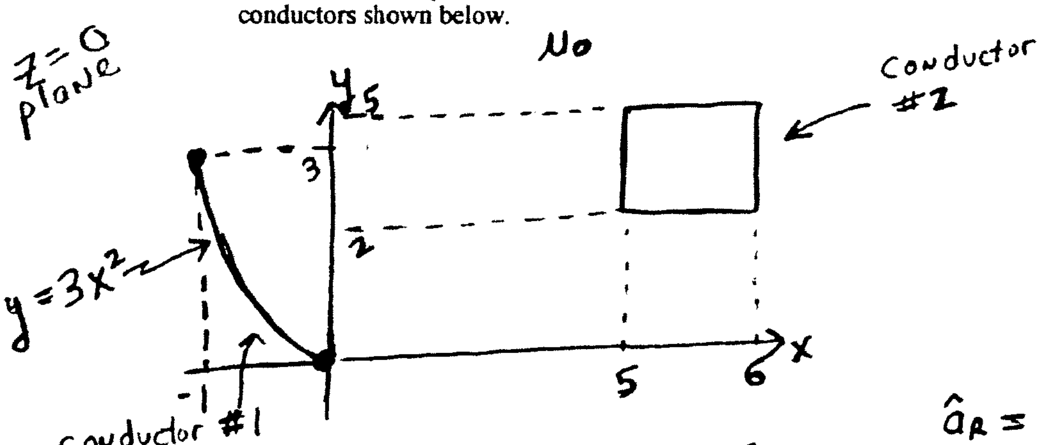
15

3. Completely setup all the integrals required to determine the mutual inductance between the two conductors shown below.



Current in #1

$$\vec{H}_2 = \int_{-1}^{0} \frac{I_1 d\vec{L} \times \hat{a}_R}{4\pi R^2}$$

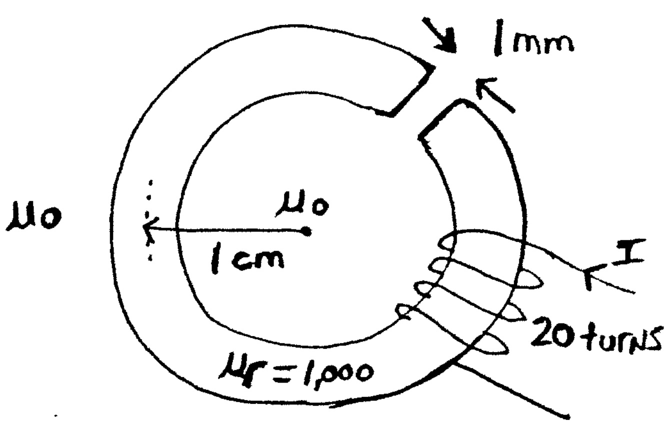
$$d\vec{L} = dx\,\hat{a}_x + dy\,\hat{a}_y = dx\,\hat{a}_x + 6x\,dx\,\hat{a}_y$$

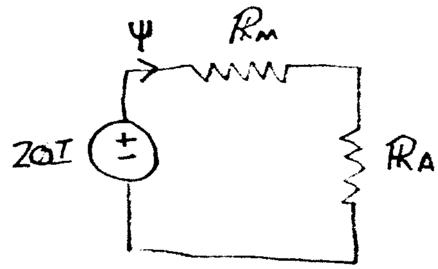
$$\hat{a}_R = \frac{(x_1 - x)\hat{a}_x + (y_1 - 3x^2)\hat{a}_y}{\sqrt{(x_1-x)^2 + (y_1-3x^2)^2}}, \quad \sqrt{(x_1-x)^2 + (y_1-3x^2)^2} = R$$

$$L_M = \frac{\Phi_2}{I_1} = \frac{\int \vec{B}_2 \cdot d\vec{S}_2}{I_1} = \frac{\mu_0 \int_2^5 \int_5^6 \vec{H}_2 \cdot dx\,dy\,\hat{a}_z}{I_1}$$

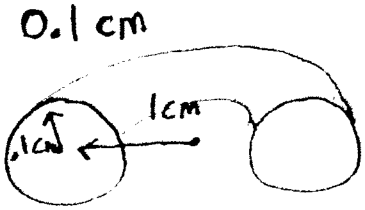
4. Determine the current required in the coil to produce a magnetic field of 100 A/m in the air gap.

10





donut-shaped object
cross-sectional radius of donut
= 0.1 cm



$$R_M = \frac{l}{\mu S} = \frac{2\pi(.01) - 1\times10^{-3}}{1{,}000\,\mu_0\,(\pi(.001)^2)}$$

$$R_A = \frac{l}{\mu S} = \frac{1\times10^{-3}}{\mu_0(\pi(.001)^2)}$$

$$\Psi = \frac{20I}{R_M + R_A} = B_A S = \mu_0 H_A S = \mu_0(100)(\pi(.001)^2)$$

$$I = \frac{(R_M + R_A)}{20}\left(\mu_0(100)\pi(.001)^2\right)$$

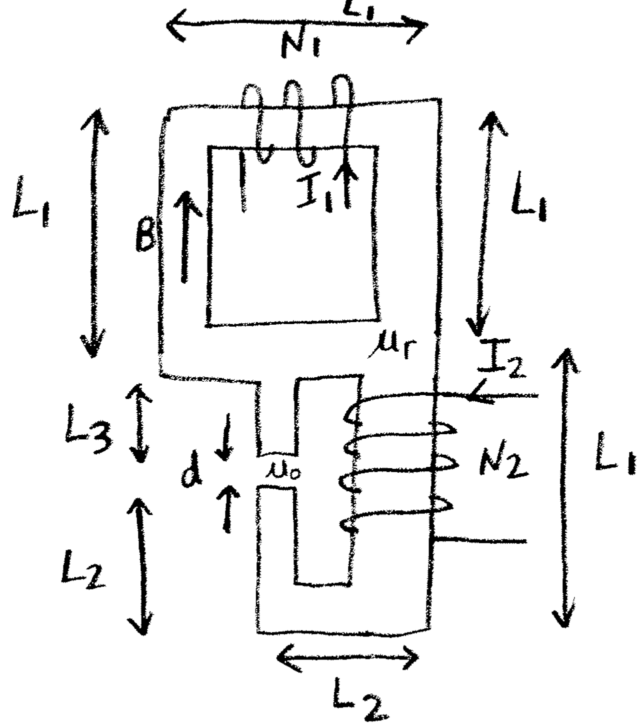
$$= 5\left[\frac{2\pi(.01) - 1\times10^{-3}}{1000} + \frac{1\times10^{-3}}{1}\right]$$

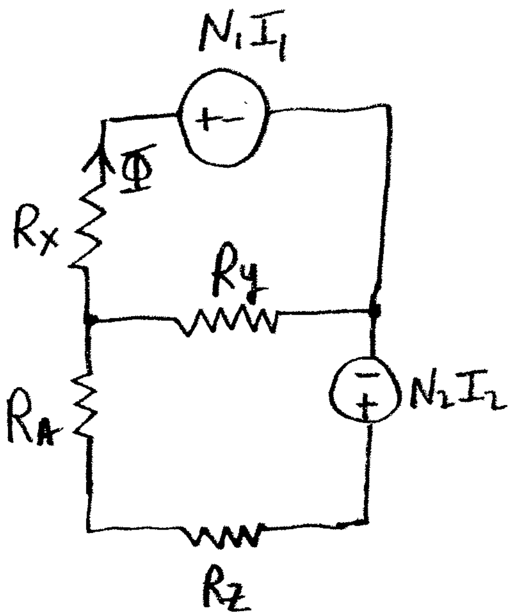
$$= 5.3 \text{ mA}$$

Ken Kaiser

ECE-240, K. Kaiser, Summer 2001

TEST 31: Determine the value of the flux density, B, for the given magnetic circuit. Carefully note the direction of the current and the orientation of the windings for each of the current-carrying coils. Assume the cross-sectional area everywhere is equal to S.





Superposition

$$\Phi = \frac{-N_1 I_1}{[(R_A+R_Z)\,//\,R_y]+R_x} + \frac{N_2 I_2}{(R_x//R_y)+R_A+R_Z} \times \frac{R_y}{R_x}$$

$$\Phi = BS$$

$$B = \frac{\Phi}{S}$$

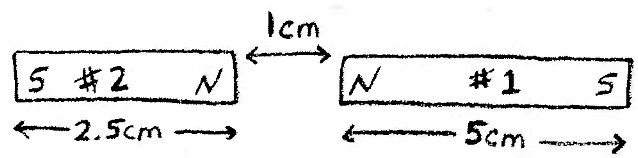
See other soln for values for these "resistances"

ASSIGNMENT #10

EE-332
K. KAISER
SPRING 90

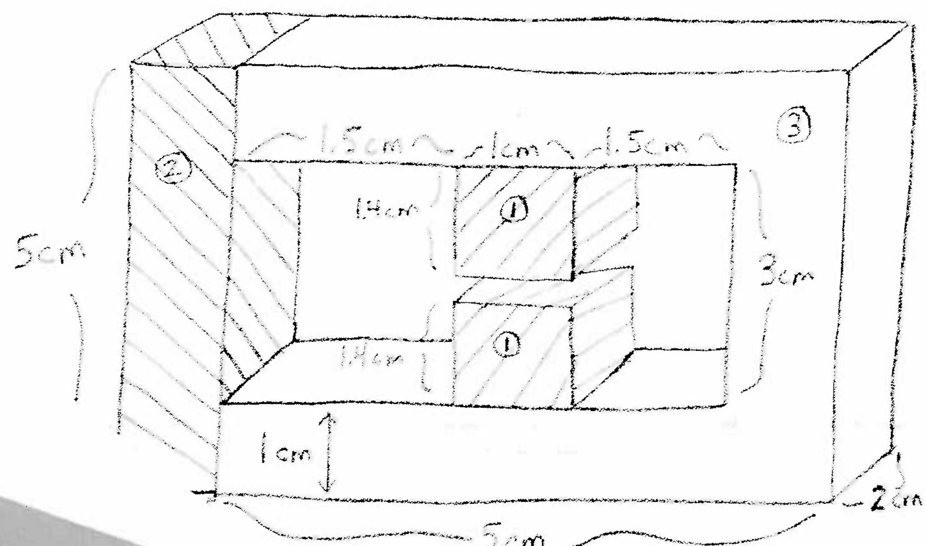
SHOW ALL WORK AND REASONING

① Determine the force between magnets #1 and #2 shown below. The pole strength of #1 is 2×10^{-6} Wb and #2 is 1×10^{-6} Wb.



② You are given one 5A current source, 20ft of copper wire, and the multi-material magnetic core shown below. Your job is to maximize the magnetic field in the air gap between the 1.4cm long segments. Assume that the core materials have a linear B-H relationship. To earn your pay (full credit) you must

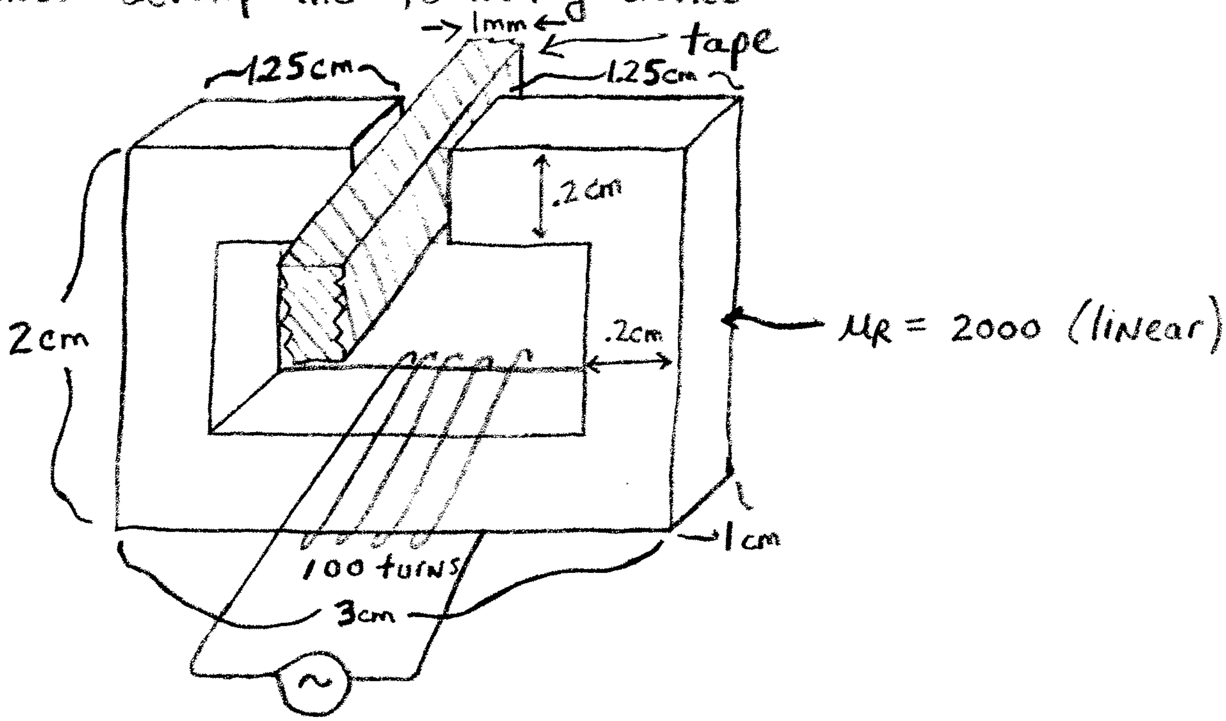
a) determine where the coil should be wound*
b) determine the B field in the air gap (including its direction as determined from your winding)



① $\mu_{R1} = 200$
② $\mu_{R2} = 300$
③ $\mu_{R3} = 100$

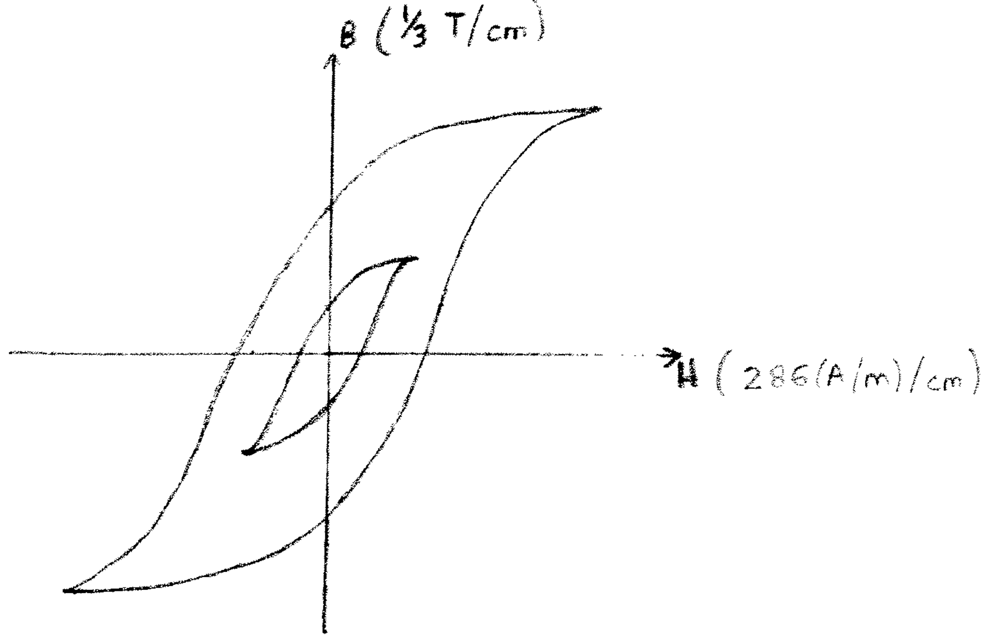
*(The coil can not be wrapped around corners.)

③ A certain high-ranking government official was concerned about the ability of her tape machine to erase "secret tape recordings. To insure that her machine "completely" erased the tapes, she had the chief engineer develop the following device:



ω is very high

The tape is slowly passed through the gap as the amplitude of the sinusoidal source is stepped from some maximum current, I_{MAX}, to some minimum current, I_{MIN}. The hysteresis curve for the tape for this maximum and minimum current follows. Determine I_{max} and I_{MIN} (amplitudes of current source).


B (1/3 T/cm)
H (286(A/m)/cm)

SOLUTIONS

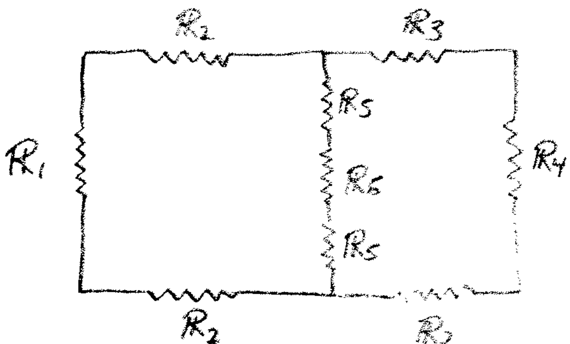
ASSIGNMENT #10, EE-332, K. KAISER, SPRING 90

① $F_{S_2N_1} = -\dfrac{(2\times10^{-6})(1\times10^{-6})}{4\pi\mu_0(.035)^2} = -.103\times10^{-3}$, $F_{S_2S_1} = \dfrac{2\times10^{-12}}{4\pi\mu_0(.085)^2} = .0175\times10^{-3}$

$F_{N_2N_1} = \dfrac{2\times10^{-12}}{4\pi\mu_0(.01)^2} = 1.26\times10^{-3}$, $F_{N_2S_1} = -\dfrac{2\times10^{-12}}{4\pi\mu_0(.06)^2} = -.0352\times10^{-3}$

total $F = (-.103 + .0175 + 1.26 - .0352)\times10^{-3} = 1.14\times10^{-3}\,N$ repulsive

② equivalent circuit without current source is $\mathcal{R} = \dfrac{l}{\mu A}$



$\mathcal{R}_1 \doteq \dfrac{.044}{300\mu_0(.02)(.01)} = 580\times10^3$ A·t/Wb

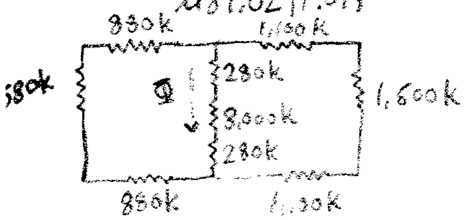
$\mathcal{R}_2 \doteq \dfrac{.022}{100\mu_0(.02)(.01)} = 880\times10^3$

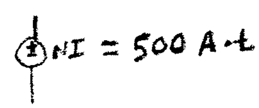
$\mathcal{R}_3 \doteq \dfrac{.027}{100\mu_0(.02)(.01)} = 1{,}100\times10^3$

$\mathcal{R}_4 = \dfrac{.04}{100\mu_0(.02)(.01)} = 1{,}600\times10^3$, $\mathcal{R}_5 = \dfrac{.014}{200\mu_0(.02)(.01)} = 280\times10^3$

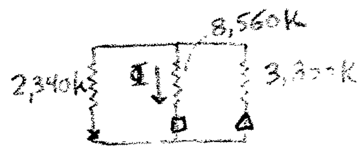
Leave a little for connection to source

$\mathcal{R}_6 = \dfrac{.002}{\mu_0(.02)(.01)} = 8{,}000\times10^3$, $N_{MAX} = \dfrac{(20\,ft)(12''/ft)(2.54\,cm/inch)}{2(2)+2(1)\ cm/turn} \doteq 100$ turns





desire max. Φ ⇒ max. voltage drop across 8,000k



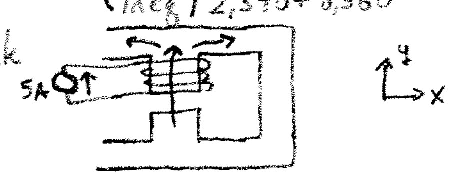
× , $\mathcal{R}_{eq} = 2{,}340k + 3{,}800k // 8{,}560k = 4{,}970k$

$\Phi_x = \left(\dfrac{NI}{\mathcal{R}_{eq}}\right)\dfrac{3{,}800}{8{,}560+3{,}800} = NI(61.9\times10^{-9})$

□ $\mathcal{R}_{eq} = 8{,}560k + 2{,}340k // 3{,}800k = 10{,}000k$, $\Phi_\square = \dfrac{NI}{\mathcal{R}_{eq}} = NI(99.9\times10^{-9})$

△ $\mathcal{R}_{eq} = 3{,}800k + 8560k // 2{,}340k = 5{,}640k$, $\Phi_\triangle = \left(\dfrac{NI}{\mathcal{R}_{eq}}\right)\dfrac{2{,}340}{2{,}340+8{,}560} = NI(38.1\times10^{-9})$

∴ Wrap the coil around either center block

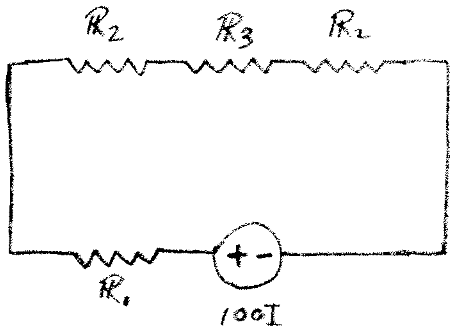


$\vec{B} = \Phi/A\,\hat{j} = \dfrac{NI(99.9\times10^{-9})}{(.02)(.01)}\hat{j} = .25\,\hat{j}$ Wb/m²

Ken Kaiser

SOLUTIONS
ASGMT. #10

(3)



$$\mathcal{R}_1 \doteq \frac{(.028 + 2(.018) + 2(.0115))}{2{,}000\mu_0(.01)(.002)} = 1.73\times10^6 \text{ A-t/Wb}$$

$$\mathcal{R}_2 \doteq \frac{(.002)}{\mu_0(.01)(.002)} = 79.6\times10^6$$

$$B_{MAX} = (3.3\text{cm})\left(\tfrac{1}{3}\text{ T/cm}\right) = 1.1 \text{ T} \quad (= \text{Wb/m}^2)$$

$$\Phi_{MAX} = B_{MAX}A = 1.1(.01)(.002) = 22\times10^{-6} \text{ Wb}$$

$$V_3 = \int \vec{H}\cdot d\vec{L} = (3.5\text{cm})(286(\text{A/m})/\text{cm})(1\times10^{-3}) = 1.001 \text{ A-t}$$

"KVL"

$$-100I_{MAX} + (1.73\times10^6)(22\times10^{-6}) + 2(79.6\times10^6)(22\times10^{-6}) + 1.001 = 0$$

$$\Rightarrow I_{MAX} = 35.4 \text{ A}$$

$$B_{MIN} = (1.3)\left(\tfrac{1}{3}\right) = .433$$

$$\Phi_{MIN} = B_{MIN}A = .433(.01)(.002) = 8.66\times10^{-6}$$

$$V_3 = (1.1)(286)(1\times10^{-3}) = .314$$

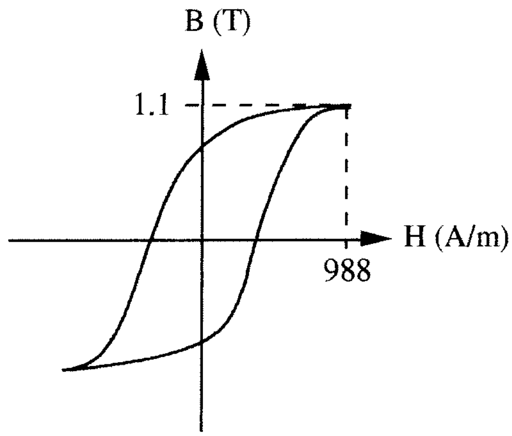
$$0 = -100I_{MIN} + (1.73\times10^6)(8.66\times10^{-6}) + 2(79.6\times10^6)(8.66\times10^{-6}) + .314$$

$$\Rightarrow I_{MIN} = 13.9 \text{ A}$$

Problem 3 (wgt = 10)

A toroid with a mean radius of 2 cm, circular cross-sectional area of 0.04 cm^2, and relative permeability of 1500 has a 0.5 cm air gap. An 100 turns coil is wrapped around the toroid. 0.4 cm thick tape is inserted in the air gap (assume all the flux passing through the air gap passes through the tape). The B-H curve of the tape corresponding to a specific current through the coil is shown below. Determine the amplitude of this specific current. DO NOT determine the permeability of the tape via the slope of the B-H curve.

$\mu_o = 4\pi \times 10^{-7}$ H/m



Problem 4 (wgt = 10)

For air the B-H curve is usually A but for most ferromagnetic materials the curve is usually B . Ferromagnetic materials are frequently utilized because they posses C relative permeability. The magnetization curve of a material before it proceeds through its hysteresis is referred to as the D B-H curve. When a needle is placed next to a strong magnet it will frequently become magnetized itself even after the magnet is removed. This common phenomena can be explained by studying the B-H curve at E and noticing that the F is nonzero. The shape of the B-H curve is very important in tape recording, permanent magnets, floppy discs, shielding, and transformers. For audio tape a low B_r will most likely result in a G signal-to-noise ratio. For permanent magnets a H H_c is desirable if the magnet is to retain its magnetization over an extended time period. To shield an object from high magnetic fields a material with a I permeability is desirable. For transformers a J is usually strived for.

Circle the *one* response following each letter below which "best fits" at the letter's underlined location in the paragraph above.

A	1) linear	2) nonlinear	3) unpredictable	4) distorted
B	1) linear	2) nonlinear	3) large	4) small
C	1)zero	2) low	3) high	4) crazy
D	1) unclean	2) pure	3) tainted	4) virgin
E	1) $H = 0$	2) $H = B$	3) $B = 0$	4) $H = -B$
F	1)remanence flux	2) coercive intensity	3) saturated flux	
G	1) zero	2) low	3) high	
H	1) zero	2) low	3) high	
I	1) zero	2) low	3) high	
J	1) small B_r	2) high B_r	3) small H_c	4) high H_c

SOLUTIONS

TEST3, EE-332, K. KAISER, FALL 90

1) a) $(4\cos 30t - 2\cos 30t)\times 10^{-3} = -d\rho_s/dt$

$\int_0^{\rho_s} d\rho_s = \int_0^t (-2\cos 30t)\times 10^{-3}dt$, $\rho_s - \rho_s(0) = (-2\times 10^{-3}\sin 30t)/30\Big|_0^t$

$\rho_s = -66.7\times 10^{-6}\sin 30t + 3\times 10^{-3}$ C/m^2

b) ω high $\rightarrow$ displacement current >> conduction current

$\frac{2\times 10^{-3}\cos\omega t}{2\times 10^{-4}} = \frac{d}{dt}(\epsilon_0 E)$, $E = \frac{10}{\omega\epsilon_0}\sin\omega t$ V/m

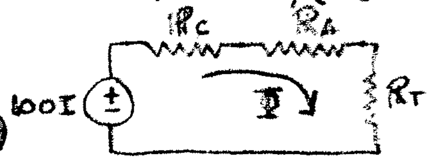
$\vec{E} = (-10/\omega\epsilon_0)\sin\omega t\,\hat{j}$ V/m

2) $\vec{H} = \int_{-\infty}^{0} \frac{I dy\,\hat{j}\times[(b-y)\hat{j}]}{4\pi[(b-y)^2]^{3/2}} + \int_0^{\infty} \frac{I dy(\hat{i}+\hat{j})\times[-y\hat{i}+(b-y)\hat{j}]}{4\pi[y^2+(b-y)^2]^{3/2}}$ (x=y)

(for x=y segment
$\vec{dL} = dx\hat{i} + dy\hat{j}$
$= dy\hat{i} + dy\hat{j}$

3) $R_C = l/\mu A = (2\pi(.02) - .005)/(\mu_0\times 1500\times .04\times 10^{-4}) = 16\times 10^6$ A·t/Wb

$R_A = (.001)/(\mu_0\times .04\times 10^{-4}) = 199\times 10^6$



$\Phi = BA = 1.1(.04\times 10^{-4}) = 4.4\times 10^{-6}$

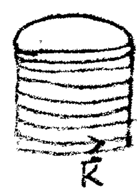
$V_T = \int \vec{H}\cdot\vec{dL} = 988(.004) = 3.95$ A·t

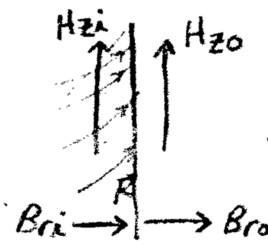
$-100I + (16\times 10^6)(4.4\times 10^{-6}) + (199\times 10^6)(4.4\times 10^{-6}) + 3.95 = 0$

$\Rightarrow I = 9.5$ A

4) A - linear, B - Nonlinear, C - high, D - virgin, E - H = 0, F - remonence flux, G - low, H - high, I - high, J - small H_c

5) a) $\frac{N}{L}dz$, b) $\vec{K} = \frac{NI}{L}\hat{\theta}$ c)





$H_{zo} - H_{zi} = -K$

d) $B_{ro} = B_{ri}$, $\mu_0 H_{ro} = \mu_c\mu_0 H_{ri}$, $H_{ro} = \mu_c H_{ri}$

e) $\vec{H}_i = -\nabla\Phi_{mi} = -C\hat{r} - A\hat{z}$

$\vec{H}_o = -\nabla\Phi_{mo} = 0$

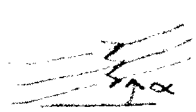
f) $0 + A = -NI/L$, $A = -NI/L$

$C = \mu_c(0)$, $C = 0$

$\vec{H}_i = \frac{NI}{L}\hat{z}$

$\vec{H}_o = 0$

g) $\vec{K} = \frac{NI}{L}(\sin\alpha\,\hat{z} + \cos\alpha\,\hat{\theta})$



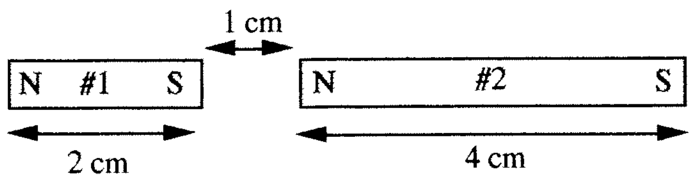
@ $\alpha = 0$ $\vec{K} = \frac{NI}{L}\hat{\theta}$ ✓

Ken Kaiser

Winter 1991

Problem 1 (wgt = 8)

Determine the force between magnets #1 and #2 shown below. The pole strength of #1 is 1.5 μWb and the pole strength of #2 is 3.0 μWb. Is this force attractive or repulsive?



$\mu_o = 4\pi \times 10^{-7}$ H/m

SOLN., TEST 3 EE-332, WINTER 91, K. KAISER

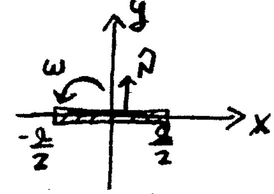
① $F_{S_1N_2} = -\frac{(1.5\times10^{-6})(3\times10^{-6})}{4\pi\mu_0(.01)^2} = -2.85\times10^{-3}$, $F_{S_1S_2} = \frac{4.5\times10^{-12}}{4\pi\mu_0(.05)^2} = 1.14\times10^{-4}$

$F_{N_1N_2} = \frac{4.5\times10^{-12}}{4\pi\mu_0(.03)^2} = 3.17\times10^{-4}$, $F_{N_1S_2} = -\frac{4.5\times10^{-12}}{4\pi\mu_0(.07)^2} = -5.82\times10^{-5}$

$F_{total} = \sum \vec{F}'s = -2.48\times10^{-3}$ N attractive

② $\Phi = \int \vec{B}\cdot d\vec{s}$

$\hat{N} = -\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}$



$\omega = 2\pi(10)$

$\ell = (15\times10^{-3})^{1/2} = .122$

$\vec{B}\cdot\hat{N} = -x^2\cos(5{,}000\pi t)\sin\omega t - x^2\sin(5{,}000\pi t)\cos\omega t$

$$\Phi = \int_{-\ell/2}^{\ell/2}\int_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t} -x^2\cos(5{,}000\pi t)\sin\omega t\,dx\,dz + \int_{-\ell/2}^{\ell/2}\int_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t} -x^2\sin(5{,}000\pi t)\cos\omega t\,dx\,dz$$

$$= \ell\left[-\frac{x^3}{3}\right]_{-\frac{\ell}{2}\cos\omega t}^{\frac{\ell}{2}\cos\omega t}\left[\cos(5{,}000\pi t)\sin\omega t + \sin(5{,}000\pi t)\cos\omega t\right]$$

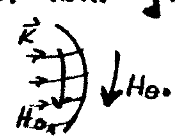
$$V_{oc} = -\frac{d\Phi}{dt} = \frac{\ell}{3}\left[\frac{\ell^3}{8}\cos^3\omega t + \frac{\ell^3}{8}\cos^3\omega t\right]\Big[\omega\cos(5{,}000\pi t)\cos\omega t +$$

$$- 5{,}000\pi\sin(5{,}000\pi t)\sin\omega t - \omega\sin(5{,}000\pi t)\sin\omega t + 5{,}000\pi\cos(5{,}000\pi t)\cos$$

$$+ \frac{\ell^4}{4}\cos^2\omega t\left[\cos(5{,}000\pi t)\sin\omega t + \sin(5{,}000\pi t)\cos\omega t\right]$$

③ a) $\Phi_{mi} = Br\cos\theta$ b) $\Phi_{mo} = \frac{A}{r^2}\cos\theta$ ($\vec{H}_o$ zero at infinity)

c) $H_{\theta o} - H_{\theta i} = -K_s\cos\theta$, $B_{ro} = B_{ri}$



d,e) $\vec{H}_i = -\nabla\Phi_{mi} = -B\cos\theta\,\hat{r} + B\sin\theta\,\hat{\theta}$

$H_o = -\nabla\Phi_{mo} = (2A/r^3)\cos\theta\,\hat{r} + (A/r^3)\sin\theta\,\hat{\theta}$

$(A/R^3)\sin\theta - B\sin\theta = -K_s\sin\theta \Rightarrow B = A/R^3 + K_s$

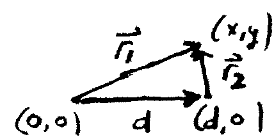
$-B\cos\theta = (2A/R^3)\cos\theta \Rightarrow B = -2A/R^3 = A/R^3 + K_s$

$A = -R^3K_s/3$, $B = 2K_s/3$

$\vec{H}_i = (-2K_s/3)\cos\theta\,\hat{r} + (2K_s/3)\sin\theta\,\hat{\theta}$

$\vec{H}_o = (-2R^3K_s/3r^3)\cos\theta\,\hat{r} + (-R^3K_s/3r^3)\sin\theta\,\hat{\theta}$

④ $\vec{A}_{total} = \left(-\frac{\mu_0 J r_1^2}{4} + \frac{\mu_0 J r_2^2}{4} + \frac{\mu_0 I_x}{2\pi}\ln r_2 + C\right)\hat{z}$



$r_1^2 = |\vec{r}_1|^2 = x^2+y^2$, $r_2^2 = |\vec{r}_2|^2 = (x-d)^2+y^2$

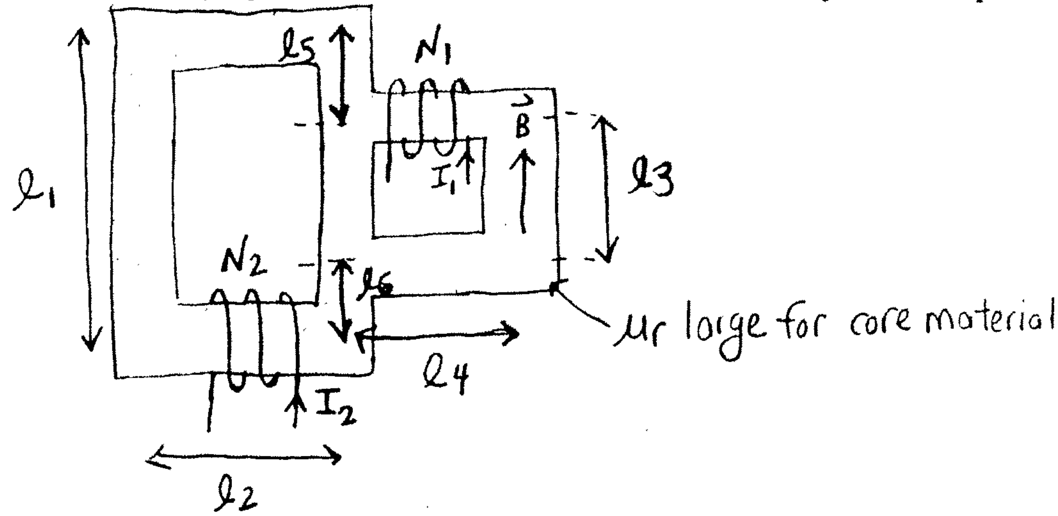
$$A_z = \frac{\mu_0}{4}\left[-Jx^2 - Jy^2 + J(x^2-2xd+d^2) + Jy^2 + \frac{2I_x}{\pi}\ln(x^2-2xd+d^2+y^2)^{1/2} + C\right.$$

$$= \frac{\mu_0}{4}\left[Jd^2 - 2Jxd + \frac{2I_x}{4}\ln(x^2-2xd+d^2+y^2)^{1/2} + C\right]$$

$\vec{B} = \nabla\times\vec{A} = \frac{dA_z}{dy}\hat{x} - \frac{dA_z}{dx}\hat{y}$

ECE-340, K. Kaiser, Summer 1999
Distant Learning Version

TEST 31: Determine the value of the flux density, B, for the given magnetic circuit. Carefully note the direction of the current and the orientation of the windings for each of the current-carrying coils. Assume the cross-sectional area everywhere is equal to S.

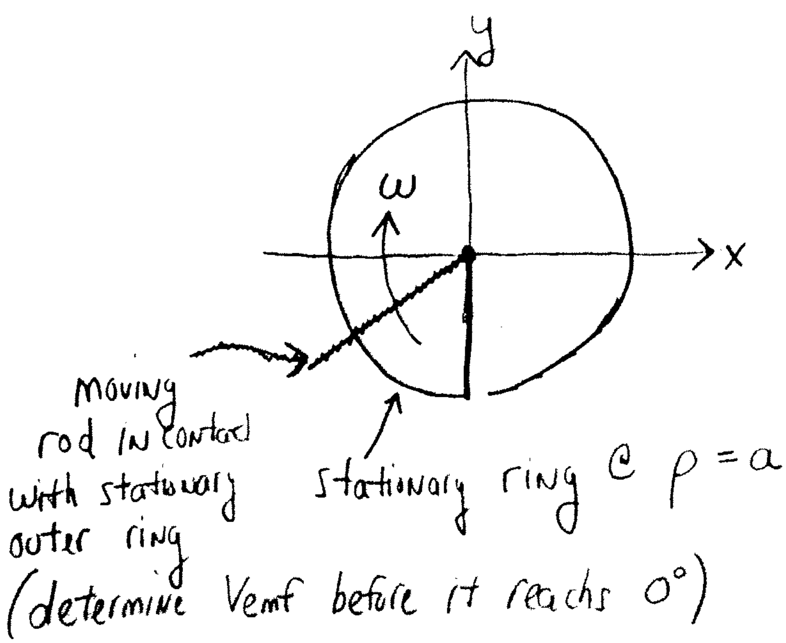


ECE-340, K. Kaiser, Summer 1999
Distant Learning Version

TEST 32: Using <u>both</u> definitions for the induced voltage

$$V_{emf} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} \quad \text{and} \quad V_{emf} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{S} + \oint_L \left(\vec{u}\times\vec{B}\right)\cdot d\vec{L}$$

determine the magnitude of the induced voltage for the given configuration. Verify that the results are the same.



initial position for moving rod is 340°

$\vec{B} = 2\cos 3t\,\hat{a}_z$

ECE-340, K. KAISER, SUMMER 99, INTERNET

(30)

$$\vec{H}_1 = \int_{\infty}^{3} \frac{I\,dx\,\hat{a}_x \times \left((0-x)\hat{a}_x + (y'-0)\hat{a}_y + (z'-0)\hat{a}_z\right)}{4\pi\left(\sqrt{(0-x)^2 + (y'-0)^2 + (z'-0)^2}\right)^3}$$

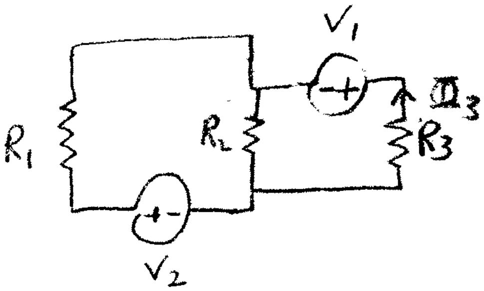
$$\vec{H}_t = \vec{H}_1 + \vec{H}_2$$

$$\vec{H}_2 = \int_{0}^{\infty} \frac{I\,dy\,\hat{a}_y \times \left((0-0)\hat{a}_x + (y'-y)\hat{a}_y + (z'-3)\hat{a}_z\right)}{4\pi\left(\sqrt{(0-0)^2 + (y'-0)^2 + (z'-0)^2}\right)^3}$$

$$\vec{B}_t = \mu_0 \vec{H}_t$$

$$M = \frac{\int \vec{B}_t \cdot d\vec{s}}{I} = \frac{\int_{-3y+1}^{7}\int_{2}^{0} \vec{B}_t \cdot dy\,dz\,\hat{a}_x}{I} + \frac{\int_{3y+1}^{7}\int_{0}^{2} \vec{B}_t \cdot dy\,dz\,\hat{a}_x}{I}$$

(31)



$$\Phi_3 = \frac{-V_2}{R_1 + R_2 // R_3} \cdot \frac{R_2}{R_2 + R_3} + \frac{-V_1}{R_1 // R_2 + R_3}$$

superposition

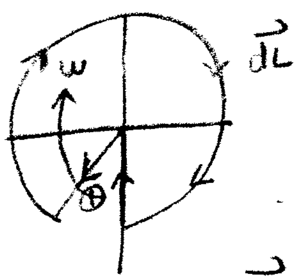
$$\Phi_3 = R_3 S$$

(32) $d\vec{s} = \rho\, d\phi\, d\rho\, \hat{a}_z$

$$V_{emf} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s} = -\frac{d}{dt}\int_{340^\circ}^{340^\circ - \omega t}\int_{0}^{a} 2\cos 3t\, \rho\, d\phi\, d\rho$$

convert to radian

$d\vec{L}$



but only rod is moving $(\vec{v}_1 \neq 0)$

$$\vec{v}_1 = -\omega\rho\hat{a}_\phi \quad , \quad \frac{d\vec{B}}{dt} = -6\sin 3t\,\hat{a}_z$$

$$d\vec{L}_r = d\rho\,\hat{a}_\rho \qquad \oint (\vec{v} \times \vec{B}) \cdot d\vec{L} = \int_0^a \left[(-\omega\rho\hat{a}_\phi) \times \vec{B}\right] \cdot d\rho\,\hat{a}_\rho$$

(33) $\left|\dfrac{\sigma \vec{E}}{\frac{d\vec{D}}{dt}}\right| \ll 1$ good insulator , $\left|\dfrac{6f\left(-4e^{-2f\cos t}\right)}{\frac{d}{dt}\left(\epsilon_0(3-f)\left(-4e^{-2f\cos t}\right)\right)}\right|$

$$= \left|\frac{-24fe^{-2f\cos t}}{\epsilon_0(3-f)\left(-4e^{-2f\cos t}\right)(2f\sin t)}\right| = \left|\frac{3}{\epsilon_0(3-f)\sin t}\right| \ll 1$$

$$3 \ll \epsilon_0 (3-f)|\sin t|$$

EE-332, Kaiser, 3/12/93

Test 23

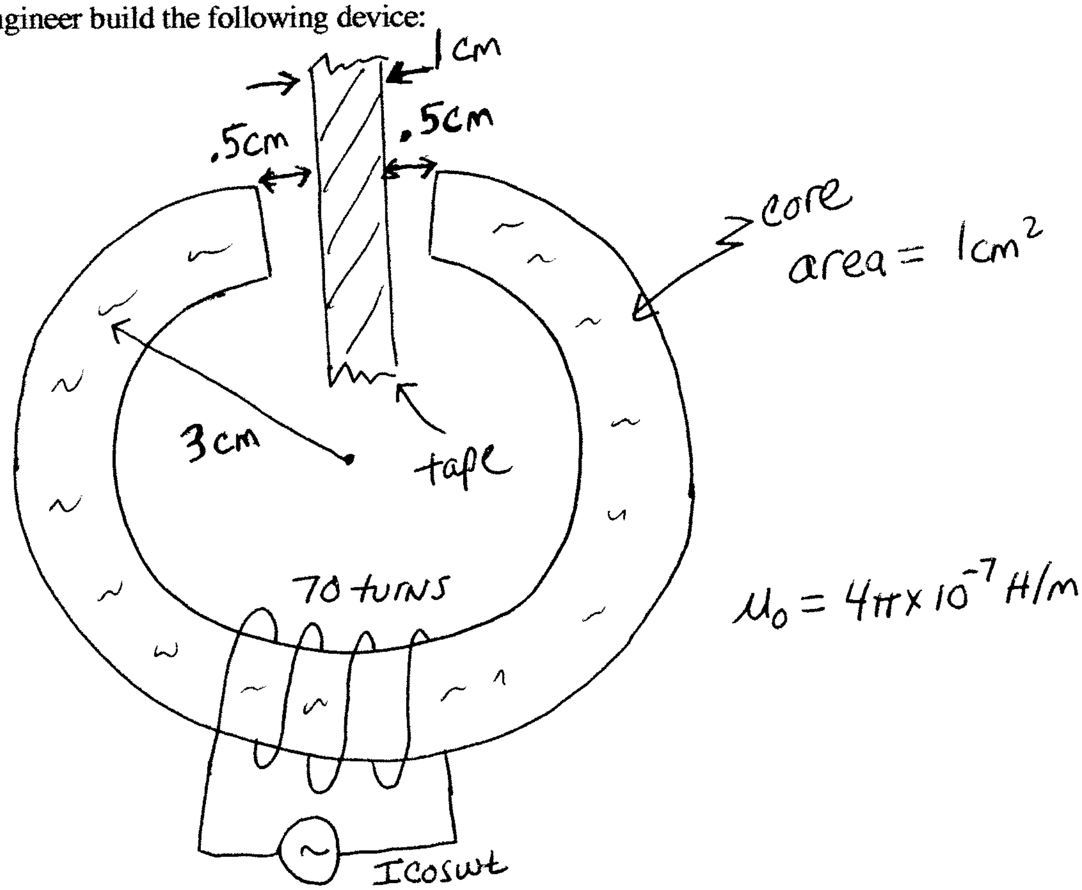
Given the flux density in a material of permeability $7\mu_o$ for $y \geq 0$

$\vec{B} = -20\hat{i} + 15\hat{j} - 30\hat{k}$ Wb/m^2

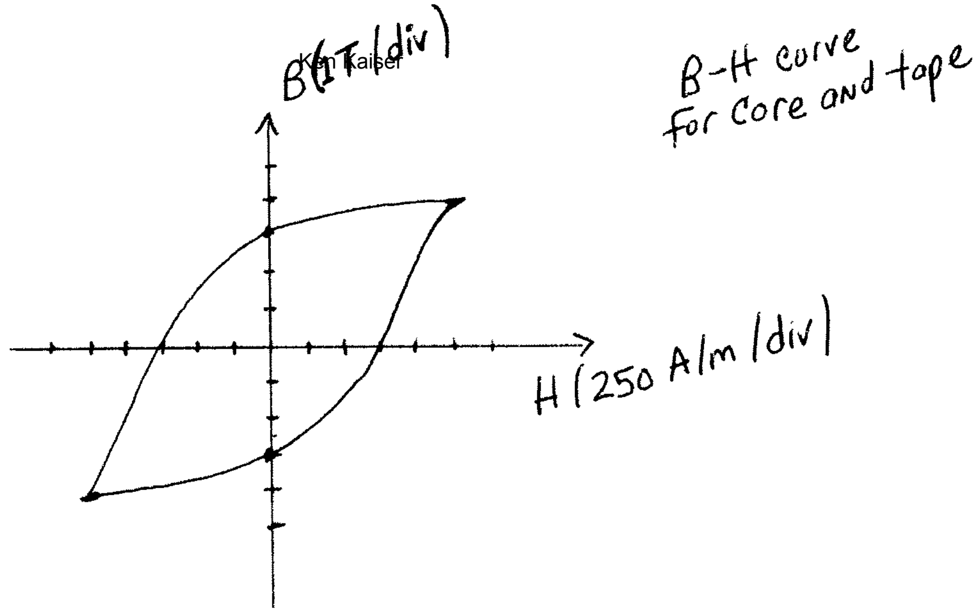
determine **H**, the magnetic field, in the region $y \leq 0$ (free space) if the interface at y = 0 carries the current 50 $\mathbf{a}_z$ A/m. Signs are important in this problem.

Test 24

A certain high-ranking government official was concerned about the ability of her tape machine to erase secret tape recordings. To insure that her machine "completely" erased the tapes, she had the chief engineer build the following device:

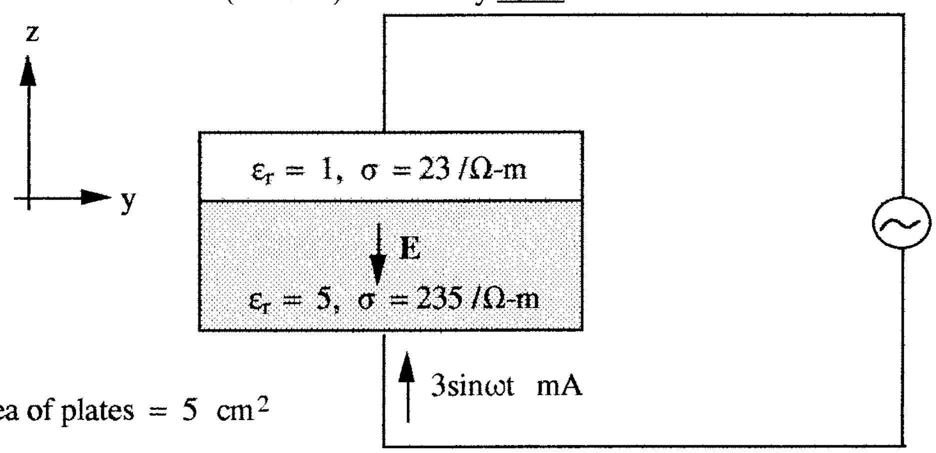


The tape is slowly passed through the gap as the amplitude of the sinusoidal source is stepped from some maximum current, I_{max}, to some minimum current, I_{min}. The hysteresis curve for the tape for this maximum current follows. Determine this maximum current only. **Note:** The core in this problem also has a nonlinear B-H relationship. Assume that this relationship can also be described by the same hysteresis curve.



Test 25

Determine **E** in V/m (a vector) if ω is very <u>small</u>



area of plates = 5 cm^2

Test 26

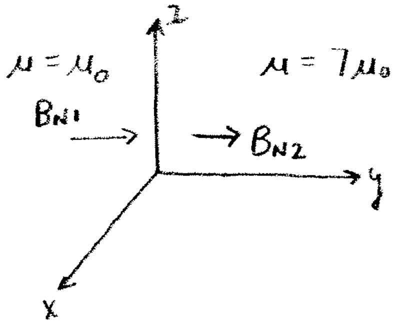
Near the tip of a lit cigarette, ions of charge density $\rho(0)$ are generated. The smoke carries these ions radially outward from the tip with a velocity distribution of

$$V_r = \frac{A}{r^3}$$

Determine the <u>differential equation</u> that that describes the steady-state distribution of charge, relative to the distance r, everywhere around the cigarette as a function of i (the <u>constant</u> steady state current from the cigarette), k the mobility of the ions in free space, $\rho(0)$, ε_o and r (the radial distance from the cigarette). (Hints: It is not necessary to directly use the charge conservation equation in this problem. The radial current density, J_r, is equal to $i/4\pi r^2$ which is equal to the sum of the conduction current and the convection current. The electric field can be solved for in this expression and substituted into Maxwell's equation.)

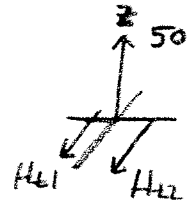
EE332, 3/12/93, SOLN., KAISER

(23)



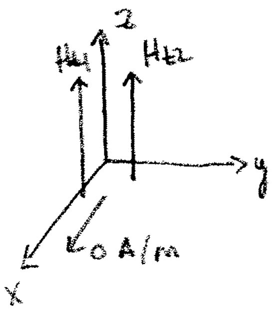
$\vec{H}_1 = a\hat{i} + b\hat{j} + c\hat{k}$

$B_{N1} = B_{N2}$
$\mu_0 b\hat{j} = 15\hat{j} \Rightarrow b = 15/\mu_0$



$\vec{B} = \mu\vec{H}$

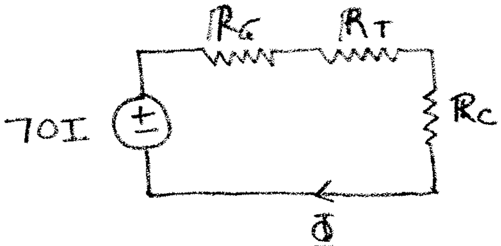
$H_{t1} - H_{t2} = K$
$a - (-20/7\mu_0) = 50$
$\Rightarrow \quad a = 50 - 20/7\mu_0$



$H_{t2} - H_{t1} = 0$
$-\frac{30}{7\mu_0} - c = 0 \Rightarrow c = -30/7\mu_0$

$\vec{H}_1 = (50 - 20/7\mu_0)\hat{i} + (15/\mu_0)\hat{j} - (30/7\mu_0)\hat{k}$

(24)



$R_G = \frac{l}{\mu A} = \frac{.01}{\mu_0 1\times10^{-4}} = 7.96\times10^{7}\ A\cdot t/Wb$

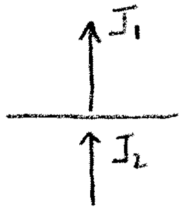
$B_{max} = 4\ T \quad (Wb/m^2)$
$\Phi_{max} = 4(1\times10^{-4}) = 4\times10^{-4}$
$H_{max} = 5(250) = 1250\,A/m$

$\int \vec{H}\, d\vec{L}$

$V_T = (1{,}250)(.01) = 12.5\ A\cdot t$
$V_C = (1{,}250)(2\pi(.03) - .02) \cong 211\ A\cdot t$
$70I = V_T + V_C + 4\times10^{-4} R_G$
$I_{max} = 458\ A$

(25) ω small ⇒ conduction current >> displacement current

$\vec{J}_t = \sigma\vec{E} + d\vec{D}/dt$
$\sigma\vec{E} + j\omega\epsilon\vec{E}$
$\sigma >> \omega\epsilon$



$J_1 = J_2 = \frac{3\sin\omega t \times 10^{-3}}{5\times10^{-4}} = 6\sin\omega t\ KA/m^2$

$\therefore\ \vec{E} = \frac{6{,}000\sin\omega t}{235}\hat{k} = -25.5\sin\omega t\,\hat{k}\ \ V/m$

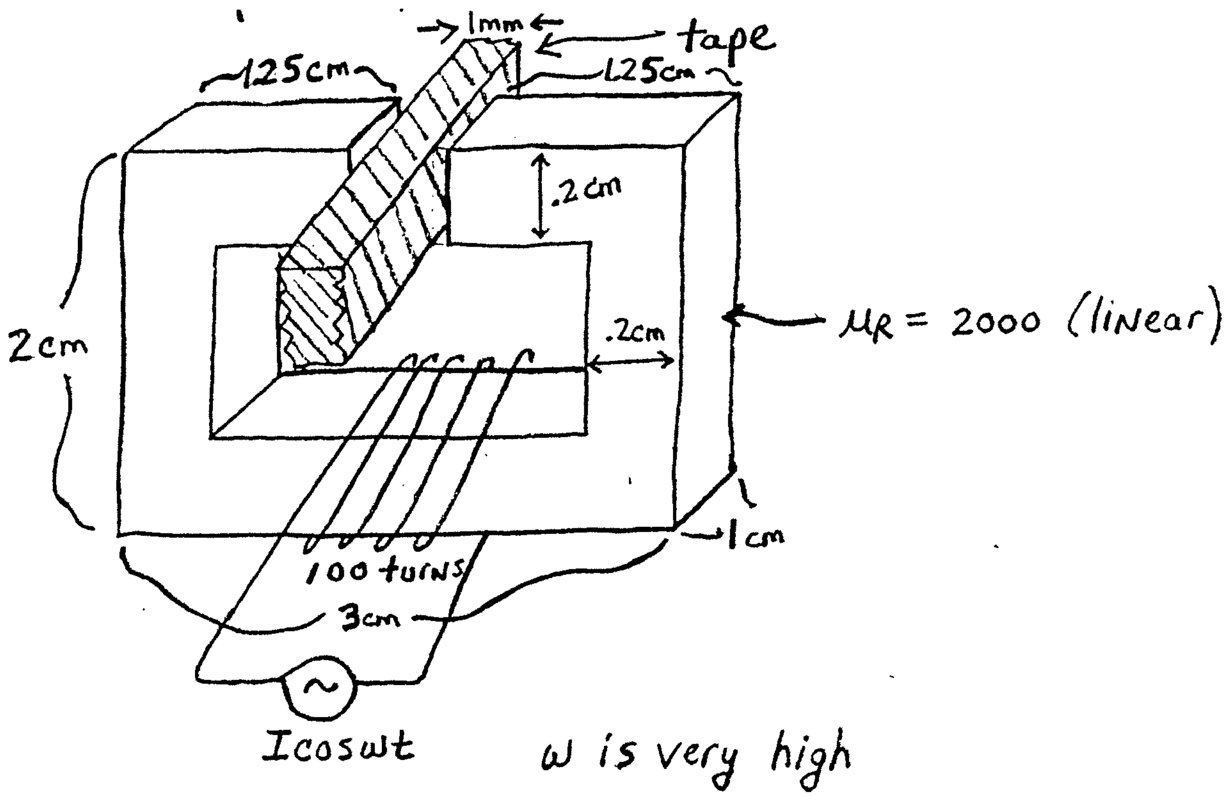
Problem K9

To remove ohmic related heat from a coaxial power line carrying a large dc current, flowing insulating oil of dielectric constant 2.5 is placed between the inner and outer conductors. The inner aluminum conductor with a radius of 2 cm carries 500 A, and the outer aluminum conductor with an inner radius of 4 cm and an outer radius of 5 cm carries the return current.

a) Use Ampere's Law to determine the magnetic field everywhere.
b) Sketch this field as a function of r, the distance from the center of the inner connector.

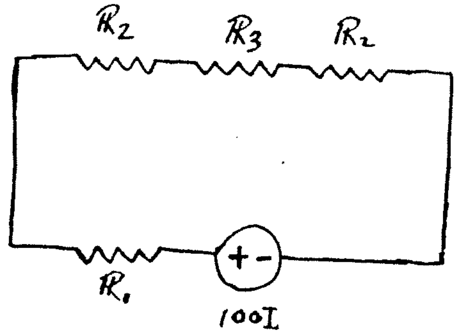
Problem K10

A certain high-ranking government official was concerned about the ability of her tape machine to erase secret tape recordings. To insure that her machine "completely" erased the tapes, she had the chief engineer build the following device:



The tape is slowly passed through the gap as the amplitude of the sinusoidal source is stepped from some maximum current, I_{max}, to some minimum current, I_{min}. The hysteresis curve for the tape for this maximum and minimum current follows. Determine this maximum and minimum current.

K-10



$$\mathcal{R}_1 \doteq \frac{(.028 + 2(.018) + 2(.0115))}{2{,}000\mu_0(.01)(.002)}$$
$$= 1.73\times10^6 \text{ A}\cdot\text{t/Wb}$$
$$\mathcal{R}_2 \doteq \frac{(.002)}{\mu_0(.01)(.002)} = 79.6\times10^6$$

$$B_{MAX} = (3.3\text{cm})(\tfrac{1}{3}\text{ T/cm}) = 1.1 \text{ T} \quad (= \text{Wb/m}^2)$$
$$\Phi_{MAX} = B_{MAX} A = 1.1(.01)(.002) = 22\times10^{-6} \text{ Wb}$$
$$V_3 = \int \vec{H}\cdot d\vec{L} = (3.5\text{cm})(286(\text{A/m})/\text{cm})(1\times10^{-3}) = 1.001 \text{ A}\cdot\text{t}$$

"KVL"

$$-100I_{MAX} + (1.73\times10^6)(22\times10^{-6}) + 2(79.6\times10^6)(22\times10^{-6}) + 1.001 = 0$$

$$\Rightarrow I_{MAX} = 35.4 \text{ A}$$

$$B_{MIN} = (1.3)(\tfrac{1}{3}) = .433$$
$$\Phi_{MIN} = B_{MIN} A = .433(.01)(.002) = 8.66\times10^{-6}$$
$$V_3 = (1.1)(286)(1\times10^{-3}) = .314$$

$$0 = -100I_{MIN} + (1.73\times10^6)(8.66\times10^{-6}) + 2(79.6\times10^6)(8.66\times10^{-6}) + .314$$

$$\Rightarrow I_{MIN} = 13.9 \text{ A}$$

K-11

a) $(4\cos30t - 2\cos30t)\times10^{-3} = -d\rho_s/dt$

$\int_0^{\rho_s} d\rho_s = \int_0^t (-2\cos30t)\times10^{-3}dt, \quad \rho_s - \rho_s(0) = (-2\times10^{-3}\sin30t)/30\Big|_0^t$

$\rho_s = -66.7\times10^{-6}\sin30t + 3\times10^{-3} \quad \text{C/m}^2$

b) ω high $\rightarrow$ displacement current >> conduction current

$\dfrac{2\times10^{-3}\cos\omega t}{2\times10^{-4}} = \dfrac{d}{dt}(\epsilon_0 E), \quad E = \dfrac{10}{\omega\epsilon_0}\sin\omega t \quad \text{V/m}$

$\vec{E}_0 = (-10/\omega\epsilon_0)\sin\omega t\,\hat{j} \quad \text{V/m}$

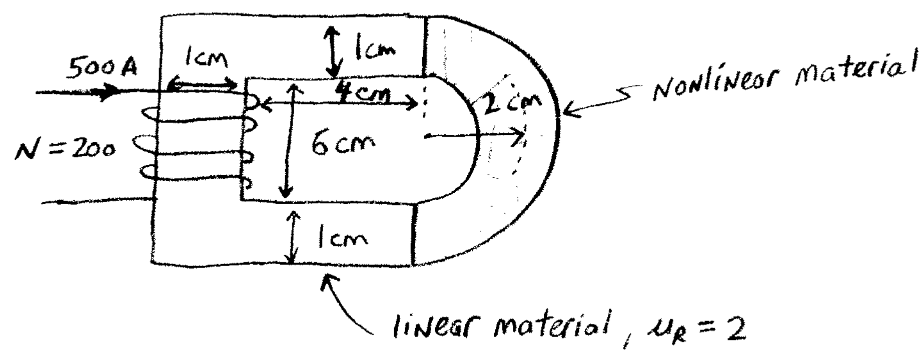
EE332, Winter 94, K. Kaiser
Tests 18-21

Test 18

A composite magnetic circuit of constant cross-sectionional area of 5 cm^2, as shown below, is made of a linear and nonlinear material. The BH characteristic of the nonlinear material is given by the equation

$$B = \frac{2H}{1 + H - 2H^2} \quad T$$

If the current through the coil is 500 A, completely setup all the equations necessary to determine the flux through the circuit.



Test 19

After participating in a spitting contest and losing, Bobby accused Sally of excessively watering down her spit. After a brief ruckus, Bobby ran home, balling all the way. A day later, in revenge, Bobby sprayed Sally down, with stick corn oil (he used his dad's painting gun system). Unfortunately, tribocharging of the oil occurred in the plastic hose. Assume the charge density in the oil at the orifice is ρ_o, the average cross-sectional area of the stream is A, the length of the stream in L, the average velocity of the stream is V, the permittivity of the oil is ε, and the conductivity of the oil is σ. Also, assume Sally is barefooted (grounded) but Bobby is not. If the charge density in the stream is given approximately by the expression

$$\rho = \rho_o \sin\left(\frac{\sigma y}{\varepsilon V}\right) + \rho_o \quad C/m^3$$

determine Bobby's potential relative to the ground.

Test 20

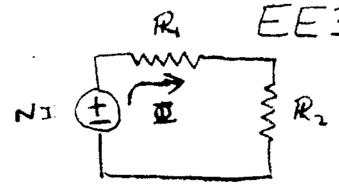
In the circuit below, a rectangular wire loop connected in series with two resistors is in a constant magnetic field given by

$$\vec{B} = 0.02\hat{i} \quad \text{Wb}/\text{m}^2$$

One side of the loop lies along the ~~x~~ z axis while another side rotates at an angular speed of w = 3.5 rad/sec. The loop is located in the yz plane at t = 0. Determine (and give reasons for) the voltages V_x and V_y.

EE332, Winter 94, K. Kaiser, Tests 18-21

(18)



$$R_1 = \frac{\ell}{\mu A} = \frac{.045+.045+.07}{2\mu_0\, 5/(100)^2}$$

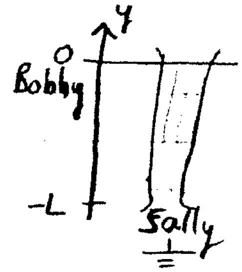
$$200(500) + R_1\Phi + H(\pi(.02)) = 0$$

(1) $$200(500) + R_1 B(5/(100)^2) + H(\pi(.02)) = 0$$

(2) $$B = \frac{2H}{1+H-2H^2}$$

Solving for B, $\Phi = B(5/(100)^2)$

(19) $J = \rho V + \sigma E = 0$ (steady state, no closed path)



$\rho(0) = \rho_0$ ✓

$$E = -\frac{\rho V}{\sigma}$$

$$V_B = -\int_{-L}^{0} \vec{E}\cdot d\vec{L} = -\int_{-L}^{0} -\frac{\rho V}{\sigma}\,dy = \frac{V}{\sigma}\int_{-L}^{0}\rho\,dy$$

$-L$ ← ref.

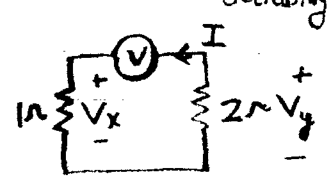
$$= \frac{V}{\sigma}\int_{-L}^{0}\left(\rho_0 \sin\frac{\sigma y}{\epsilon V} + \rho_0\right)dy = \frac{V}{\sigma}\left[-\frac{\rho_0 V\epsilon}{\sigma}\cos\frac{\sigma y}{\epsilon V} + \rho_0 y\right]_{-L}^{0}$$

$$= \frac{V}{\sigma}\left[-\frac{\rho_0 V\epsilon}{\sigma} - \frac{\rho_0 V\epsilon}{\sigma}\cos\frac{\sigma L}{\epsilon V} + \rho_0 L\right] \quad V$$

(20) $V = -\dfrac{d\Phi}{dt}$, $\Phi = BA = (.02\cos\omega t)(.03\cdot.015) = 9\times10^{-6}\cos\omega t$ Wb

decreasing with time from $t=0$

$$V = 9\times10^{-6}\,\omega\sin\omega t = 3.15\times10^{-5}\sin 3.5t$$



$$|I| = \frac{V}{3} = 1.05\sin 3.5t \quad A$$

$I = 1.05\sin 3.5t$ A produces $\vec{H}$ in opposite direction to $\vec{B} = .02\hat{i}$

$$V_x = I(1) = 1.05\sin 3.5t \quad V$$

$$V_y = -I(2) = -2.1\sin 3.5t \quad V$$

(21) $\nabla\cdot\vec{B} = 0$ ✓

$$\nabla\times\vec{H} = \vec{J} + \frac{d\vec{D}}{dt} = \epsilon_0\frac{d\vec{E}}{dt}$$

$$10e^{-2z}\sin\omega t\,\hat{i} = \epsilon_0\frac{d\vec{E}}{dt}, \quad \frac{d\vec{E}}{dt} = \frac{10}{\epsilon_0}e^{-2z}\sin\omega t\,\hat{i}$$

$$\vec{E} = -\frac{10e^{-2z}}{\omega\epsilon_0}\cos\omega t\,\hat{i} \quad \text{steady state}$$

$\nabla\cdot D = \rho$, $\nabla\times\vec{E} = -d\vec{B}/dt$

$0 = \rho$, $\dfrac{20}{\omega\epsilon_0}e^{-2z}\cos\omega t\,\hat{j} = ?\; -\mu_0\omega 5e^{-2z}\cos\omega t$

$\therefore \dfrac{20}{\omega\epsilon_0} = -\mu_0\omega 5$, $-4 = \omega^2\mu_0\epsilon_0$ NOT POSSIBLE