Name (Printed): SOLUTION

14 pts total

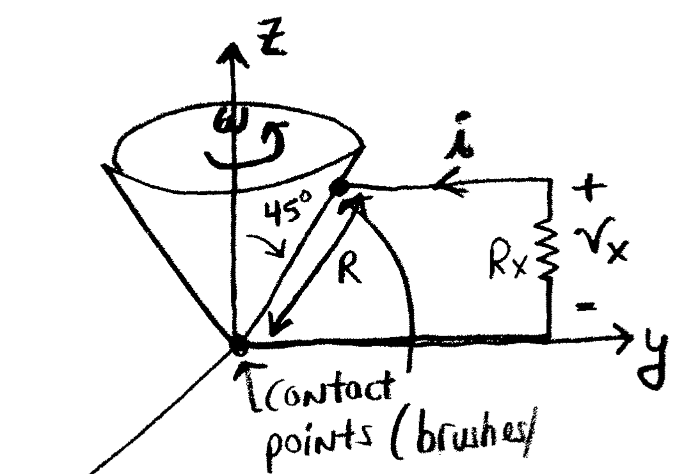
Signature:_____________________________________

Exam #5
Electromagnetic Fields and Applications, EE240
K. Kaiser, Winter 2011
This is a closed notes and open book test. Also, no calculators or other electronic devices are allowed.

Using both (equivalent) definitions for the induced voltage

$$V_{emf} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{s} \quad \text{and} \quad V_{emf} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} + \oint_L \left(\vec{v}\times\vec{B}\right)\cdot d\vec{L}$$

determine the current, i, and the voltage, v_x, for the given configuration. Verify that the results are the same. Show all steps (even the obvious ones) and clearly define all variables given in the previous definitions. Assume the resistor R_x is sufficiently large to limit the induced magnetic field to a level well below the applied magnetic field.

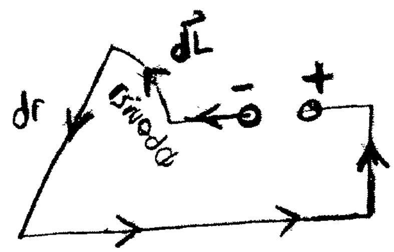


R_x located in yz plane

rotating conducting cone
ω rad/s about z axis

$\vec{B} = B_0 \sin(\omega_x t)\hat{a}_y$
present everywhere

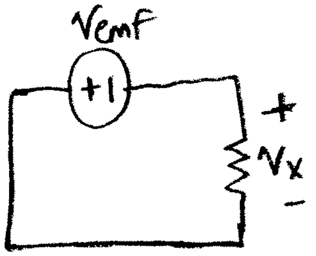
Note: ω not necessarily equal to ω_x



$d\vec{s} = -dr\, r\sin\theta\, d\phi\, \hat{a}_\theta$ +1 $\theta = 45^\circ = \pi/4$

(no lead contribution)

$V_x = -iR_x$ +1



$-V_x - V_{emf} = 0$

$V_x = -V_{emf}$ +1

$\hat{a}_y = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$ +1 $\sin 45^\circ = 1/\sqrt{2}$ $\cos 45^\circ = 1/\sqrt{2}$

+.5 +.5

① $$V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{s} = -\frac{d}{dt}\int_{\pi/2}^{\pi/2+\omega t}\int_0^R B_0\sin(\omega_x t)\frac{1}{\sqrt{2}}\sin\phi\left(-dr\, r\frac{1}{\sqrt{2}}d\phi\right)$$

+.5

$$= \frac{d}{dt}\left[\frac{B_0\sin(\omega_x t)}{2}\int_{\pi/2}^{\pi/2+\omega t}\int_0^R \sin\phi\, r\,dr\,d\phi\right]$$

$$= \frac{d}{dt}\left[\frac{B_0\sin(\omega_x t)}{2}\int_{\pi/2}^{\pi/2+\omega t}\frac{R^2}{2}\sin\phi\, d\phi\right]$$

+2 $$= -\frac{d}{dt}\left[\frac{B_0\sin(\omega_x t)R^2}{4}\cos\phi\Big|_{\pi/2}^{\pi/2+\omega t}\right]$$

$$= -\frac{B_0 R^2}{4}\frac{d}{dt}\left[\sin(\omega_x t)\cos(\omega t+\pi/2) - 0\right]$$ since $\cos\frac{\pi}{2} = 0$

$$= -\frac{B_0 R^2}{4}\left[\omega_x\cos(\omega_x t)\cos\left(\omega t+\frac{\pi}{2}\right) - \sin(\omega_x t)\,\omega\sin\left(\omega t+\frac{\pi}{2}\right)\right]$$

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② $V_{emf} = -\iint \frac{d\vec{B}}{dt}\cdot d\vec{S} + \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$

$+1$

$= -\int_{\pi/2}^{\pi/2+\omega t}\int_0^R \omega_x B_0 \cos(\omega_x t)\frac{1}{\sqrt{2}}\sin\phi\left(-dr\, r\frac{1}{\sqrt{2}}d\phi\right) + \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$

$= -\frac{\omega_x B_0 \cos(\omega_x t)}{2}\frac{R^2}{2}\cos\left(\frac{\pi}{2}+\omega t\right) + \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$

✓

$= -\frac{B_0\omega_x R^2}{4}\cos(\omega_x t)\cos\left(\omega t+\frac{\pi}{2}\right) + \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$

$v = \omega\rho = \omega r\sin\theta = \omega r/\sqrt{2}$ $+1$, $\vec{v} = \frac{\omega r}{\sqrt{2}}\hat{a}_\phi$

$\vec{B} = B_0\sin(\omega_x t)\left[\frac{1}{\sqrt{2}}\sin\phi\,\hat{a}_r + \frac{1}{\sqrt{2}}\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi\right]$

$r \Rightarrow \theta \Rightarrow \phi$

$d\vec{L} = dr\,\hat{a}_r$ $+1$

$\hat{a}_\phi\times\hat{a}_\phi = 0$

$\hat{a}_\phi\times\hat{a}_r = \hat{a}_\theta$

$\hat{a}_\phi\times\hat{a}_\theta = -\hat{a}_r$

since $d\vec{L} = dr\,\hat{a}_r$

$+1$

$\oint(\vec{v}\times\vec{B})\cdot d\vec{L} = \int_R^0 -\frac{\omega r}{\sqrt{2}}B_0\sin(\omega_x t)\frac{1}{\sqrt{2}}\sin\phi\, dr$

$+.5$ $\quad \phi = \omega t+\frac{\pi}{2}$ $\quad +1$

$= -\frac{\omega}{2}B_0\sin(\omega_x t)\sin\left(\omega t+\frac{\pi}{2}\right)\left(0-\frac{R^2}{2}\right)$

$+1$

$= \frac{\omega R^2 B_0}{4}\sin(\omega_x t)\sin\left(\omega t+\frac{\pi}{2}\right)$ ✓

these two results, when summed, are equal to the expression given for ①

Ken Kaiser

Let's verify that the "other" moving side results in 0 extra voltage:

$$\vec{dL} = r\sin\theta\, d\phi\, \hat{a}_\phi = \frac{r}{\sqrt{2}}\, d\phi\, \hat{a}_\phi$$

$$\vec{v} = \frac{\omega R}{\sqrt{2}}\, \hat{a}_\phi$$

$$\to r \to \theta \to \phi \to$$

$$\hat{a}_\phi \times \hat{a}_\phi = 0$$

$$\hat{a}_\phi \times \hat{a}_r = \hat{a}_\theta$$

$$\hat{a}_\phi \times \hat{a}_\theta = -\hat{a}_r$$

Since $\vec{dL}$ is in the $\hat{a}_\phi$ direction, and $\vec{v}\times\vec{B}$ only has $\hat{a}_\theta$ and $\hat{a}_r$ components

$$(\vec{v}\times\vec{B})\cdot\vec{dL} = 0$$

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K. Kaiser, Winter 2010
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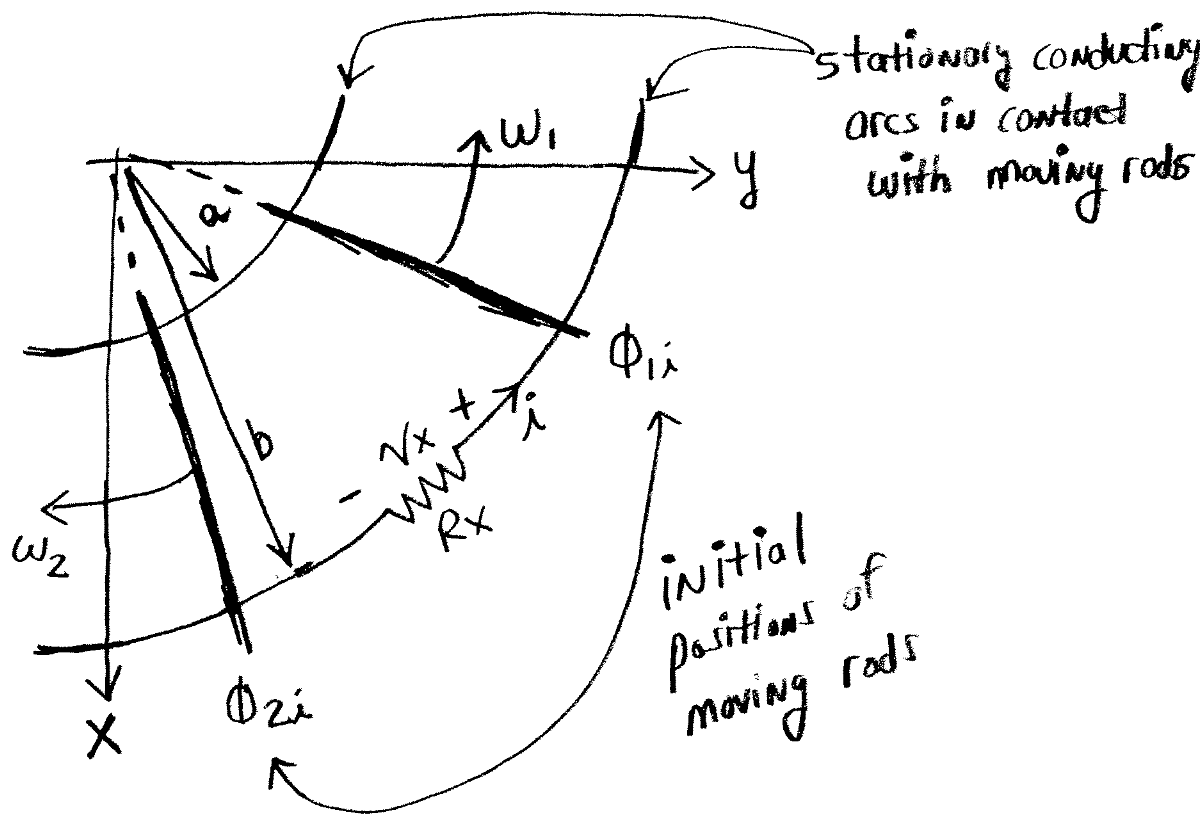
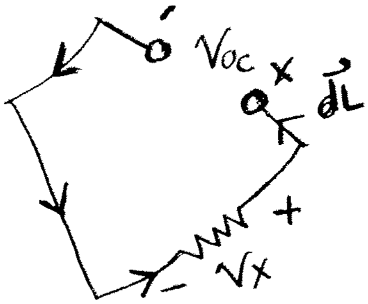
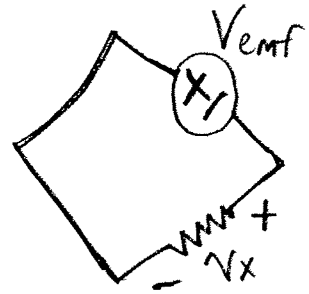


$$\vec{B} = (\rho-\phi)\cos\omega_x t\,\hat{a}_\rho + (3\rho\phi)\sin\omega_x t\,\hat{a}_\phi + (2\rho+\phi)\sin\omega_x t\,\hat{a}_z$$

14 pts total

$$v_x = -iR_x \quad +1$$





$$-v_x - V_{emf} = 0$$

$$v_x = -V_{emf}$$

$+1$

(1)

$+.5$ $\quad \phi_{1i} + \omega_1 t \quad +.5 \quad \vec{ds} = d\rho\,\rho\, d\phi\, \hat{a}_z \quad +1$

$$V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{ds} = -\frac{d}{dt}\int_{\phi_{2i}-\omega_2 t}^{\phi_{1i}+\omega_1 t}\int_a^b (2\rho + \phi)\sin\omega_x t\, d\rho\,\rho\, d\phi \Big|_{z=0}$$

$\phi_{2i} - \omega_2 t \quad +.5$

$(2\rho^2 + \phi\rho)\sin\omega_x t$

$$= -\frac{d}{dt}\int_{\phi_{2i}-\omega_2 t}^{\phi_{1i}+\omega_1 t}\left(\frac{2\rho^3}{3} + \frac{\phi\rho^2}{2}\right)\Big|_a^b \sin\omega_x t\, d\phi$$

$$= -\frac{d}{dt}\left[\sin\omega_x t\int_{\phi_{2i}-\omega_2 t}^{\phi_{1i}+\omega_1 t}\left(\frac{2b^3}{3} + \frac{\phi b^2}{2} - \frac{2a^3}{3} - \frac{\phi a^2}{2}\right) d\phi\right]$$

$+2$

$$= -\frac{d}{dt}\left[\sin\omega_x t\left(\frac{2}{3}(b^3 - a^3)\phi + \left(\frac{b^2}{2} - \frac{a^2}{2}\right)\frac{\phi^2}{2}\right)\Bigg|_{\phi_{2i}-\omega_2 t}^{\phi_{1i}+\omega_1 t}\right]$$

$$= -\frac{d}{dt}\Big[\sin\omega_x t\Big\{\frac{2}{3}(b^3 - a^3)(\phi_{1i} + \omega_1 t - \phi_{2i} - \omega_2 t)$$

$$+ \left(\frac{b^2}{4} - \frac{a^2}{4}\right)\left[(\phi_{1i} + \omega_1 t)^2 - (\phi_{2i} - \omega_2 t)^2\right]\Big\}\Big]$$

$$= -\Big[\omega_x \cos\omega_x t\,\{\quad\} + \sin\omega_x t\Big(\frac{2}{3}(b^3 - a^3)(\omega_1 - \omega_2)$$

$$+ \left(\frac{b^2}{4} - \frac{a^2}{2}\right) 2\omega_1(\phi_{1i} + \omega_1 t) + 2\omega_2(\phi_{2i} + \omega_2 t)$$

(2)

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$$V_{emf} = -\iint \frac{d\vec{B}}{dt}\cdot d\vec{s} + \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$$

$$\vec{B} = B_\rho\hat{a}_\rho + B_\phi\hat{a}_\phi + B_z\hat{a}_z$$

+1

$$= -\int_{\phi_{2i}-\omega_2 t}^{\phi_{1i}+\omega_1 t}\int_a^b (2\rho+\phi)\omega_x\cos(\omega_x t)\,d\rho\,\rho\,d\phi$$

since

$\vec{v} = \omega\rho\hat{a}_\phi$ and

$d\vec{L} = d\rho\hat{a}_\rho$

$\therefore \vec{v}\times\vec{B}$

$= -\omega\rho B_\rho\hat{a}_z + \omega\rho B_z$

and

$(\vec{v}\times\vec{B})\cdot d\vec{L}$

$= \underbrace{\omega\rho B_z\, d\rho}_{+1}$

+.5

$$+ \sin\omega_x t\int_b^a \omega_1\rho(2\rho+\phi)\,d\rho\Big/_{\substack{z=0\\ \phi=\phi_{1i}+\omega_1 t}}$$

+1 +1

$$+ \sin\omega_x t\int_a^b -\omega_2\rho(2\rho+\phi)\,d\rho\Big/_{\substack{z=0\\ \phi=\phi_{2i}-\omega_2 t}}$$

+1

$$\rightarrow \omega_1\sin\omega_x t\left[\frac{2\rho^3}{3}+\frac{\rho^2\phi}{2}\right]_b^a \quad \phi=\phi_{1i}+\omega_1 t$$

12

$$= \omega_1\sin\omega_x t\left[\frac{2a^3}{3}+\frac{a^2}{2}(\phi_{1i}+\omega_1 t)-\frac{2b^3}{3}-\frac{b^2}{2}(\phi_{1i}+\omega_1 t)\right]$$

$$\rightarrow -\omega_2\sin\omega_x t\left[\frac{2\rho^3}{3}+\frac{\rho^2\phi}{2}\right]_a^b \quad \phi=\phi_{2i}-\omega_2 t$$

$$= -\omega_2\sin\omega_x t\left[\frac{2b^3}{3}+\frac{b^2}{2}(\phi_{2i}-\omega_2 t)-\frac{2a^3}{3}-\frac{a^2}{2}(\phi_{2i}-\omega_2 t)\right]$$

$$-\int_{\phi_{2i}-\omega_2 t}^{\phi_{1i}+\omega_1 t}\int_a^b (2\rho+\phi)\omega_x\cos(\omega_x t)\,d\rho\,\rho\,d\phi$$ integrated as before

Name (Printed): SOLN

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K. Kaiser, Winter 2008
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Using both (equivalent) definitions for the induced voltage

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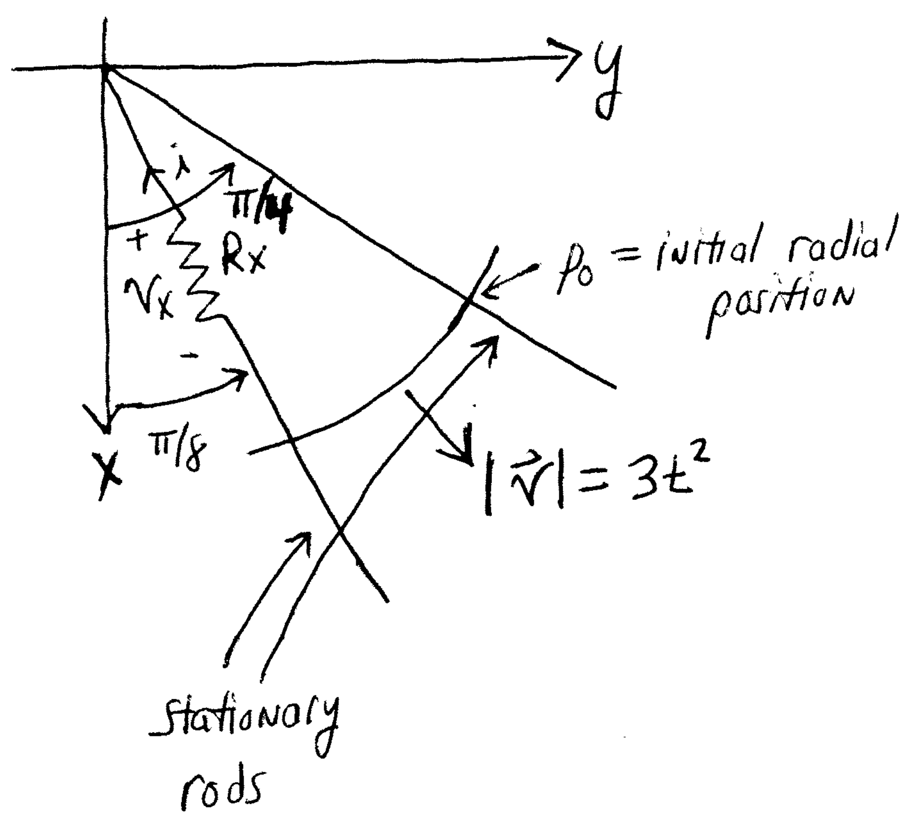


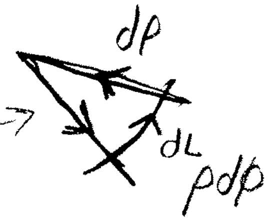
$$\vec{B} = (\rho+\phi^2)\cos(\omega t)\,\hat{a}_\rho - 3\rho\phi\sin(\omega t)\,\hat{a}_z + (6z^2+\rho)\sin(\omega t)\,\hat{a}_\phi$$

14 pts total

Ken Kaiser

1

① $$V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{S} \quad , \quad d\vec{S} = \overbrace{d\rho\,\rho\, d\phi\,\hat{a}_z}^{+1}$$



$$= -\frac{d}{dt}\int_{\pi/8}^{\overbrace{\pi/4}^{+.5}}\int_{\underbrace{0}_{+.5}}^{\overbrace{\rho_0+t^3}^{+1}} \underbrace{-3\rho\phi\sin(\omega t)}_{+1}\,\rho\, d\rho\, d\phi$$

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$x = \int_0^t 3t^2 dt + x_0$$

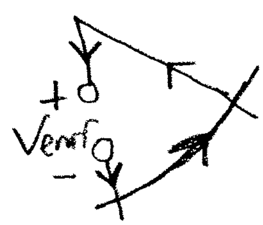
$$= t^3 + x_0$$

$$= +\frac{d}{dt}\left[3\sin(\omega t)\int_{\pi/8}^{\pi/4}\int_0^{\rho_0+t^3} \rho^2 d\rho\, \phi\, d\phi\right]$$

$$= \frac{d}{dt}\left[3\sin(\omega t)\left(\frac{(\pi/4)^2}{2} - \frac{(\pi/8)^2}{2}\right)\left(\frac{(\rho_0+t^3)^3}{3} - 0\right)\right]$$

+2 $$= 3\left[\frac{(\pi/4)^2}{2} - \frac{(\pi/8)^2}{2}\right]\left\{\omega\cos(\omega t)\left(\frac{(\rho_0+t^3)^3}{3}\right) + \sin(\omega t)\,\frac{3}{3}(\rho_0+t^3)^2\, 3t^2\right\} \quad V$$

② $$V_{emf} = -\iint \frac{d\vec{B}}{dt}\cdot d\vec{S} + \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$$



$$= -\overbrace{\int_{\pi/8}^{\pi/4}\int_0^{\rho_0+t^3} -3\rho\phi\,\omega\cos(\omega t)\,\rho\, d\rho\, d\phi}^{+1}$$

↓ ONLY NONZERO term (other sides $\vec{v} = 0$)

$$+ \int_{\pi/8}^{\pi/4}\left(\underbrace{3t^2\hat{a}_\rho}_{+1}\times\vec{B}\right)\cdot\underbrace{\rho\, d\phi\,\hat{a}_\phi}_{+1}\Bigg/_{\rho=\rho_0+t^3}$$

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$$= 3\omega\cos(\omega t)\left\{\left[\frac{\left(\frac{\pi}{4}\right)^2}{2} - \frac{\left(\frac{\pi}{8}\right)^2}{2}\right]\left[\frac{(\rho_0+t^3)^3}{3}\right]\right.$$

✓ agrees with ①

+1

$$\rho\ \phi\ z$$

$$\hat{a}_\rho \times \hat{a}_\rho = 0$$

$$\boxed{\hat{a}_\rho \times \hat{a}_z = -\hat{a}_\phi}$$

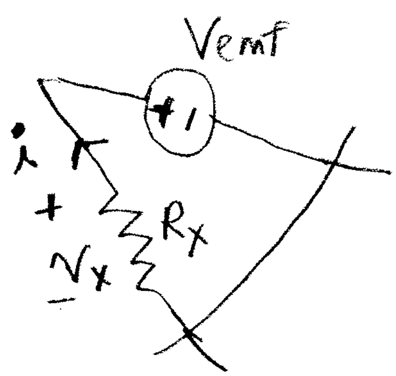
$$\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_z$$

$$\left. + \int_{\pi/8}^{\pi/4} 3t^2 \left(\overbrace{3\rho\phi\sin(\omega t)}^{+1}\right)\rho\, d\phi \ \right|_{\rho=\rho_0+t^3}$$

$$\downarrow$$

✓ agrees with ①

$$+1 \quad 9t^2\sin(\omega t)\,(\rho_0+t^3)^2\left[\frac{(\pi/4)^2}{2} - \frac{(\pi/8)^2}{2}\right]$$



$V_x = V_{emf}$ +1

$i = \dfrac{-V_{emf}}{R_x}$ +1

16pts

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Signature:

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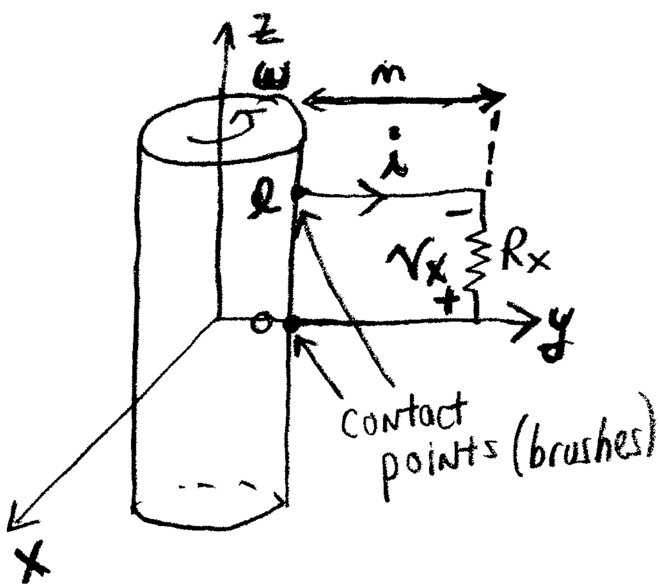
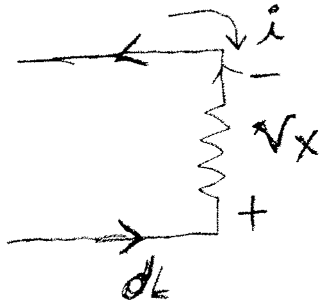


radius = R

rotating conducting cylinder
ω rad/s about z axis

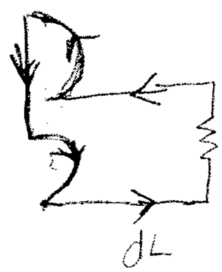
$\vec{B} = B_0 \sin(\omega_x t)\hat{a}_y$
present everywhere

Note: ω not necessarily equal to ω_x



$$V_x = -iR \quad +1$$

$$V_x = V_{emf} \quad +1$$



(1)

$$V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{ds}$$

i.e. No field passing through leads to Rx

$$= -\frac{d}{dt}\Bigg[\underbrace{\int_0^{\ell}\int_{R_x+1}^{m+R} B_0 \sin(\omega_x t)\hat{a}_y \cdot dy\,dz\,\hat{a}_x}_{=0} + \int_0^{\ell}\int_{\pi/2}^{\pi/2+\omega t} B_0 \sin(\omega_x t)\hat{a}_y \cdot \rho\, d\phi\, dz\, \hat{a}_\rho\Bigg]\Bigg/_{\rho=R}$$

$+1$ $+1$ $+1$

$$= \frac{d}{dt}\left[\int_0^{\ell}\int_{\pi/2}^{\pi/2+\omega t} B_0 \sin(\omega_x t)\left[\sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi\right]\cdot R\,d\phi\,dz\,\hat{a}_\rho\right]$$

$+1$ $+1$

$$= \frac{d}{dt}\left[\ell B_0 \sin(\omega_x t) R \int_{\pi/2}^{\pi/2+\omega t} \sin\phi\, d\phi\right]$$

$$= -\ell B_0 R\,\frac{d}{dt}\left[\sin(\omega_x t)\left(\cos\left(\frac{\pi}{2}+\omega t\right) - \cos\left(\frac{\pi}{2}\right)\right)\right]$$

$$= -\ell B_0 R\,\frac{d}{dt}\left[\sin(\omega_x t)\cos\left(\frac{\pi}{2}+\omega t\right)\right]$$

$$= -\ell B_0 R\left[\omega_x \cos(\omega_x t)\cos\left(\frac{\pi}{2}+\omega t\right) - \omega \sin(\omega_x t)\sin\left(\frac{\pi}{2}+\omega t\right)\right]$$

$+2$

② $$V_{emf} = -\iint \frac{d\vec{B}}{dt}\cdot d\vec{S} \quad + \quad \oint(\vec{v}\times\vec{B})\cdot d\vec{L}$$

$$= -\int_0^{\ell}\int_{\pi/2}^{\pi/2+\omega t} B_0\omega_x\cos(\omega_x t)\hat{a}_y \cdot -\rho\, d\phi\, dz\, \hat{a}_\rho \Big|_{\rho=R} \Big] \; +1$$

$$+ \int_{\pi/2}^{\pi/2+\omega t}(\omega\rho\hat{a}_\phi \times B_0\sin(\omega_x t)\hat{a}_y)\cdot \rho\, d\phi\, \hat{a}_\phi \Big|_{\rho=R}$$

will cancel

$$+ \int_{\ell}^{0 \;\; +1}(\underbrace{\omega\rho}_{+1}\,\hat{a}_\phi \times B_0\sin(\omega_x t)\hat{a}_y)\cdot \overbrace{dz\,\hat{a}_z}^{+1} \Big|_{\rho=R,\; \underline{\phi=\omega t+\pi/2}_{\;+1}}$$

$$+ \int_{\pi/2+\omega t}^{\pi/2}(\omega\rho\hat{a}_\phi \times B_0\sin(\omega_x t)\hat{a}_y)\cdot \rho\, d\phi\, \hat{a}_\phi \Big|_{\rho=R}$$

$$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi$$

$$= \int_0^{\ell}\int_{\pi/2}^{\pi/2+\omega t} B_0\omega_x\cos(\omega_x t)\sin\phi\, R\, d\phi\, dz$$

$\rho \quad \phi \quad z$

$$+ \int_{\ell}^{0} -\omega R\, B_0\sin(\omega_x t)\sin\phi\, dz \Big|_{\phi=\omega t+\pi/2}$$

$$= R B_0\omega_x\ell\cos(\omega_x t)\left[\cos\left(\frac{\pi}{2}+\omega t\right) - \cos\left(\frac{\pi}{2}\right)\right] + \omega R\ell B_0\sin(\omega_x t)\sin(\omega t +$$

$$= -\ell B_0 R\,\omega_x\cos(\omega_x t)\cos\left(\frac{\pi}{2}+\omega t\right) + \ell B_0 R\,\omega\sin(\omega_x t)\sin(\omega t+$$

✓ ✓ +2

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Signature:_______________________________

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K. Kaiser, Spring 2007
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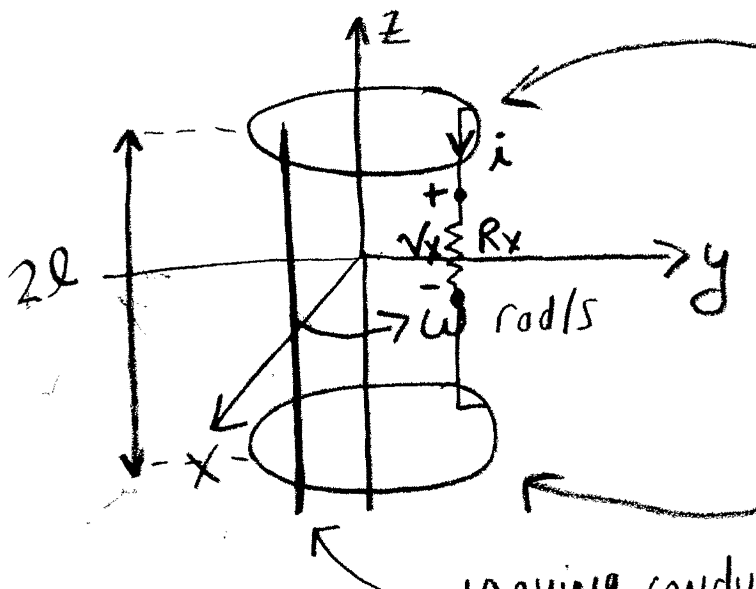
determine the current, i, and the voltage, v_x, for the given configuration. Verify that the results are the same. Show all steps (even the obvious ones) and clearly define all variables given in the previous definitions. Assume the resistor R_x is sufficiently large to limit the induced magnetic field to a level well below the applied magnetic field.



two stationary ring conductors each of radius R centered about the z axis

moving conducting bar in contact with both rings, the bar is initially in the xz plane

the resistor R_x is located in the yz plane

$\vec{B} = \frac{B_0}{\rho}\cos(\omega t)\hat{a}_\rho$ present everywhere

NOTE: ω's the same in cos(ωt) and rod movement

Ken Kaiser

13 pts

①

$$V_{emf} = -\frac{d}{dt}\int_{-\ell}^{\overset{+1}{\ell}}\int_{\underbrace{\omega t}_{+1}}^{\overset{+1}{\pi/2}} \frac{B_0}{\rho}\cos(\omega t)\hat{a}_\rho \cdot \overbrace{\rho\, d\phi\, dz\, \hat{a}_\rho}^{+1}\Big|_{\rho=R}$$

$$= -\frac{d}{dt}\left[B_0 2\ell\left(\frac{\pi}{2}-\omega t\right)\cos(\omega t)\right] \quad +1$$

$$= -\frac{d}{dt}\left[B_0\ell\pi\cos(\omega t) - B_0 2\ell\omega t\cos(\omega t)\right]$$

$\downarrow +2$

$$= -\left[-B_0\ell\pi\omega\sin(\omega t) - B_0 2\ell\omega\cos(\omega t) + B_0\omega^2 2\ell t\sin(\omega t)\right]$$

$$= B_0\ell\pi\omega\sin(\omega t) + B_0 2\ell\omega\cos(\omega t) - B_0\omega^2 2\ell t\sin(\omega t)$$

②

$$V_{emf} = -\int_{-\ell}^{\ell}\int_{\omega t}^{\pi/2} -\frac{B_0}{\rho}\omega\sin(\omega t)\hat{a}_\rho\cdot\rho\, d\phi\, dz\,\hat{a}_\rho\Big|_{\rho=R} \quad +1$$

$$+\int_{\underbrace{\ell}_{+1}}^{-\ell}\left(\overbrace{\omega R\hat{a}_\phi}^{+1} \times \frac{B_0}{\rho}\cos(\omega t)\hat{a}_\rho\right)\cdot \overbrace{dz\,\hat{a}_z}^{+1}\Big|_{\rho=R}$$

$$= 2\ell\left(\frac{\pi}{2}-\omega t\right)B_0\omega\sin(\omega t) \quad +1$$

$\rho\ \phi\ z$

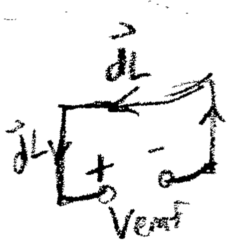
$+1 \quad \hat{a}_\phi\times\hat{a}_\rho = -\hat{a}_z$

$$-\ \omega B_0(-2\ell)\cos(\omega t)$$

$$= \ell\pi B_0\omega\sin(\omega t) - 2\ell\omega^2 t B_0\sin(\omega t) + \omega B_0 2\ell\cos(\omega t)$$

eqn. same

eqn. valid from $0 \le \omega t \le \pi/2$



$i = V_x/R_x \quad +1$

$V_x = -V_{emf} \quad +1$

$-V_{emf} - V_x = 0$

Name (Printed): SOLUTION

Signature:______________________________

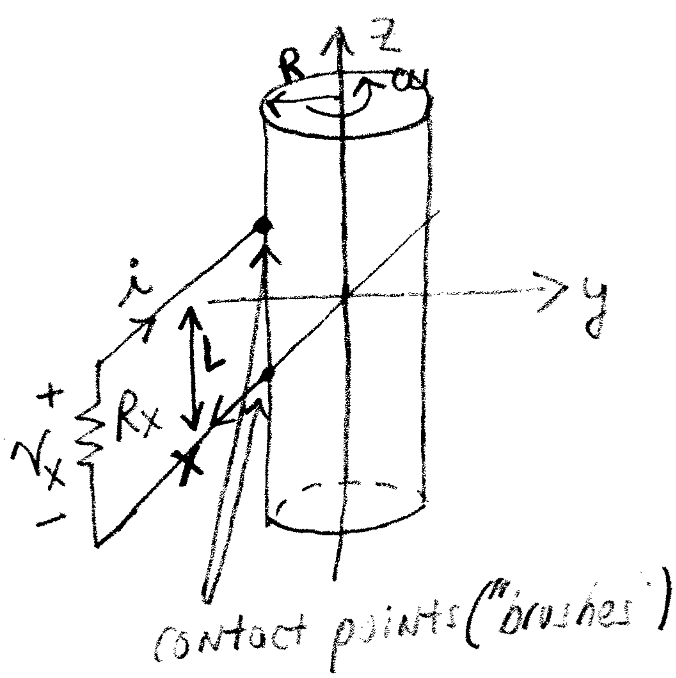
Exam #5
Electromagnetic Fields and Applications, EE240
K. Kaiser, Summer 2008

This is a closed notes and open book test. Also, no calculators or other electronic devices are allowed.

Using both (equivalent) definitions for the induced voltage

$$V_{emf} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{s} \quad \text{and} \quad V_{emf} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} + \oint_L \left(\vec{v}\times\vec{B}\right)\cdot d\vec{L}$$

determine the current, i, and the voltage, v_x, for the given configuration. Verify that the results are the same. Show all steps (even the obvious ones) and clearly define all variables given in the previous definitions. Assume the resistor R_x is sufficiently large to limit the induced magnetic field to a level well below the applied magnetic field.

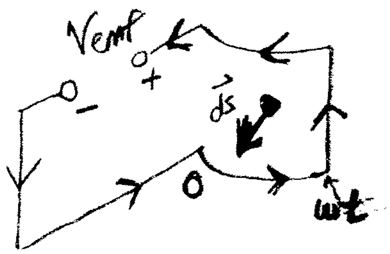


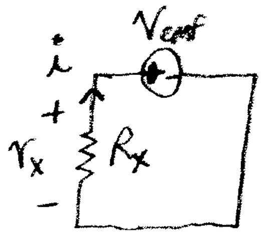
rotating conducting cylinder
ω rad/s about z axis

$\vec{B} = B_0 \cos\omega_x t\, \hat{a}_x$ present everywhere

Note: ω not necessarily equal to ω_x

$v = \omega R \ \ m/s$ Ken Kaiser

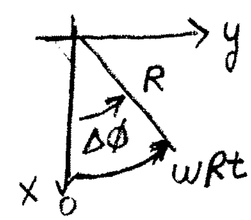




$v_x = -iR_x$ +1 , $-V_{emf} + v_x = 0$

$V_{emf} = v_x$ +1

① $$V_{emf} = -\frac{d}{dt}\int_0^{\omega t}\int_0^{L} B_0 \cos\omega_x t\, \hat{a}_x \cdot \rho\, d\phi\, dz\, \hat{a}_\rho \Big/_{\rho = R}$$



$\Delta\phi R = \omega R t$

$\Delta\phi = \omega t$

(rad/s)s = rad ✓

$$= -\frac{d}{dt}\int_0^{\omega t}\int_0^{L} B_0 \cos\omega_x t\left[\cos\phi\, \hat{a}_\rho - \sin\phi\, \hat{a}_\phi\right] \cdot \rho\, d\phi\, dz\, \hat{a}_\rho \Big/_{\rho = R}$$

$$= -\frac{d}{dt}\int_0^{\omega t}\int_0^{L} B_0 \cos\omega_x t \cos\phi\, R\, d\phi\, dz$$

$$= -\frac{d}{dt}\left[B_0(\cos\omega_x t) RL \left(\sin\phi\right)\Big/_0^{\omega t}\right]$$

$$= -B_0 RL \frac{d}{dt}\left[\cos\omega_x t \sin\omega t\right]$$

$$= -B_0 RL\left[-\omega_x \sin\omega_x t \sin\omega t + \omega \cos\omega_x t \cos\omega t\right]$$

$$= B_0 RL\left[\omega_x \sin\omega_x t \sin\omega t - \omega \cos\omega_x t \cos\omega t\right]$$ +2

Ken Kaiser

②
$$V_{emf} = -\int_0^{\omega t}\int_0^{L}\left(-\omega_x B_0 \sin\omega_x t\,\hat{a}_x\right)\cdot \rho\, d\phi\, dz\, \hat{a}_\rho \Big/_{\rho=R}$$
$$+ \int_0^{\omega t}\left(\omega\rho\,\hat{a}_\phi \times \vec{B}\right)\cdot \rho\, d\phi\, \hat{a}_\phi \Big/_{\substack{\rho=R\\ z=L}}$$
$$+ \int_0^{L}\left(\omega\rho\,\hat{a}_\phi \times \vec{B}\right)\cdot dz\, \hat{a}_z \Big/_{\substack{\rho=R\\ \phi=\omega t}}$$
$$+ \int_{\omega t}^{0}\left(\omega\rho\,\hat{a}_\phi \times \vec{B}\right)\cdot \rho\, d\phi\, \hat{a}_\phi \Big/_{\substack{\rho=R\\ z=0}}$$

cancel

ⓐ
$$-\int_0^{\omega t}\int_0^{L}\left(-\omega_x B_0 \sin\omega_x t\left[\cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi\right]\cdot R\, d\phi\, dz\, \hat{a}_\rho\right.$$

$\rho\,\phi\,z$

$$= L R\omega_x B_0 \sin\omega_x t \int_0^{\omega t}\cos\phi\, d\phi = L R\omega_x B_0 \sin\omega_x t\,\sin\omega t$$

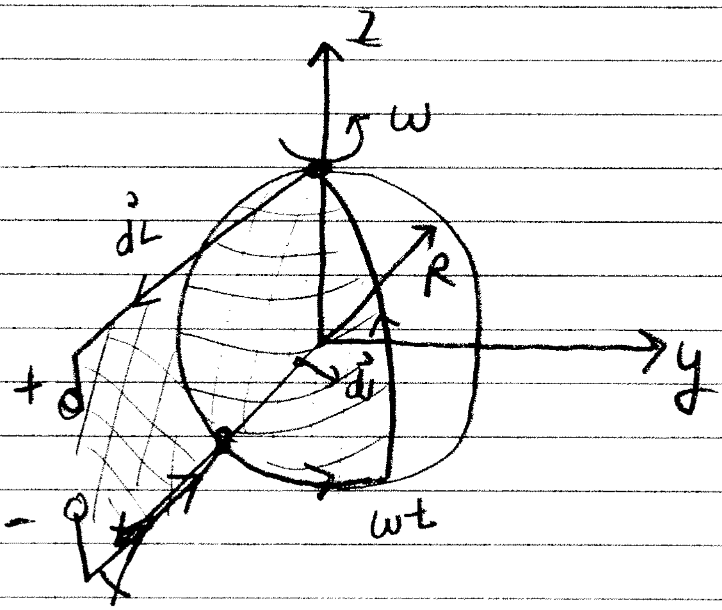
ⓑ
$$\int_0^{L}\omega R\,\hat{a}_\phi \times \left\{B_0\cos\omega_x t\left[\cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi\right]\right.\cdot dz\,\hat{a}_z \Big/_{\phi=\omega t}$$

$$= \omega R B_0 \cos\omega_x t \int_0^{L} -\cos\phi\,\hat{a}_z\cdot dz\,\hat{a}_z \Big/_{\phi=\omega t}$$

$$= L\omega R B_0 \cos\omega_x t\,\sin\omega t$$

adding ⓐ and ⓑ we get the final result, which is equal to the previous expression

Spring 2007 FINAL PART B
K. Kaiser



$\vec{B} = B_0 \hat{a}_z$

rotating conducting sphere - hemisphere

$d\vec{s} = r\,d\theta\, r\sin\theta\, d\phi\, \hat{a}_\rho$

$v = \omega\rho$ in cylindrical coordinates

$= \omega r \sin\theta$ in spherical coordinates

@ $\theta = 0°$ $v = 0$ ✓

@ $\theta = \pi/2$ $v = \omega r = \omega R$ ✓

(2) $$V_{emf} = \iint -\frac{d\vec{B}}{dt} \cdot d\vec{s} + \oint (\vec{v} \times \vec{B}) \cdot d\vec{L}$$

$$= 0 + \oint (v \times B_0 \hat{a}_z) \cdot d\vec{L}$$

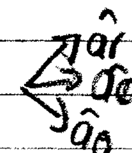
$$= \int_0^{\omega t} (\omega R \hat{a}_\phi \times B_0 \hat{a}_z) \cdot R \sin\theta\, d\phi\, \hat{a}_\phi \Big|_{\theta = \pi/2}$$

$$+ \int_{\pi/2}^{0} (\omega r \sin\theta\, \hat{a}_\phi \times B_0 \hat{a}_z) \cdot R\, d\theta\, \hat{a}_\theta$$

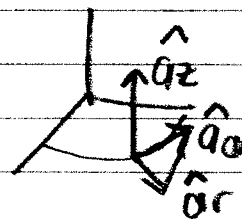
$$B_0\hat{a}_z = B_0\cos\theta\,\hat{a}_r - B_0\sin\theta\,\hat{a}_\theta$$

$r\theta\phi$

$$\omega R\sin\theta\,\hat{a}_\phi \times \left[B_0\cos\theta\,\hat{a}_r - B_0\sin\theta\,\hat{a}_\theta\right]$$
$$= \omega R B_0 \sin\theta\left[\cos\theta\,\hat{a}_\theta + \sin\theta\,\hat{a}_r\right]$$



Note that @ $\theta = \pi/2$ this cross product is entirely in the $\hat{a}_r$ direction



$$V_{emf} = 0 + \int_{\pi/2}^{0} \omega R B_0 \sin\theta\cos\theta\, R\, d\theta$$

$$= \int_{\pi/2}^{0} \omega R^2 B_0 \frac{\sin 2\theta}{2}\, d\theta = -\frac{\omega R^2 B_0}{2}\,\frac{\cos 2\theta}{2}\Bigg|_{\pi/2}^{0}$$

$$= -\frac{\omega R^2 B_0}{4}\left(1 - \cos\pi\right) = -\frac{\omega R^2 B_0}{2}$$

① $$V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{s} = -\frac{d}{dt}\int_0^{\omega t}\int_0^{\pi/2} \vec{B} \cdot R^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_\phi$$

$$= -\frac{d}{dt}\int_0^{\omega t}\int_0^{\pi/2} B_0 \cos\theta \, R^2 \sin\theta \, d\theta \, d\phi$$

$$= -\frac{d}{dt}\int_0^{\omega t}\int_0^{\pi/2} B_0 R^2 \frac{\sin 2\theta}{2} \, d\theta \, d\phi$$

$$= -\frac{B_0 R^2}{2}\frac{d}{dt}\left[-\omega t\left(\frac{\cos 2\theta}{2}\Big|_0^{\pi/2}\right)\right]$$

$$= \frac{B_0 R^2}{4}\frac{d}{dt}\left[\omega t\left(\cos\pi - \cos 0\right)\right]$$

$$= -\frac{B_0 R^2}{2}\omega$$

Name (Printed): SOLUTION

Signature: ______________________

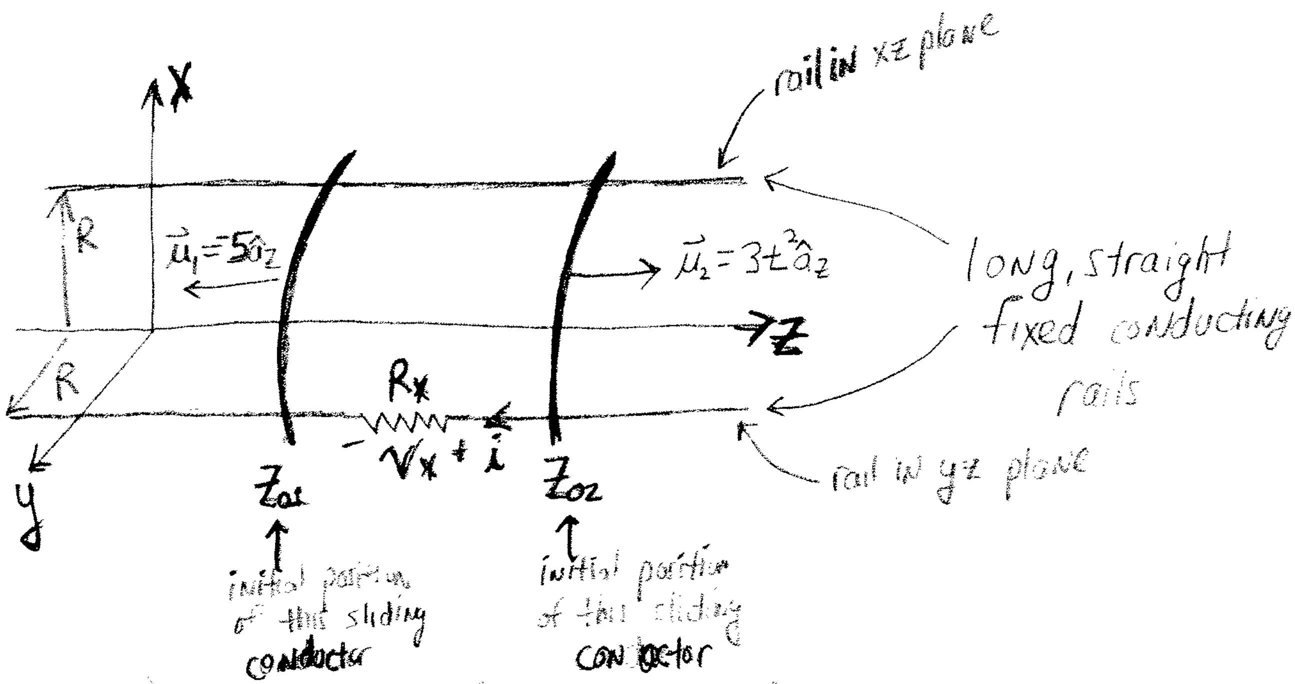
Exam #5
Electromagnetic Fields and Applications, EE240
K. Kaiser, Winter 2005

This is a closed notes and closed book test. Also, no calculators or other electronic devices are allowed.

Using both (equivalent) definitions for the induced voltage

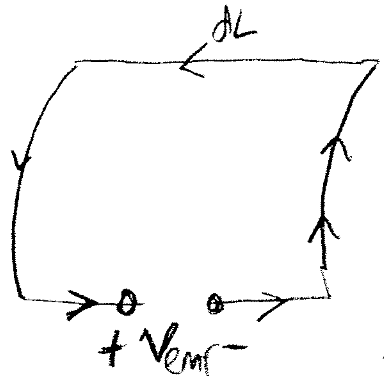
$$V_{emf} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} \quad \text{and} \quad V_{emf} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{s} + \oint_L \left(\vec{u}\times\vec{B}\right)\cdot d\vec{L}$$

determine the current, i, and the voltage, v_x, for the given configuration. Verify that the results are the same. Show all steps (even the obvious ones) and clearly define all variables given in the previous definitions. Assume the resistor R_x is sufficiently large to limit the induced magnetic field to a level well below the applied magnetic field.



$$\vec{B} = (\rho+\phi^2)\cos(\omega t)\hat{a}_\rho + 3\rho^2\cos(\omega t)\hat{a}_\phi - 6z^2\cos(\omega t)\hat{a}_z$$

The two sliding conductors, which are in constant contact with the rails, are arc-shaped with a radius of curvature of R (centered about the z axis).



$$\vec{dS} = \rho\, d\phi\, dz\, \hat{a}_\rho$$ +1.5 sign part(s on)

$$v = \frac{dz}{dt}$$

$$\int_{z_{02}}^{z} dz = \int_0^t v\,dt = \int_0^t 3t^2\,dt$$

$$z = t^3 + z_{02}$$

+1 +2

(1) $$V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{dS} = -\frac{d}{dt}\int_0^{\pi/2}\int_{z_{01}-5t}^{z_{02}+t^3} (\rho+\phi^2)\cos(\omega t)\,\rho\, dz\, d\phi \Big/_{\rho=R}$$

$$= -\frac{d}{dt}\int_0^{\pi/2}\int_{z_{01}-5t}^{z_{02}+t^3} (\rho^2+\underbrace{\rho\phi^2}_{\text{dot product } +1})\cos(\omega t)\,dz\,d\phi \Big/_{\rho=R}$$ +1

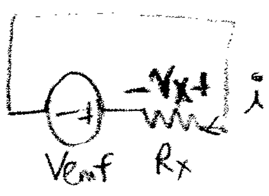
$$= -\frac{d}{dt}\left[\cos(\omega t)\left[z_{02}+t^3-z_{01}+5t\right]\left(\rho^2\phi+\frac{\rho\phi^3}{3}\right)\Big|_0^{\pi/2}\Big/_{\rho=R}\right]$$

$$= -\frac{d}{dt}\left[\cos(\omega t)\left[z_{02}-z_{01}+t^3+5t\right]\left(R^2\frac{\pi}{2}+\frac{R\pi^3}{24}\right)\right]$$

chain rule +1

$$= -\left(\frac{R^2\pi}{2}+\frac{R\pi^3}{24}\right)\left(-\omega\sin(\omega t)\left(z_{02}-z_{01}+t^3+5t\right) + \cos(\omega t)\left[3t^2+5\right]\right) \quad V$$

$$\left(\frac{R^2\pi}{2}+\frac{R\pi^3}{24}\right)\omega\sin(\omega t)\left(z_{02}-z_{01}+t^3+5t\right) - \left(\frac{R^2\pi}{2}+\frac{R\pi^3}{24}\right)\left(3t^2+5\right)\cos(\omega t)$$



$$V_x = -V_{emf}$$ +1

$$i = -\frac{V_{emf}}{R_x}$$ +1

1.0

(2) $$V_{emf} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{s} + \oint (\vec{v}\times\vec{B})\cdot d\vec{L}$$

$$= -\int_0^{\pi/2}\int_{z_{01}-5t}^{z_{02}+t^3} -(\rho+\phi^2)\,\omega\sin(\omega t)\,\rho\, d\phi\, dz\Big|_{\rho=R}$$

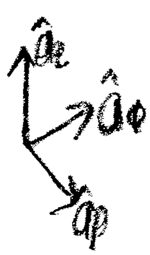
$$+ \int_{\pi/2}^{0} (3t^2\hat{a}_z \times \vec{B})\cdot \rho\, d\phi\, \hat{a}_\phi\Big|_{\substack{\rho=R\\ z=z_0+\omega}}$$

+1
+.5 remove in 3rd case

$$+ \int_0^{\pi/2} (-5\hat{a}_z \times \vec{B})\cdot \rho\, d\phi\, \hat{a}_\phi\Big|_{\substack{\rho=R\\ z=+}}$$

$$= \omega\sin(\omega t)\,(z_{02}+t^3-z_{01}+5t)\int_0^{\pi/2}(R^2+R\phi^2)\,d\phi$$

+1 cross product / dot product



$$3t^2\int_{\pi/2}^{0}(\rho+\phi^2)\cos(\omega t)\,\rho\, d\phi\Big|_{\rho=R} - 5\int_0^{\pi/2}(\rho+\phi^2)\cos(\omega t)\,\rho\, d\phi\Big|_{\rho=R}$$

$$= \omega\sin(\omega t)\,(z_{02}-z_{01}+3t^3+5t)\left(R^2\frac{\pi}{2}+R\frac{\pi^3}{24}\right)$$

$$+ 3t^2\cos(\omega t)\int_{\pi/2}^{0}(R^2+R\phi^2)\,d\phi - 5\cos(\omega t)\int_0^{\pi/2}(R^2+R\phi^2)\,d\phi$$

+1 work ok

$$= \omega\sin(\omega t)\,(z_{02}-z_{01}+t^3+5t)\left(R^2\frac{\pi}{2}+R\frac{\pi^3}{24}\right)$$

$$+ 3t^2\cos(\omega t)\left[R^2\phi+\frac{R\phi^3}{3}\right]_{\pi/2}^{0} - 5\cos(\omega t)\left[R^2\phi+\frac{R\phi^3}{3}\right]_0^{\pi/2}$$

$$= \omega\sin(\omega t)\,(z_{02}-z_{01}+t^3+5t)\left(R^2\frac{\pi}{2}+R\frac{\pi^3}{24}\right)$$

agrees with first method

$$+3t^2\cos(\omega t)\left[-R^2\frac{\pi}{2}-\frac{R\pi^3}{24}\right] - 5\cos(\omega t)\left[R^2\frac{\pi}{2}+\frac{R\pi^3}{24}\right]\ V$$

Name (Printed): SOLUTION

Signature:________________________________

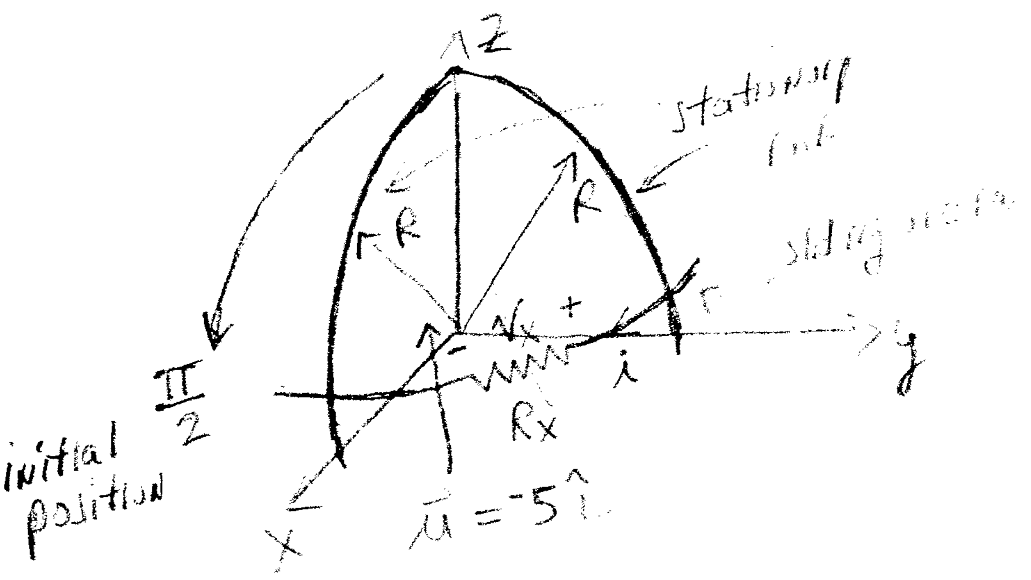
"Bonus" Question

The following question will be given on 3/21/05 at 4:40 PM. Only 30 minutes will be given to answer this question. Notes, books, or electronic computing devices may not be used. There will be little partial credit when grading this question. The approximate value of correctly and completely working this question is 5% added to your final grade.

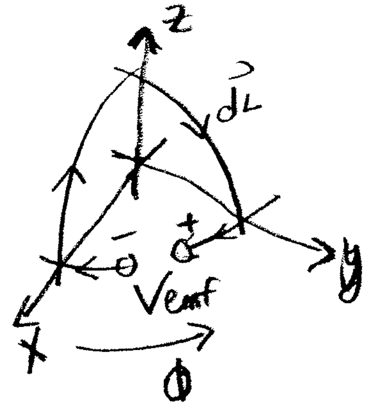
Using both (equivalent) definitions for the induced voltage

$$V_{emf} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} \quad \text{and} \quad V_{emf} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{s} + \oint_L \left(\vec{u}\times\vec{B}\right)\cdot d\vec{L}$$

determine the current, i, and the voltage, v_x, for the given configuration. Verify that the results are the same. Show all steps (even the obvious ones) and clearly define all variables given in the previous definitions. Assume the resistor R_x is sufficiently large to limit the induced magnetic field to a level well below the applied magnetic field.



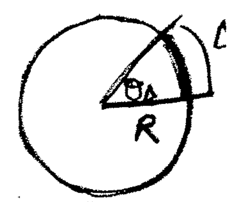
$$\vec{B} = (r+\phi)\sin(\omega t)\,\hat{a}_r - (r\theta+\phi)\cos(\omega t)\,\hat{a}_\theta + (r+2\phi)\sin(\omega t)\,\hat{a}_\phi$$



① $V_{emf} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{S}$

$d\vec{S} = r\,d\theta\, r\sin\theta\, d\phi\,(-\hat{a}_r)$

$$= +\frac{d}{dt}\int_0^{\pi/2}\int_0^{\frac{\pi}{2}-\frac{5t}{R}} (r^2+\phi)\sin(\omega t)\, r^2\sin\theta\, d\theta\, d\phi \Big|_{r=R}$$



$\Delta_R = \theta_\Delta R = 5t$

$\Rightarrow \theta_\Delta = 5t/R$

Note that

$\vec{v} = -5\hat{a}_\theta = -5(\cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z)$

$v = |\vec{v}| = 5\sqrt{\cos^2\theta\cos^2\phi + \cos^2\theta\sin^2\phi + \sin^2\theta}$

$= 5\sqrt{\cos^2\theta(\cos^2\phi+\sin^2\phi)+\sin^2\theta} = 5\sqrt{\cos^2\theta+\sin^2\theta} = 5$

(of course since $|\hat{a}_\theta| = 1$ by definition)

$$V_{emf} = \frac{d}{dt}\int_0^{\pi/2}\int_0^{\frac{\pi}{2}-\frac{5t}{R}} (R^4+R^2\phi)\sin(\omega t)\sin\theta\, d\theta\, d\phi$$

$$= -\frac{d}{dt}\left[\sin(\omega t)\int_0^{\pi/2}(R^4+R^2\phi)\cos\theta\Big|_0^{\frac{\pi}{2}-\frac{5t}{R}} d\phi\right]$$

$$= -\frac{d}{dt}\left[\sin(\omega t)\int_0^{\pi/2}(R^4+R^2\phi)\left\{\cos\left(\frac{\pi}{2}-\frac{5t}{R}\right)-1\right\}d\phi\right]$$

$$= -\frac{d}{dt}\left[\sin(\omega t)\left\{\cos\left(\frac{\pi}{2}-\frac{5t}{R}\right)-1\right\}\left(R^4\phi+\frac{R^2\phi^2}{2}\right)\Big|_0^{\pi/2}\right]$$

$$= -\frac{d}{dt}\left[\sin(\omega t)\left\{\cos\left(\frac{\pi}{2} - \frac{5t}{R}\right) - 1\right\}\left(\frac{R^4\pi}{2} + \frac{R^2\pi^2}{8}\right)\right]$$

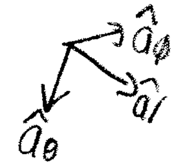
$$= -\left[\omega\cos(\omega t)\left\{\cos\left(\frac{\pi}{2} - \frac{5t}{R}\right) - 1\right\}\left(\frac{R^4\pi}{2} + \frac{R^2\pi^2}{8}\right) + \sin(\omega t)\left\{\frac{5}{R}\sin\left(\frac{\pi}{2} - \frac{5t}{R}\right)\right\}\left(\frac{R^4\pi}{2} + \frac{R^2\pi^2}{8}\right)\right]$$

$$= -\left(\frac{R^4\pi}{2} + \frac{R^2\pi^2}{8}\right)\left[\omega\cos(\omega t)\left\{\cos\left(\frac{\pi}{2} - \frac{5t}{R}\right) - 1\right\} + \sin(\omega t)\left\{\frac{5}{R}\sin\left(\frac{\pi}{2} - \frac{5t}{R}\right)\right\}\right]$$

(2) $$V_{emf} = -\iint \frac{d\vec{B}}{dt} \cdot d\vec{s} + \oint (\vec{u} \times \vec{B}) \cdot d\vec{L}$$

$$d\vec{L} = r \sin\theta \, d\phi \, \hat{a}_\phi$$

$$= + \int_0^{\pi/2} \int_0^{\pi/2 - \frac{5t}{R}} (r^2 + \phi)\, \omega \cos(\omega t)\, r\, d\theta\, r \sin\theta\, d\phi$$



$$+ \int_{\pi/2}^{0} \Big[-5(r^2 + \phi) \sin(\omega t)(-\hat{a}_\phi)$$
$$- 5(r\theta + \phi^2) \cos(\omega t)(0)$$
$$- 5(r + 2\phi) \sin(\omega t)(\hat{a}_r) \Big] \cdot r \sin\theta \, d\phi \, \hat{a}_\phi \Big|_{r=R}$$

and $\theta = f(t)$

$$= \int_0^{\pi/2} \int_0^{\pi/2 - \frac{5t}{R}} (R^4 + R^2\phi)\, \omega \cos(\omega t) \sin\theta \, d\theta \, d\phi$$

$$+ \int_{\pi/2}^{0} 5(R^3 + R\phi) \sin(\omega t) \sin\theta \, d\phi$$

$$= -\omega \cos(\omega t) \int_0^{\pi/2} (R^4 + R^2\phi) \cos\theta \Big|_0^{\pi/2 - \frac{5t}{R}} d\phi$$

$$+ \sin(\omega t) \sin\theta \; 5(R^3\phi + R\phi^2/2) \Big|_{\pi/2}^{0}$$

$$= -\omega \cos(\omega t) \int_0^{\pi/2} (R^4 + R^2\phi) \left\{ \cos\left(\frac{\pi}{2} - \frac{5t}{R}\right) - 1 \right\} d\phi$$

$$- \sin(\omega t) \sin\theta \; 5\left(R^3 \frac{\pi}{2} + \frac{R\pi^2}{8}\right)$$

$$= -\omega\cos(\omega t)\left\{\cos\left(\frac{\pi}{2}-\frac{5t}{R}\right)-1\right\}\left(R^4\phi+\frac{R^2\phi^2}{2}\Big|_0^{\pi/2}\right)$$

$$-\sin(\omega t)\sin\theta\, 5\left(R^3\frac{\pi}{2}+\frac{R\pi^2}{8}\right)$$

$$= -\omega\cos(\omega t)\left\{\cos\left(\frac{\pi}{2}-\frac{5t}{R}\right)-1\right\}\left(R^4\frac{\pi}{2}+\frac{R^2\pi^2}{8}\right)$$

$$-\sin(\omega t)\sin\theta\, 5\left(R^3\frac{\pi}{2}+\frac{R\pi^2}{8}\right)$$

where $\theta = \frac{\pi}{2}-\frac{5t}{R}$

$$= -\left(\frac{R^4\pi}{2}+\frac{R^2\pi^2}{8}\right)\left[\omega\cos(\omega t)\left\{\cos\left(\frac{\pi}{2}-\frac{5t}{R}\right)-1\right\}\right.$$

$$\left.+\sin(\omega t)\sin\left(\frac{\pi}{2}-\frac{5t}{R}\right)\cdot\frac{5}{R}\right]$$

where $5\left(R^3\frac{\pi}{2}+\frac{R\pi^2}{8}\right)$
$= 5\left(R^4\frac{\pi}{2}+\frac{R^2\pi^2}{8}\right)\frac{1}{R}$

$$V_{emf} = V_X$$

$$i = \frac{V_X}{R_X}$$