{-10}$$

$r_1 := 0, .05 .. 5 \quad r_2 := 5.0, 5.05 .. 5.1 \quad r_3 := 5.1, 5.15 .. 10$

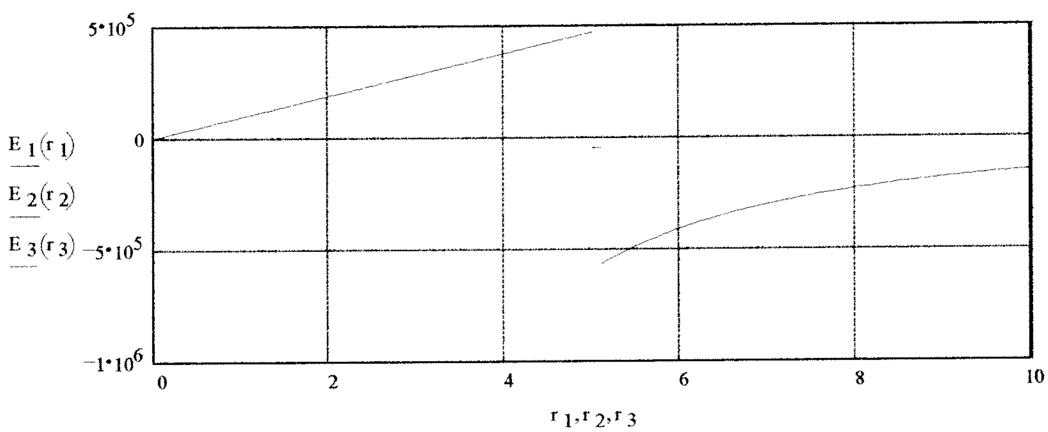
$V_1(r_1) := \frac{-\rho_v \cdot r_1^2}{6 \cdot \varepsilon_1} + D_2$ Volts $\qquad E_1(r_1) := -\frac{d}{dr_1} V_1(r_1)$ Volts/meter

$V_2(r_2) := \frac{-D_3}{r_2} + D_4$ Volts $\qquad E_2(r_2) := -\frac{d}{dr_2} V_2(r_2)$ Volts/meter

$V_3(r_3) := -\frac{D_5}{r_3} + D_6$ Volts $\qquad E_3(r_3) := -\frac{d}{dr_3} V_3(r_3)$ Volts/meter



The above graph shows that the potential is continuous in the region 0 < r < 10 meters.



The above graph shows that the electric field is not continuous in the region 0 < r < 10 meters.

In conclusion, as the permittivity increased in the region from 5 to 5.1 meters, the E-field approached zero. This implied the slope of the electric potential decreased. Although the electric potential was not greatly affected by the increasing permittivity, the E-field was affected. Therefore, a cushion with high permittivity should be used.

[redacted]

Problem Number 3B
Date: November 9, 1993
Problem Originator: [redacted]
Problem Reviewer: [redacted]

Student Problem

Title: Guess the weight of the particle

Charged particles of unknown weight are shot between two charged parallel plates. The particles are given a constant initial velocity. The plates are at different electric potentials, which create an electric field. The electric field is calculated and then the electrophoretic force is determined. By adding the electrophoretic force to the force of gravity, the equations for the trajectory of the particle were calculated. The different weights of the particle affected the trajectory of the particle in different ways. By placing a photo sensitive film over the bottom plate and measuring the distance the particle traveled, the weight of the particle could be determined. It was found that the range of weights of particles measurable in the experiment was from $1.2*10^{-2}$ to 30.4 Kg. The lightest particles traveled the shortest distances before striking the photo sensitive film. The heaviest particles traveled the greater distances. This shows that the lighter particles are affected more by the electrophoretic force than the heavier particles.

Part I:

Constants:

$$q := 14.5\cdot 10^{-12}\ \ C \qquad m_1 := 1\ \ g$$

$$v_o := 0.67\ \ m/s \qquad g := -9.810\ \ \frac{m}{s^2}$$

The electric potential is defined as:

$$\Phi(x,y) := \frac{100}{0.5\cdot x + y^2}\quad V$$

The electric potential then becomes:

$$E(x,y) := -\mathrm{del}\cdot\left[\frac{100}{\left(0.5\cdot x + y^2\right)}\right]\text{V/m}$$

$$E_x(x,y) := -\left[\frac{d}{dx}\left[\frac{100}{\left(0.5\cdot x + y^2\right)}\right]\right] \qquad E_y(x,y) := -\left[\frac{d}{dy}\left[\frac{100}{\left(0.5\cdot x + y^2\right)}\right]\right]$$

$$E_x(x,y) := \frac{50.0}{\left(.5\cdot x + y^2\right)^2} \qquad E_y(x,y) := \frac{200}{\left(.5\cdot x + y^2\right)^2}\cdot y$$

The electrophoretic force is then calculated:

$$F_x(x,y) := q\cdot E_x(x,y) \qquad F_y(x,y) := q\cdot E_y(x,y)$$

The force on the particle due to gravity is:

$$F_g := m_1\cdot g$$

The trajectory equation for the particle is found:

$$\frac{dy}{dx} := \frac{E_y}{E_x} \qquad m\cdot\left(\frac{d}{dt}\frac{d}{dt}y\right) := q\cdot E(x,y) \qquad \frac{d}{dt}\frac{d}{dt}y := \frac{q\cdot E(x,y)}{m}$$

$$\frac{d}{dt}y := \frac{q\cdot E(x,y)}{m}\cdot t + A \qquad y := \frac{q\cdot E(x,y)}{2\cdot m}\cdot t^2 + A\cdot t + B$$

$$V_i := \frac{d}{dt}y \text{ at } t=0 \quad V_i(0) := 0.67 \quad A := 0.67 - \frac{q\cdot E(x,y)}{m}$$

$$y(0) := .9 \qquad B := 0.9 - \frac{q\cdot E(x,y)}{m} - \left(0.67 - \frac{q\cdot E(x,y)}{m}\right)$$

The trajectory is then computed for the greatest measureable mass particle which will strike the lower plate:

$m_1 := 30.4$

$i := 1..100$ $\quad x_{a_0} := 0$ $\quad y_{a_0} := 0.9$ $\quad a_{x1}(x,y) := \dfrac{F_x(x,y)}{m_1}$

$t := 0.025$ $\quad v_{x_0} := v_o$ $\quad v_{y_0} := 0$ $\quad a_{y1}(x,y) := \dfrac{F_y(x,y) + F_g}{m_1}$

$$\begin{bmatrix} v_{x_i} \\ v_{y_i} \\ x_{a_i} \\ y_{a_i} \end{bmatrix} := \begin{bmatrix} v_{x_{i-1}} + a_{x1}\left(x_{a_{i-1}}, y_{a_{i-1}}\right)\cdot t \\ v_{y_{i-1}} + a_{y1}\left(x_{a_{i-1}}, y_{a_{i-1}}\right)\cdot t \\ x_{a_{i-1}} + v_{x_{i-1}}\cdot t + \frac{1}{2}\cdot a_{x1}\left(x_{a_{i-1}}, y_{a_{i-1}}\right)\cdot t^2 \\ y_{a_{i-1}} + v_{y_{i-1}}\cdot t + \frac{1}{2}\cdot a_{y1}\left(x_{a_{i-1}}, y_{a_{i-1}}\right)\cdot t^2 \end{bmatrix}$$

The trajectory for the lightest mass which can be measured at the lowest plate is shown below:

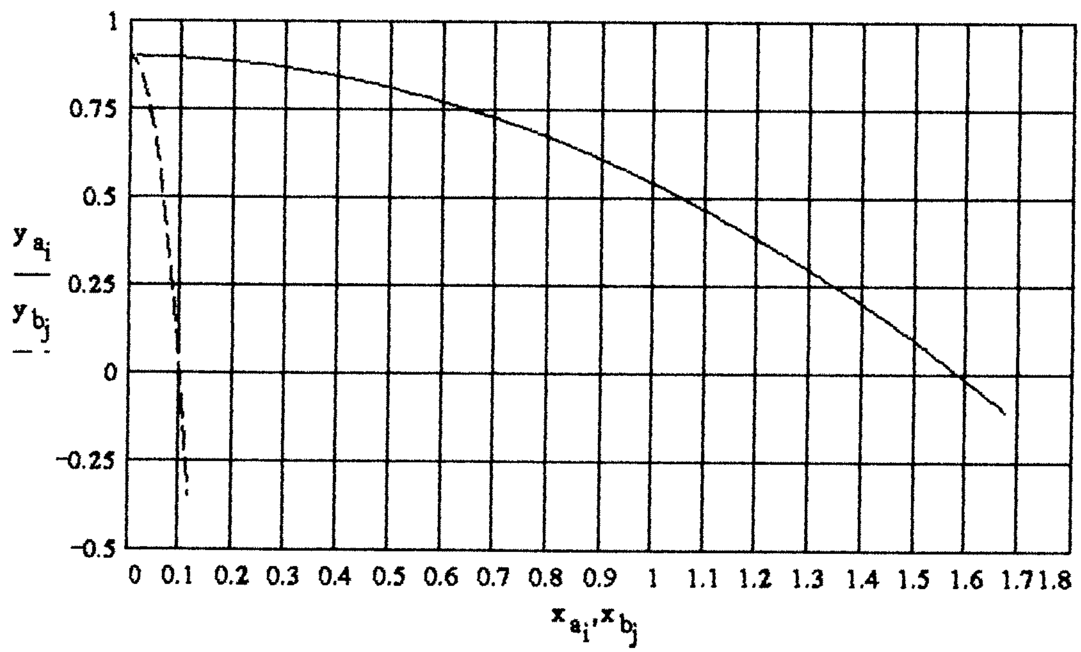
$m_2 := 1.2 \cdot 10^{-1}$

$j := 1..700 \qquad x_{b_0} := 0 \qquad y_{b_0} := 0.9 \qquad a_{x2}(x,y) := \frac{F_x(x,y)}{m_2}$

$t := 0.00025 \qquad v_{x_0} := v_o \qquad v_{y_0} := 0 \qquad a_{y2}(x,y) := \frac{F_y(x,y) + F_g}{m_2}$

$$\begin{bmatrix} v_{x_j} \\ v_{y_j} \\ x_{b_j} \\ y_{b_j} \end{bmatrix} := \begin{bmatrix} v_{x_{j-1}} + a_{x2}\left(x_{b_{j-1}}, y_{b_{j-1}}\right) \cdot t \\ v_{y_{j-1}} + a_{y2}\left(x_{b_{j-1}}, y_{b_{j-1}}\right) \cdot t \\ x_{b_{j-1}} + v_{x_{j-1}} \cdot t + \frac{1}{2} \cdot a_{x2}\left(x_{b_{j-1}}, y_{b_{j-1}}\right) \cdot t^2 \\ y_{b_{j-1}} + v_{y_{j-1}} \cdot t + \frac{1}{2} \cdot a_{y2}\left(x_{b_{j-1}}, y_{b_{j-1}}\right) \cdot t^2 \end{bmatrix}$$

The trajectory of the heaviest and lightest mass are plotted as follows:

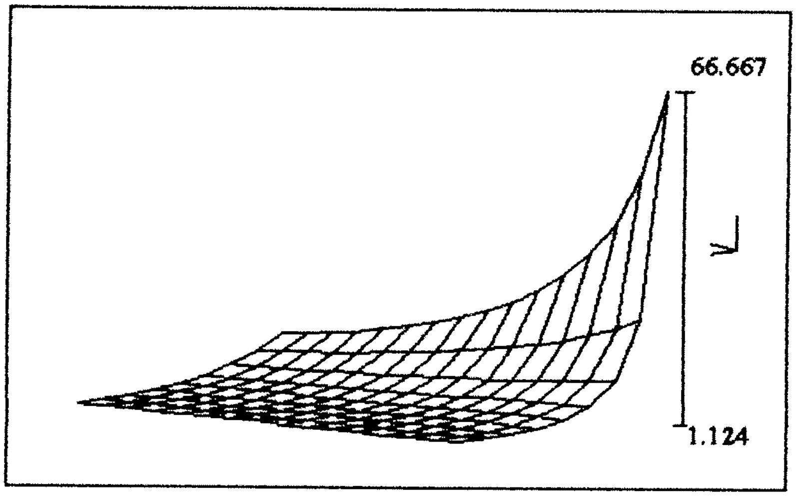


The potential is plotted below:

ORIGIN := 1

$x := 1..16$ $y := 1..9$

$V_{x,y} := \mathrm{if}(x \neq 0, \Phi(x,y), \mathrm{if}(y \neq 0, \Phi(x,y), 100))$



V

The results of this problem show that the electric field generated by the potential is the most influential force acting on the particles. The particles with the most mass (which will accelerate quicker in gravity) traveled the greatest distance before striking the lower plate. The particles with the least amount of mass (which acclerate slower in gravity) traveled the least distance prior to striking the lower plate. This shows that the particles with the least mass are affected more by the electrophoretic force than by gravity. The final graph shown illustrates the the magnatude of the potential. At the point where the particles enter the field the electrophoretic force is the strongest, and as the trajectory proceeds the force due to the electric field decreases.

Problem 3B

Date : 11/24/93

Submitted by : [redacted]

Submitted to : Professor Ken Kaiser

Student Problem

Reviewer : [redacted]

Title : Response of Charged Particles to Electric and Gravitational Fields

Abstract:

A certain potential was assumed to exist in 2-D space. The electric field due to this potential was calculated. The trajectory equation for a particle in this field, ignoring the effects of mass and inertia, was calculated and plotted. This trajectory was then compared with those that do not ignore the effects of gravity.

It was found that the trajectory of a charged particle through the electric field did vary significantly with the mass of the particle.

[redacted]

The trajectory of a particle is computed and plotted, ignoring the effects of mass and inerta on its path. Then a new trajectory is plotted that accurately represents the effects of gravity on the particle trajectory. These two plots are compared. The effect of mass on the particle trajectory is also investigated.

If the potential is..

$$\Phi := x^2 \cdot y^2$$

Then

$$E = -Del \cdot \Phi$$

$$E_x := -\frac{d}{dx}\Phi \quad \textbf{and} \quad E_y := -\frac{d}{dy}\Phi$$

$$E_x(x,y) := -2 \cdot x \cdot y^2 \qquad E_y(x,y) := -2 \cdot y \cdot x^2$$

Therefore, the trajectory of the field is given by

$$\frac{d}{dx}y = \frac{E_y}{E_x}$$

$$\frac{d}{dx}y = \frac{x}{y}$$

$$\int y\, dy = \int x\, dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + A$$

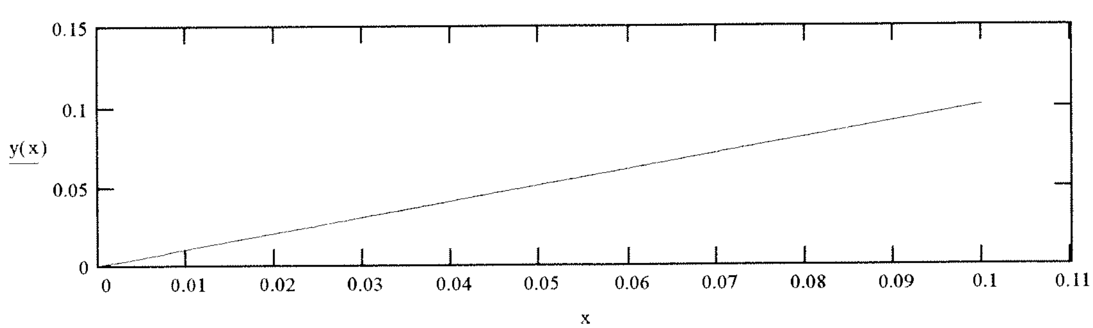
$$y = \sqrt{(x^2) + 2 \cdot A}$$

Given that the intial conditions are y=0, x=0, if follows that A=0. The equation of motion then becomes:

$$y(x) := x$$

The particle trajectory following this equation is plotted on the graph below.

$$x := 0, 0.001 .. 0.1$$



$g := 9.81 \qquad q := 1\cdot 10^{-12}$ Ken Kaiser

Acceleration in the x and y directions, as a function of position, is developed from the sum of the forces in those directions.

$$a_x(x,y,m) := \frac{q\cdot E_x(x,y)}{m} \qquad a_y(x,y,m) := \frac{q\cdot E_y(x,y)}{m} - g$$

The following kinematic equations dictate a particles motion.

$$v(t) := v_i + a\cdot t$$

$$x(t) := x_i + v_i\cdot t + \frac{a\cdot t^2}{2}$$

Case I

$i := 1 .. 3000 \qquad t := .00005 \qquad m := 1\cdot 10^{-16}$ **kg**

The initial conditions are

$V_{x_0} := 1 \qquad V_{y_0} := 1 \qquad x_{a_0} := 0 \qquad y_{a_0} := 0$

$$\begin{bmatrix} V_{x_i} \\ V_{y_i} \\ x_{a_i} \\ y_{a_i} \end{bmatrix} := \begin{bmatrix} a_x\left(x_{a_{i-1}}, y_{a_{i-1}}, m\right)\cdot t + V_{x_{i-1}} \\ a_y\left(x_{a_{i-1}}, y_{a_{i-1}}, m\right)\cdot t + V_{y_{i-1}} \\ \dfrac{a_x\left(x_{a_{i-1}}, y_{a_{i-1}}, m\right)\cdot t^2}{2} + V_{x_{i-1}}\cdot t + x_{a_{i-1}} \\ \dfrac{a_y\left(x_{a_{i-1}}, y_{a_{i-1}}, m\right)\cdot t^2}{2} + V_{y_{i-1}}\cdot t + y_{a_{i-1}} \end{bmatrix}$$

Case II

$i := 1..3000 \qquad t := .00005 \qquad m := 10 \cdot 10^{-16}$ **kg**

The initial conditions are

$V_{x_0} := 1 \qquad V_{y_0} := 1 \qquad x_{b_0} := 0 \qquad y_{b_0} := 0$

$$\begin{bmatrix} V_{x_i} \\ V_{y_i} \\ x_{b_i} \\ y_{b_i} \end{bmatrix} := \begin{bmatrix} a_x\left(x_{b_{i-1}}, y_{b_{i-1}}, m\right) \cdot t + V_{x_{i-1}} \\ a_y\left(x_{b_{i-1}}, y_{b_{i-1}}, m\right) \cdot t + V_{y_{i-1}} \\ \dfrac{a_x\left(x_{b_{i-1}}, y_{b_{i-1}}, m\right) \cdot t^2}{2} + V_{x_{i-1}} \cdot t + x_{b_{i-1}} \\ \dfrac{a_y\left(x_{b_{i-1}}, y_{b_{i-1}}, m\right) \cdot t^2}{2} + V_{y_{i-1}} \cdot t + y_{b_{i-1}} \end{bmatrix}$$

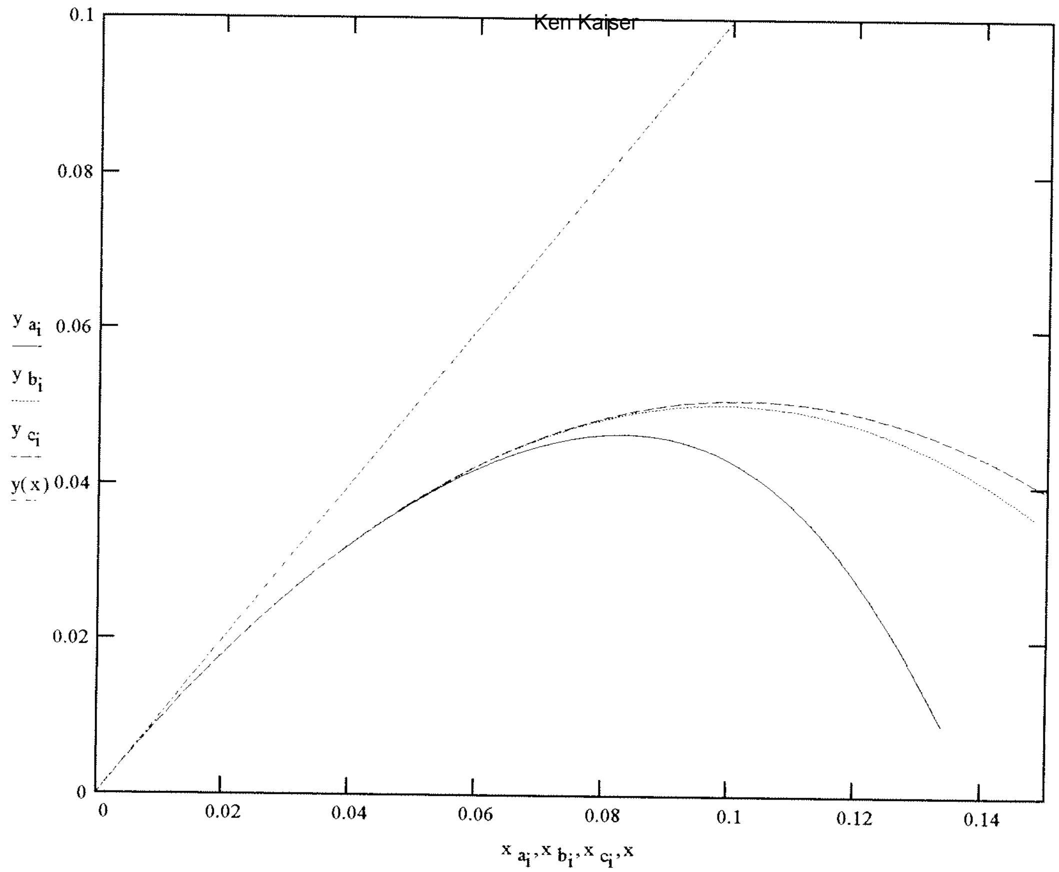
Case III

$i := 1..3000 \qquad t := .00005 \qquad m := 100 \cdot 10^{-16}$ **kg**

The initial conditions are

$V_{x_0} := 1 \qquad V_{y_0} := 1 \qquad x_{c_0} := 0 \qquad y_{c_0} := 0$

$$\begin{bmatrix} V_{x_i} \\ V_{y_i} \\ x_{c_i} \\ y_{c_i} \end{bmatrix} := \begin{bmatrix} a_x\left(x_{c_{i-1}}, y_{c_{i-1}}, m\right) \cdot t + V_{x_{i-1}} \\ a_y\left(x_{c_{i-1}}, y_{c_{i-1}}, m\right) \cdot t + V_{y_{i-1}} \\ \dfrac{a_x\left(x_{c_{i-1}}, y_{c_{i-1}}, m\right) \cdot t^2}{2} + V_{x_{i-1}} \cdot t + x_{c_{i-1}} \\ \dfrac{a_y\left(x_{c_{i-1}}, y_{c_{i-1}}, m\right) \cdot t^2}{2} + V_{y_{i-1}} \cdot t + y_{c_{i-1}} \end{bmatrix}$$



The trajectory of the particle does depend on its mass. As the graph suggests, the trajectory changes with respect to the electric field and with particle mass. The larger the mass, the smaller the range. This is due to the fact that the expression for displacement involves the ratio of mass to electric field.

Ken Kaiser

Problem Number 5

Date: 15 November 1993

Problem Originator: [redacted] B

Reviewer: [redacted] A

Student Problem

Dielectrophoretic Forces   The Orbiting Glass Partical

Abstract:

The total dielectrophoretic force required to force a glass particle to orbit the conductor on the z-axis was determined. By using the equation for the electric field due to an infinite line charge and the electric field inside the glass particle, the charge density was calculated as a function of the dielectrophoretic force. Using the calculated value for charge density, the force on the particle in the x and y directions was then found and the trajectory of the particle was determined. The trajectory of the particle followed a helical orbit. The trajectory was then plotted in twoplanes.

Given:

$m_g := 10^{-11}$ kg ...mass of the glass particle

$k := 6$ ...dielectric constant of glass

$\varepsilon_r := 6$ ...permittivity of glass

$\varepsilon_o := 8.854 \cdot 10^{-12}$ F/m ...permittivity of free space

$\rho := 2200$ kg/m^3 ...density of glass

If $v_r := 1.0$ m/s

and $r := 0.03$ m,

then the radial force required for the particle to orbit the z-axis is:

$$F(r) := \frac{m_g \cdot v_r^2}{r} \quad \text{N}$$

$$F(r) = 3.333 \cdot 10^{-10} \quad \text{N}$$

The required dielectrophoretic force is:

$$F_r := F(r) \cdot \frac{\rho}{m_g} \quad \text{N/m}^3$$

$$F_r = 7.333 \cdot 10^{4} \quad \text{N/m}^3$$

The equation for dielectrophoretic force is:

$$\vec{F} := \left(\frac{1}{2}\right) \cdot \varepsilon_o \cdot (k - 1) \cdot \Delta \cdot \left(\vec{E}\right)^2 \quad \text{N/m}^3$$

The electric field for an infinite line charge is:

$$\vec{E}_o := \frac{\rho_L}{2 \cdot \pi \cdot \varepsilon_o \cdot r} \cdot \vec{r} \quad \text{V/m}$$

where ρ_L is the line charge density.

The electric field inside the glass particle:

$$\vec{E}_{in} := \vec{E}_o \cdot \left(\frac{3}{\varepsilon_r + 2}\right) \quad \text{V/m}$$

Since the electric field is only in the radial direction, the equation can be written as:

$$F_r := \left(\frac{1}{2}\right) \cdot \varepsilon_o \cdot (k - 1) \cdot \frac{d}{dr}\left[\left(\frac{\rho_L}{2 \cdot \pi \cdot \varepsilon_o \cdot r}\right) \cdot \left(\frac{3}{\varepsilon_r + 2}\right)\right]^2 \quad \text{N/m}^3$$

$$\frac{d}{dr}\left[\left(\frac{\rho_L}{2 \cdot \pi \cdot \varepsilon_o \cdot r}\right) \cdot \left(\frac{3}{\varepsilon_r + 2}\right)\right]^2 := \frac{-9}{2} \cdot \frac{\rho_L^2}{\left[\pi^2 \cdot \left[\varepsilon_o^2 \cdot \left[r^3 \cdot \left(\varepsilon_r + 2\right)^2\right]\right]\right]}$$

Solving for ρ_L :

$$\rho_L := \sqrt{\frac{F_r \cdot 4 \cdot \left[\pi^2 \cdot \left[\varepsilon_o^2 \cdot \left[r^3 \cdot (\varepsilon_r + 2)^2\right]\right]\right]}{\varepsilon_o \cdot (k-1) \cdot 9}}$$

$\rho_L = 3.137 \cdot 10^{-5}$ C/m

The force on the particle in the cartesian coordinate system is:

$$F_x(x,y) := \left[\left(\frac{1}{2}\right) \cdot \varepsilon_o \cdot (k-1) \cdot \frac{d}{dx}\left[\left(\frac{\rho_L}{2 \cdot \pi \cdot \varepsilon_o \cdot \sqrt{x^2+y^2}}\right) \cdot \left(\frac{3}{\varepsilon_r + 2}\right)\right]^2\right] \cdot \left(\frac{m_g}{\rho}\right) \quad \text{N}$$

$$F_y(x,y) := \left[\left(\frac{1}{2}\right) \cdot \varepsilon_o \cdot (k-1) \cdot \frac{d}{dy}\left[\left(\frac{\rho_L}{2 \cdot \pi \cdot \varepsilon_o \cdot \sqrt{x^2+y^2}}\right) \cdot \left(\frac{3}{\varepsilon_r + 2}\right)\right]^2\right] \cdot \left(\frac{m_g}{\rho}\right) \quad \text{N}$$

Since F=ma:

$$a_x(x,y) := \frac{F_x(x,y)}{m_g} \quad \text{m/s}^2$$

$$a_y(x,y) := \frac{F_y(x,y)}{m_g} \quad \text{m/s}^2$$

To determine the trajectory of the particle, the following dynamics equations will be applied:

$$v = v_0 + at$$

$$x = x_0 + v_0 t + (1/2)at^2$$

The acceleration is assumed to be to be constant if a small time interval is used. If t=0.00005 seconds, it will take 4000 time intervals to achieve a time of .20 seconds.

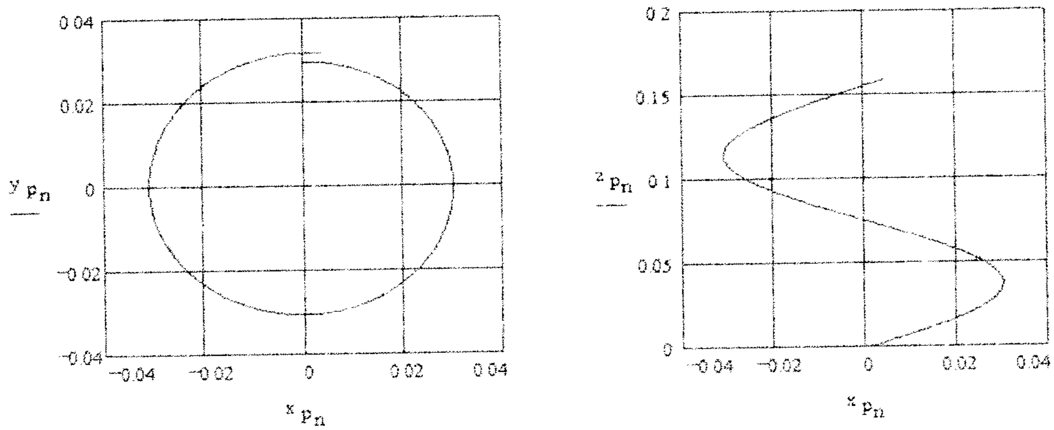
Initial conditions: $x_{P_0} := 0.0$ m $\quad y_{P_0} := 0.03$ m $\quad v_{x_0} := 1.0$ m/s $\quad v_{y_0} := 0.0$ m/s

$n := 1..4000 \quad t := 0.00005$ sec $\quad v_{z_0} := 0.8$ m/s $\quad z_{P_0} := 0.0$ m

Dynamic equations:

$$\begin{bmatrix} v_{z_n} \\ z_{p_n} \\ v_{x_n} \\ v_{y_n} \\ x_{p_n} \\ y_{p_n} \end{bmatrix} := \begin{bmatrix} v_{z_{n-1}} \\ z_{p_{n-1}} + v_{z_{n-1}} \cdot t \\ v_{x_{n-1}} + \left(a_x\left(x_{p_{n-1}}, y_{p_{n-1}}\right)\right) \cdot t \\ v_{y_{n-1}} + \left(a_y\left(x_{p_{n-1}}, y_{p_{n-1}}\right)\right) \cdot t \\ x_{p_{n-1}} + v_{x_{n-1}} \cdot t + \frac{1}{2} \cdot \left(a_x\left(x_{p_{n-1}}, y_{p_{n-1}}\right)\right) \cdot t^2 \\ y_{p_{n-1}} + v_{y_{n-1}} \cdot t + \frac{1}{2} \cdot \left(a_y\left(x_{p_{n-1}}, y_{p_{n-1}}\right)\right) \cdot t^2 \end{bmatrix}$$

Trajectory plot:



From the equations for dielectrophoretic force and the electric field due to an infinite line charge, the required value of the line charge density was found to be 1.177×10^{-5} C/m. When the calculated line charge density was used to calculate the trajectory, the particle followed a helical orbit around the z-axis.

Problem Number **5B**
Date: November 29, 1993
Problem Originator: [redacted]
Problem Reviewer: [redacted]

Student Problem

Title: The oil canon

Abstract:

An oil "canon" is formed by placing two oppositely charged coned plates, touching an insulator, at varying angles. The plates are connected to a dc voltage source that provides the charge and the source of the electric field. The electric field results in a dielectrophoretic force on the oil. This force accelerates the oil out of the "canon" and produces a stream of oil. It was found that the smaller the angle was between the plates, the greater the acceleration of the oil and, therefore the best "canon."

[redacted]

[redacted]

Constants:

$\varepsilon_r := 2.2$ (The relative permittivity of oil)

$\varepsilon_o := 8.854 \cdot 10^{-12}$ F/m

$k := \varepsilon_r$

$V := 40 \cdot 10^5$ V

$\theta_1 := 30 \qquad \theta_2 := 45 \qquad \theta_3 := 60 \qquad y := 1..3$

$\theta_{r_y} := \theta_y \cdot \frac{\pi}{180}$ radians $\qquad r_y := \frac{0.25}{\sin\left(\frac{\pi}{2} - \theta_{r_y}\right)}$

θ_{r_y}	r_y
0.524	0.289
0.785	0.354
1.047	0.5

$\rho := 910$ kg/m^3 (density of oil)

$g := -9.81$ m/s^2 (acceleration due to gravity)

The dielectrophoretic force is defined as:

$$F := \frac{1}{2} \cdot \varepsilon_o \cdot (k - 1) \cdot \Delta \cdot \left(\left|\vec{E}\right|\right)^2 \cdot \frac{\text{newton}}{m^3}$$

The electric field in spherical coordinates is defined as:

$$E := \frac{V}{d} \cdot \frac{\text{volts}}{m} \qquad E := \frac{V}{r \cdot \theta_r} \cdot \left(\vec{r}\right) \qquad E^2 := \frac{V^2}{r^2 \cdot \theta_r^{\,2}}$$

$$\Delta \cdot E^2 := \frac{d}{dr} \frac{V^2}{r^2 \cdot \theta_r^{\,2}} \qquad \Delta \cdot E^2 := \frac{-2 \cdot V^2}{r^3 \cdot \theta_r^{\,2}}$$

The dielectrophoretic force then becomes:

$$F := -(k - 1) \cdot \varepsilon_o \cdot \frac{V^2}{\left(r^3 \cdot \theta_r^{\,2}\right)} \cdot \left(\vec{r}\right) \text{N/cu m} \qquad \text{TOL} \equiv 1$$

A voltage of 4000 kV is applied across the plates. The total dielectrophoretic force on the oil is then calculated.

$$F_y := \int_{0.1}^{\theta_{r_y}} \int_{0.1}^{2 \cdot \pi} \int_{0.03}^{r_y} \frac{(k - 1) \cdot \varepsilon_o \cdot (V)^2}{\tau^3 \cdot \alpha^2} \cdot \tau^2 \cdot \sin(\phi) \, d\tau \, d\alpha \, d\phi$$

where $\alpha := \theta_r$ and $\tau := r$

$$F_1 = 556.882 \qquad F_2 = 1.354 \cdot 10^3 \qquad F_3 = 2.656 \cdot 10^3$$

In order to determine the total force acting on the oil, the mass must be determined for each of the nozzles.

$$m_y := \int_0^{\theta_{r_y}} \int_0^{2\cdot\pi} \int_{0.03}^{r_y} \rho\cdot\tau^2\cdot\sin(\phi)\,d\tau\,d\alpha\,d\phi \quad \text{kg} \qquad vol_y := \frac{m_y}{\rho} \quad m^3$$

$m_1 = 6.136$ $vol_1 = 0.007$ $m_2 = 24.655$ $vol_2 = 0.027$ $m_3 = 119.093$ $vol_3 = 0.131$

The force due to gravity is found to be:

$$F_{g_y} := m_y\cdot g \quad \text{kg m/s}^2 \qquad F_{g_1} = -60.191 \qquad F_{g_2} = -241.868 \qquad F_{g_3} = -1.168\cdot 10^3$$

The total force on the oil is determined:

$$F_{tot_y} := F_y + F_{g_y} \quad \text{kg m/s}^2 \qquad F_{tot_1} = 496.691 \qquad F_{tot_2} = 1.112\cdot 10^3 \qquad F_{tot_3} = 1.488\cdot 10^3$$

The initial acceleration of the oil is found to be:

$$a_y := \frac{F_{tot_y}}{m_y} \quad \text{m/s}^2 \qquad a_1 = 80.951 \qquad a_2 = 45.122 \qquad a_3 = 12.493$$

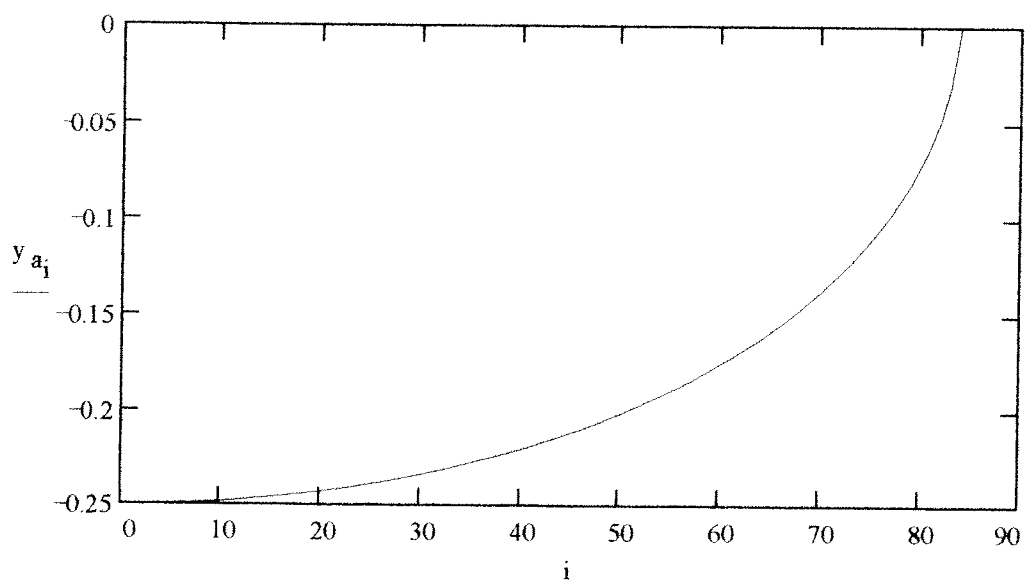
The trajectory of the oil inside the nozzle from t = .001 to .084 seconds at angle of 30 degrees:

$$i := 1..84 \qquad v_{a_0} := 0 \quad \text{m/s}^2 \qquad y_{a_0} := -0.25 \text{ m} \qquad t := 0.001 \text{ s}$$

$$F_t(r) := -\frac{(k-1)\cdot\varepsilon_o\cdot(V)^2}{r^3\cdot\left(\theta_{r_1}\right)^2}\cdot vol_1 \qquad a_t(r) := \frac{F_t(r)}{m_1} + g$$

$$\begin{pmatrix} v_{a_i} \\ y_{a_i} \end{pmatrix} := \begin{bmatrix} v_{a_{i-1}} + a_t\left(y_{a_{i-1}}\right)\cdot t \\ y_{a_{i-1}} + v_{a_{i-1}}\cdot t + \frac{1}{2}\cdot a_t\left(y_{a_{i-1}}\right)\cdot t^2 \end{bmatrix}$$

The y-axis on the graph is the nozzle height with zero being the nozzle tip. The negative values are the distance below the nozzle tip.



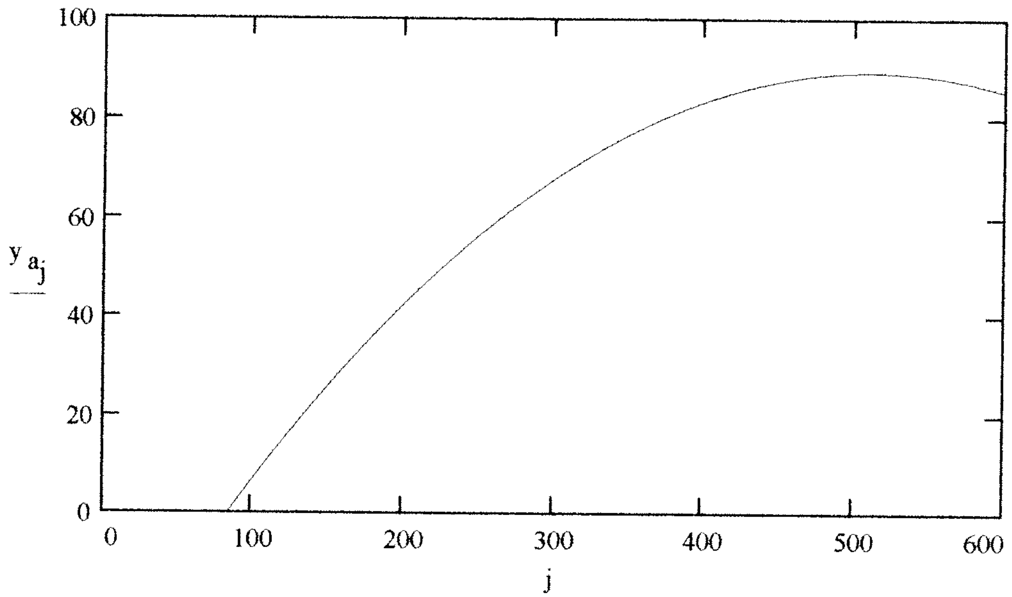
The velocity of the oil at t = .84 seconds at the nozzle tip is: $v_{a_{84}} = 41.85$ m/s

The trajectory of the oil, from t = .85 to .6 seconds, after it leaves the nozzle is determined:

$$j := 85..600 \qquad t := 0.01$$

$$\begin{pmatrix} v_{a_j} \\ y_{a_j} \end{pmatrix} := \begin{bmatrix} v_{a_{j-1}} + g \cdot t \\ y_{a_{j-1}} + v_{a_{j-1}} \cdot t + \frac{1}{2} \cdot g \cdot t^2 \end{bmatrix}$$

The y-axis on the graph is the nozzle height with zero being the nozzle tip.



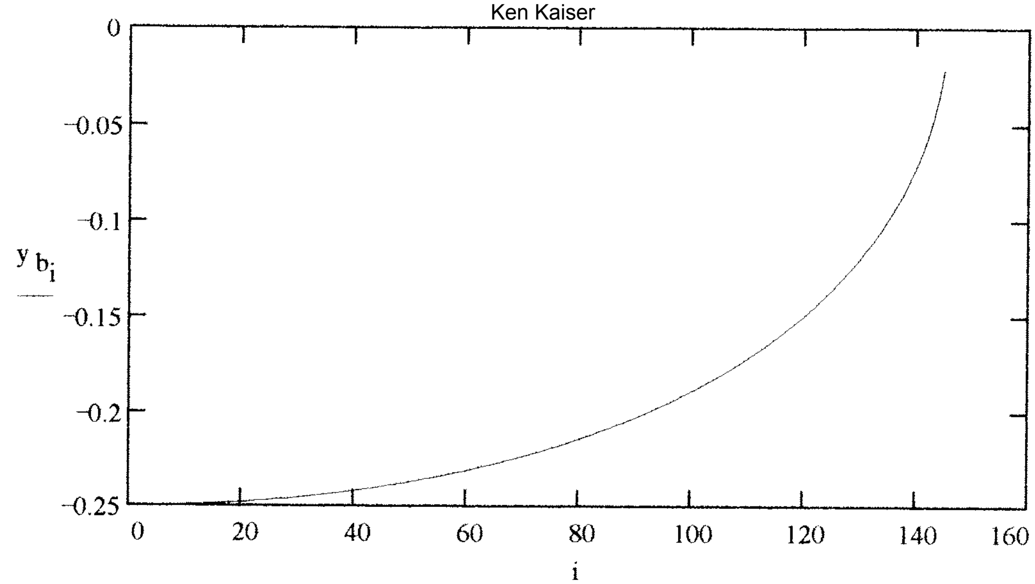
The trajectory of the oil inside the nozzle from t = .001 to .145 seconds at angle of 45 degrees:

$$i := 1..145 \qquad v_{b_0} := 0 \qquad y_{b_0} := -0.25 \qquad t := 0.001$$

$$F_t(r) := -\frac{(k-1) \cdot \varepsilon_o \cdot (V)^2}{r^3 \cdot \left(\theta_{r_2}\right)^2} \cdot vol_2 \qquad a_t(r) := \frac{F_t(r)}{m_2} + g$$

$$\begin{pmatrix} v_{b_i} \\ y_{b_i} \end{pmatrix} := \begin{bmatrix} v_{b_{i-1}} + a_t\left(y_{b_{i-1}}\right) \cdot t \\ y_{b_{i-1}} + v_{b_{i-1}} \cdot t + \frac{1}{2} \cdot a_t\left(y_{b_{i-1}}\right) \cdot t^2 \end{bmatrix}$$

The y-axis on the graph is the nozzle height with zero being the nozzle tip.



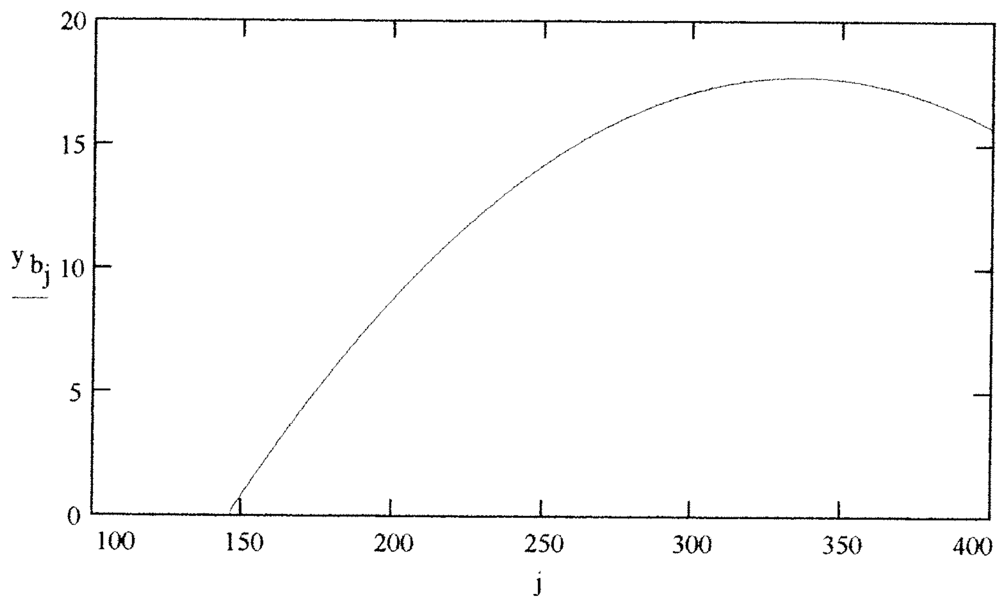
The velocity of the oil at .145 seconds at the nozzle tip is: $v_{b_{145}} = 18.662\,\mathrm{m/s}$

The trajectory of the oil from t = .146 seconds to .4 seconds after it leaves the nozzle is determined to be:

$j := 146..400 \qquad t := 0.01$

$$\begin{pmatrix} v_{b_j} \\ y_{b_j} \end{pmatrix} := \begin{bmatrix} v_{b_{j-1}} + g \cdot t \\ y_{b_{j-1}} + v_{b_{j-1}} \cdot t + \frac{1}{2} \cdot g \cdot t^2 \end{bmatrix}$$

The y-axis on the graph is the nozzle height with zero being the nozzle tip.



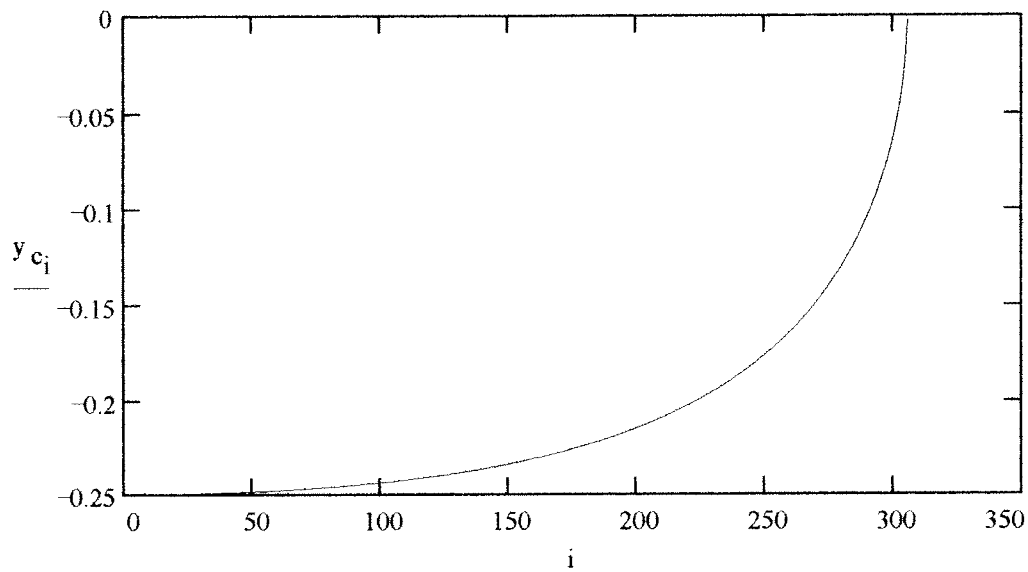
The trajectory of the oil inside the nozzle from t = .001 to .306 seconds at angle of 60 degrees:

$$i := 1..306 \qquad v_{c_0} := 0 \qquad y_{c_0} := -0.25 \qquad t := 0.001$$

$$F_t(r) := -\frac{(k-1)\cdot\varepsilon_o\cdot(V)^2}{r^3\cdot\left(\theta_{r_3}\right)^2}\cdot vol_3 \qquad a_t(r) := \frac{F_t(r)}{m_3} + g$$

$$\begin{pmatrix} v_{c_i} \\ y_{c_i} \end{pmatrix} := \begin{bmatrix} v_{c_{i-1}} + a_t\left(y_{c_{i-1}}\right)\cdot t \\ y_{c_{i-1}} + v_{c_{i-1}}\cdot t + \frac{1}{2}\cdot a_t\left(y_{c_{i-1}}\right)\cdot t^2 \end{bmatrix}$$

The y-axis on the graph is the nozzle height with zero being the nozzle tip.



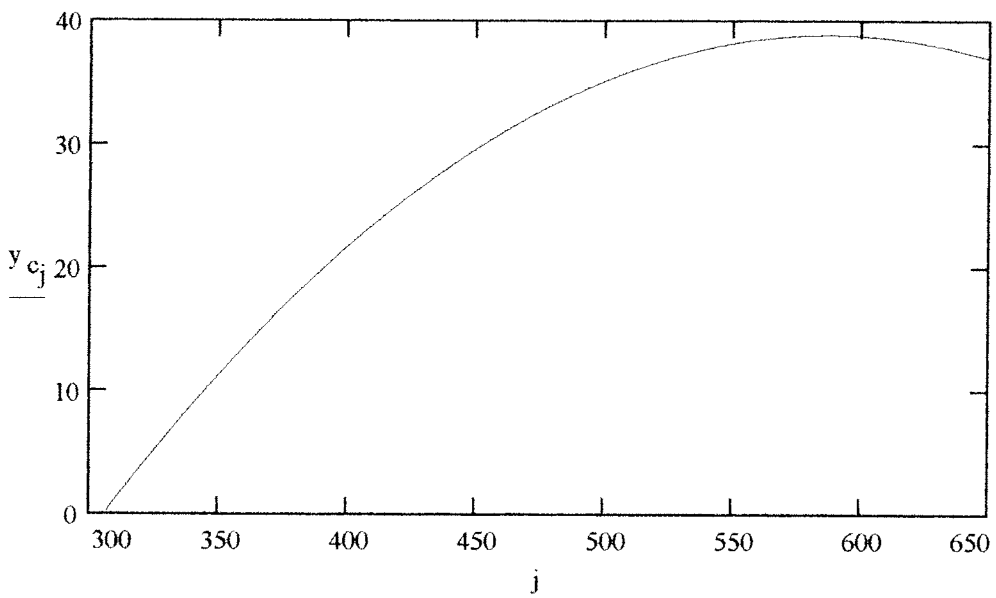
The velocity of the oil at t = .306 seconds at the nozzle tip is: $v_{c_{306}} = 27.607$ m/s

The trajectory of the oil, from t = .307 to .65 seconds, after it leaves the nozzle is determined:

$j := 307..650 \qquad t := 0.01$

$$\begin{pmatrix} v_{c_j} \\ y_{c_j} \end{pmatrix} := \begin{bmatrix} v_{c_{j-1}} + g \cdot t \\ y_{c_{j-1}} + v_{c_{j-1}} \cdot t + \frac{1}{2} \cdot g \cdot t^2 \end{bmatrix}$$

The y-axis on the graph is the nozzle height with zero being the nozzle tip.



This problem demonstrates that the total force on the oil increases as the angle between the plates increases. This is due to the fact that the volume of the oil between the plates increases while all other parameters remain constant. The plots for the oil inside the nozzle do not reflect the true acceleration of the oil. However, without any experience in fluid mechanics, it provides an approximation from which the trajectory of the oil after leaving the nozzle can be determined.