a) Determine the expression for the electric field.

b) Clearly sketch the equipotentials and the flux lines (including their direction) for $0 \leq x \leq \pi$. Label all the 0 V contours. (Suggestion: Use the electric field expression to help plot the flux lines.)

Exam 2L

A small piece of Goldenrod pollen is floating above the surface of the earth. It has a dielectric constant of 2.5 and a radius of 0.01m. The electric field in this pollen due to both the atmospheric electric field and the internal charge is in spherical coordinates approximately

$$\vec{E} = K\left[r^2\vec{r} + r\vec{\theta}\right] \text{ V/m}$$

where r is the distance from the <u>center of the pollen</u> and K is a constant.

a) For this piece of pollen, determine the <u>total</u> dielectrophoretic force in Newtons (<u>not</u> force density in N/m^3)

The gradient of a function f in spherical coordinates is given by the expression

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{\phi}$$

The divergence of a vector F in spherical coordinates is given by the expression

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\, F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

The differential volume element in spherical coordinates is

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

Exam 3P

A plane wave of the form (at x = 0)

$$\vec{E}_i = E_m \cos\omega t\, \hat{k}$$

$$\vec{H}_i = \frac{E_m}{\eta_o}\cos\omega t\, \hat{j} \quad \text{where } \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

strikes a pair of "patent pending" sunglasses. The glasses are unique in that they have a unique response to the incoming electric field. The <u>amplitude</u> of the output response is

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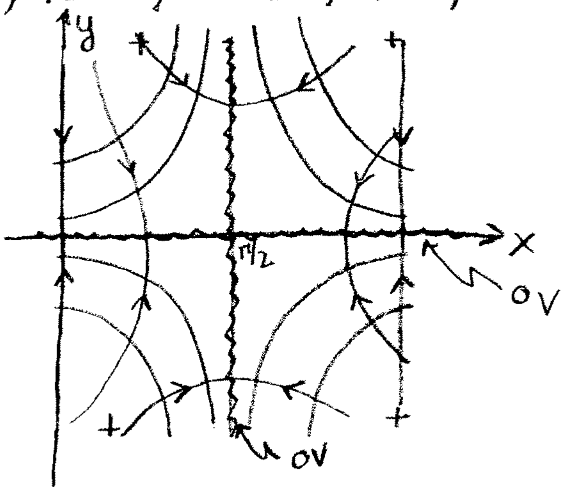
$\Phi = 100(y\cos x)^2$ V



$\Phi \geq 0$ for all x, y

$y^2\cos^2 x = K$

$y = \sqrt{\dfrac{K}{\cos^2 x}}$

@ $x = \pi/2$ $\vec{E} = 0$
@ $y = 0$ $\vec{E} = 0$
@ $x = 0$ $\vec{E} = -200y\hat{j}$
@ $x = \pi$ $\vec{E} = -200y\hat{j}$

2L) $\vec{F} = \frac{1}{2}\epsilon_0(k-1)\nabla|\vec{E}|^2$ N/m^3 $\quad |\vec{E}|^2 = \vec{E}\cdot\vec{E} = K^2(r^4 + r^2)$

$\nabla|\vec{E}|^2 = K^2(4r^3 + 2r)\hat{r}$

$$\vec{F} = \tfrac{1}{2}\epsilon_0(1.5)K^2\left[(4r^3 + 2r)\hat{r}\right]$$

$$\vec{f}_t = \int_V \vec{F}\,dv \quad , \quad f_{rt} = \int_0^{2\pi}\int_0^{\pi}\int_0^{.01} \tfrac{1}{2}\epsilon_0(1.5)K^2 2(2r^3 + r)r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$f_{rt} = 4\pi\epsilon_0(1.5)K^2\left[\frac{r^6}{3} + \frac{r^4}{4}\right]_0^{.01} = 4.17\times10^{-19}K^2 \text{ N}$$

3P) a) $\vec{E}_o = (E_m\cos\omega t - E_m^4\cos^4\omega t)\hat{j}$

$\vec{H}_o = -\dfrac{|\vec{E}_o|}{\eta_o}\hat{k}$

$$\vec{P}_o = \vec{E}_o \times \vec{H}_o = -\frac{1}{\eta_o}\left(E_m^2\cos^2\omega t - 2E_m^5\cos^5\omega t + E_m^8\cos^8\omega t\right)\hat{i}$$

b) $\vec{P}_i = \vec{E}_i \times \vec{H}_i = -\dfrac{E_m^2}{\eta_o}\cos^2\omega t\,\hat{i}$ $\qquad \circlearrowright\hat{i}\times\hat{j}\times\hat{k}$

$$\left|\frac{\vec{P}_o}{\vec{P}_i}\right| = 1 - 2E_m^3\cos^3\omega t + E_m^6\cos^6\omega t$$

$$\hat{P} = \frac{1}{T}\int_0^T\left(1 - \underset{\nwarrow \text{odd function}}{2E_m^3\cos^3\omega t} + E_m^6\cos^6\omega t\right)dt = 1 - \frac{2E_m^3}{T}\int_0^T\cos\omega t\left(\frac{1-\cos 2\omega}{2}\right.$$

$$+ \frac{E_m^6}{T}\int_0^T\cos^2\omega t\cos^4\omega t\,dt$$

$$= 1 - \frac{2E_m^3}{T}\int^T\left(\overset{0}{\cancel{\frac{\cos\omega t}{2}}} - \overset{0}{\cancel{\frac{\cos\omega t\cos 2\omega t}{2}}}\right)dt + \frac{E_m^5}{T}\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)\cos^4\omega t\,dt$$

$$= 1 + \frac{E_m^6}{T}\int^{T}\left(\frac{\cos^4\omega t}{2} - \frac{\cos 2\omega t}{2}\cos^4\omega t\right)dt$$

③ A proposed spherical fluid tank for outer space consists of an inner, metallic, spherical electrode of radius .1 m surrounded by an outer metallic, symmetric spherical electrode of radius 1m. A voltage of

$$V = 150 \times 10^3 \cos \omega t \text{ volts}$$

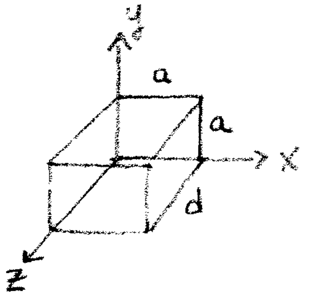
is applied between the electrodes. The fluid inlets are along the inner electrode. If the fluid is initially "pressed" against the inner electrode with the fluid surface located at $r = .5\,m$, and the dielectric constant of the fluid is 3, find the total force on the fluid and its direction. If the mass density of the liquid is 1,000 kg/m³, determine the weight of this fluid on the surface of the earth.

④ The E & M fields inside a microwave oven of length d and cross-sectional area a^2 are

$$E_y = E_o \sin\frac{\pi x}{a} \sin\frac{\pi z}{d} \cos\omega t$$

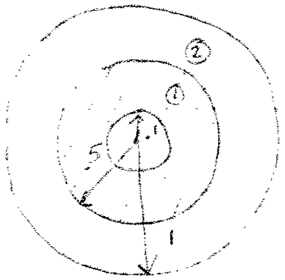
$$H_x = \frac{E_o}{2d} \sin\frac{\pi x}{a} \cos\frac{\pi z}{d} \sin\omega t$$

$$H_z = -\frac{E_o}{2a} \cos\frac{\pi x}{a} \sin\frac{\pi z}{d} \sin\omega t$$



Find $\vec{P}$ and time average $\vec{P}$. If a hot dog of $\sigma = 3 \times 10^{-2}/\Omega\text{-m}$ with a volume of $22 \times 10^{-6}\,m^3$ ("small" compared to $a^2 d$) is placed at the center of the oven, find the time average thermal power delivered to this tasty treat. Assume $\underline{E_o} = 21\,kV/m$.

③



$$\nabla^2\Phi_1 = 0 \quad , \quad \nabla^2\Phi_2 = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 0 \quad , \quad r^2\frac{d\Phi}{dr} = C \quad , \quad \frac{d\Phi}{dr} = \frac{C}{r^2}$$

$$\Phi = -C/r + D$$

Let $\Phi_1 = -A/r + B$, $\Phi_2 = -C/r + D.$, $V = 150\times10^3\cos 600t$ Volts

B.C. $\Phi_1(.1) = 0$, $\Phi_2(1) = V$, $\Phi_1(.5) = \Phi_2(.5)$, $D_{1N}(.5) = D_{2N}(.5)$

or $-3\epsilon_0\frac{d\Phi_1}{dr} = -\epsilon_0\frac{d\Phi_2}{dr}\Big|_{r=.5}$.

$-A/.1 + B = 0 \Rightarrow A = B/10$, $-C/1 + D = V \Rightarrow D = V + C$

$-A/.5 + B = -C/.5 + D$, $-2A + B = -2C + D$, $-B/5 + B = -2C + V + C$

$\Rightarrow B = \frac{5}{4}(V - C)$, $-3\epsilon_0(A/.5^2) = -\epsilon_0(C/.5^2) \Rightarrow A = C/3$

$A = B/10 = 5/40(V - C) = 5/40(V - 3A) \Rightarrow A = V/11$

$E_{r1} = -dV_1/dr = -A/r^2 = -V/11r^2 = 13.6\times10^3\cos 600t/r^2$ radial direction

$\vec{F} = \frac{1}{2}\epsilon_0(\epsilon - 1)\nabla E^2 = \epsilon_0\nabla\left(\frac{185\times10^6\cos^2 600t}{r^4}\right) = 1.64\times10^{-3}\cos^2 600t\,\nabla(r^{-4})$

[illegible] $= -6.55\times10^{-3}\cos^2 600t/r^5\,\hat{r}$

$$\vec{F} = \int_0^{2\pi}\int_0^{\pi}\int_{.1}^{.5}\left(-6.55\times10^{-3}\cos^2 600t\right)\frac{1}{r^5}\,r^2\sin\theta\,dr\,d\theta\,d\phi = .041\cos^2 600t\left(\frac{1}{r^2}\right)\Big|_{.1}^{.5}$$

$= -3.94\cos^2 600t$ N $\quad |F_{max}| = 3.94$N inward radial force

$F_g = mg = \frac{4\pi}{3}(.5^3 - .1^3)(1000)\,9.8 = 5{,}090$ N !

④ $\vec{P} = \vec{E}\times\vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & E_y & 0 \\ H_x & 0 & H_z \end{vmatrix} = E_yH_z\hat{i} - E_yH_x\hat{k}$

$$= -\frac{E_0^2}{2a}\cos\frac{\pi x}{a}\sin\frac{\pi x}{a}\sin^2\frac{\pi z}{d}\cos\omega t\sin\omega t\,\hat{i} - \frac{E_0^2}{2d}\sin^2\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin\frac{\pi z}{d}\cos\omega t\sin\omega t\,\hat{k}$$

$$\frac{1}{T}\int_0^T\cos\omega t\sin\omega t\,dt = \frac{1}{T}\int_0^T\left(\frac{1}{2}\sin 2\omega t\right)dt = \frac{1}{2T}\,\frac{-\cos 2\omega t}{2\omega}\Big|_0^{\frac{2\pi}{\omega}} = 0\,!$$

$\therefore \hat{P} = 0$

$P_T = \int_V\vec{E}\cdot\vec{J}\,dV = \int_V\sigma E^2\,dV \quad (\vec{J} = \sigma\vec{E})$

$$\doteq \sigma E^2(22\cdot10^{-6}) = \frac{\sigma E_0^2\sin^2\frac{\pi x}{a}\sin^2\frac{\pi z}{d}\cos^2\omega t}{45.5\times10^3}\Big|_{(\frac{a}{2},\frac{b}{2},\frac{d}{2})} = \frac{\sigma E_0^2\cos^2\omega t}{45.5\times10^3}$$

$$\hat{P} = \frac{1}{T}\int_0^T\frac{\sigma E_0^2\cos^2\omega t}{45.5\times10^3}\,dt = \frac{\sigma E_0^2}{90.9\times10^3} = 146\text{ W}$$

this assumes the dog does not influence the field ($\vec{E}$ unchanged with dog addition)

Problem 5 (wgt = 7)

A small piece of Goldenrod pollen is floating at a height of 8 m above the ground. It has a dielectric constant of k, and it is uniformly charged. The electric field in this pollen due to both the atmospheric electric field and the internal, uniformly-distributed charge is approximately

$$\vec{E} = \frac{-100}{y^2}\,\vec{j}\ \text{V/m}$$

where y is the distance from the ground. For this piece of pollen, determine the ratio of the dielectrophoretic force to the electrophoretic force ($\rho\vec{E}$). (Hint: Use one of Maxwell's equations.)

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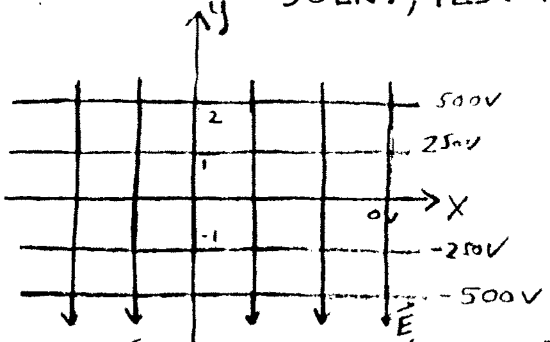
①



$E = -250\hat{a}_y$

$d = 1,\ a = .5$

OVEN center $= (.25, .25, .5)$

$.25 + .1/2 = .3,\ .25 - .1/2 = .2$

$.5 + .08/2 = .54,\ .5 - .08/2 = .4$

② $P_D = \int \vec{J}\cdot\vec{E}\,dV = \int \sigma\vec{E}\cdot\vec{E}\,dv = \sigma\int|\vec{E}|^2dV = \sigma\int\int\int(2{,}500)^2\sin^2\frac{\pi x}{a}\sin^2(\frac{\pi z}{d})\cos^2\omega$

$= (2{,}500)^2(5\times10^{-3})\cos^2\omega t\int_{.46}^{.54}\int_{.2}^{.3}\int_{.2}^{.3}\sin^2\frac{\pi x}{a}\sin^2\frac{\pi z}{d}\,dx\,dy\,dz$

$= 3.13\times10^3\cos^2\omega t\int_{.46}^{.54}\int_{.2}^{.3}\sin^2\frac{\pi z}{d}\left(\frac{1-\cos\frac{2\pi x}{a}}{2}\right)dx\,dz$

don't forget to use radians

$= 1.56\times10^3\cos^2\omega t\int_{.46}^{.54}\sin^2\frac{\pi z}{d}\left[x - \frac{\sin\frac{2\pi x}{a}}{2\pi/a}\right]\Big|_{.2}^{.3}dz = 302\cos^2\omega t\int_{.46}^{.54}\left(\frac{1-\cos\frac{2\pi z}{d}}{2}\right)d$

$= 151\cos^2\omega t\left[z - \frac{\sin\frac{2\pi z}{d}}{2\pi/d}\right]_{.46}^{.54} = 24.0\cos^2\omega t\ \ W$

$\therefore\ \hat{P}_D = \frac{1}{T}\int_0^T 24\cos^2\omega t\,dt = 12\ W$

③ $\Phi_i(r = 1m) = 0V? = -2\times10^{-9}/6\epsilon_0 - 200 + 254.4 = 16.8$ Not satisfied

$\Phi_o(r = 5m) = 20V? = 35/5 + 13 = 20V$ ✓ satisfied

$\Phi_i(r = 1.5) = ?\ \Phi_o(r = 1.5),\ \Phi_i = 36.4V,\ \Phi_o = 36.3$ which is very close ✓ satisfied

$D_{No} - D_{Ni} = \rho_s = 0,\quad D_{No} = D_{Ni},\ -\epsilon_0\frac{d\Phi_o}{dr} = ?\ -\epsilon_0\frac{d\Phi_i}{dr}\Big/_{r=1.5}$

$\left.\begin{array}{l} d\Phi_i/dr = -2\times10^{-9}r/3\epsilon_0 + 200/r^2\big/_{1.5} = -24.1 \\ d\Phi_o/dr = -35/r^2\big/_{1.5} = -15.6 \end{array}\right\}$ Not satisfied

④ a) $Q_C = C_{CE}V_{CE} + C_{CY}(V_{CE} - V_{YE})$

$Q_Y = 0 = C_{YE}V_{YE} + C_{CY}(V_{YE} - V_{CE}) \Rightarrow V_{YE} = C_{CY}V_{CE}/(C_{YE} + C_C$

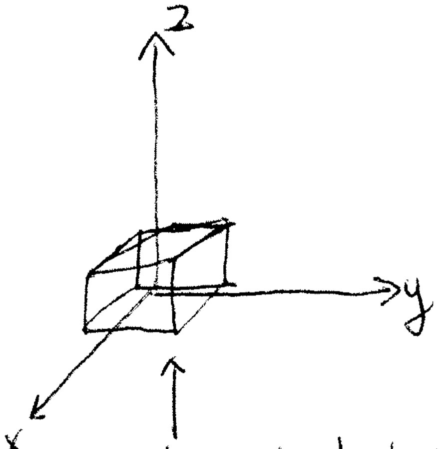
$Q_C = C_{CE}V_{CE} + C_{CY}\left[V_{CE} - \frac{C_{CY}V_{CE}}{C_{YE} + C_{CY}}\right]$

$\therefore V_{YG} = 6.3\,kV,\ Q_C = 2.7\,\mu C$

⑤ $\left|\frac{\frac{1}{2}\epsilon_0(k-1)\nabla|E|^2}{\rho\vec{E}}\right| = \left|\frac{\frac{1}{2}\epsilon_0(k-1)\nabla|\vec{E}|^2}{k\epsilon_0(\nabla\cdot\vec{E})\vec{E}}\right| = \frac{1}{2}\frac{(k-1)}{k}\left|\frac{\nabla\left(\frac{10{,}000}{y^4}\right)}{\left(\frac{200}{y^3}\right)\left(\frac{-100}{y^2}\right)\hat{j}}\right|$

$= \frac{1}{2}\left(\frac{k-1}{k}\right)\frac{(-40{,}000/y^5)}{(-20{,}000/y^5)} = \frac{k-1}{k}$

FIELDS II QUIZ KAISER



1×1×1 cube centered about the origin, $k=3$, Neutral

$\vec{E} = 2x\hat{i} + 3y^2\hat{j}$ V/m
inside the cube

Setup the integral to determine the total force (dielectrophoretic force)

$$\vec{F} = \frac{1}{2}\epsilon_0(k-1)\nabla E^2 \qquad k=3$$

$$E^2 = \vec{E}\cdot\vec{E} = 4x^2 + 9y^4$$

$$\nabla E^2 = 8x\hat{i} + 36y^3\hat{j}$$

$$\vec{f}_T = \int_{-1/2}^{1/2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2}(\epsilon_0 8x\hat{i} + \epsilon_0 36y^3\hat{j})\,dx\,dy\,dz$$

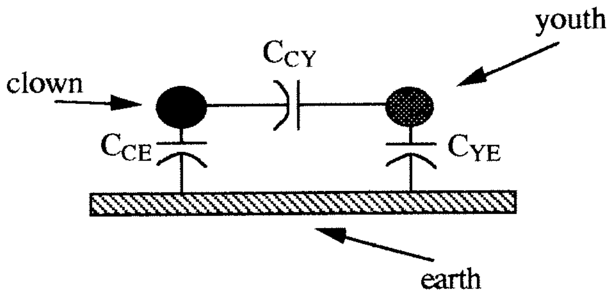
Exam 1L

Pennywise is a very, very bad clown. He frequently zaps unsuspecting children with jolts of electrostatic energy. Imagine that Pennywise is charged to 3 mC and that he is alongside an innocent charge-free youth (who is not grounded). Assume that the mutual capacitance between the clown and the earth is $C_{CE} = 250$ pF, between the clown and the youth is $C_{CY} = 30$ pF, and between the youth and the earth is $C_{YE} = 170$ pF, as shown below.

1) What is the potential of the clown relative to the earth?

2) What is the potential of the youth **relative to the clown**?

(Hint: The charge on the clown is distributed between the capacitors C_{CE} and C_{CY} and is also a function of the voltage of the clown (relative to the earth) and the voltage between the clown and youth.)



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1P) a) $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = \frac{-\rho}{\epsilon_0} = \frac{6r^2 \times 10^{-6}}{\epsilon_0}$, $\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = \frac{6r^4 \times 10^{-5}}{\epsilon_0}$

$r^2\frac{d\Phi}{dr} = \frac{6r^5}{5\epsilon_0}\times 10^{-6} + A$, $\frac{d\Phi}{dr} = \frac{6r^3 \times 10^{-6}}{5\epsilon_0} + \frac{A}{r^2}$

$\Phi_i = \frac{6r^4 \times 10^{-6}}{20\epsilon_0} - \frac{A}{r} + B \qquad .5 \le r \le .8 \text{ m}$

$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 0$, $\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 0$, $r^2\frac{d\Phi}{dr} = C$

$\frac{d\Phi}{dr} = \frac{C}{r^2}$, $\Phi_o = -\frac{C}{r} + D \qquad .8 \le r \le 8 \text{ m}$

b) $\Phi_i(.5) = 0$, $\Phi_o(8) = 3$, $\Phi_i(.8) = \Phi_o(.8)$

$D_{ri}(.8) = D_{ro}(.8)$

$-\epsilon_0\frac{d\Phi_i}{dr}(.8) = -\epsilon_0\frac{d\Phi}{dr}(.8)$, $\left.\frac{d\Phi_i}{dr} = \frac{d\Phi_o}{dr}\right|_{r=.8}$

1L) $Q_C = C_{CE}V_{CE} + C_{CY}(V_{CE} - V_{YE})$

$0 = C_{YE}V_{YE} + C_{CY}(V_{YE} - V_{CE}) \Rightarrow V_{YE} = \frac{C_{CY}V_{CE}}{C_{YE} + C_{CY}}$

$Q_C = C_{CE}V_{CE} + C_{CY}V_{CE} - \frac{C_{CY}^2 V_{CE}}{C_{YE} + C_{CY}}$

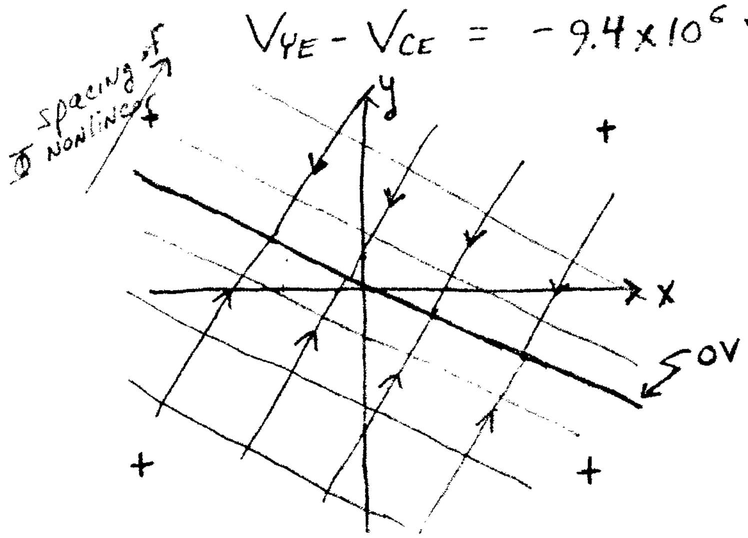
$V_{CE}\left(C_{CE} + C_{CY} - \frac{C_{CY}^2}{C_{YE}+C_{CY}}\right) = Q_C = 3\times 10^{-3}$

$\therefore V_{CE} = 1.1\times 10^7 \text{ V}$! /

$V_{YE} = 1.6\times 10^6 \text{ V}$

$V_{YE} - V_{CE} = -9.4\times 10^6 \text{ V}$ /

2P)



$\Phi = 500(x+2y)^2 \ge 0$

$= 0$ when $x = -2y$, $y = -x/2$

$K = (x+2y)$

$y = (K-x)/2$

$\vec{E} = -\nabla\Phi = \frac{\partial\Phi}{\partial x}\hat{x} - \frac{\partial\Phi}{\partial y}\hat{y}$

$= -1{,}000(x+2y)\hat{x} - 2{,}000(x+2y)\hat{y}$

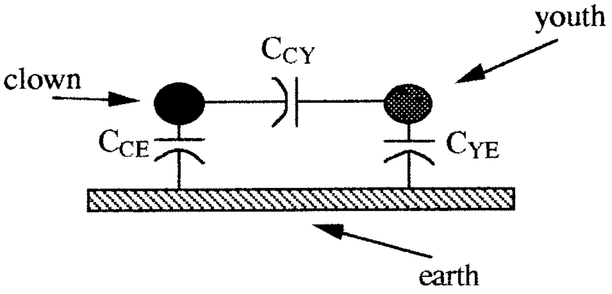
$\frac{dy}{dx} = \frac{E_y}{E_x} = 2$, $y = 2x + C$, $.2 = 2(.1) + C \Rightarrow C = 0$

constant slope $\qquad y = 2x$ trajectory

Problem 4 (wgt =10)

Pennywise is a very, very bad clown. He frequently zaps unsuspecting children with jolts of electrostatic energy. Imagine that Pennywise is charged to a voltage of 15 kV (relative to the earth) when he is alongside an innocent charge-free youth (who is not grounded). Assume that the mutual capacitance between the clown and the earth is $C_{CE} = 150$ pF, between the clown and the youth is $C_{CY} = 50$ pF, and between the youth and the earth is $C_{YE} = 70$pF, as shown below.

What is the charge on the clown and the potential of the youth? (Hint: The charge on the clown is distributed between the capacitors C_{CE} and C_{CY} and is also a function of the voltage on the clown and the voltage between the clown and youth.)



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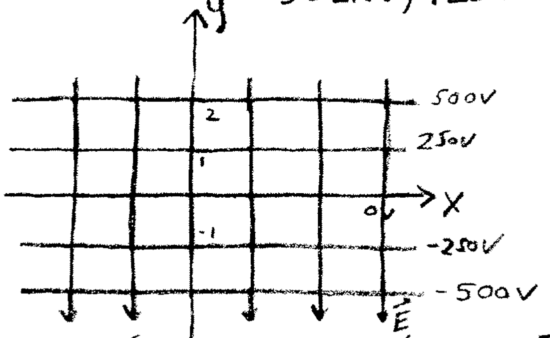
①

$\vec{E} = -250\hat{a}_y$



$d=1$, $a=.5$

oven center $= (.25, .25, .5)$

$.25 + .1/2 = .3$, $.25 - .1/2 = .2$

$.5 + .08/2 = .54$, $.5 - .08/2 = .46$

② $P_D = \int \vec{J}\cdot\vec{E}\,dv = \int \sigma\vec{E}\cdot\vec{E}\,dv = \sigma\int|\vec{E}|^2 dv = \sigma\int\int\int (2{,}500)^2 \sin^2\frac{\pi y}{a}\sin^2(\frac{\pi z}{d})\cos^2\omega t\,dx\,dy$

$$= (2{,}500)^2(5\times10^{-3})\cos^2\omega t \int_{.46}^{.54}\int_{.2}^{.3}\int_{.2}^{.3} \sin^2\frac{\pi y}{a}\sin^2\frac{\pi z}{d}\,dx\,dy\,dz$$

$$= 3.13\times10^3\cos^2\omega t \int_{.46}^{.54}\int_{.2}^{.3}\sin^2\frac{\pi z}{d}\left(\frac{1-\cos\frac{2\pi y}{a}}{2}\right)dx\,dz$$

don't forget to use radians!

$$= 1.56\times10^3\cos^2\omega t\int_{.46}^{.54}\sin^2\frac{\pi z}{d}\left[x - \frac{\sin\frac{2\pi x}{a}}{2\pi/a}\right]\Big|_{.2}^{.3}dz = 302\cos^2\omega t\int_{.46}^{.54}\left(\frac{1-\cos\frac{2\pi z}{d}}{2}\right)dz$$

$$= 151\cos^2\omega t\left[z - \frac{\sin\frac{2\pi z}{d}}{2\pi/d}\right]_{.46}^{.54} = 24.0\cos^2\omega t \quad W$$

$$\therefore \quad \hat{P}_D = \frac{1}{T}\int_0^T 24\cos^2\omega t\,dt = 12\ W$$

③ $\Phi_i(r=1m) = 0V? = -2\times10^{-9}/6\epsilon_0 - 200 + 254.4 = 16.8$ Not satisfied

$\Phi_o(r=5m) = 20V? = 35/5 + 13 = 20V$ ✓ satisfied

$\Phi_i(r=1.5) = ?\ \Phi_o(r=1.5)$, $\Phi_i = 36.4V$, $\Phi_o = 36.3$ which is very close ✓ satisfied

$D_{No} - D_{Ni} = \rho_s = 0$, $D_{No} = D_{Ni}$, $-\epsilon_0\frac{d\Phi_o}{dr} = ?\ -\epsilon_0\frac{d\Phi_i}{dr}\Big|_{r=1.5}$

$$\left.\begin{array}{l} d\Phi_i/dr = -2\times10^{-9}r/3\epsilon_0 + 200/r^2\big|_{1.5} = -24.1 \\ d\Phi_i/dr = -35/r^2\big|_{1.5} = -15.6 \end{array}\right\} \text{Not satisfied}$$

④ a) $Q_C = C_{CE}V_{CE} + C_{CY}(V_{CE} - V_{YE})$

$Q_Y = 0 = C_{YE}V_{YE} + C_{CY}(V_{YE} - V_{CE}) \Rightarrow V_{YE} = C_{CY}V_{CE}/(C_{YE}+C_{CY})$

$$Q_C = C_{CE}V_{CE} + C_{CY}\left[V_{CE} - \frac{C_{CY}V_{CE}}{C_{YE}+C_{CY}}\right]$$

$\therefore V_{YE} = 6.3\ kV$, $Q_C = 2.7\ \mu C$

⑤ $$\left|\frac{\frac{1}{2}\epsilon_0(k-1)\nabla|\vec{E}|^2}{\rho\vec{E}}\right| = \left|\frac{\frac{1}{2}\epsilon_0(k-1)\nabla|\vec{E}|^2}{k\epsilon_0(\nabla\cdot\vec{E})\vec{E}}\right| = \frac{1}{2}\frac{(k-1)}{k}\frac{\left|\nabla\left(\frac{10{,}000}{y^4}\right)\right|}{\left|\left(\frac{200}{y^3}\right)\left(\frac{-100}{y^2}\right)\hat{j}\right|}$$

$$= \frac{1}{2}\left(\frac{k-1}{k}\right)\frac{(-40{,}000/y^5)}{(-20{,}000/y^5)} = \frac{k-1}{k}$$

Exam 1P

Research into lightning rods has revealed that a possible function of a rod is not to attract lightening but to repel it! In order to understand this, one must realize that the sharp nature of the rod usually causes corona discharge, or breakdown of the air, in the vicinity of the rod. Thus, ions are produced in the vicinity of the rod which provide a protective dome around the structure to be safeguarded from the lightning. To determine the effect of these ions on the potential and the electric field, the following configuration is to be analyzed: An outer spherical conductor of 10 m radius is at 20 V, an inner, concentric spherical conductor of 0.2 m radius is at ground potential, and a thin 0.1 m layer of ions with a space-varying volume charge density of - $6r^3$ $\mu C/m^3$ is along the inner conductor (r is the distance from the center of the spheres–the radius). **A uniform surface-charge density** of 2 $\mu C/m^2$ exists between the space-varying volume charge density and the free space surrounding it.

a) Using Poisson's equation

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_o}$$

in the Spherical coordinate system, determine the general expressions for the potential distribution everywhere between the two electrodes (do not solve for the constants of integration).

b) State all boundary conditions necessary to solve for all the integration constants. These boundary conditions must be in terms of the potential (not the electric field)! Do not solve for the potential or electric field.

The Laplacian of a function f in spherical coordinates is given by the expression

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

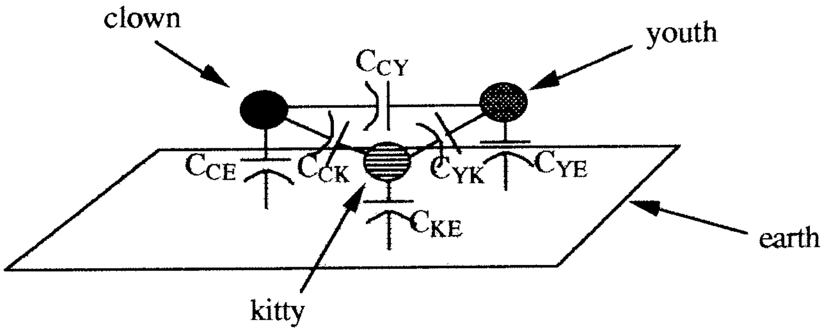
The gradient of a function f in spherical coordinates is given by the expression

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi}$$

Exam 1L

Pennywise is a very, very bad clown. He frequently zaps unsuspecting children and cats with jolts of electrostatic energy. Imagine that Pennywise is charged to a voltage of 25 kV (relative to the earth) when he is alongside an innocent charge-free youth (who is not grounded) and a grounded kitty. Assume that the mutual capacitance between the clown and the earth is $C_{CE} = 130$ pF, between the clown and the youth is $C_{CY} = 40$ pF, between the youth and the earth is $C_{YE} = 90$ pF, between the clown and the kitty is $C_{CK} = 30$ pF, between the youth and the kitty is $C_{YK} = 10$pF,and between the kitty and the earth is $C_{KE} =$ 50 pF, as shown below.

1) Set up (but do not solve or simplify) all the equations required to determine all the unknown charges and voltages in this scenario. Clearly indicate the value of all the known variables. Do not solve for the unknowns in these equations.



Exam 2P

A small charged paint droplet of mass 10 × 10^{-10} kg and charge - 10^{-14} C is in an electroquasistatic field with a potential distribution

$$\Phi(x,y) = 100y\sin x \text{ volts}$$

a) Determine the expression for the electric field.

b) Clearly sketch the equipotentials and the electric field for $0 \leq x \leq \pi$. Label all the 0 V contours.

Exam 2L

A small piece of Goldenrod pollen is floating above the surface of the earth. It has a dielectric constant of 2.5. The electric field in this pollen due to both the atmospheric electric field and the internal charge is in spherical coordinates approximately

$$\vec{E} = K\left[\frac{\vec{r}}{r^4} + \frac{\vec{\phi}}{r\theta}\right] \text{ V/m}$$

where r is the distance from the center of the earth and K is a constant.

a) For this piece of pollen, determine the ratio of the dielectrophoretic force to the electrophoretic force ($\rho\vec{E}$). (Hint: Use one of Maxwell's equations.) Do not simplify the expression.

The Laplacian of a function f in spherical coordinates is given by the expression

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

The gradient of a function f in spherical coordinates is given by the expression

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi}$$

The divergence of a vector F in spherical coordinates is given by the expression

$$\nabla\cdot\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$$

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1P) a) $.2 < r < .3\ m$

$$\nabla^2 \Phi_i = -\rho/\epsilon_0 \quad , \quad \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi_i}{dr}\right) = +\frac{6r^3\times10^{-6}}{\epsilon_0} \quad , \quad \frac{d}{dr}\left(r^2\frac{d\Phi_i}{dr}\right) = \frac{6r^5\times10^{-5}}{\epsilon_0}$$

$$r^2\frac{d\Phi_i}{dr} = \frac{r^6\times10^{-5}}{\epsilon_0} + A \quad , \quad \frac{d\Phi_i}{dr} = \frac{r^4\times10^{-6}}{\epsilon_0} + \frac{A}{r^2}$$

$$\Phi_i = \frac{r^5\times10^{-6}}{5\epsilon_0} - \frac{A}{r} + B$$

$.3 < r < 10m$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi_o}{dr}\right) = 0 \quad , \quad \frac{d}{dr}\left(r^2\frac{d\Phi_o}{dr}\right) = 0 \quad , \quad r^2\frac{d\Phi_o}{dr} = C \quad , \quad \frac{d\Phi_o}{dr} = \frac{C}{r^2}$$

$$\Phi_o = -\frac{C}{r} + D$$

b) $\Phi_i(.2) = 0\ V \quad , \quad \Phi_o(10) = 20\ V \quad , \quad \Phi_i(.3) = \Phi_o(.3)$

$$D_{No} - D_{Ni} = \rho_s\Big|_{r=.3} \quad , \quad \epsilon_0 E_{No} - \epsilon_0 E_{Ni} = \rho_s\Big|_{r=.3}$$

$$\epsilon_0\left(-\frac{d\Phi_o}{dr}\right) - \epsilon_0\left(-\frac{d\Phi_i}{dr}\right) = \rho_s\Big|_{r=.3} \quad , \quad -\frac{d\Phi_o}{dr} + \frac{d\Phi_i}{dr} = \frac{\rho_s}{\epsilon_0}\Big|_{r=.3}$$

where $\rho_s = 2\times10^{-5}\ C/m^2$

1L)

$$Q_C = C_{CE}V_{CE} + C_{CK}(V_{CE} - V_{KE}) + C_{CY}(V_{CE} - V_{YE})$$

$$Q_Y = C_{YE}V_{YE} + C_{YK}(V_{YE} - V_{KE}) + C_{CY}(V_{YE} - V_{CE})$$

$$Q_K = C_{KE}V_{KE} + C_{YK}(V_{KE} - V_{YE}) + C_{CK}(V_{KE} - V_{CE})$$

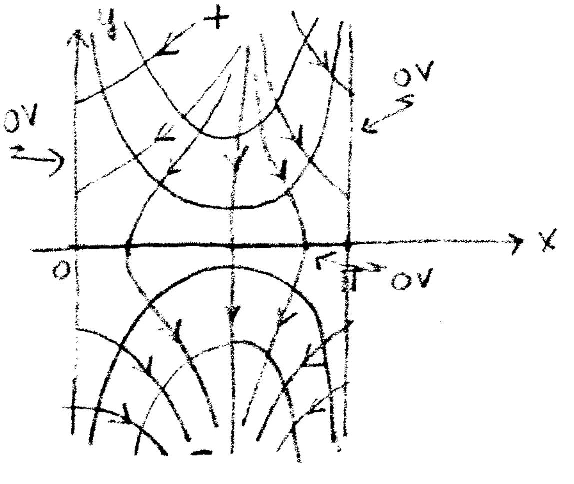
C's defined as in problem statement

$Q_Y = 0 \quad , \quad V_{KE} = 0$

2P) a) $\Phi(x,y) = 100y\sin x \quad , \quad \vec{E} = -\nabla\Phi = -\frac{d\Phi}{dx}\hat{i} - \frac{d\Phi}{dy}\hat{j}$

$$\vec{E} = -100y\cos x\,\hat{i} - 100\sin x\,\hat{j}$$

b)



$100y\sin x = 0$

$@\ y = 0 \ , \ x = 0, \pi$

$$y = \frac{K}{\sin x}$$

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2L) $$\left|\frac{F_D}{F_E}\right| = \frac{\left|\frac{1}{2}\epsilon_0(K-1)\nabla\vec{E}^2\right|}{|\rho\vec{E}|}$$

$$\vec{E}^2 = \vec{E}\cdot\vec{E} = K\left[\frac{\vec{r}}{r^4} + \frac{\vec{\Phi}}{r\theta}\right]\cdot K\left[\frac{\vec{r}}{r^4} + \frac{\vec{\Phi}}{r\theta}\right] = K^2\left[\frac{1}{r^8} + \frac{1}{r^2\theta^2}\right]$$

$$\nabla\vec{E}^2 = K^2\left[\left(-\frac{8}{r^9} - \frac{2}{r^3\theta^2}\right)\vec{r} + \frac{1}{r}\left(\frac{-2}{r^2\theta^3}\right)\vec{\theta}\right]$$

$$\rho = \nabla\cdot\vec{E} = \frac{1}{r^2}\frac{d}{dr}\left(\frac{K}{r^2}\right) = \frac{1}{r^2}\,\frac{-2K}{r^3} = \frac{-2K}{r^5}$$

$$\left|\frac{F_D}{F_E}\right| = \frac{\left|\frac{1}{2}\epsilon_0(1.5)K^2\left[\left(\frac{-8}{r^9} - \frac{2}{r^3\theta^2}\right)\vec{r} + \frac{-2}{r^3\theta^3}\vec{\theta}\right]\right|}{\left|\frac{-2K}{r^5}K\left[\frac{\vec{r}}{r^4} + \frac{\vec{\Phi}}{r\theta}\right]\right|}$$

Problem 3 (wgt = 12)

A proposed spherical fluid tank for outerspace consists of an inner, metallic, spherical electrode of radius 0.2 m surrounded by an outer, metallic, spherical electrode of radius 0.9 m. A time varying voltage is applied across the electrodes. The fluid inlets are along the inner electrode. The fluid is initially "pressed" against the inner electrode with the fluid/air interface located at r = 0.4 m. The dielectric constant of the fluid is equal to 5. The potential distribution inside the fluid is determined to be

$$\Phi(r) = \frac{-11 \times 10^3 \cos 120t}{r} + 55 \times 10^3 \cos 120t \quad \text{V}$$

Find the total force on the fluid and its direction.

$\varepsilon_o = 8.854 \times 10^{-12}$ F/m

The gradient of a function f in spherical coordinates is given by the expression

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{\phi}$$

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1) a)  No movement, $\frac{dE_y}{dz} < 0$

b)  , $\frac{dE_x}{dy} > 0$, $\frac{dE_x}{dx} < 0$

2) $|B| = .05\times10^{-6}\cos 94.2t \ \ T$, $\vec{B} = |B|(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}$, $d\vec{S}_1 = \hat{i}\,dy\,dz$, $d\vec{S}_2 = \hat{j}\,dx\,dz$

$$V = -N\frac{d\Phi}{dt} \approx -\frac{10(.001)}{\sqrt{3}}\frac{d}{dt}(|B|) - \frac{20(.003)}{\sqrt{3}}\frac{d}{dt}(|B|)$$

$$= 1.91\times10^{-7}\sin 94.2t \ \ V$$

3) $\vec{E} = -\nabla\Phi = -\frac{d}{dr}(\Phi)\hat{r} = 11\times10^{3}\cos 120t\left(\frac{1}{r^2}\right)$

$$\vec{F} = \frac{1}{2}\epsilon_0(5-1)\nabla E^2 = 2\epsilon_0\nabla\left(\frac{1.2\times10^{8}\cos^2 120t}{r^4}\right) = 2.1\times10^{-3}\cos^2 120t\,\frac{d}{dr}(r^{-4})\hat{r}$$

$$= (-8.6\times10^{-3}\cos^2 120t)/r^5\ \hat{r}\ N/m^3$$

$$f = \int_0^{2\pi}\int_0^{\pi}\int_{.2}^{.4} -\frac{8.6\times10^{-3}\cos^2 120t}{r^5}\, r^2\sin\theta\, dr\, d\theta\, d\phi = .054\cos^2 120t\left(\frac{1}{r^2}\right)\Big|_{.2}^{.4}$$

$$= -\cos^2 120t \ \ N$$

4) a) $\frac{d^2\Phi}{dy^2} = -\frac{5+10y}{\epsilon_0}$, $\frac{d\Phi}{dy} = -\frac{5y}{\epsilon_0} + \frac{5y^2}{\epsilon_0} + C$, $\Phi = -\frac{5y^2}{2\epsilon_0} + \frac{5y^3}{3\epsilon_0} + Cy + D$

b) $\Phi(0) = 0$, $\Phi(.05) = 0$

c) $\Phi(0) = 0 = D$, $\Phi(.05) = 0 = -\frac{5(.05)^2}{2\epsilon_0} + \frac{5(.05)^3}{3\epsilon_0} + C(.05) \Rightarrow C = 1.36\times10^{1}$

$$\Phi = \left(-\frac{5y^2}{2\epsilon_0} + \frac{5y^3}{3\epsilon_0} + 1.36\times10^{10}y\right)\times\underset{\substack{\uparrow\\ \rho(y)\ \text{IN NC}/m^3}}{10^{-9}} \ \ V$$

$$\doteq -280y^2 + 190y^3 + 14y \ \ V \quad (\Phi(0) = 0 \checkmark,\ \Phi(.05) \approx 0 \checkmark)$$

d) $\vec{E} = -\nabla\Phi = -\frac{d\Phi}{dy}\hat{j} = (560y - 570y^2 - 14\big/_{y=.05})\hat{j} = 13\hat{j} \ \ V/m$

⑤ $\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt} = \sigma(10\cos 20t)\hat{i} + \epsilon\frac{d}{dt}(10\cos 20t)\hat{i} = \sigma 10\cos 20t\,\hat{i} - \epsilon 200\sin 20t\,\hat{i}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ H_x & H_y & H_z \end{vmatrix} = \hat{i}\left(\frac{dH_z}{dy} - \frac{dH_y}{dz}\right) = \hat{i}\frac{dH_z}{dy} = (\sigma 10\cos 20t - \epsilon 200\sin 20t)\hat{i}$$

$$\therefore H_z = (\sigma 10\cos 20t - \epsilon 200\sin 20t)y + C$$

The constant C could be a function of x. Both the divergence and curl of $\vec{H}$ are required to completely describe $\vec{H}$. (Curious Note: Why is $\nabla\times\vec{E} = 0 = -d\vec{B}/dt$? Does this imply that $\vec{B}$ = constant?)

Exam 2L

A young, associate professor "hard-up" for cash decides to send a proposal to the Environmental Protection Agency (EPA) in hopes of obtaining grant money. The professor proposes that bees can be utilized to pick-up undesirable lint throughout the land. The professor believes that each time a bee pollinates a flower it picks up a net charge of +2 pC. On the average a bee pollinates 10 flowers before picking up lint for a bird's nest. Assume the lint is "small" with a dielectric constant of 3.1 and radius of 1 mm, the bee can be modeled a spherical conductor of radius 0.2 cm, and the lint and the bee are far from the ground. The electric field inside the lint is approximately

$$\vec{E}_{int} = \frac{3}{5}\vec{E}_{ext}$$

where $\vec{E}_{ext}$ is the applied external field.

1) Determine the average lint <u>mass</u> the bee can levitate if the bee and lint are separated by 0.5 cm (center-to-center distance).
2) At what distance from the center of the bee is the charge in the lint <u>not</u> approximately uniformly distributed? Use one of Maxwell's equations and approximate numerical results to substantiate your claim. (Carefully use the equation for the divergence given below.)

$$\oint_s \vec{D} \cdot \vec{ds} = Q$$

$\varepsilon_o = 8.854 \times 10^{-12}$ F/m

$g = 9.8$ m/s^2

The gradient of a function f in the spherical coordinate system is

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{\phi}$$

The divergence of a vector F in the spherical coordinate system is

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

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2L) $\vec{E}_{ext} = \frac{Q\hat{r}}{4\pi\epsilon_0 r^2} = \frac{(2\times10^{-12})(10)\hat{r}}{4\pi\epsilon_0 r^2} = \frac{.18}{r^2}\hat{r}$

$\vec{E}_{int} = \frac{3}{5}\left(\frac{.18}{r^2}\right)\hat{r} = \frac{.11}{r^2}\hat{r}$, $\vec{F} = \frac{1}{2}\epsilon_0(3.1-1)\nabla E^2$

$\vec{F} = 1.05\epsilon_0 \nabla\left(\frac{.0121}{r^4}\right) = 1.05\epsilon_0(.0121)\frac{(-4)\hat{r}}{r^5} = \frac{-.0508\epsilon_0\hat{r}}{r^5}$

$r = 0.5\times10^{-2}$, $\vec{F} = -.144\,\hat{r}$ N/m^3

$|F|\left(\frac{4}{3}\pi r_L^3\right) = mg = m(9.8)$, $r_L = 1\times10^{-3}$ m

$\therefore\ m = 6.2\times10^{-11}$ Kg very light mass

$\nabla\cdot\vec{D} = \rho$ (C/m^3)

$3.1\epsilon_0 \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{.18}{r^2}\right) = 3.1\epsilon_0\frac{1}{r^2}\frac{d}{dr}(.18) = 0$ of course! The lint is charge neutral.

3P) $\vec{E}_o = E_0\cos\omega t\,\hat{j} - 3\alpha\cos\omega t(E_0\cos\omega t\,\hat{j})$

$= E_0\cos\omega t\,\hat{j} - 3\alpha E_0\cos^2\omega t\,\hat{j}$ @ $x = 0$

$\vec{H}_o = \frac{E_0}{\eta_0}\cos\omega t\,\hat{k} - \frac{3\alpha E_0}{\eta_0}\cos^2\omega t\,\hat{k}$

$$\vec{P}_o = \vec{E}_o\times\vec{H}_o = \left[\frac{E_0^2}{\eta_0}\cos^2\omega t - \frac{3\alpha E_0^2}{\eta_0}\cos^3\omega t - \frac{3\alpha E_0^2}{\eta_0}\cos^3\omega t + \frac{9\alpha^2E_0^2}{\eta_0}\cos^4\omega t\right]\hat{i}$$

$\vec{P}_i = \vec{E}_i\times\vec{H}_i = \frac{E_0^2}{\eta_0}\cos^2\omega t\,\hat{i}$

$\left|\frac{P_o}{P_i}\right| = 1 - 6\alpha\cos\omega t + 9\alpha^2\cos^2\omega t$

$$\frac{1}{T}\int_0^T\left(1 - 6\alpha\cos\omega t + 9\alpha^2\cos^2\omega t\right)dt = 1 + 0 + \frac{9\alpha^2}{2} = \frac{2+9\alpha^2}{2}$$

3L) $E_z = E_0(x,y)\cos(\omega t+\theta) = Re[E_0(x,y)e^{j(\omega t+\theta)}] = Re[E_0(x,y)e^{j\theta}e^{j\omega t}]$

$E_{zs} = E_0(x,y)e^{j\theta}$

$\frac{d^2E_z}{dx^2} \Leftrightarrow \frac{d^2E_{zs}}{dx^2}$, $-\frac{dE_z}{dt} \Leftrightarrow -j\omega E_{zs}$, $\frac{dE_z}{dy} \Leftrightarrow \frac{dE_{zs}}{dy}$

$5\sin(\omega t) = 5\cos(\omega t - 90°) = Re[5e^{j(\omega t-90°)}] = Re[5e^{-j90°}e^{j\omega t}]$

$-2\cos(\omega t) = Re[-2e^{j\omega t}]$

$\frac{d^2E_{zs}}{dx^2}e^{j\omega t} - j\omega E_{zs}e^{j\omega t} + \frac{dE_{zs}}{dy}e^{j\omega t} = 5e^{-j90°}e^{j\omega t} - 2e^{j\omega t}$

$\frac{d^2E_{zs}}{dx^2} - j\omega E_{zs} + \frac{dE_{zs}}{dy} = -5j - 2$

Exam 2L

A small piece of Goldenrod pollen is floating at a height of 10 m above the ground. It has a dielectric constant of 2.5. The electric field in this pollen due to both the atmospheric electric field and the internal charge is approximately

$$\vec{E} = \frac{-200}{y^3}\,\vec{j}\ \text{V/m}$$

where y is the distance from the ground.

1) For this piece of pollen, determine the ratio of the dielectrophoretic force to the electrophoretic force ($\rho\vec{E}$). (Hint: Use one of Maxwell's equations.)

2) Is the charge in the pollen uniformly distributed? Use one of Maxwell's equations and approximate numerical results to substantiate your claim. The pollen is located 10 m from the ground, and the pollen is small.

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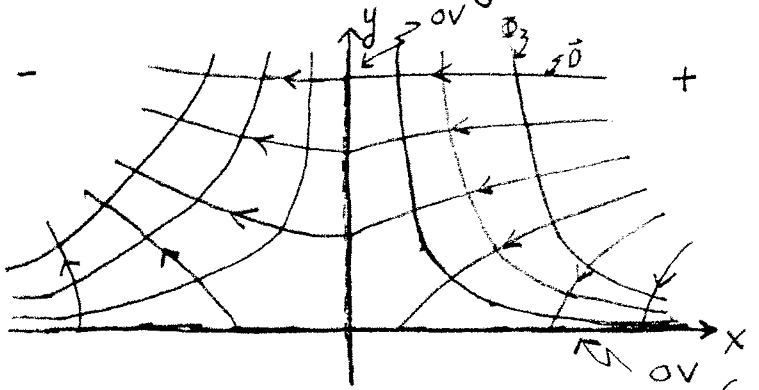
2P) $\vec{E} = -\nabla\Phi = -\frac{d\Phi}{dx}\hat{i} + -\frac{d\Phi}{dy}\hat{j} = -500\ln(y+1)\hat{i} - \frac{500x}{y+1}\hat{j}$ V/m



$\Phi = 500x\ln(y+1)$ V

$K = x\ln(y+1)$

$y+1 = e^{K/x}$

$y = e^{K/x} - 1$

$(\ln(0+1) = 0)$

2L) $\left|\frac{F_D}{F_E}\right| = \frac{\left|\frac{1}{2}\epsilon_0(k-1)\nabla\vec{E}^2\right|}{|\rho\vec{E}|} = \frac{\left|\frac{1}{2}\epsilon_0(1.5)\nabla\left(\frac{40000}{y^6}\right)\right|}{|(\nabla\cdot\vec{D})\vec{E}|} = \frac{\left|.75\epsilon_0(40000)\nabla\left(\frac{1}{y^6}\right)\right|}{\left|2.5\epsilon_0\nabla\cdot(-200/y^3\hat{j})(-200/y^3\hat{j}\right|}$

$$= \frac{.75(40{,}000)\left|\nabla\left(\frac{1}{y^6}\right)\right|}{2.5(40{,}000)\left|\nabla\cdot(\hat{j}/y^3)(\hat{j}/y^3)\right|} = \frac{.3\left|-6y^{-7}\hat{j}\right|}{\left|-3y^{-4}\cdot y^{-3}\hat{j}\right|}$$

$$= \frac{(.3)6/y^7}{3/y^7} = .6$$

In the pollen

$$\rho = \nabla\cdot\vec{D} = \nabla\cdot\left(k\epsilon_0\frac{(-200)\hat{j}}{y^3}\right) = -200k\epsilon_0\nabla\cdot\left(\frac{\hat{j}}{y^3}\right)$$

$$= -200(2.5)\epsilon_0\left(-\frac{3}{y^4}\right)$$

$$= 1.33\times10^{-8}/y^4$$

$\nabla = \frac{d}{dy}\hat{j}$

$\nabla\cdot(-\hat{j}/y^3) = -\frac{d}{dy}(y^{-3})$

Since the particle is "small", we can assume that the above charge variation near y = 10m is relatively small - uniform distribution.

e.g. If pollen spherical with 1mm radius, the charge variation is

$1.32\times10^{-8} \leq \rho \leq 1.34\times10^{-8}$ c/m^3

Exam 2L

A small piece of Goldenrod pollen is floating at a height of 10 m above the surface of the earth. It has a dielectric constant of 3.0. The electric field in this pollen due to both the atmospheric electric field and the internal charge is approximately

$$\vec{E} = \frac{K}{r^2}\vec{r} \text{ V/m}$$

where r is the distance from the center of the earth and K is a constant. The radius of the earth is 6.37 Mm.

1) For this piece of pollen, determine the ratio of the dielectrophoretic force to the electrophoretic force ($\rho\vec{E}$). (Hint: Use one of Maxwell's equations.)
2) If K = 25,000, is the charge in the pollen approximately uniformly distributed?

The Laplacian of a function f in spherical coordinates is given by the expression

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

The gradient of a function f in spherical coordinates is given by the expression

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi}$$

The divergence of a vector F in spherical coordinates is given by the expression

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$$

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2P) $\nabla^2 V = \frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \quad , \quad \frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \quad , \quad r\frac{dV}{dr} = A$

$\frac{dV}{dr} = \frac{A}{r} \quad , \quad V = A\ln r + B$

$V_1 = A\ln r + B \qquad .05 < r < 1$

$V_2 = C\ln r + D \qquad 1 < r < 2$

$0 = A\ln(.5) + B \quad , \quad A = 1.44B$

$25{,}000 = C\ln(2) + D \quad , \quad C = -1.44D + 36{,}100$

$A\ln(1) + B = C\ln(1) + D \quad , \quad B = D$

$D_{N1} = D_{N2}\Big|_{r=1} \quad , \quad -4\epsilon_0\frac{dV}{dr} = -\epsilon_0\frac{dV}{dr}\Big|_{r=1} \quad , \quad 4\left(\frac{A}{r}\right) = \frac{C}{r}\Big|_{r=1}$

$A = C/4 = (-1.44B + 36{,}100)/4 = 1.44B$

$\Rightarrow B = 5{,}010 \quad , \quad A = 7{,}200$

$\vec{E}_1 = -\nabla V_1 = -\frac{dV_1}{dr}\hat{r} = -\frac{A}{r}\hat{r} = -\frac{7{,}200}{r}\cos 130t\,\hat{r}$

$|\vec{E}_1| = \frac{7{,}200}{r}\cos 130t \quad V/m$

2L) 1) $R = \frac{\left|\frac{1}{2}\epsilon_0(K-1)\nabla|\vec{E}|^2\right|}{|\rho\vec{E}|} = \frac{\left|\frac{1}{2}\epsilon_0(2)\nabla|\vec{E}|^2\right|}{|(\nabla\cdot\vec{D})\vec{E}|} = \frac{|\epsilon_0\nabla|\vec{E}|^2|}{|\epsilon_0(\nabla\cdot\vec{E})\vec{E}|}$

$\nabla|\vec{E}|^2 = \nabla\left(\frac{K^2}{r^4}\right) = \frac{-4K^2}{r^5}$

$\nabla\cdot\vec{E} = \frac{1}{r^2}\frac{d}{dr}(r^2E_r) = \frac{1}{r^2}\frac{d}{dr}(K) = 0$

$R = \frac{-\frac{4K^2}{r^5}}{0} \rightarrow \infty$ No net charge on pollen

2) It's always zero - Neutral.

3P) 1) a) $\rho = \nabla\cdot\vec{D} = 75\epsilon_0\nabla\cdot\vec{E} = 75\epsilon_0(0) = 0$

$-j\omega\mu_0\vec{H}_s = \nabla\times\vec{E}_s = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ E_x & E_y & E_z \end{vmatrix} = -\hat{j}\frac{dE_{zs}}{dx} + -\hat{k}\frac{dE_{xs}}{dy}$ $\qquad \omega = 377$

$= E_a\left[-\hat{j}(-6x^2) - \hat{k}(6y)\right] = E_a\left[6x^2\hat{j} - 6y\hat{k}\right]$

$\frac{1}{-j} = -(-j)$

$\frac{1}{-j} = 1 \qquad \vec{H}_s = E_a\left[\frac{6x^2}{\omega\mu_0}\angle 90^\circ\hat{j} - \frac{6y}{\omega\mu_0}\angle 90^\circ\hat{k}\right]$

Problem 5 (wgt = 8)

A small insulating ball of radius b, mass density 0.9 kg/m^3, and dielectric constant 4 is placed in an electric field at the location (0, -2) m described by the expression

$$\vec{E} = \frac{1}{y^2}\,\vec{j}\ \text{V/m}$$

The electric field inside the ball is approximately

$$\overrightarrow{E_{int}} = \frac{3}{k+2}\overrightarrow{E_{ext}}$$

where $\overrightarrow{E_{ext}}$ is the applied external field. If gravity is acting in the -y direction, determine the maximum radius of the ball for electrical levitation to occur.

$\varepsilon_o = 8.854 \times 10^{-12}$ F/m

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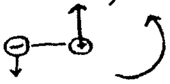
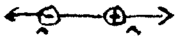
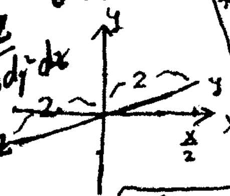
① a) $dE_z/dx > 0$  initial rotation direction

b) $dE_x/dy < 0$  No motion since center-of-charge fixed

② $g = x - 2y$, $\nabla g = \hat{i} - 2\hat{j}$, $|\nabla g| = \sqrt{5}$, $\hat{n} = (\hat{i} - 2\hat{j})/\sqrt{5}$

$\vec{B}\cdot\hat{n} = \frac{1}{\sqrt{5}}(y^2\cos 120\pi t + 2x^2\cos 120\pi t)$

*see back



$y = x/2$

$V = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = -\frac{d}{dt}\left[\frac{1}{\sqrt{5}}\int_{-2}^{2}\int_{-1.79}^{1.79}\left(\frac{9x^2}{4}\cos 120\pi t\right)\cancel{dx}\,dz\right]$

$dS = dL\,dz$

$= \sqrt{dx^2+dy^2}\,dx$

$= \sqrt{5}\,dy\,dz$

$= -\frac{d}{dt}\left[\cancel{\frac{9}{\sqrt{5}}\cos 120\pi t\,\frac{x^3}{3}\Big|_{-1.79}^{1.79}}\right] \cancel{= 5.8\sin 120\pi t\ kV}$

$\sqrt{16} = 4$

$= 2\cdot 2$

$2 = \sqrt{x^2+y^2} = \sqrt{x^2+\frac{x^2}{4}} = \frac{x}{2}\sqrt{}$

$\Rightarrow x \doteq 1.79$

③ a) $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -\rho/\epsilon$, $\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -\rho r^2/\epsilon_0$, $r^2\frac{dV}{dr} = -\rho r^3/3\epsilon_0 + C$

$\frac{dV}{dr} = -\rho r/3\epsilon_0 + C/r^2$, $V = -\rho r^2/6\epsilon_0 - C/r + D$

b) $V_i(r=0) = \text{finite}$, $V_o(r=100b) = 0$, $V_i(r=b) = V_o(r=b)$,

$dV_i/dr = dV_o/dr\,/_{r=b}$

c) $V_i(r=0)$ is finite $\Rightarrow C = 0$ $\qquad V_i = -\rho r^2/6\epsilon_0 - C/r + D$

$V_o(r=100b) = 0 = -E/100b + F$ $\qquad V_o = -E/r + F$

$\Rightarrow F = E/100b$

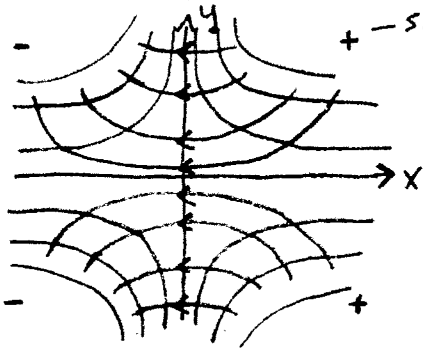
$-\rho b^2/6\epsilon_0 + D = -E/b + F \Rightarrow D = \rho b^2/6\epsilon_0 - E\left(\frac{99}{100b}\right)$

$-\rho b/3\epsilon_0 = E/b^2 \Rightarrow E = -\rho b^3/3\epsilon_0$, $D = \rho b^2/6\epsilon_0 + 99\rho b^2/300\epsilon_0$

$D = 149\rho b^2/300\epsilon_0$

$\therefore V_i = -\rho r^2/6\epsilon_0 + 149\rho b^2/300\epsilon_0 = \frac{\rho}{300\epsilon_0}(149b^2 - 50r^2)$

④



$10{,}000xy^2 = K$

$y^2 = \frac{K}{10{,}000}\left(\frac{1}{x}\right)$, $y \propto \frac{1}{\sqrt{x}}$

—— equipotentials

→ $\vec{E}$

$\vec{E} = -\nabla\Phi = -(10{,}000y^2\hat{i} + 20{,}000xy\hat{j})$ V/m

$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{2x}{y}$, $y\,dy = 2x\,dx \Rightarrow \frac{y^2}{2} = x^2 + K$

c $(0.1, 0.1)$ $\frac{(.1)^2}{2} = (.1)^2 + K \Rightarrow K = -.005$ $\qquad x^2 - \frac{y^2}{2} = .005$

⑤ $\vec{F_e} = \frac{1}{2}\epsilon_0(k-1)\nabla E^2$, $\vec{E} = \frac{1}{2}\frac{1}{y^2}\hat{j}$, $E^2 = \frac{1}{4y^4}$ $\qquad \nabla E^2 = \frac{d}{dy}\left(\frac{1}{4y^4}\right)\hat{j} = \frac{-\hat{j}}{y^5}$

$F_e = \frac{3}{2}\epsilon_0(-\hat{j}/y^5) = 415\times 10^{-15}$ N/m^3

m

$(415\times 10^{-15})\frac{4}{3}\pi b^3 > \left(\frac{4}{3}\pi b^3\rho_m\right)g$

$415\times 10^{-15} > \rho_m g = 8.82$ No

Not a function of b — above work not necessary to determine this — this ball will not levitate