Problem 1 (wgt = 7)

A young seismologist believes that a strong magnetic field is emitted prior to an earthquake. In order to test this theory she constructs a coil and places it in a hole deep in the ground. The coil consists of ten closely spaced turns (each turn has an area of 0.001 m^2). Assume the plane of the coil coincides with the $x = 0$ plane in the Cartesian coordinate system. If a 20 Hz magnetic field is emitted in a direction along the diagonal $x = y = z$ (with an amplitude of 0.02 μT at the coil), determine the voltage picked-up by the coil.

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① $V = -N\frac{d\Phi}{dt} = -10\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = -10\frac{d}{dt}\int (.02\times10^{-6}\cos 126t)(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}\cdot \hat{i}\,dy\,dz$

$\approx (-.2\times10^{-6}/\sqrt{3})(.001)\frac{d}{dt}(\cos 126t) = 1.45\times10^{-8}\sin 126t \;\; V$

② $P_D = \int_V \sigma E_T^2\, dV \doteq (5\times10^{-2})(10{,}000\cos\omega t)^2(25\times10^{-6})$ $\quad$ (E_y at $(\frac{a}{2},\frac{a}{2},\frac{d}{2})$)

$\hat{P}_D = \frac{1}{T}\int_0^T P_D\, dt = \frac{1}{2}(125) = 62.5 \text{ W}$

③ $E_y = -dV/dy = -(x^2+1)10{,}000\cos 377t \;\; V/m$

$i = dq/dt = \frac{d}{dt}\int \epsilon_0 E_y\, dx\,dz = \frac{d}{dt}\left(-10{,}000\,\epsilon_0\cos 377t \int_{-.5}^{.5}\int_x^{x+1}(x^2+1)\,dx\,dz\right)$ ← one possible set of limits (e.g. $\int_{x-.5}^{x+.5}$ OK)

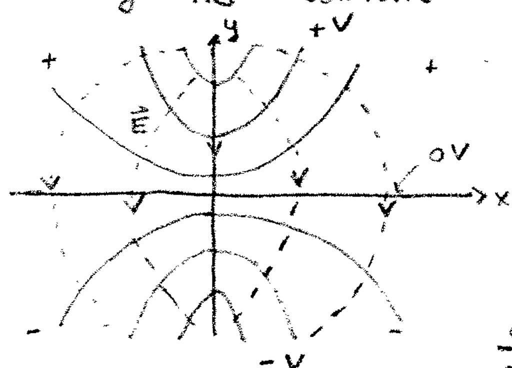
$= 3.34\times10^{-5}\sin 377t\left[\frac{x^3}{3}+x\right]_x^{x+1}$

$= 3.34\times10^{-5}\sin 377t\left[\frac{(x+1)^3}{3}+x+1-\frac{x^3}{3}-x\right] = 3.34\times10^{-5}\sin 377t\left[\frac{(x+1)^3}{3}-\frac{x^3}{3}+1\right]$

$di/dx \propto 3(x+1)^2/3 - 3x^2/3 = x^2+2x+1-x^2 = 2x+1 = 0 \Rightarrow x = -1/2$

current a minimum at $x=-1/2$ $\quad (i(-1/2) < i(0))$

④ $ye^{-x} = K \Rightarrow y = Ke^{x}$ contours $\qquad \Phi = 10{,}000\,y e^{-|x|}$



$- \rightarrow -$ $\vec{E}$

—— equipotentials

$(1,2)\;\; x>0 \quad \Phi = 10{,}000\,ye^{-x}$

$\vec{E} = -\nabla\Phi = 10{,}000\,ye^{-x}\hat{i} - 10{,}000\,e^{-x}\hat{j}$

$\frac{dy}{dx} = \frac{E_y}{E_x} = -\frac{1}{y}$, $\quad y\,dy = -dx$

$\frac{y^2}{2} = -x + C$

$\frac{(2)^2}{2} = -1 + C \Rightarrow C = 3 \qquad y^2/2 + x = 3$ (negative y direction)

⑤ $\vec{E} = \frac{Q\,\hat{r}}{4\pi\epsilon_0 r^2} = \frac{(-1\times10^{-12})(12)}{4\pi\epsilon_0 r^2}\hat{r} = \frac{-.108}{r^2}\hat{r}\;V/m$, $\quad \vec{E}_{int} \doteq \frac{-.0647}{r^2}\hat{r}\;\;V/m$

$\vec{F} = \frac{1}{2}\epsilon_0(k-1)\nabla E^2 = \epsilon_0\nabla(.00419/r^4) = \epsilon_0(.00419)(-4/r^5)\hat{r}$

$= (-1.48\times10^{-13}/r^5)\hat{r}\,\big|_{r=.02} = -4.64\times10^{-5}\,\hat{r}\;\; N/m^3$

$|\vec{F}|(\frac{4}{3}\pi r_L^3) = (\rho_m \frac{4}{3}\pi r_L^3)g$

$\Rightarrow \rho_m = 4.73\times10^{-6}\;kg/m^3$!

ASSIGNMENT #1

EE-432
K. Kaiser

1) A young seismologist believes that a strong 15 Hz magnetic field is emitted prior to an earthquake. In order to test this theory she constructs an elaborate pick-up "coil" and places it in a hole deep in the ground. The "coil" consists of three perpendicular coils each of 10 turns (each turn has an area of 0.01 m^2). Assume the plane of each of the three coils coincides with the three planes in the Cartesian coordinate system. If a magnetic field is emitted in a direction along the diagonal $x = y = z$ (with an amplitude of 0.01 μT at the coils), determine the voltage picked-up by the "coil". Also, assume each of the coils are connected in series for maximum voltage and there is no interaction between the coils.

2) Research into lightning rods has revealed that the function of a rod is not to attract lightening but to repel it! In order to understand this one must realize that the sharp nature of the rod usually causes a corona discharge, or breakdown of the air, in the vicinity of the rod. Thus, ions are produced in the vicinity of the rod which provide a protective dome around the structure to be safeguarded from the lightning. Let's determine the effect of these ions by studying the following problem (the actual magnitudes have been altered to simplify the numerics). Assume that an outer spherical conductor (the clouds) with a 10 m radius is at a potential of 100 volts, and that an inner, concentric spherical conductor with a 1 m radius (the ground) is at ground potential. What is the electric field near the surface? Now, assuming that a thin, 1 meter layer of ions with an uniform charge density of -0.1 nC/m^3 exists along the ground, what is the electric field near the surface? Did the magnitude of the electric field increase or decrease?

3) Johnny loved pushing his baby brother in his wagon but frequently noticed a tingling sensation when passing near a local power line. Assuming that Johnny, his brother, and the wagon are good conductors, the total resistance from the ground through Johnny to the wagon is 2 kΩ, and the "cross-sectional area" of the group including the wagon is 3 m^2, determine the induced current through and the voltage across Johnny as a function of their distance along the ground from the power line. The voltage of the overhead power line is

$$V_o = 7.6\cos 377t \text{ kV}$$

the height of the line is 10 m, and the radius of the line is 1 cm (assume a low current, electroquasistatic situation). They travel perpendicular to the line. How would you reduce this tingling? The potential distribution between a cylindrical line of radius a at a potential of V_o and a ground plane at a distance h from the center of the line is given by

$$V = \frac{V_o}{\ln\left[\frac{h}{a} + \sqrt{\left(\frac{h}{a}\right)^2 - 1}\right]} \ln\left[\frac{\sqrt{(b+x)^2 + y^2}}{\sqrt{(b-x)^2 + y^2}}\right]$$

$$\text{where } b = \sqrt{h^2 - a^2}$$

The x axis passes directly through the center of the power line and the y axis is tangent to the ground.

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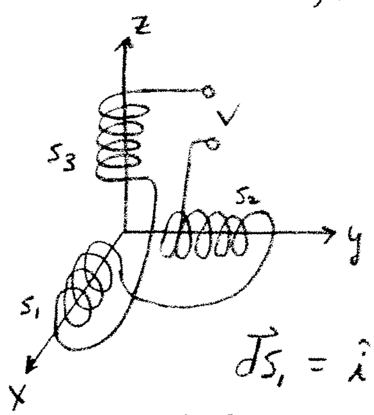
①



$$|B| = .01\times10^{-6}\cos 94.2t \quad T$$

$$V = -\frac{d}{dt}\int \vec{B}\cdot\vec{ds} = -N\frac{d\Phi}{dt}$$

$$\vec{B} = |B|(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}$$

$$\vec{ds_1} = \hat{i}\,dy\,dz\ ,\quad \vec{ds_2} = \hat{j}\,dx\,dz\ ,\quad \vec{ds_3} = \hat{k}\,dx\,dy$$

$$V = \underset{\substack{\uparrow\\ 10\text{ turns}}}{10}\left\{-\frac{d}{dt}\int_{s_1}|B|/\sqrt{3}\,dy\,dz - \frac{d}{dt}\int_{s_2}|B|/\sqrt{3}\,dx\,dz - \frac{d}{dt}\int_{s_3}|B|/\sqrt{3}\,dx\,dy\right\}$$

Assuming that $|B|$ is approximately constant over the pick-up coils

$$V = -\frac{10}{\sqrt{3}}\frac{d}{dt}\left(3\times.01\times10^{-6}(.01)\cos 94.2t\right) = 1.63\times10^{-7}\sin 94.2t \quad V$$

② $\nabla^2 V = -\rho/\epsilon_0$, spherical symmetry , $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -\rho/\epsilon_0$

$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -\rho r^2/\epsilon_0$, $r^2\frac{dV}{dr} = -\rho r^3/3\epsilon_0 + C$, $\frac{dV}{dr} = -\rho r/3\epsilon_0 + C/r^2$

$V = -\rho r^2/6\epsilon_0 - C/r + D$

a) $\rho = 0$, $V = -C/r + D$

B.C. , $V(1) = 0$, $V(10) = 100\,V$, $V(1) = 0 = -C + D \Rightarrow C = D$

$V(10) = 100 = -C/10 + D \Rightarrow C = D = 111$

$\vec{E} = -\nabla V$, $E_r = -dV/dr = -C/r^2\big|_{r=1m} = -111\ V/m$

b) $V_1 = -\rho r^2/6\epsilon_0 - C_1/r + D_1$, $V_2 = -C_2/r + D_2$, $\rho = .1\times10^{-9}$

V_1 valid for $1 \le r \le 2m$, V_2 for $2 \le r \le 10\,m$

B.C. $V_1(1) = 0$, $V_2(10) = 100V$, $V_1(2) = V_2(2)$, $D_{N1}(2) = D_{N2}(2)$

(assumes $\rho_s = 0$ along ionized layer interface)

$V_1(1) = 0 = -\rho/6\epsilon_0 - C_1 + D_1 \Rightarrow C_1 = D_1 + 1.88$, $D_1 = C_1 - 1.88$

$V_2(10) = 100 = -C_2/10 + D_2 \Rightarrow C_2 = -1{,}000 + 10D_2$, $D_2 = 100 + C_2/10$

$V_1(2) = 7.53 - C_1/2 + D_1 = -C_2/2 + D_2 = V_2(2)$

$7.53 - C_1/2 + C_1 - 1.88 = -C_2/2 + 100 + C_2/10$

$C_1/2 = -4C_2/10 + 94.4$, $C_1 = -8C_2/10 + 189$

$D_{N1}(2) = D_{N2}(2)$, $\epsilon_0 -\frac{dV_1}{dr}\big|_2 = \epsilon_0 -\frac{dV_2}{dr}\big|_2$, $-\rho 2/3\epsilon_0 + C_1/4 = C_2/4$

$\Rightarrow C_1 = C_2 - 30.1$, $C_2 - 30.1 = -8C_2/10 + 189$ $\therefore$ $C_2 = 122$

$C_1 = 91.6$

$E_r = \rho r/3\epsilon_0 - C_1/r^2\big|_{r=1} = -95.4\ V/m$ The electric field magnitude has decreased near the ground!

<u>Problem 2 (wgt = 10)</u>

A young seismologist believes that a strong magnetic field is emitted prior to an earthquake. In order to test this theory she constructs <u>two</u> coils and places them in a hole deep in the ground. The first coil consists of 10 closely spaced turns (each turn has an area of 0.001 m^2), and the plane of this coil coincides with the $x = 0$ plane in the Cartesian coordinate system. The second coil consists of 20 closely spaced turns (each turn has an area of 0.003 m^2), and the plane of this coil coincides with the $y = 0$ plane in the Cartesian coordinate system. If a 15 Hz magnetic field is emitted in a direction along the diagonal $x = y = z$ (with an amplitude of 0.05 μT at the coils), determine the voltage picked-up by the coils. Assume the coils are connected in series for maximum voltage and there is no interaction between the coils.

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① a)  No movement, $\frac{dE_y}{dz} < 0$

b)  , $\frac{dE_x}{dy} > 0$, $\frac{dE_x}{dx} < 0$

② $|B| = .05\times10^{-6}\cos 94.2t \ \ T$, $\vec{B} = |B|(\hat{i}+\hat{j}+\hat{k})/\sqrt{3}$, $d\vec{s}_1 = \hat{i}\,dy\,dz$, $d\vec{s}_2 = \hat{j}\,dx\,dz$

$$V = -N\frac{d\Phi}{dt} \approx -\frac{10(.001)}{\sqrt{3}}\frac{d}{dt}(|B|) - \frac{20(.003)}{\sqrt{3}}\frac{d}{dt}(|B|)$$

$$= 1.91\times10^{-7}\sin 94.2t \ \ V$$

③ $\vec{E} = -\nabla\Phi = -\frac{d}{dr}(\Phi)\hat{r} = 11\times10^{3}\cos 120t\left(\frac{1}{r^2}\right)$

$$\vec{F} = \frac{1}{2}\epsilon_0(5-1)\nabla E^2 = 2\epsilon_0\nabla\left(\frac{1.2\times10^{8}\cos^2 120t}{r^4}\right) = 2.1\times10^{-3}\cos^2 120t\,\frac{d}{dr}(r^{-4})\,\hat{r}$$

$$= (-8.6\times10^{-3}\cos^2 120t)/r^5\ \hat{r}\ N/m^3$$

$$f = \int_0^{2\pi}\int_0^{\pi}\int_{.2}^{.4} -\frac{8.6\times10^{-3}\cos^2 120t}{r^5}\, r^2\sin\theta\, dr\, d\theta\, d\phi = .054\cos^2 120t\left(\frac{1}{r^2}\right)\Big|_{.2}^{.4}$$

$$= -\cos^2 120t \ \ N$$

④ a) $\frac{d^2\Phi}{dy^2} = -\frac{5+10y}{\epsilon_0}$, $\frac{d\Phi}{dy} = -\frac{5y}{\epsilon_0} + \frac{5y^2}{\epsilon_0} + C$, $\Phi = -\frac{5y^2}{2\epsilon_0} + \frac{5y^3}{3\epsilon_0} + Cy + D$

b) $\Phi(0) = 0$, $\Phi(.05) = 0$

c) $\Phi(0) = 0 = D$, $\Phi(.05) = 0 = -\frac{5(.05)^2}{2\epsilon_0} + \frac{5(.05)^3}{3\epsilon_0} + C(.05) \Rightarrow C = 1.36\times10^{1}$

$$\Phi = \left(-\frac{5y^2}{2\epsilon_0} + \frac{5y^3}{3\epsilon_0} + 1.36\times10^{10}y\right)\times10^{-9} \ \ V$$

$\uparrow$ $\rho(y)$ in nC/m³

$$\doteq -280y^2 + 190y^3 + 14y \ \ V \quad (\Phi(0) = 0 \checkmark,\ \Phi(.05) \approx 0 \checkmark)$$

d) $\vec{E} = -\nabla\Phi = -\frac{d\Phi}{dy}\hat{j} = (560y - 570y^2 - 14\big|_{y=.05})\hat{j} = 13\hat{j}\ \ V/m$

⑤ $\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt} = \sigma(10\cos 20t)\hat{i} + \epsilon\frac{d}{dt}(10\cos 20t)\hat{i} = \sigma 10\cos 20t\,\hat{i} - \epsilon 200\sin 20t\,\hat{i}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ H_x & H_y & H_z \end{vmatrix} = \hat{i}\left(\frac{dH_z}{dy} - \frac{dH_y}{dz}\right) = \hat{i}\frac{dH_z}{dy} = (\sigma 10\cos 20t - \epsilon 200\sin 20t)\hat{i}$$

$$\therefore\ H_z = (\sigma 10\cos 20t - \epsilon 200\sin 20t)y + C$$

The constant C could be a function of x. Both the divergence and curl of $\vec{H}$ are required to completely describe $\vec{H}$. (Curious Note: Why is $\nabla\times\vec{E} = 0 = -d\vec{B}/dt$? Does this imply that $\vec{B}$ = constant?)

Exam 1P

Johnny loved pushing his baby brother in his wagon but frequently noticed a tingling sensation when passing nearby an experimental, over-head, high-voltage generator. Assuming a low current, electroquasistatic situation, the voltage distribution near the ground is approximately

$$V = [x^2(z^2 - 1)]\, 10\cos 377t \ \text{kV}$$

Johnny is traveling in the positive z direction, the ground is tangent to the z axis, and the x axis is in the vertical direction (toward the clouds) intersecting the over-head generator. The wagon is insulated from the ground via the wheels, and the total resistance from the ground through Johnny to the wagon is 2.5 kΩ. Assume that Johnny, his brother, and the wagon are good conductors, and the "square, cross-sectional area" of the group including the wagon is 0.16 m^2. Note: The average height of the wagon/Johnny/baby group above ground is 0.3 m.

1) Determine the induced current through Johnny as a function of their distance along the ground near this experimental device.

2) Where is the surface-charge density induced on Johnny a maximum (let t = 0)?

$\varepsilon_o = 8.854 \times 10^{-12}$ F/m

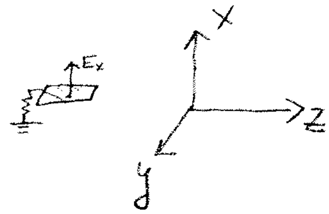
Exam 1L

A loop is 0.16 m^2 square, centered at (0,0,0), and in the x = 5y plane. Determine the voltage, V, measured across the loop (induced emf) if the magnetic flux density through it is approximately

$$\vec{B} = x^3\cos 120\pi t\,\vec{i} - y\cos 120\pi t\,\vec{j} \ \ \text{Wb/m}^2$$

SOLN., TESTS, OCTOBER 9, 1992, EE432, K. KAISER

1P) 1)



$\hat{x} = .3\ m$

$$E_x = -dV/dx$$
$$= -2x(z^2-1)10\cos 377t \ kV/m$$
$$i = dq/dt$$
$$= \frac{d}{dt}\int \epsilon_0 E_x \, dy\, dz = \frac{d}{dt}\left(-20{,}000\,\epsilon_0 \cos 377t \ \hat{x} \int_{z}^{z+.4}\int_{-.2}^{.2} (z^2-1)\,dy\,dz\right)$$
$$= 8.01\times 10^{-6}\sin 377t \left[\frac{z^3}{3} - z\right]_z^{z+.4}$$
$$= 8.01\times 10^{-6}\sin 377t \left[\frac{(z+.4)^3}{3} - (z+.4) - \frac{z^3}{3} + z\right]$$
$$= 8.01\times 10^{-6}\sin 377t \left[\frac{(z+.4)^3}{3} - \frac{z^3}{3} - .4\right] \quad A$$

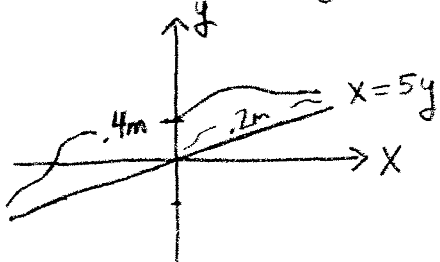
2) $\rho_s = \epsilon_0 E_x = -2\hat{x}\epsilon_0(z^2-1)\,10\cos 377t \ \ kC/m^2$

$\frac{d\rho_s}{dz} \propto -2(.3)\epsilon_0(2z) = 0 \quad @\ \underline{\underline{z = 0}}$, $\rho_s(0) \geq \rho_s(z)\Big|_{t=0}$ ✓ MAX.

1L) $g = x - 5y$, $\nabla g = \hat{i} - 5\hat{j}$, $|\nabla g| = \sqrt{1+25} = \sqrt{26}$

$\hat{N} = \nabla g / |\nabla g| = (\hat{i} - 5\hat{j})/\sqrt{26}$

$\vec{B}\cdot\hat{N} = (x^3\cos 120\pi t + 5y\cos 120\pi t)/\sqrt{26}$



$.4\times .4 = .16\ m^2$

$\frac{.4}{2} = .2 = \sqrt{x^2+y^2} = \sqrt{25y^2+y^2}$

$\Rightarrow y = .0392$

$x = .196$

$$dS = d\ell\, dz = \sqrt{dx^2+dy^2}\,dz = \sqrt{25dy^2+dy^2}\,dz = \sqrt{26}\,dy\,dz$$

$$V = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = -\frac{d}{dt}\int\int\left[(x^3\cos 120\pi t + 5y\cos 120\pi t)/\sqrt{26}\right]\sqrt{26}\,dy\,dz$$

$$= -\frac{d}{dt}\left\{\cos 120\pi t \int_{-.2}^{.2}\int_{-.0392}^{.0392}(125y^3 + 5y)\,dy\,dz\right\}$$

$$= 120\pi \sin 120\pi t\,(.4)\left[\frac{125y^4}{4} + \frac{5y^2}{2}\right]_{-.0392}^{.0392}$$

$$= 0$$

Exam 1L

For the following loop determine the voltage, V, measured across the loop (induced emf) if

$$\vec{B} = y^2\cos 120\pi t\,\vec{i} - x^2\cos 120\pi t\,\vec{j} \quad \mathrm{Wb/m^2}$$

The loop is a 0.25 m^2 square loop centered at (0,0,0) and in the y = 3x plane.

SOLN., EXAMS, JULY 31, 1992, EE-432, K. KAISER

1P) a) $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -\frac{\rho}{\epsilon_0} = -\frac{3\times10^{-6}}{\epsilon_0 r}$, $\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -\frac{3\times10^{-6}r}{\epsilon_0}$

$r^2\frac{d\Phi}{dr} = -\frac{3\times10^{-6}r^2}{2\epsilon_0} + C$, $\frac{d\Phi}{dr} = -\frac{3\times10^{-6}}{2\epsilon_0} + \frac{C}{r^2}$

$\Phi_i = -\frac{3\times10^{-6}}{2\epsilon_0}r - \frac{C}{r} + D \qquad 1 \le r \le 1.2m$

$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 0$, $r^2\frac{d\Phi}{dr} = E$, $\frac{d\Phi}{dr} = \frac{E}{r^2}$

$\Phi_o = -\frac{E}{r} + F \qquad 1.2 \le r \le 7m$

b) $\Phi_i(1) = -30$, $\Phi_o(7) = 0$, $\Phi_i(1.2) = \Phi_o(1.2)$

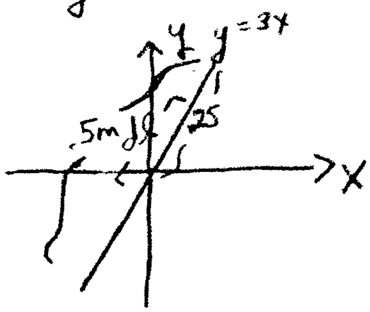
$D_{or} - D_{ir} = \rho_s = 0\big|_{r=1.2m}$, $\epsilon_0 E_{or} = \epsilon_0 E_{ir}\big|_{r=1.2m}$

$\frac{d\Phi_i}{dr} = \frac{d\Phi_o}{dr}\Big|_{r=1.2m}$

1L) $g = y - 3x$, $\nabla g = -3\hat{i} + \hat{j}$, $|\nabla g| = \sqrt{9+1} = \sqrt{10}$

$\hat{N} = \nabla g/|\nabla g| = (-3\hat{i} + \hat{j})/\sqrt{10}$

$\vec{B}\cdot\hat{N} = (-3y^2\cos120\pi t - x^2\cos120\pi t)/\sqrt{10}$



$.5 \times .5 = .25\ m^2$

$\frac{.5}{2} = .25 = \sqrt{x^2+y^2} = \sqrt{x^2+9x^2}$

$\Rightarrow x = .079m$

$y = 3x = .24$

$dS = dl\,dz = \sqrt{dx^2+dy^2}\,dz = \sqrt{dx^2+9dx^2}\,dz = \sqrt{10}\,dx\,dz$

$$V = -\frac{d}{dt}\int\vec{B}\cdot d\vec{S} = -\frac{d}{dt}\int_{-.25}^{.25}\int_{-.079}^{+.079}\left[(-27x^2\cos120\pi t - x^2\cos120\pi t)/\sqrt{10}\right]\sqrt{10}\,dx\,dz$$

$$= -\frac{d}{dt}\left\{-.5\cos120\pi t\int_{-.079}^{.079}-28x^2\,dx\right\} = 28(.5)(120\pi)\sin120\pi t\ \frac{x^3}{3}\Big|_{-.079}^{.079}$$

$$= 1.75\sin120\pi t \quad \text{Volts}$$

Problem 2 (wgt = 11)

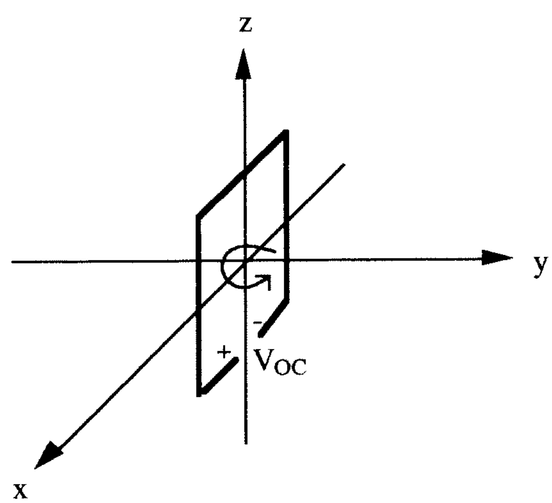
A DEA (Drug Enforcement Agency) Van, cleverly disguised as a bug elimination vehicle, contains a rotating 15×10^{-3} m^2 square r.f. pickup loop on its roof (concealed inside a model of large plastic roach). The loop rotates at a frequency of 10 Hz. If the magnetic field of the criminal radio activity near the loop is approximately

$$\overline{B} = x^2\cos(5000\pi t)\hat{i} - x^2\sin(5000\pi t)\hat{j}$$

determine the magnitude of the open circuit voltage, V_{OC}, across the pickup loop as a function of time. Use Faraday's Law

$$V_{OC} = -\frac{d\Phi}{dt}$$

in this analysis (At t = 0 the loop is oriented in the **xz** plane, centered at (0, 0, 0), and rotating as shown below around the z axis.)



SOLN., TEST 3 EE-332, WINTER 91, K. KAISER

① $F_{S_1N_2} = -\dfrac{(1.5\times10^{-6})(3\times10^{-6})}{4\pi\mu_o(.01)^2} = -2.85\times10^{-3}$, $F_{S_1S_2} = \dfrac{4.5\times10^{-12}}{4\pi\mu_o(.05)^2} = 1.14\times10^{-4}$

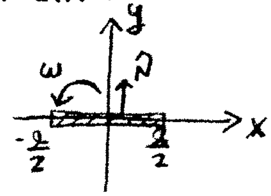
$F_{N_1N_2} = \dfrac{4.5\times10^{-12}}{4\pi\mu_o(.03)^2} = 3.17\times10^{-4}$, $F_{N_1S_2} = -\dfrac{4.5\times10^{-12}}{4\pi\mu_o(.07)^2} = -5.82\times10^{-5}$

$F_{total} = \sum F's = -2.48\times10^{-3}\ N$ attractive

② $\Phi = \int \vec{B}\cdot d\vec{s}$

$\hat{N} = -\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}$

$\vec{B}\cdot\hat{N} = -x^2\cos(5{,}000\pi t)\sin\omega t - x^2\sin(5{,}000\pi t)\cos\omega t$



$\omega = 2\pi(10)$

$\ell = (15\times10^{-3})^{1/2} = .122$

$$\Phi = \int_{-\ell/2}^{\ell/2}\int_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t} -x^2\cos(5{,}000\pi t)\sin\omega t\,dx\,dz + \int_{-\ell/2}^{\ell/2}\int_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t} -x^2\sin(5{,}000\pi t)\cos\omega t\,dx\,dz$$

$$= \ell\left[-\frac{x^3}{3}\right]_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t}\left[\cos(5{,}000\pi t)\sin\omega t + \sin(5{,}000\pi t)\cos\omega t\right]$$

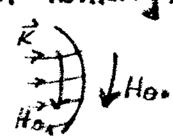
$$V_{oc} = -\frac{d\Phi}{dt} = \frac{\ell}{3}\left[\frac{\ell^3}{8}\cos^3\omega t + \frac{\ell^3}{8}\cos^3\omega t\right]\Big[\omega\cos(5{,}000\pi t)\cos\omega t + $$

$$- 5{,}000\pi\sin(5{,}000\pi t)\sin\omega t - \omega\sin(5{,}000\pi t)\sin\omega t + 5{,}000\pi\cos(5{,}000\pi t)\cos$$

$$+ \frac{\ell^4}{4}\cos^2\omega t\left[\cos(5{,}000\pi t)\sin\omega t + \sin(5{,}000\pi t)\cos\omega t\right]$$

③ a) $\Phi_{mi} = Br\cos\theta$ b) $\Phi_{mo} = \frac{A}{r^2}\cos\theta$ ($\vec{H}_o$ zero at infinity)

c) $H_{\theta o} - H_{\theta i} = -K_s\cos\theta$, $B_{ro} = B_{ri}$



d,e) $\vec{H}_i = -\nabla\Phi_{mi} = -B\cos\theta\,\hat{r} + B\sin\theta\,\hat{\theta}$

$\vec{H}_o = -\nabla\Phi_{mo} = (2A/r^3)\cos\theta\,\hat{r} + (A/r^3)\sin\theta\,\hat{\theta}$

$(A/R^3)\sin\theta - B\sin\theta = -K_s\sin\theta \Rightarrow B = A/R^3 + K_s$

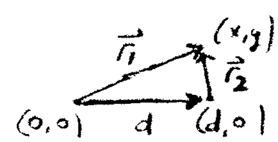
$-B\cos\theta = (2A/R^3)\cos\theta \Rightarrow B = -2A/R^3 = A/R^3 + K_s$

$A = -R^3K_s/3$, $B = 2K_s/3$

$\vec{H}_i = (-2K_s/3)\cos\theta\,\hat{r} + (2K_s/3)\sin\theta\,\hat{\theta}$

$\vec{H}_o = (-2R^3K_s/3r^3)\cos\theta\,\hat{r} + (-R^3K_s/3r^3)\sin\theta\,\hat{\theta}$

④ $\vec{A}_{total} = \left(-\dfrac{\mu_o J r_1^2}{4} + \dfrac{\mu_o J r_2^2}{4} + \dfrac{\mu_o I_x}{2\pi}\ln r_2 + C\right)\hat{z}$



$r_1^2 = |\vec{r}_1|^2 = x^2+y^2$, $r_2^2 = |\vec{r}_2|^2 = (x-d)^2+y^2$

$$A_z = \frac{\mu_o}{4}\left[-Jx^2 - Jy^2 + J(x^2-2xd+d^2) + Jy^2 + \frac{2I_x}{\pi}\ln(x^2-2xd+d^2+y^2)^{1/2} + C\right.$$

$$= \frac{\mu_o}{4}\left[Jd^2 - 2Jxd + \frac{2I_x}{4}\ln(x^2-2xd+d^2+y^2)^{1/2} + C\right]$$

$\vec{B} = \nabla\times\vec{A} = \dfrac{dA_z}{dy}\hat{x} - \dfrac{dA_z}{dx}\hat{y}$

ASSIGNMENT #11

EE-332
K. KAISER
SPRING 90

SHOW ALL WORK AND REASONING

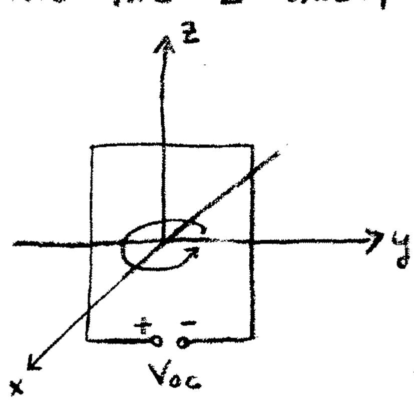
① A DEA (Drug Enforcement Agency) Van, cleverly disguised as a bug elimination vehicle, contains a rotating $10 \times 10^{-3}\ m^2$ r.f. pickup, square loop on its roof (concealed inside a model of a large plastic roach). The loop rotates at a frequency of 5 Hz. If the magnetic field of the criminal radio activity near the loop is approximately

$$\vec{B} = x\cos(4{,}000\pi t)\hat{i} - y\cos(4{,}000\pi t)\hat{j}$$

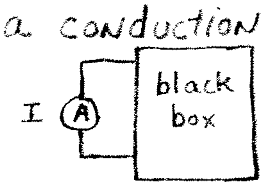
determine the open circuit voltage, Voc, across the pickup loop as a function of time. Use Faraday's law

$$V = -\frac{d\Phi}{dt}$$

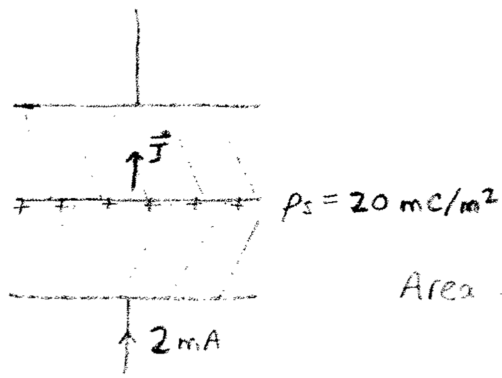
in this analysis. (At t=0 the loop is oriented in the yz plane, centered at (0,0,0), and rotating as shown below around the z-axis.)



② A constant current I is measured in the circuit below over some finite time interval. Is the current flowing <u>in</u> the black box a conduction or displacement current? Explain

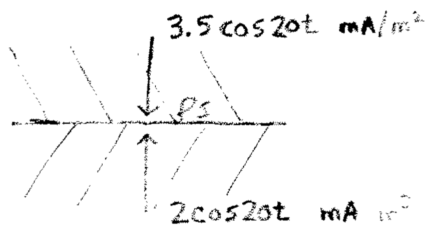


③ a) Determine $\vec{J}$

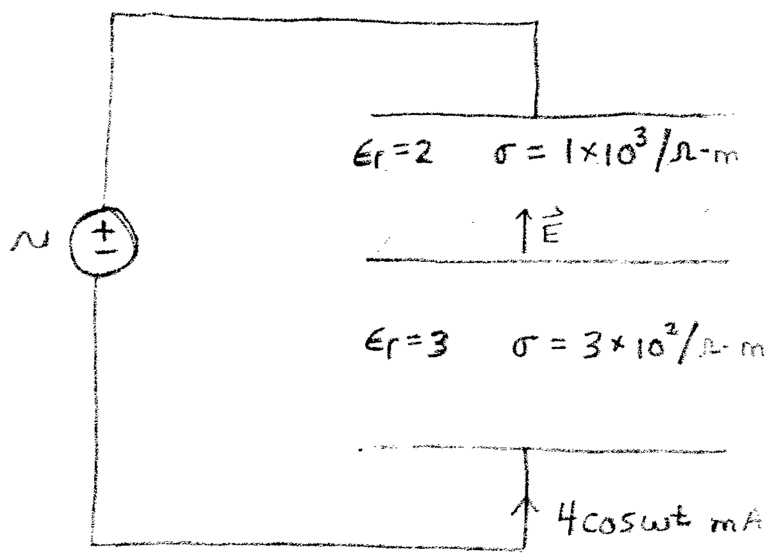


Area of plates = $2\,cm^2$

b) Determine ρ_s ($\rho_s(0) = 0$)



c) Determine $\vec{E}$ (ω is very high)

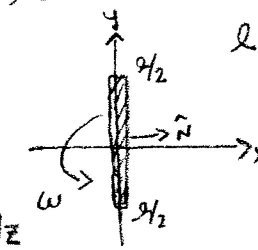


Area of plates = $2\,cm^2$

SOLUTIONS

ASSIGNMENT #11, EE-332, K. KAISER, SPRING 90

$\ell = (10 \times 10^{-3})^{1/2} = .1$



① $\Phi = \int \vec{B} \cdot d\vec{s}$, $\hat{N} = \cos\omega t\, \hat{i} + \sin\omega t\, \hat{j}$ $\quad (|\hat{N}| = 1)$

$\vec{B} \cdot \hat{N} = x\cos(4{,}000\pi t)\cos\omega t - y\cos(4{,}000\pi t)\sin\omega t$

$$\Phi = \int_{-\frac{\ell}{2}}^{\ell/2} \int_{\frac{\ell}{2}\sin\omega t}^{-\frac{\ell}{2}\sin\omega t} \left[x\cos(4{,}000\pi t)\cos\omega t\right] dx\,dz \; - \int_{-\ell/2}^{\ell/2} \int_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t} \left[y\cos(4{,}000\pi t)\sin\omega t\right] dy\,dz$$

$\omega = 2\pi(5)$

$$= \ell\cos(4{,}000\pi t)\cos\omega t \left.\frac{x^2}{2}\right|_{\frac{\ell}{2}\sin\omega t}^{-\frac{\ell}{2}\sin\omega t} - \ell\cos(4{,}000\pi t)\sin\omega t \left.\frac{y^2}{2}\right|_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t}$$

$= 0 \Rightarrow V_{oc} = 0!$ (total flux through loop is zero)

② Need more information, could either be a steady flow of charge (conduction) or a constantly increasing $\vec{E}$ field near stationary charges ($\frac{dD}{dt}$ displacement) (or even convection or diffusion)

③ a) $J_{1N} - J_{2N} = -\frac{d\rho_s}{dt} = -\frac{d}{dt}(20) = 0 \Rightarrow J_{1N} = J_{2N}$

$J = 2\times10^{-3}/2\times10^{-4} = 10\ A/m^2$, $\vec{J} = 10\hat{j}\ A/m^2$

b) $-3.5\cos 20t - 2\cos 20t = -d\rho_s/dt$

$$\int_0^{\rho_s} d\rho_s = -\int_0^t -5.5\cos 20t\, dt = 5.5 \left.\frac{\sin 20t}{20}\right|_0^t = \frac{5.5}{20}\sin 20t$$

$$\rho_s = .275 \sin 20t \quad mC/m^2$$

c) because ω is "very high" the current flow across the plates is displacement current ($\vec{J}_T = \sigma\vec{E} + \frac{dD}{dt}$)

$$\frac{4\times10^{-3}\cos\omega t}{2\times10^{-4}} = \frac{d}{dt}(2\epsilon_0 E)$$

$$20\cos\omega t = \frac{d}{dt}(2\epsilon_0 E)$$

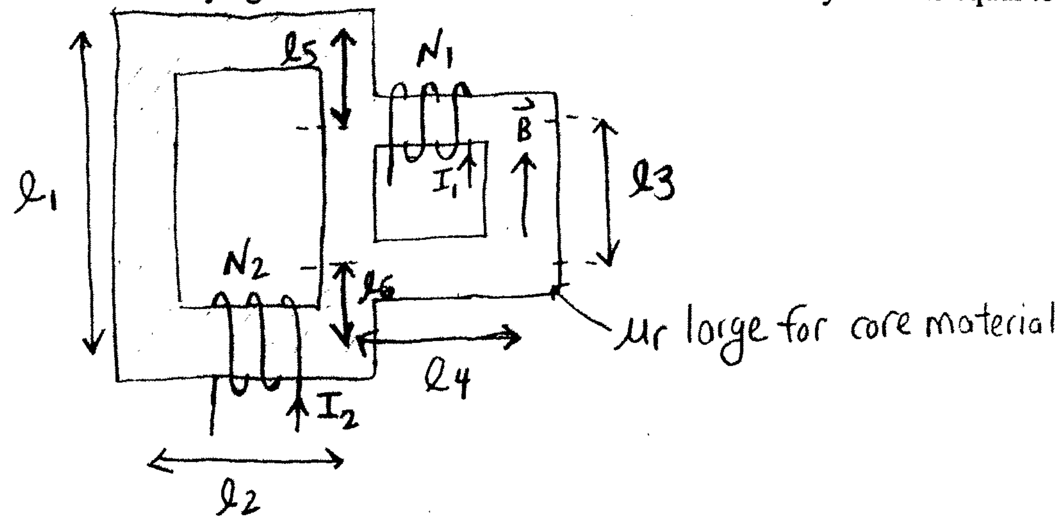
$$\epsilon_0 E = \int_0 10\cos\omega t\, dt = \frac{10}{\omega}\sin\omega t$$

$$\vec{E} = \frac{10}{\omega\epsilon_0}\sin\omega t\, \hat{j} \quad V/m$$

Ken Kaiser

ECE-340, K. Kaiser, Summer 1999
Distant Learning Version

TEST 31: Determine the value of the flux density, B, for the given magnetic circuit. Carefully note the direction of the current and the orientation of the windings for each of the current-carrying coils. Assume the cross-sectional area everywhere is equal to S.

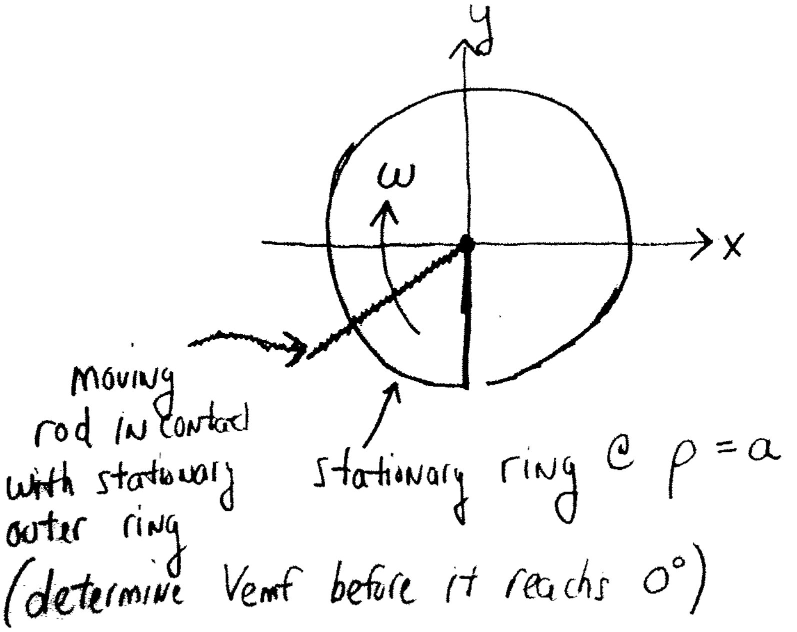


ECE-340, K. Kaiser, Summer 1999
Distant Learning Version

TEST 32: Using <u>both</u> definitions for the induced voltage

$$V_{emf} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} \quad \text{and} \quad V_{emf} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{S} + \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L}$$

determine the magnitude of the induced voltage for the given configuration. Verify that the results are the same.



initial position for moving rod is 340°

$\vec{B} = 2\cos 3t\, \hat{a}_z$

ECE-340, K. Kaiser, Summer 1999
Distant Learning Version

TEST 33: Given

$$\vec{E} = -4e^{-2f\cos t}\hat{a}_z, \quad \sigma = 6f, \quad \varepsilon_r = 3 - f$$

determine the values for f and t that this material is considered a good insulator.

TEST 34: Given

$$\vec{E} = -7e^{-4x}\cos(\omega t - 7z)a_y, \quad \sigma = 4/\Omega - \text{m}, \quad \varepsilon_r = 3, \quad \mu_r = 1$$

determine the magnetic field in the time domain first using one of Maxwell's equations

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

and then using another of Maxwell's equation:

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

TEST 35:

a) Convert the following complex number to the form $A + jB$, where A and B are real numbers. You may use the calculator as a check, but all intermediate rationalizing steps must be shown.

$$\frac{(j7-5)}{(2-3j)(j-1)}$$

b) Convert the following phasor to the time domain (assuming that ω is the radian frequency).

$$E_y = (6-3j)\cos(\omega)$$

c) Convert the following time-domain equation to a phasor.

$$\vec{E} = -2e^{-7y}\sin(10t)\hat{a}_x + 3e^{-6x}\cos(10t)\hat{a}_y$$

ECE-340, K KAISER, SUMMER 99, [illegible]

(30)

$$\vec{H}_1 = \int_{\infty}^{0} \frac{I\,dx\,\hat{a}_x \times \left((0-x)\hat{a}_x + (y'-0)\hat{a}_y + (z'-0)\hat{a}_z\right)}{4\pi\left(\sqrt{(0-x)^2 + (y'-0)^2 + (z'-0)^2}\right)^3}$$

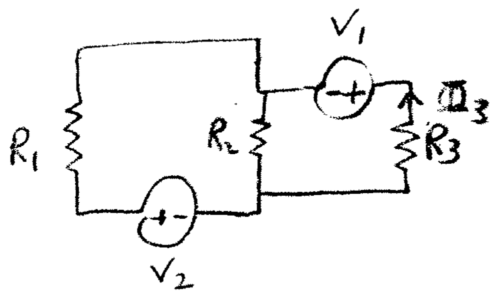
$$\vec{H}_2 = \int_{0}^{\infty} \frac{I\,dy\,\hat{a}_y \times \left((0-0)\hat{a}_x + (y'-y)\hat{a}_y + (z'-0)\hat{a}_z\right)}{4\pi\left(\sqrt{(0-0)^2 + (y'-0)^2 + (z'-0)^2}\right)^3}$$

$$\vec{H}_t = \vec{H}_1 + \vec{H}_2$$

$$\vec{B}_t = \mu_0 \vec{H}_t$$

$$M = \frac{\int \vec{B}_t \cdot d\vec{s}}{I} = \frac{\int_{-3y+1}^{7}\int_{2}^{0} \vec{B}_t \cdot dy\,dz\,\hat{a}_x}{I} + \frac{\int_{3y+1}^{7}\int_{0}^{2} \vec{B}_t \cdot dy\,dz\,\hat{a}_x}{I}$$

(31)



$$\Phi_3 = \frac{-V_2}{R_1 + R_2 \| R_3} \cdot \frac{R_2}{R_2 + R_3} + \frac{-V_1}{R_1 \| R_2 + R_3}$$

superposition

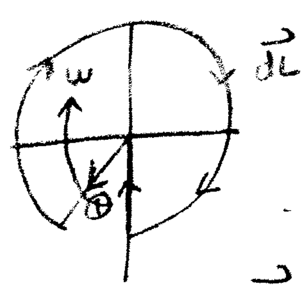
$$\Phi_3 = R_3 5$$

(32) $d\vec{s} = \rho\, d\phi\, d\rho\, \hat{a}_z$

$$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} = -\frac{d}{dt}\int_{340^\circ}^{340-\omega t}\int_0^a 2\cos 3t\, \rho\, d\phi\, d\rho$$

convert to radian

$d\vec{L}$



but only rod is moving ($\vec{N}_1 \neq 0$)

$\vec{N}_1 = -\omega\rho\hat{a}_\phi$, $\frac{d\vec{B}}{dt} = -6\sin 3t\,\hat{a}_z$

$d\vec{L}_r = d\rho\,\hat{a}_\rho$ $\quad \oint (\vec{N} \times \vec{B}) \cdot d\vec{L} = \int_0^a [(-\omega\rho\hat{a}_\phi) \times \vec{B}] \cdot d\rho\,\hat{a}_\rho$

(33) $\left|\frac{\sigma\vec{E}}{\frac{d\vec{D}}{dt}}\right| \ll 1$ good insulator , $\left|\frac{6f(-4e^{-2f\cos t})}{\frac{d}{dt}\left(\epsilon_0(3-f)(-4e^{-2f\cos t})\right)}\right|$

$$= \left|\frac{-24f e^{-2f\cos t}}{\epsilon_0(3-f)(-4e^{-2f\cos t})(2f\sin t)}\right| = \left|\frac{3}{\epsilon_0(3-f)\sin t}\right| \ll 1$$

$$3 \ll \epsilon_0(3-f)|\sin t|$$

ECE-340, K. KAISER, SUMMER 99, INTERNET

(34) $\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$ $\quad \nabla \times \vec{H} = \sigma\vec{E}_s + j\omega\epsilon\vec{E}_s = (\sigma + j\omega\epsilon)\vec{E}_s$

$$\vec{E}_s = -7e^{-4x}e^{-j7z}\hat{a}_y$$

$$\vec{H}_s = \frac{\nabla \times \vec{E}_s}{-j\omega\mu} = \frac{j}{\omega\mu}\left[-j49e^{-4x}e^{-j7z}\hat{a}_x + 28e^{-4x}e^{-j7z}\hat{a}_z\right]$$

$$= \frac{49}{\omega\mu}e^{-4x}e^{-j7z}\hat{a}_x + \frac{28}{\omega\mu}e^{j\frac{\pi}{2}}e^{-4x}e^{-j7z}\hat{a}_z$$

$$\vec{H}(t) = \frac{49}{\omega\mu}e^{-4x}\cos(\omega t - 7z)\hat{a}_x + \frac{28}{\omega\mu}e^{-4x}\cos(\omega t - 7z + \tfrac{\pi}{2})\hat{a}_z$$

$$\nabla \times \vec{H}_s = (4 + j\omega 3\epsilon_0)(-7e^{-4x}e^{-j7z}\hat{a}_y)$$

$$\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = (4 + j\omega 3\epsilon_0)(-7e^{-4x}e^{-j7z}) \quad \text{etc.!}$$

(35) a) $\dfrac{\sqrt{7^2+5^2}\angle 126^\circ}{\sqrt{2^2+3^2}\angle -56^\circ \sqrt{1^2+1^2}\angle 135^\circ} = 1.69\angle 47^\circ = 1.2 + j1.2$

b) $E_{ys} = 6\cos\omega - 3\cos(\omega)e^{j90^\circ}$

$E_y(t) = 6\cos(\omega)\cos\omega t - 3\cos(\omega)\cos(\omega t + \frac{\pi}{2})$

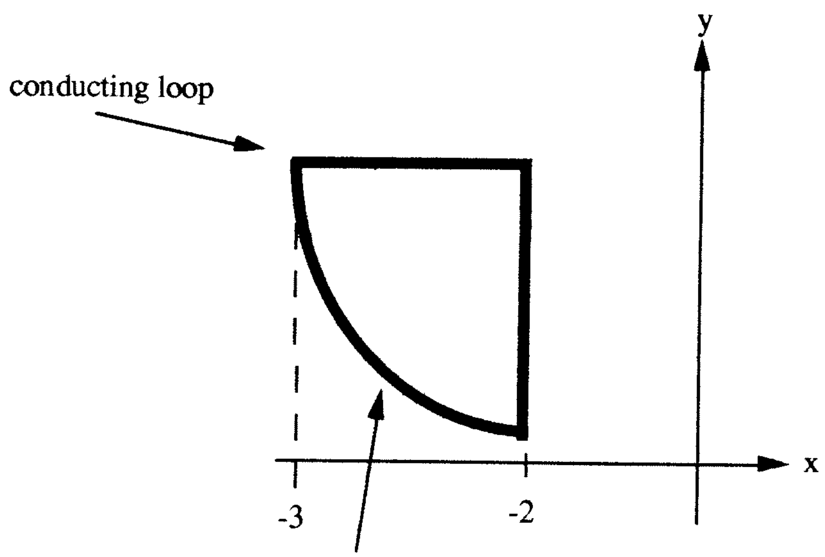
c) $\vec{E}_s = -2e^{-7y}e^{-j\frac{\pi}{2}}\hat{a}_x + 3e^{-6x}\hat{a}_y$ $\qquad \sin(10t) = \cos(10t - \frac{\pi}{2})$

EE-332, Kaiser, 3/25/93

Test 27

A conducting loop shown below lies in the z = 0 plane. If the loop is in the magnetic field density

$\vec{B} = xy^2\cos(\omega t)\hat{x} + x\cos(\omega t)\hat{z}$ Wb/m^2



the equation for this curve is $y = x^2$

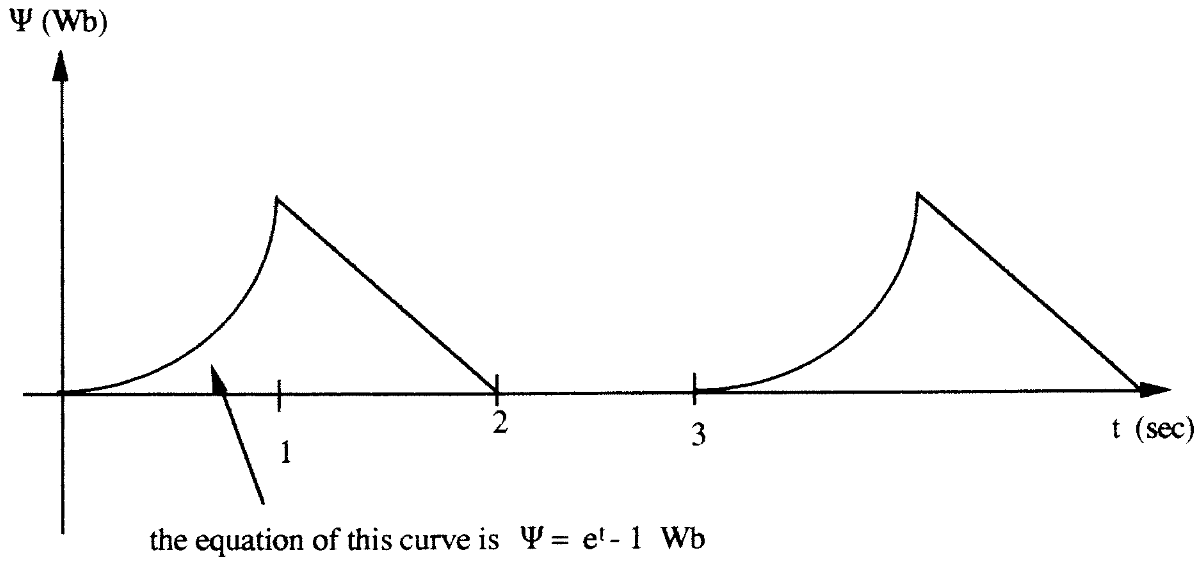
and has a resistance of 12 ohms, calculate the current in the loop. Specify its direction for increasing magnetic field density.

Test 28

The following time-varying flux links 23 turns of a coil.

a) Calculate the induced emf at t = 0.3 seconds.

b) Sketch the induced emf for $0 \le t \le 3$ seconds. Label maximums and minimums.



<u>Test 29</u>

State whether the following field satisfies Maxwell's equations

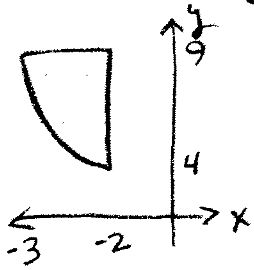
$\vec{E} = \left(\rho\hat{\rho} + \cos\phi\hat{\phi} - \rho z^2\hat{z}\right)\cos(\omega t)$ V/m

in free space. Show all work. If it does, find the corresponding magnetic field and current density.

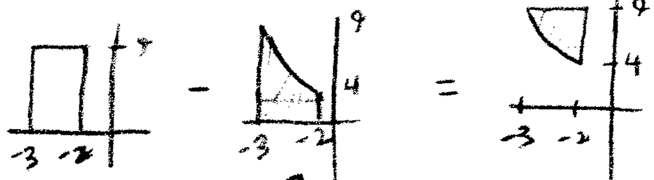
EE332, 3/25/93, KAISER, SOLN.

(27) $V = -N\frac{d\psi}{dt} = -\frac{d\psi}{dt}$, $\psi = \int \vec{B}\cdot d\vec{s}$

$d\vec{s} = dx\,dy\,\hat{z}$, $\vec{B}\cdot d\vec{s} = x\cos\omega t\,dx\,dy$



$$A = \int_{-3}^{-2} 9\,dx - \int_{-3}^{-2} x^2\,dx \quad \left(= 9 - 6.\bar{3} = 2.\bar{6}\right)$$



$$= \int_{x^2}^{9}\int_{-3}^{-2} dx\,dy$$

$$\psi = \int_{-3}^{-2} 9x\cos\omega t\,dx - \int_{-3}^{-2} x^3\cos\omega t\,dx$$

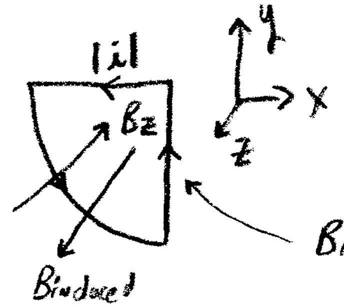
$$= \cos\omega t\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_{-3}^{-2} = -6.25\cos\omega t \quad \text{Wb}$$

$\therefore \quad V = -6.25\,\omega\sin\omega t \quad V$

$i = \frac{V}{R} = -.521\,\omega\sin\omega t$

@ $t=0$

$B_z = xy < 0$ in the loop ($x<0$, $y>0$)



$B_{induced}$ opposite to B_z

(28)

a) $V = -N\frac{d\psi}{dt} = -23\frac{d}{dt}\left(e^t - 1\right) = -23e^t \qquad 0<t<1$

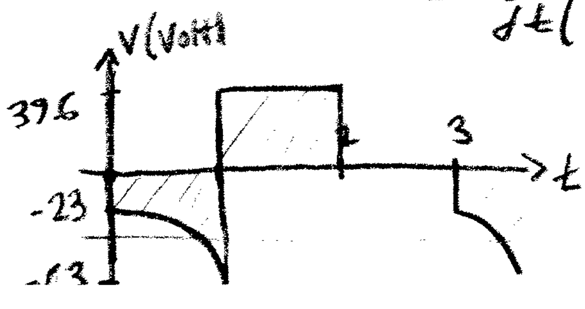
@ $t = .3$, $V = -23e^{.3} = -31$ V

b) @ $t = 1$, $\psi = e^1 - 1 = 1.72$ Wb

@ $t = 2$, $\psi = 0$

$\psi = mt + b$, $m = \frac{1.72 - 0}{1 - 2} = -1.72$

$1<t<2 \qquad V = -23\frac{d}{dt}\left(-1.72t + b\right) = -23(-1.72) = 39.6$ V



EE332, 3/25/93, KAISER, SOLN.

29

$$\nabla \cdot \vec{D} = \rho$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$
$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

Free space $\Rightarrow$ $\rho = 0$, $\vec{J} = 0$
$\epsilon = \epsilon_0$
$\mu = \mu_0$

$$\nabla \cdot \vec{E} = 0 \qquad \vec{E} = [\rho\hat{\rho} + \cos\phi\hat{\phi} - \rho z^2\hat{z}]\cos\omega t \quad V/m$$

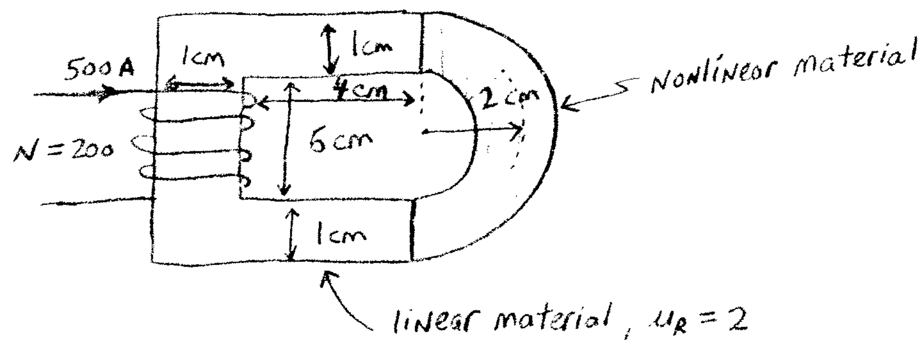
$$\nabla \cdot \vec{E} = \frac{1}{\rho}\frac{d}{d\rho}(\rho E_\rho) + \frac{1}{\rho}\frac{dE_\phi}{d\phi} + \frac{dE_z}{dz}$$
$$= \left[\frac{1}{\rho}\frac{d}{d\rho}(\rho^2) + \frac{1}{\rho}(-\sin\phi) - 2\rho z\right]\cos\omega t$$
$$= \left[\frac{2\rho}{\rho} - \frac{\sin\phi}{\rho} - 2\rho z\right]\cos\omega t = \left(2 - \frac{\sin\phi}{\rho} - 2\rho z\right)\cos\omega t$$
$$\neq 0 \quad \text{Not satisfied for all } t$$

EE332, Winter 94, K. Kaiser
Tests 18-21

Test 18
A composite magnetic circuit of constant cross-sectionional area of 5 cm^2, as shown below, is made of a linear and nonlinear material. The BH characteristic of the nonlinear material is given by the equation

$$B = \frac{2H}{1 + H - 2H^2} \quad T$$

If the current through the coil is 500 A, completely setup all the equations necessary to determine the flux through the circuit.



Test 19
After participating in a spitting contest and losing, Bobby accused Sally of excessively watering down her spit. After a brief ruckus, Bobby ran home, balling all the way. A day later, in revenge, Bobby sprayed Sally down, with stick corn oil (he used his dad's painting gun system). Unfortunately, tribocharging of the oil occurred in the plastic hose. Assume the charge density in the oil at the orifice is ρ_o, the average cross-sectional area of the stream is A, the length of the stream in L, the average velocity of the stream is V, the permittivity of the oil is ε, and the conductivity of the oil is σ. Also, assume Sally is barefooted (grounded) but Bobby is not. If the charge density in the stream is given approximately by the expression

$$\rho = \rho_o \sin\left(\frac{\sigma y}{\varepsilon V}\right) + \rho_o \quad C/m^3$$

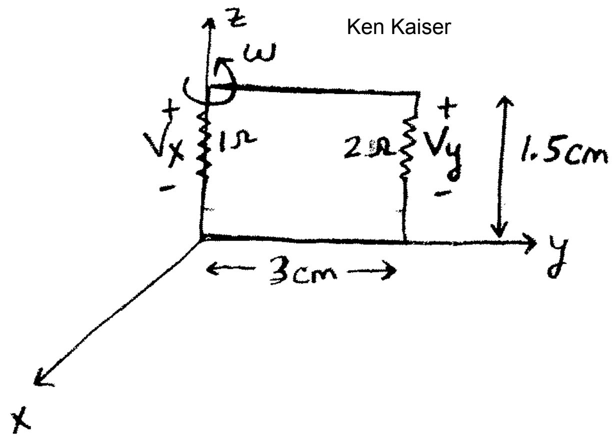
determine Bobby's potential relative to the ground.

Test 20
In the circuit below, a rectangular wire loop connected in series with two resistors is in a constant magnetic field given by

$$\vec{B} = 0.02\hat{i} \quad \text{Wb / m}^2$$

One side of the loop lies along the z axis while another side rotates at an angular speed of w = 3.5 rad/sec. The loop is located in the yz plane at t = 0. Determine (and give reasons for) the voltages V_x and V_y.

Ken Kaiser



Test 21

A magnetic field of the form

$$\vec{H} = 5e^{-2z} \sin \omega t \, \hat{j} \quad \text{A / m}$$

exists in free space. Determine

a) the associated electric field consistent with all of Maxwell's equations

b) and the corresponding charge density.

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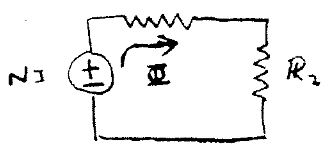
(18)



$$R_1 = \frac{l}{\mu A} = \frac{.045 + .045 + .07}{2\mu_0\, 5/(100)^2}$$

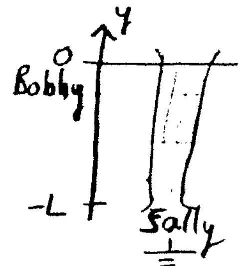
$$200(500) + R_1\Phi + H(\pi(.02)) = 0$$

(1) $$200(500) + R_1 B(5/(100)^2) + H(\pi(.02)) = 0$$

(2) $$B = \frac{2H}{1 + H - 2H^2}$$

Solving for B, $\Phi = B(5/(100)^2)$

(19) $J = \rho V + \sigma E = 0$ (steady state, no closed path)



$\rho(0) = \rho_0$ ✓

$$E = -\frac{\rho V}{\sigma}$$

$$V_B = -\int_{-L\,\leftarrow\,\text{ref.}}^{0} \vec{E}\cdot d\vec{L} = -\int_{-L}^{0} -\frac{\rho V}{\sigma}\,dy = \frac{V}{\sigma}\int_{-L}^{0}\rho\,dy$$

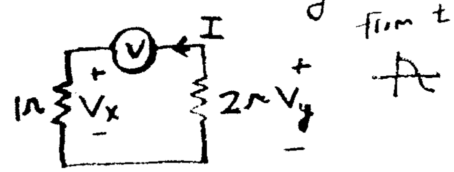
$$= \frac{V}{\sigma}\int_{-L}^{0}\left(\rho_0 \sin\frac{\sigma y}{\epsilon V} + \rho_0\right)dy = \frac{V}{\sigma}\left[-\frac{\rho_0 V\epsilon}{\sigma}\cos\frac{\sigma y}{\epsilon V} + \rho_0 y\right]_{-L}^{0}$$

$$= \frac{V}{\sigma}\left[-\frac{\rho_0 V\epsilon}{\sigma} - \frac{\rho_0 V\epsilon}{\sigma}\cos\frac{\sigma L}{\epsilon V} + \rho_0 L\right] \quad V$$

(20) $V = -\frac{d\Phi}{dt}$, $\Phi = BA = (.02\cos\omega t)(.03\cdot.015) = 9\times10^{-6}\cos\omega t$ Wb

decreasing with time from $t=0$

$$V = 9\times10^{-6}\,\omega\sin\omega t = 3.15\times10^{-5}\sin 3.5t$$



$$|I| = \frac{V}{3} = 1.05\sin 3.5t \quad A$$

$I = 1.05\sin 3.5t$ A produces $\vec{H}$ in opposite direction to $\vec{B} = .02\hat{i}$

$V_x = I(1) = 1.05\sin 3.5t$ V

$V_y = -I(2) = -2.1\sin 3.5t$ V

(21) $\nabla\cdot\vec{B} = 0$ ✓

$$\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt} = \epsilon_0\frac{d\vec{E}}{dt}$$

$$10e^{-2z}\sin\omega t\,\hat{i} = \epsilon_0\frac{d\vec{E}}{dt}, \quad \frac{d\vec{E}}{dt} = \frac{10}{\epsilon_0}e^{-2z}\sin\omega t\,\hat{i}$$

$$\vec{E} = -\frac{10e^{-2z}}{\omega\epsilon_0}\cos\omega t\,\hat{i} \quad \text{steady state}$$

$\nabla\cdot\vec{D} = \rho$, $\nabla\times\vec{E} = -d\vec{B}/dt$

$0 = \rho$, $\frac{20}{\omega\epsilon_0}e^{-2z}\cos\omega t\,\hat{j} = ?\ -\mu_0\omega 5e^{-2z}\cos\omega t$

$\therefore \frac{20}{\omega\epsilon_0} = -\mu_0\omega 5$, $-4 = \omega^2\mu_0\epsilon_0$ NOT POSSIBLE

Test 19

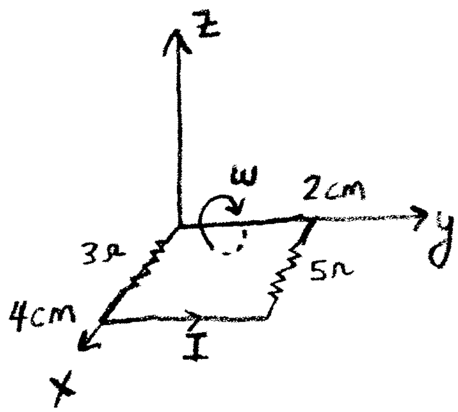
A dielectric filled (k = 3) parallel-plate capacitor has plates of 5 cm^2 that are separated by a distance of 1 mm. If the capacitance is connected to a 20 V 2 MHz sinusoidal voltage source, calculate the magnitude of the displacement current, neglecting fringing. If the dielectric is imperfect with a conductivity of $\sigma = 10^{-6}$ S/m, determine the magnitude of the conduction current, neglecting fringing.

Test 20

In the circuit below, a rectangular wire loop connected in series with two resistors is in a magnetic field given by

$$\vec{B} = 0.045\cos 20t\,\hat{i} \quad \mathrm{Wb/m^2}$$

One side of the loop lies along the axis while another side rotates at an angular speed of w = 2 rad/sec. The loop is located in the xy plane at t = 0. Determine (and give a reason for) the magnitude of the current I.



Test 21

An electric and a magnetic field of the form

$\vec{E} = 3e^{-4z}\cos 20t\,\hat{k} \quad \mathrm{V/m}$

$\vec{H} = -4e^{-2x}\sin 20t\,\hat{j} \quad \mathrm{A/m}$

exists in free space.

a) Determine the charge density.

b) Determine which of Maxwell's equations are satisfied and not satisfied.

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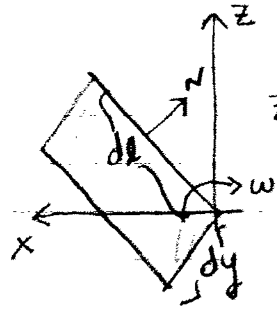
(20) $V = -\frac{d\psi}{dt}$, $\psi = \int \vec{B} \cdot d\vec{s}$

$$\psi = (.02)(.04)(.045\cos 20t)\sin \omega t$$
$$= 3.6\times10^{-5}\cos 20t \sin \omega t$$

$$V = -3.6\times10^{-5}\left[-20\sin 20t \sin\omega t + \omega\cos 20t\cos\omega t\right] \text{ V}$$

$$|I| = \frac{|V|}{8\Omega}$$



$z = mx$ $\qquad dz = m\,dx$

$g = z - mx$

$\nabla g = -m\hat{i} + \hat{k}$

$\hat{N} = \frac{\nabla g}{|\nabla g|} = \frac{-m\hat{i} + \hat{k}}{\sqrt{m^2+1}}$

$$d\vec{s} = dl\,dy\,\hat{N}$$
$$= \sqrt{dx^2 + dz^2}\,dy\left(\frac{-m\hat{i}+\hat{k}}{\sqrt{m^2+1}}\right) = dx\sqrt{1+m^2}\,dy\left(\frac{-m\hat{i}+\hat{k}}{\sqrt{m^2+1}}\right)$$
$$= dx\,dy\left(-m\hat{i} + \hat{k}\right)$$

Need to relate m to ωt?

OR

$$\hat{N} = -\sin\omega t\,\hat{i} + \cos\omega t\,\hat{k}$$
$$\vec{B}\cdot\hat{N} = -.045\cos 20t \sin\omega t$$

(21) a) $\nabla\cdot\vec{D} = \rho$

$$\epsilon_0\nabla\cdot\vec{E} = \epsilon_0\frac{dE_z}{dz} = -12\epsilon_0 e^{-4z}\cos 20t \quad C/m^3$$

b) $\nabla\times\vec{E} = 0 \overset{?}{=} -\frac{d\vec{B}}{dt} = -\mu_0\frac{d\vec{H}}{dt} = +\mu_0 4e^{-2x}20\cos 20t\,\hat{j}$ Not Satisfied

$\nabla\cdot\vec{B} \overset{?}{=} 0$

$\mu_0 0 = 0$ Satisfied

$\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt} = \frac{d\vec{D}}{dt}$

$$\nabla\times\vec{H} = \frac{dH_y}{dx}\hat{k} = 8e^{-2x}\sin 20t\,\hat{k} \overset{?}{=} \frac{d\vec{D}}{dt} = \epsilon_0\frac{d\vec{E}}{dt} = -\epsilon_0 60e^{-4z}\sin 20t\,\hat{k}$$

Not Satisfied

9. Given the magnetic flux density $\vec{B}_s$ in the frequency domain (i.e., phasor form), determine the electric field, $\vec{E}(t)$, <u>in the time domain</u> using the expression

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

Assume a frequency of ω rad/sec and free space conditions. Do not use any other expression. (The magnetic flux density field provided may actually only be an approximation to the actual field.) Your result can contain the parameters ε_0 and μ_0.

$$\vec{B}_s = \left(\frac{-1}{j}e^{xy}\right)\hat{a}_y$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

NONZERO

$$\nabla \times \vec{H}_s = j\omega\epsilon_0 \vec{E}_s \qquad \vec{J} = 0,\ \epsilon = \epsilon_0,\ \mu = \mu_0$$

free space

$$\vec{E}_s = \frac{1}{j\omega\epsilon_0}\nabla \times \vec{H}_s$$

$$= \frac{1}{j\omega\epsilon_0\mu_0}\left(\frac{-y}{j}e^{xy}\hat{a}_z\right) = \frac{y}{\omega\epsilon_0\mu_0}e^{xy}\hat{a}_z$$

$$\vec{E}(t) = Re\left[E_s e^{j\omega t}\right] = Re\left[\frac{y}{\omega\epsilon_0\mu_0}e^{xy}\hat{a}_z\left(\cos(\omega t) + j\sin(\omega t)\right)\right]$$

$$= \frac{y}{\omega\epsilon_0\mu_0}e^{xy}\hat{a}_z\cos(\omega t)$$

9. Given the magnetic flux density $\vec{B}_s$ in the frequency domain (i.e., phasor form), determine the electric field vector, $\vec{E}(t)$, <u>in the time domain</u> using the expression

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

Assume a frequency of ω rad/sec and free-space conditions. Do not use any other expression. (The magnetic flux density provided may actually only be an approximation to the actual field.) Your result can contain the parameters ε_o and μ_o.

$$\vec{B}_s = (2+2j)e^{xy}\hat{a}_y$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

$$\vec{B}_s = \mu_0 \vec{H}_s \qquad (\vec{J} = 0)$$

$$\nabla \times \vec{H}_s = j\omega \vec{D}_s = j\omega\varepsilon_0 \vec{E}_s$$

$$\vec{E}_s = \frac{1}{j\omega\varepsilon_0} \nabla \times \vec{H}_s = \frac{1}{j\omega\varepsilon_0} \nabla \times \left[\frac{(2+2j)e^{xy}\hat{a}_y}{\mu_0}\right]$$

$$= \frac{1}{j\omega\varepsilon_0\mu_0}(2+2j)\left[y e^{xy}\hat{a}_z\right]$$

$$= \frac{-j}{\omega\varepsilon_0\mu_0}(2+2j)y e^{xy}\hat{a}_z = \frac{1}{\omega\varepsilon_0\mu_0}(-2j+2)y e^{xy}\hat{a}_z$$

$$\vec{E}(t) = Re\left[\vec{E}_s e^{j\omega t}\right] = \frac{2y e^{xy}\hat{a}_z}{\omega\varepsilon_0\mu_0}\left[(1-j)(\cos(\omega t) + j\sin(\omega t))\right]$$

$$= \frac{2y e^{xy}\hat{a}_z}{\omega\varepsilon_0\mu_0}\left[\cos(\omega t) + \sin(\omega t)\right]$$

OR

$$= \frac{2y e^{xy}\hat{a}_z}{\omega\varepsilon_0\mu_0}\left[\sqrt{2}e^{-j45°}e^{j\omega t}\right] = \frac{2\sqrt{2}y e^{xy}\hat{a}_z}{\omega\varepsilon_0\mu_0}\cos(\omega t - 45°)$$

Problem 3

Given the magnetic flux density $\vec{B}_s$ in the frequency domain (i.e., phasor form), determine the electric field vector, $\vec{E}(t)$, in the time domain using the expression

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

Assume a frequency of ω rad/sec and free-space conditions. Do not use any other expression. (The magnetic flux density provided may actually only be an approximation to the actual field.) Your result can contain the parameters ε_o and μ_o.

$$\vec{B}_s = 3xe^{y}e^{j100^\circ}\hat{a}_z$$

$$\nabla \times \vec{H}_s = j\omega \vec{D}_s$$

$$\nabla \times \frac{\vec{B}_s}{\mu_0} = j\omega\varepsilon_0 \vec{E}_s$$

$$\vec{E}_s = \frac{1}{j\omega\varepsilon_0\mu_0} \nabla \times \vec{B}_s$$

$$= \frac{-j}{\omega\varepsilon_0\mu_0} \nabla \times \left(3xe^{y}e^{j100}\hat{a}_z\right)$$

$$= \frac{-3je^{j100}}{\omega\varepsilon_0\mu_0} \nabla \times \left(xe^{y}\hat{a}_z\right)$$

$$= \frac{-3je^{j100}}{\omega\varepsilon_0\mu_0}\left[xe^{y}\hat{a}_x - e^{y}\hat{a}_y\right]$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \hat{a}_x\left(\frac{dH_z}{dy} - \frac{dH_y}{dz}\right) - \hat{a}_y\left(\frac{dH_z}{dx} - \frac{dH_x}{dz}\right) + \hat{a}_z\left(\frac{dH_y}{dx} - \frac{dH_x}{dy}\right)$$

$$\vec{E}(t) = Re\left[E_s e^{j\omega t}\right] = \frac{-3xe^{y}\hat{a}_x}{\omega\varepsilon_0\mu_0} Re\left[je^{j(\omega t+100^\circ)}\right] + \frac{3e^{y}\hat{a}_y}{\omega\varepsilon_0\mu_0} Re\left[je^{j(\omega t+100^\circ)}\right]$$

$$= \frac{3xe^{y}\hat{a}_x}{\omega\varepsilon_0\mu_0}\sin(\omega t+100^\circ) - \frac{3e^{y}\hat{a}_y}{\omega\varepsilon_0\mu_0}\sin(\omega t+100^\circ)$$

$$j^2 = -1$$

$$e^{j(\omega t+190^\circ)}$$

$$= je^{j(\omega t+100^\circ)} = j\left[\cos(\omega t+100^\circ) + j\sin(\omega t+100^\circ)\right]$$