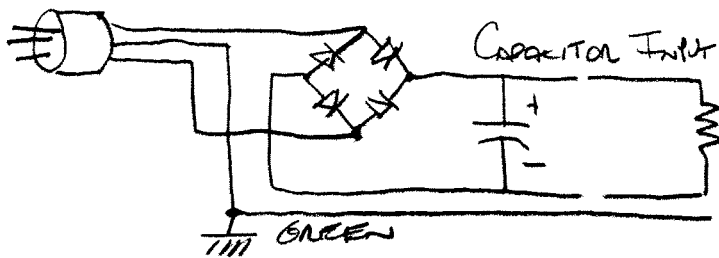


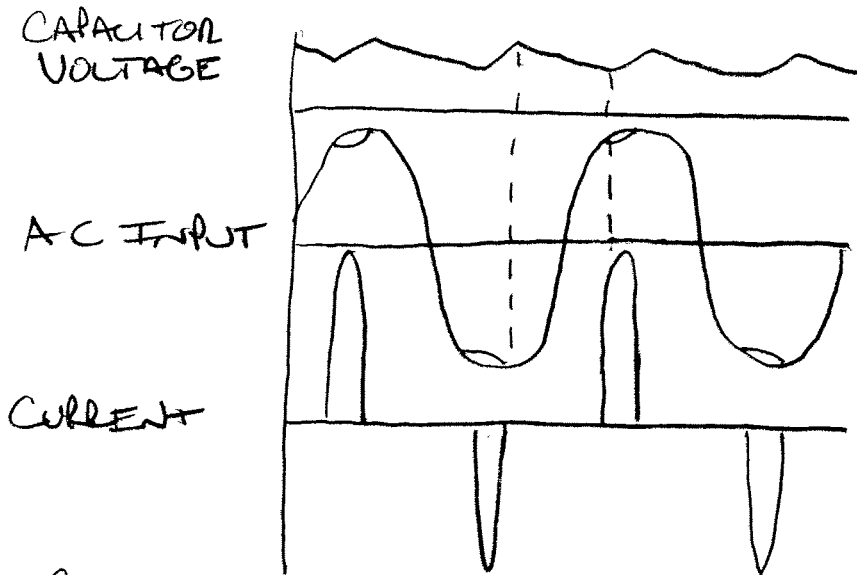


WITH A SWITCHING POWER SUPPLY THERE ARE PERIODS OF CURRENT SURGES, WHY? THE POWER FACTOR IS DEFINED TO BE THE RATIO OF THE REAL POWER TO THE APPARENT POWER. ALTHOUGH THE CURRENT AND VOLTAGE ARE IN-PHASE WITH THE SWITCHING POWER SUPPLY, THE POWER FACTOR IS NOT ONE. WHY? WHAT IS ANOTHER MAJOR CONSEQUENCE OF THE CURRENT SURGES [CORP]?

- A SUDDEN INCREASE IN CURRENT CAUSES A RESULTING CHANGE IN THE MAGNETIC FIELD SURROUNDING THE CONDUCTOR, WHICH CAN INDUCE VOLTAGE CHANGES IN NEARBY CONDUCTORS.
- CONDUCTED VOLTAGE SPIKES $V = L di/dt$, A FAST RISE TIME (di/dt) IT CAN BE RADIATED, CAUSING AN RF SPIKE.
- CURRENT SURGES ARE PRESENT TO KEEP THE CAPACITOR CHARGED TO A MINIMUM VALUE.



LOAD
POWER
SUPPLY SWITCHER

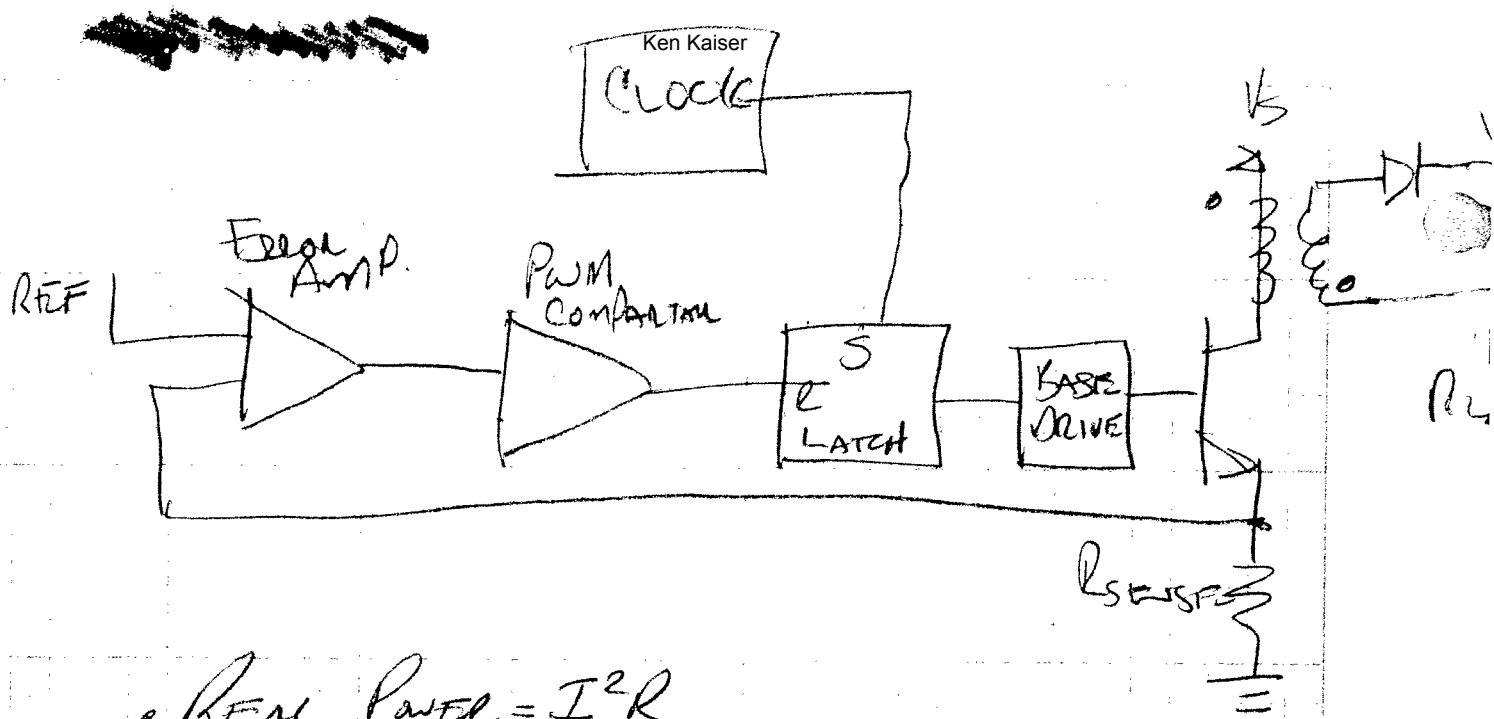


• THIS RESULTS IN CURRENT PEAKS THAT ARE SEVERAL TIMES HIGHER THAN A NORMAL CONSTANT IMPEDANCE LOAD RESISTIVE LOAD WHOULD DRAW, BUT THEY ARE SHORT DURATION.

• Power Factor = $\frac{P_r}{P_a} = \frac{\text{REAL POWER}}{\text{APPARENT POWER}}$ HENCE, THE APPARENT POWER IS MUCH HIGHER THAN THE REAL INPUT CURRENTS, EVEN THOUGH THEY ARE IN PHASE. SINCE POWER FACTOR IS DEFINED AS P_r/P_a , THIS EFFECT RESULTS IN LOW POWER FACTOR.

PF = 1
ALL RESISTIVE

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
ANSI



• REAL POWER = $I^2 R$
 $= E I_{RMS} \cos \theta$ WATT (RESERVED FOR REAL POWER)

• APPARENT POWER = $E I$ (VA) E & I are OUT OF PHASE

$$\frac{P_R}{P_A} = \cos \theta$$



- ANY DISTORTION OF THE AC VOLTAGE OR CURRENT SINE WAVE RESULTS IN A LESS THAN UNITY POWER FACTOR. (A POWER FACTOR OF 1 IS PERFECT)
- " TRADITIONALLY A POOR POWER FACTOR WAS THE RESULT OF A PHASE SHIFT BETWEEN VOLTAGE AND CURRENT DUE TO INDUCTIVE OR CAPACITIVE LOADS. HOWEVER, IN THE CASE OF SWITCHING POWER SUPPLIES, THE POOR POWER FACTOR IS USUALLY THE RESULT OF A LOAD CURRENT / WAVE SHAPE THAT IS NOT SINUSOIDAL WAVE YET IS STILL IN PHASE WITH THE INPUT. THE PROBLEM IS THAT IN THE CASE OF INPUT AC POWER TO A SWITCHING POWER SUPPLY, THE APPARENT CIRCUIT LOAD IMPEDANCE APPEARS TO CHANGE DURING AN AC CYCLE, RESULTING IN VERY COMPLEX PROBLEMS.

IF I_p & V_p ARE CONSTANT, THEN THE POWER UNDER THE CURVE IS, (PEAK POWER)

$$P_p = E_p I_p$$

- AVE POWER, $P_{AV} = \frac{1}{t} \int_0^t i v dt$

AVE POWER \neq APPARENT POWER

- REAL POWER = $P_p = I_{r,ic} E_p$

WHERE $I_{r,ic}$ = REAL INPUT CURRENT (APPARENT)
 E_p = PEAK INSTAN. VOLTAGE AT THAT MOMENT IN TIME.

- APPARENT POWER = $I_{L,APPARENT} V_{INSTAN.}$
 NO DUTY CYCLE (ACCOUNT)

- SWITCHING CHANGES THE LOAD CONDITIONS AND MUST ALWAYS PRODUCE TRANSIENTS.

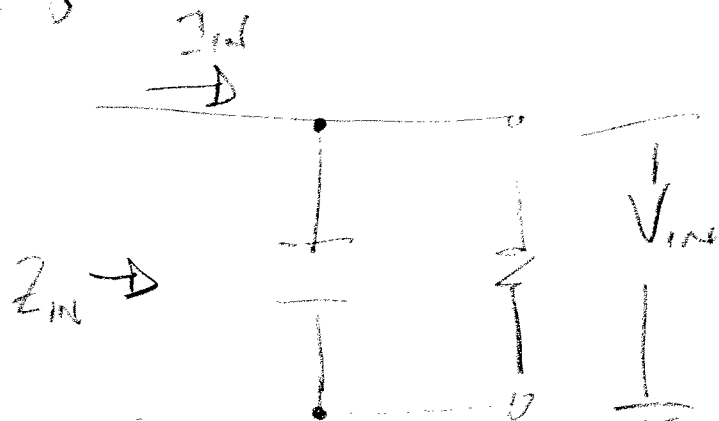
22-141 50 SHEETS
 22-142 100 SHEETS
 22-144 200 SHEETS



33-14)

Power line sags can be caused intentionally when the power company must shut down certain generators but consumption rate stays the same causing a general shortage of power.

This lowered voltage in devices that operate on a fixed amount of power requires that more current must flow into the device. In the case of computers which have active voltage regulation circuitry the additional current may cause more radiated magnetic noise from this circuitry.



$$Z_{in} = \frac{V_{in}}{I_{in}}$$

NOTE: UNLESS CERTAIN THAT NETWORK IS WHOLLY RESISTIVE, SHORT/OPEN CIRCUIT TESTS DO NOT YIELD MAX/MIN VALUES OF $\text{Re}\{Z_{in}\}$

SHORT: $Z_{in} = 0$

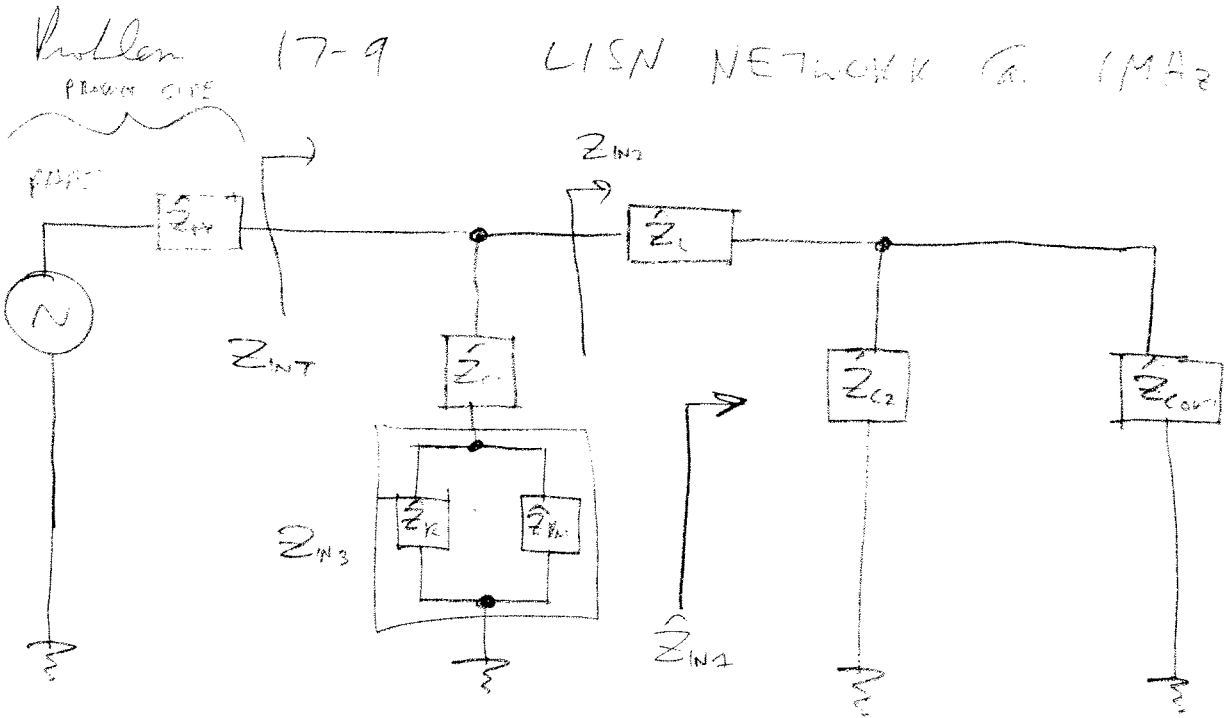
OPEN: I IS WHOLLY IMAGINARY $\text{Re}\{Z_{in}\} = 0$
THEREFORE Z_{in} IS TOTALLY REACTIVE.

WITH REAL LOAD, Z HAS REAL & IMAGINARY COMPONENT.

Z_{in} HAS REAL COMPONENT.

Z_{in} REAL WILL BE GREATEST WITH

SOME FINITE, REAL LOAD.



Problem 17-9: LISN Network @ 1 MHz

$$f := 1 \cdot 10^6 \quad \omega := 2 \cdot \pi \cdot f \quad L := 50 \cdot 10^{-6} \quad C_2 := 1 \cdot 10^{-6} \quad C_1 := 1 \cdot 10^{-7}$$

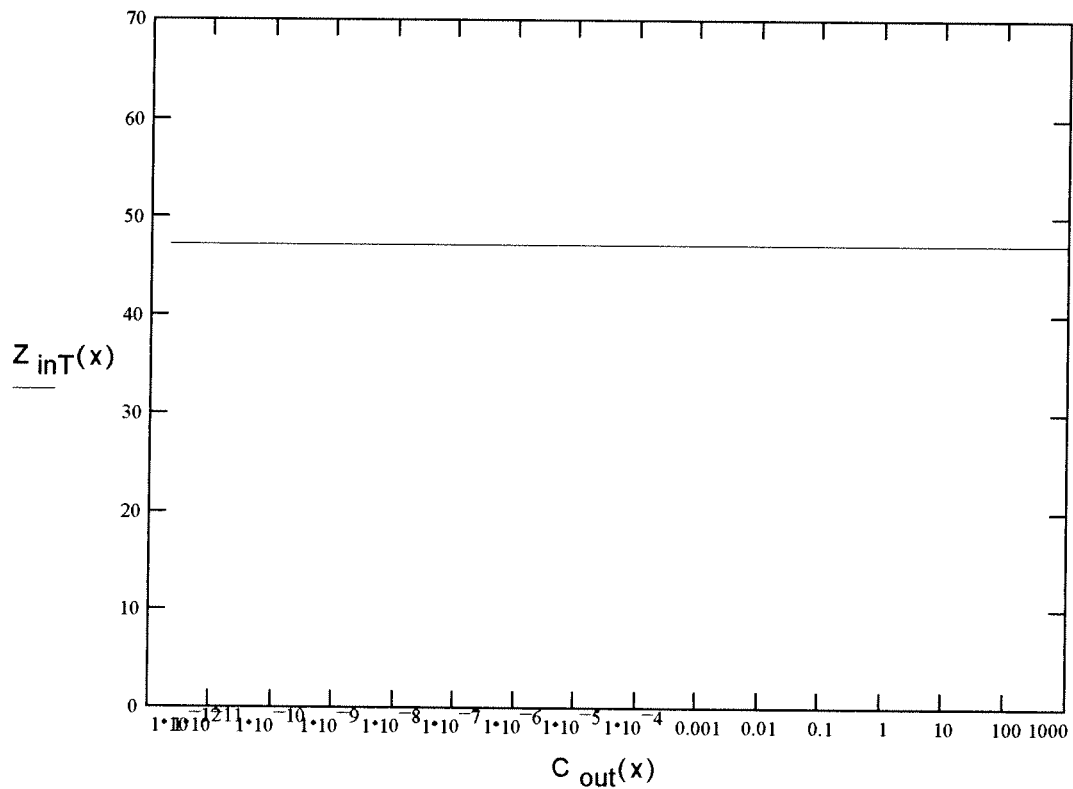
$$R_m := 50 \quad R := 1000$$

$$x := 1, 2 \dots 150 \quad C_{out}(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right) - 12} j := \sqrt{-1}$$

$$Z_{in1}(x) := \frac{1}{j \cdot \omega \cdot (C_{out}(x) + C_2)} \quad Z_{in2}(x) := j \cdot \omega \cdot L + Z_{in1}(x)$$

$$Z_{c1} := \frac{1}{j \cdot \omega \cdot C_1} \quad Z_{in3} := \frac{R \cdot R_m}{R + R_m}$$

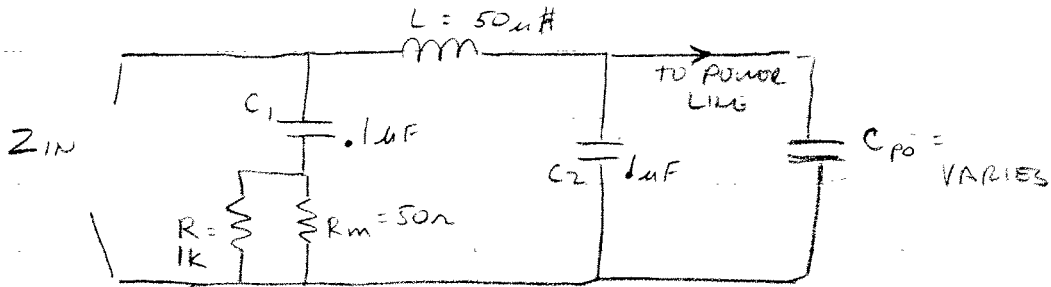
$$Z_{inT}(x) := \frac{(Z_{in3} + Z_{c1}) \cdot Z_{in2}(x)}{Z_{in3} + Z_{c1} + Z_{in2}(x)} \quad |Z_{inT}(70)| = 47.342$$



17-9 MAXIMUM INPUT IMPEDANCE OF LISN

- DETERMINE INPUT IMPEDANCE OF LISN @ 1MHz FROM THE PROBE SIDE BETWEEN HOT & GND IF THE IMPEDANCE OF THE POWER OUTLET IS CAPACITIVE.

LISN (HOT & GROUND ONLY) CIRCUIT



$$Z_{in} = [(Z_{C2} \parallel Z_{Cpo}) + Z_L] \parallel [C1 + (R \parallel R_m)]$$

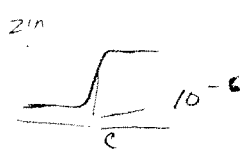
SIMPLIFIED IN MATHECAD,

EQU & PLOT ATTACHED

=

- Z_{in} VARIES FROM:

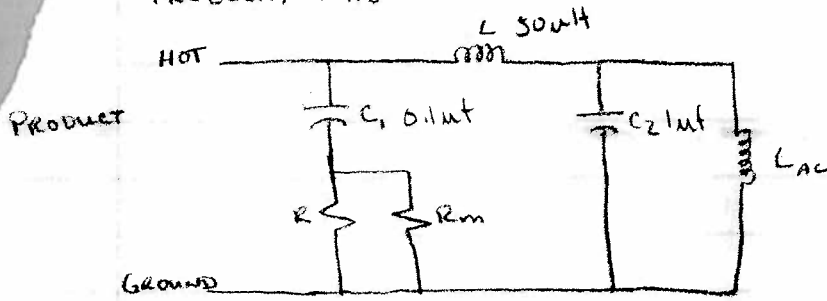
C _{po} (FARAD)	Z _{in} (OHM)
10 ⁻¹² < C _{po} < 10 ⁻⁹	≈ 47.34156
10 ⁻⁸ < C _{po} < 10 ⁻⁴	≈ 47.34156 - 47.34199
10 ⁻⁴ < C _{po}	≈ 47.34199



REASON, L ≈ 315 nH AT 1MHz

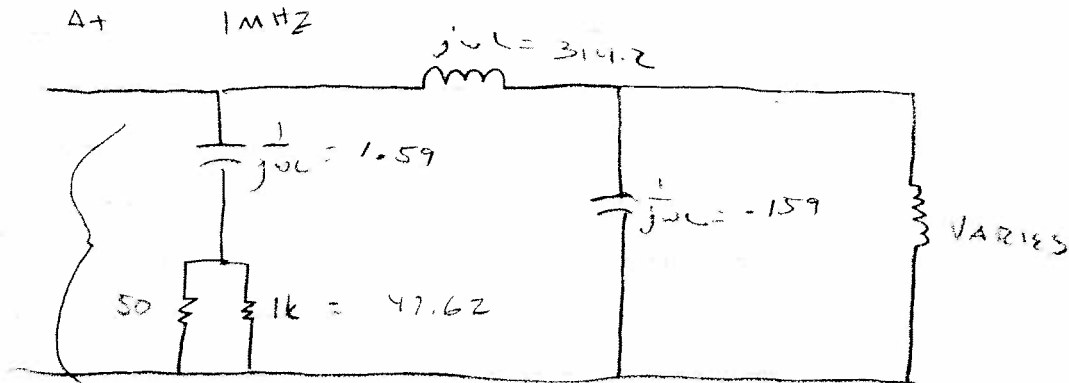
1 MHz RESONANCE? $(10^6)^2 (H) = \frac{1}{LC}$ $C = \frac{1}{(L)(10^6 2\pi)^2}$ $C = 5 \times 10^{-12}$

PROBLEM 17.10



$$\frac{R \cdot R_m}{R + R_m} + \frac{1}{j\omega C_1} \parallel j\omega L + \left(\frac{1}{j\omega C_2} \parallel j\omega L_{AC} \right)$$

$L = 50 \mu H$ $C_2 = 1 \mu F$ $C_1 = 0.1 \mu F$ $R_m = 50 \Omega$ $R = 1 k \Omega$ L_{AC} VARIES



THE FIRST GROUP OF MATHCAD PLOTS SHOW THE INPUT IMPEDANCE ^{at 1 MHz}, Looking in at the product side, as the Inductor Load is VARIED THROUGH A LARGE RANGE OF FREQUENCIES

THE FINAL 3 PAGES SHOW THE INPUT IMPEDANCE AS A FUNCTION OF FREQUENCY WITH A ~~50 μH~~ INDUCTOR LOAD OF 50 μH; NOTICE AT LOW FREQUENCIES HOW THE INPUT IMPEDANCE VARIES SIGNIFICANTLY AND IS UNPREDICTABLE.

$$j := \sqrt{-1} \quad L := 50 \cdot 10^{-6} \quad \omega := 2 \cdot \pi \cdot 1 \cdot 10^6 \quad \text{Ken Kaiser}$$

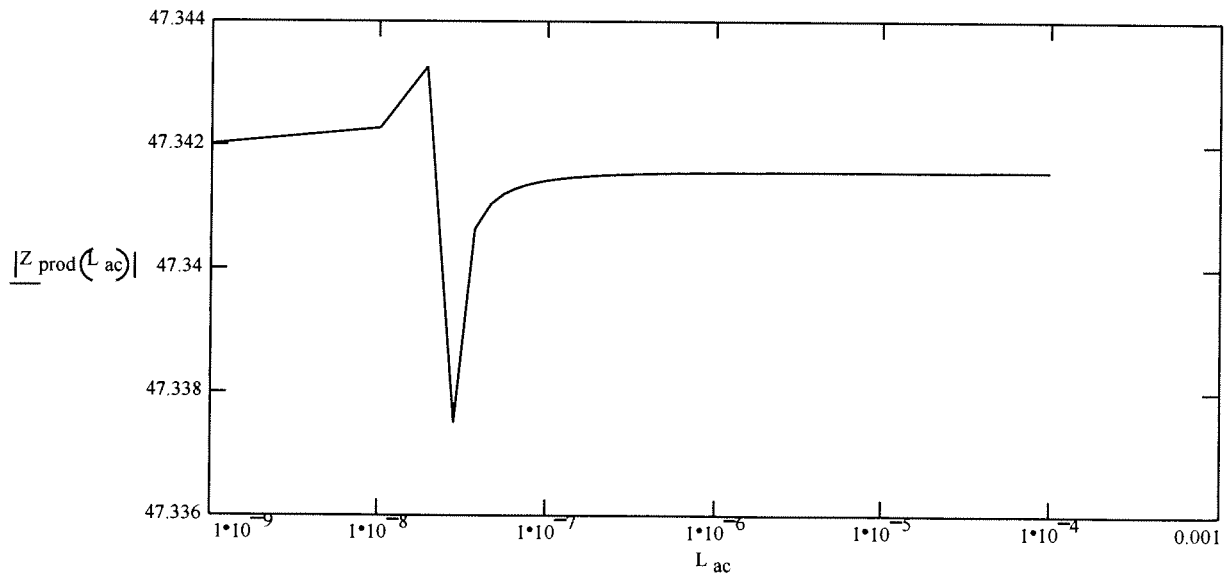
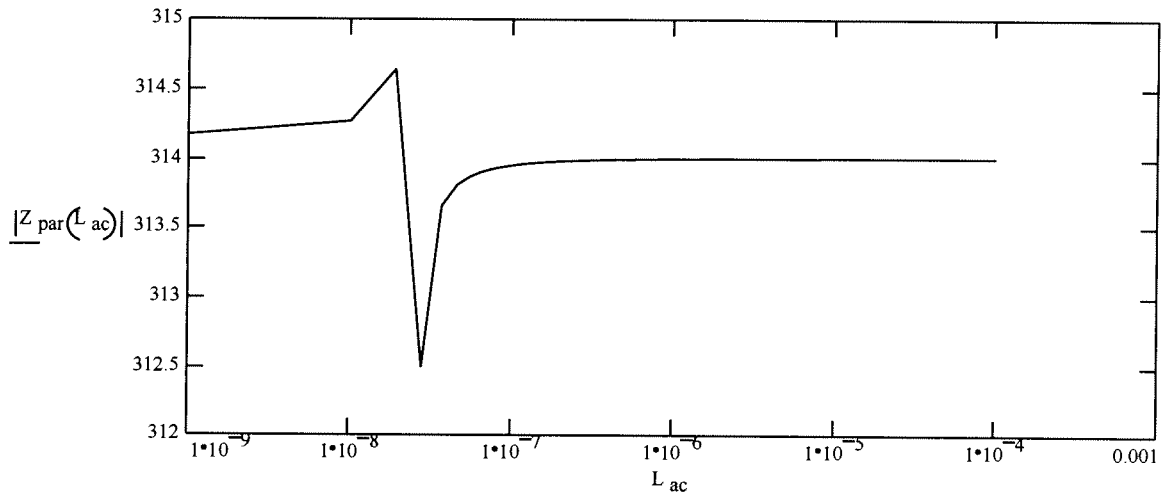
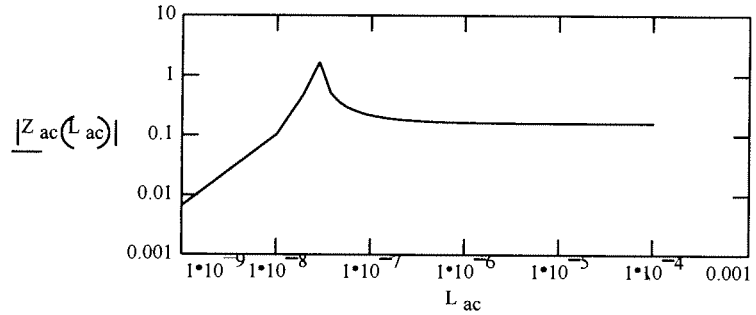
$$C_1 := 1 \cdot 10^{-6} \quad C_2 := 1 \cdot 10^{-6} \quad L_{ac} := 1 \cdot 10^{-9}, 10 \cdot 10^{-9} .. 100 \cdot 10^{-6}$$

$$R := 1000 \quad R_m := 50$$

$$Z_r := \left(\frac{R \cdot R_m}{R + R_m} + \frac{1}{j \cdot \omega \cdot C_1} \right) \quad Z_{ac}(L_{ac}) := \frac{j \cdot \omega \cdot L_{ac} \cdot \frac{1}{j \cdot \omega \cdot C_2}}{j \cdot \omega \cdot L_{ac} + \frac{1}{j \cdot \omega \cdot C_2}} \quad Z_{par}(L_{ac}) := j \cdot \omega \cdot L + Z_{ac}(L_{ac})$$

$$|Z_r| = 47.646$$

$$Z_{prod}(L_{ac}) := \frac{Z_r \cdot Z_{par}(L_{ac})}{Z_r + Z_{par}(L_{ac})}$$



$$j := \sqrt{-1} \quad L := 50 \cdot 10^{-6} \quad C_1 := .1 \cdot 10^{-6} \quad C_2 := 1 \cdot 10^{-6} \quad \text{Kaiser}$$

$$R = 1000 \quad R_m := 50$$

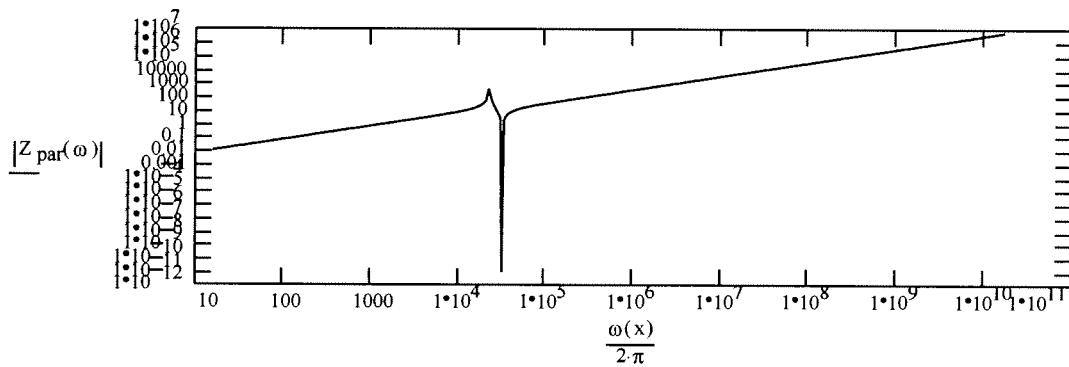
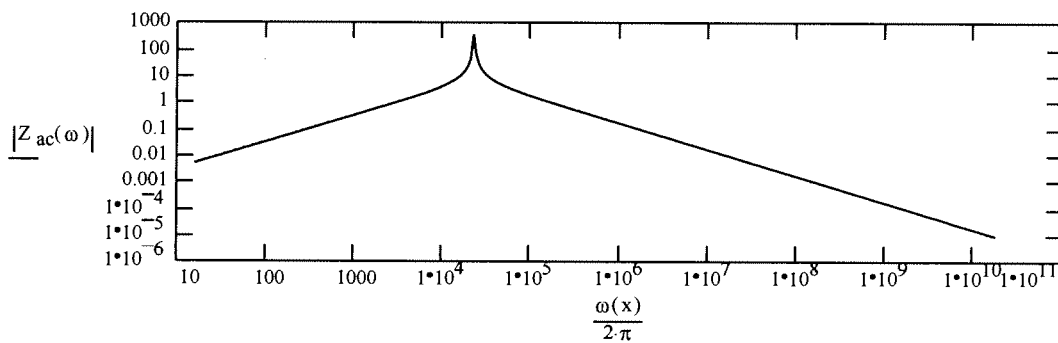
$$x := 20, 20.1..110$$

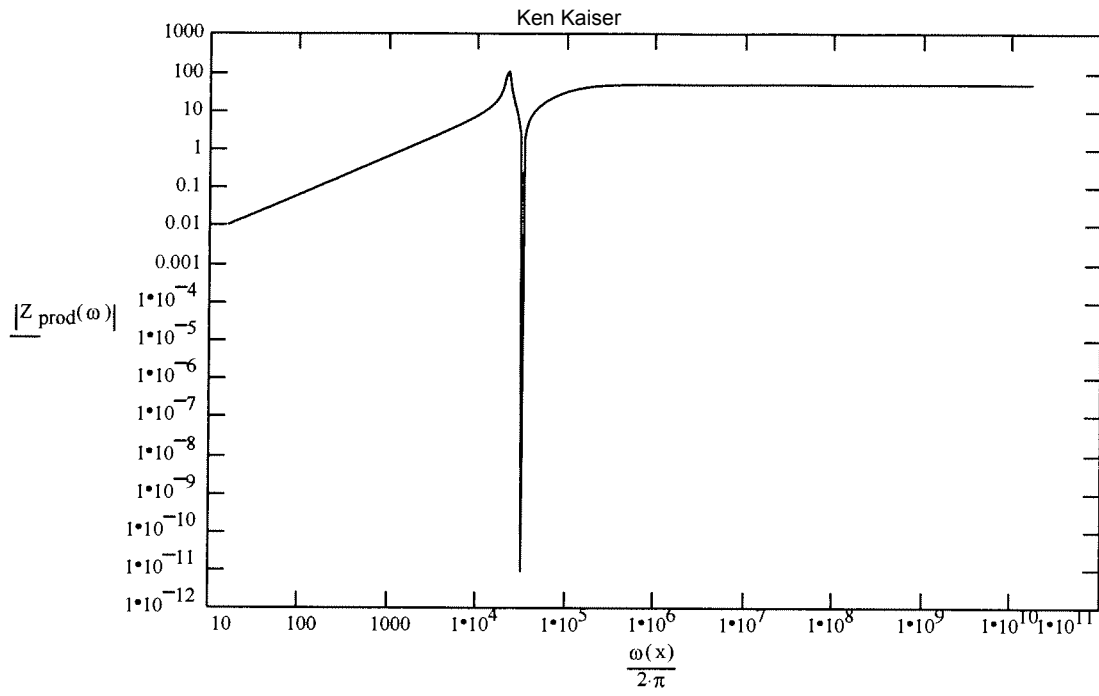
$$\omega(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$Z_r(\omega) := \left(\frac{R \cdot R_m}{R + R_m} + \frac{1}{j \cdot \omega(x) \cdot C_1} \right) \quad Z_{ac}(\omega) := \frac{j \cdot \omega(x) \cdot L \cdot \frac{1}{j \cdot \omega(x) \cdot C_2}}{j \cdot \omega(x) \cdot L + \frac{1}{j \cdot \omega(x) \cdot C_2}}$$

$$Z_{par}(\omega) := j \cdot \omega(x) \cdot L + Z_{ac}(\omega)$$

$$Z_{prod}(\omega) := \frac{Z_r(\omega) \cdot Z_{par}(\omega)}{Z_r(\omega) + Z_{par}(\omega)}$$





$$j := \sqrt{-1} \quad L := 50 \cdot 10^{-6} \quad C_1 := .1 \cdot 10^{-6} \quad C_2 := .1 \cdot 10^{-6}$$

$$R := 1000 \quad R_m := 50$$

$$x := 40, 40.1 \dots 60$$

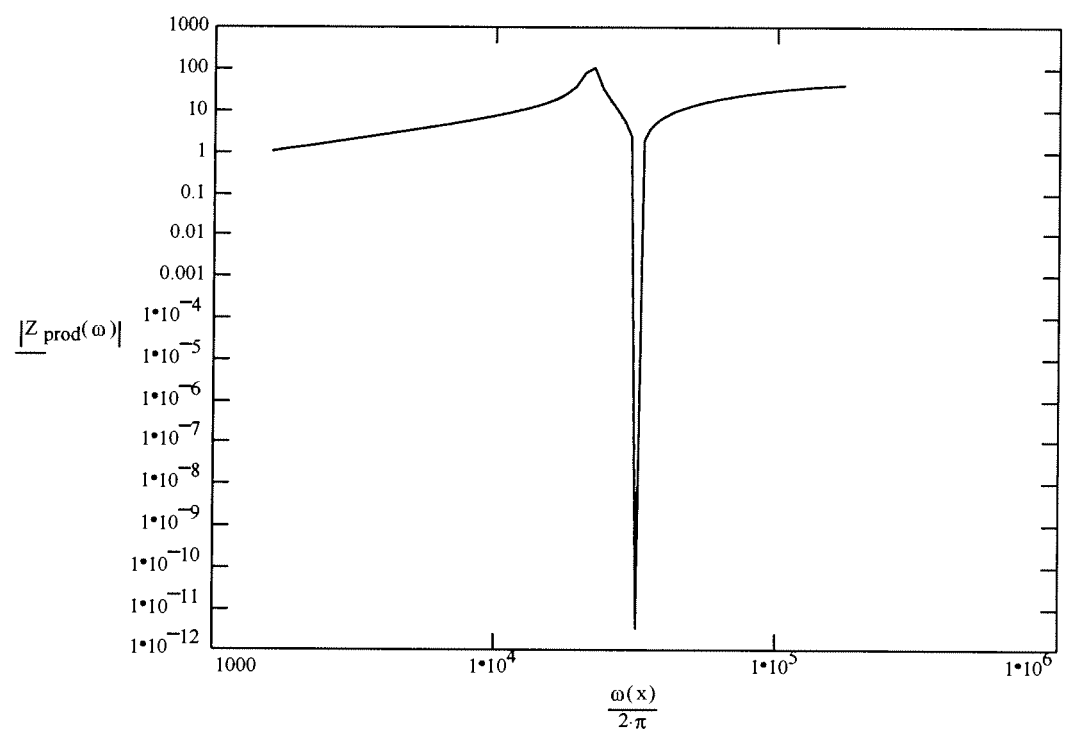
$$\omega(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$Z_r(\omega) := \left(\frac{R \cdot R_m}{R + R_m} + \frac{1}{j \cdot \omega(x) \cdot C_1} \right)$$

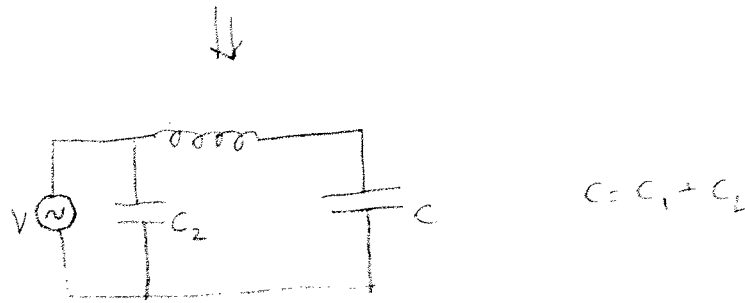
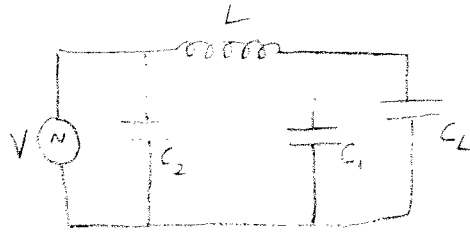
$$Z_{ac}(\omega) := \frac{j \cdot \omega(x) \cdot L \cdot \frac{1}{j \cdot \omega(x) \cdot C_2}}{j \cdot \omega(x) \cdot L + \frac{1}{j \cdot \omega(x) \cdot C_2}}$$

$$Z_{par}(\omega) := j \cdot \omega(x) \cdot L + Z_{ac}(\omega)$$

$$Z_{prod}(\omega) := \frac{Z_r(\omega) \cdot Z_{par}(\omega)}{Z_r(\omega) + Z_{par}(\omega)}$$



3.) The equivalent circuit between hot and ground is



Equivalent Impedance is $[j\omega L + \frac{1}{j\omega C}] \parallel (\frac{1}{j\omega C_2})$

$$= \left(\frac{-\omega^2 LC + 1}{j\omega C} \right) \parallel \left(\frac{1}{j\omega C_2} \right)$$

$$= \left(\frac{1 - \omega^2 LC}{j\omega C} \right) \left(\frac{1}{j\omega C_2} \right)$$

$$\frac{\left(\frac{1 - \omega^2 LC}{j\omega C} \right) \left(\frac{1}{j\omega C_2} \right)}{\left(\frac{1 - \omega^2 LC}{j\omega C} \right) + \frac{1}{j\omega C_2}}$$

$$= \frac{1 - \omega^2 LC / -\omega^2 / C C_2}{(1 - \omega^2 LC) j\omega C_2 + j\omega C / -\omega^2 / C C_2}$$

$$= \frac{1 - \omega^2 LC}{j\omega C_2 - j\omega^2 LC C_2 + j\omega C} = \frac{1 - \omega^2 LC}{j(\omega C_2 - \omega^2 LC C_2 + \omega C)}$$

$$= \frac{-j C_2 (1 - \omega^2 LC)}{\omega C_2 - \omega^2 LC C_2 + \omega C}$$

Equating imaginary parts to zero
Ken Kaiser

$$1 - \omega^2 LC = 0$$

$$1 = \omega^2 LC$$

$$\omega^2 = \frac{1}{LC}$$

$$\therefore \text{resonant frequency } f = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi\sqrt{L(C_1 + C_2)}}$$

$$\Rightarrow C_2 = \frac{1}{4\pi^2 f^2 L} - C_1$$

$$C_1 = 0.1 \times 10^{-6} \quad \& \quad L = 50 \times 10^{-6}$$

Since the whole range is negative, we can say that there are no ^{positive} real values of C_2 that can cause a resonance over the 450 kHz to 30 MHz operating range.

Taking the resistances into consideration we see that the input impedance is $\frac{1}{j\omega C_2} \parallel \left[j\omega L + \left[\frac{1}{j\omega C_1} + R_1 \right] \parallel \frac{1}{j\omega C_2} \right]$ where $R_1 = \frac{R R_n}{R + R_n}$

When this input impedance was plotted along with ω and the resistance in a three-dimensional plot, it was found that R didn't have a significant effect on the resonant frequency, but it did have a pronounced effect on the magnitude & phase.

Ken Kaiser

```
C1=0.1*10^(-6);  
C2=0.1*10^(-6);  
LL=1.1*10^(-6);  
L=50*10^(-6);  
Z=zeros(99,100);
```

```
for w=10^4+10000:10000:10^6
```

```
    for R=100:100:10^4
```

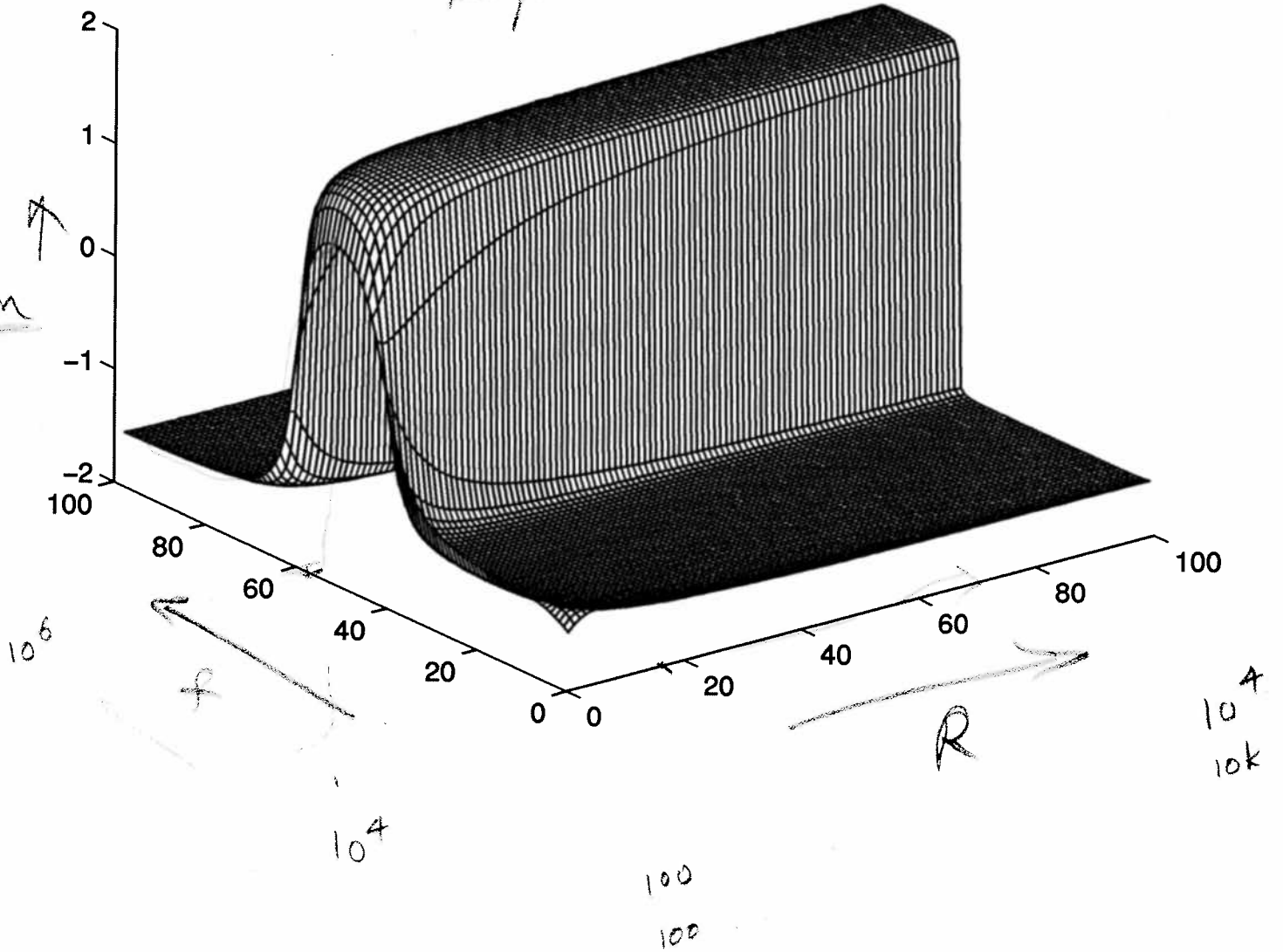
```
        Z( (w-10^4)/10000 , R/100 )=( ( (1/(i*w*C1))+R)*(i*w*LL) )/( (1/(i*w*C1))+
```

```
    end
```

```
w  
end
```

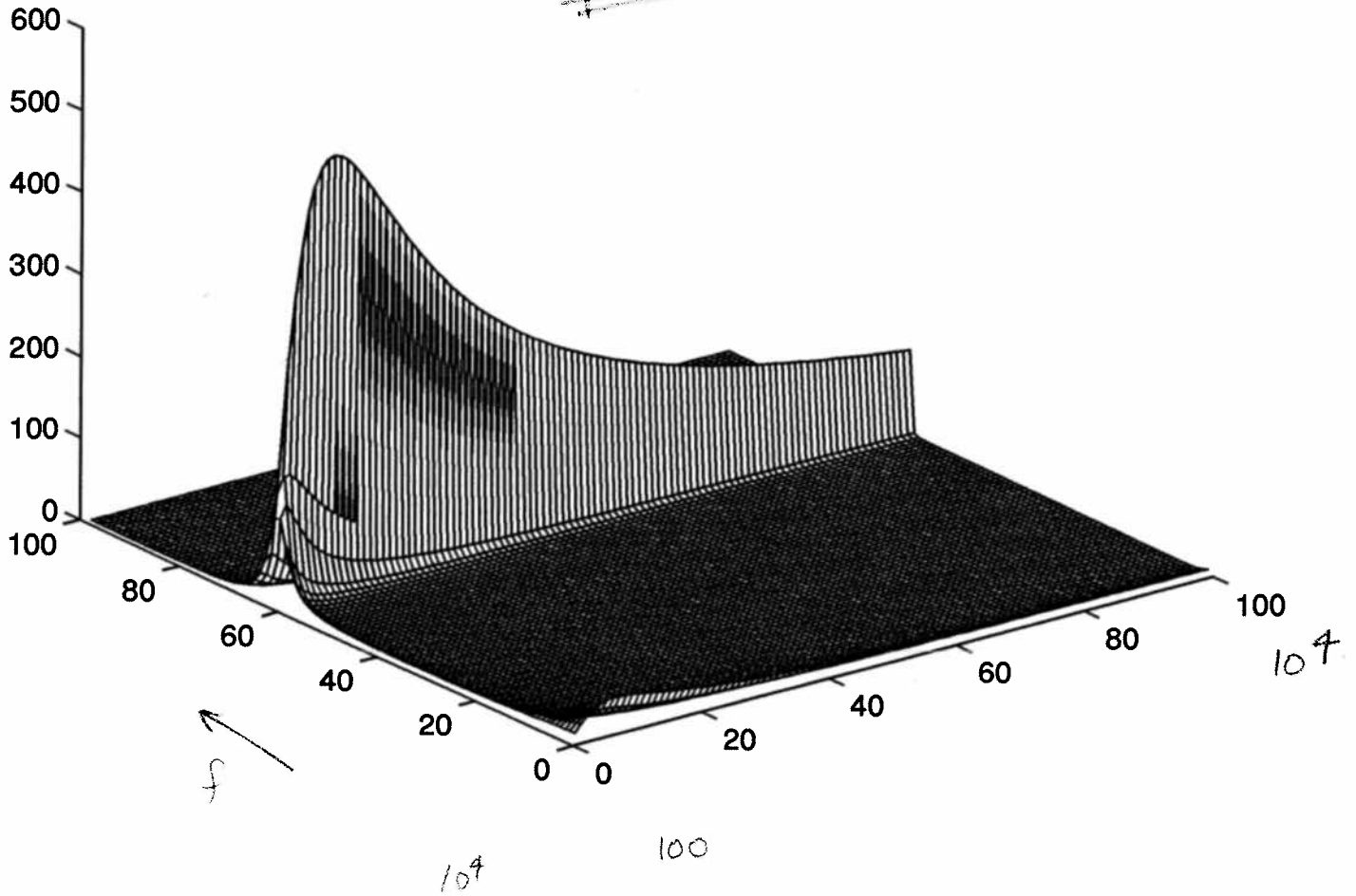
Magnitude (Z_{in})

magn



phase(Z in)

~~phase~~



$$C1 := .1 \cdot 10^{-6} \quad C2 := .1 \cdot 10^{-6}$$

$$R := 1 \cdot 10^3 \quad R_m := 50 \quad \text{Ken Kaiser} \quad j := \sqrt{-1}$$

$$x := 50, 50.5 \dots 100$$

Vi Luu
sec17-6

$$Zl := 0$$

$$w(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$L := \frac{9400}{(60 \cdot 10^6 \cdot 2 \cdot \pi)} \quad L = 2.493 \cdot 10^{-5}$$

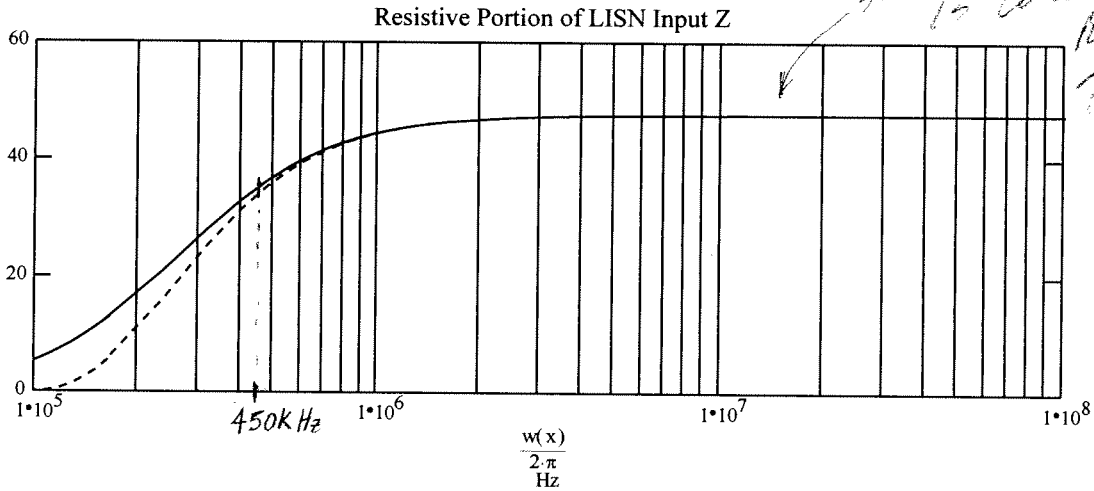
Block out frequency range up to 60 MHz L must be:

Increase the frequency (from 30 MHz) range of impedance by 2 reduces the inductance by 2.

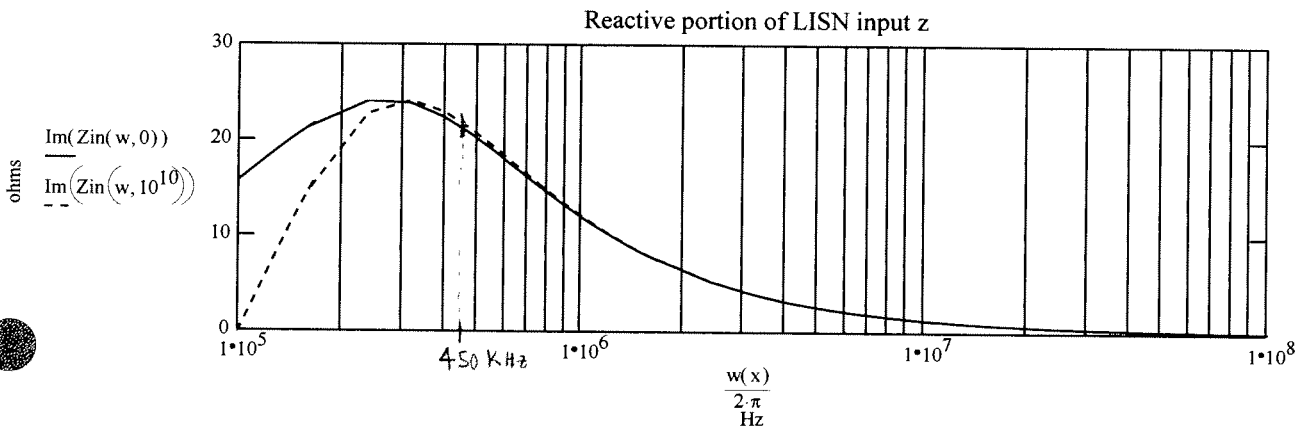
$$L = \frac{50 \cdot 10^{-6}}{2}$$

Analysis using section 17-6 for frequency range 450 kHz to 60 MHz; Impedance looking from the product side. Short and open circuit analysis of the load at the output supply side.

$$Z_{in}(w, Zl) := \frac{\left[\frac{Zl}{j \cdot w(x) \cdot C2} + j \cdot w(x) \cdot L \right] \cdot \left(\frac{1}{j \cdot w(x) \cdot C1} + R \cdot \frac{R_m}{R + R_m} \right)}{\left[\frac{Zl}{j \cdot w(x) \cdot C2} + j \cdot w(x) \cdot L \right] + \left(\frac{1}{j \cdot w(x) \cdot C1} + R \cdot \frac{R_m}{R + R_m} \right)}$$



The Impedance gives little variance from under open and short circuit output load for the real part.



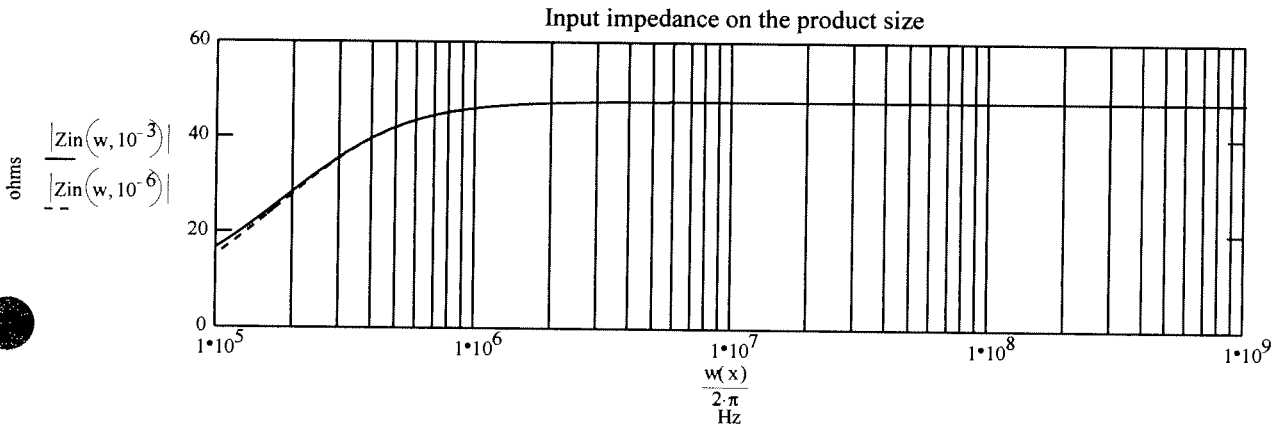
The Impedance gives little variance (frequency greater than 300 kHz) for the imaginary part.

$C1 := .1 \cdot 10^{-6}$ $C2 := .1 \cdot 10^{-6}$ $R := 1 \cdot 10^3$ $Rm := 50$ Ken Kaiser $j := \sqrt{-1}$ $x := 50, 50.5 .. 100$
 $L := \frac{50 \cdot 10^{-6}}{2}$ $Zl(w, C1) := \frac{1}{j \cdot w(x) \cdot C1}$ $C1 := 1 \cdot 10^{-6}$ $w(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$

Analysis of section 17-8

Similar analysis is done with an capacitive load.

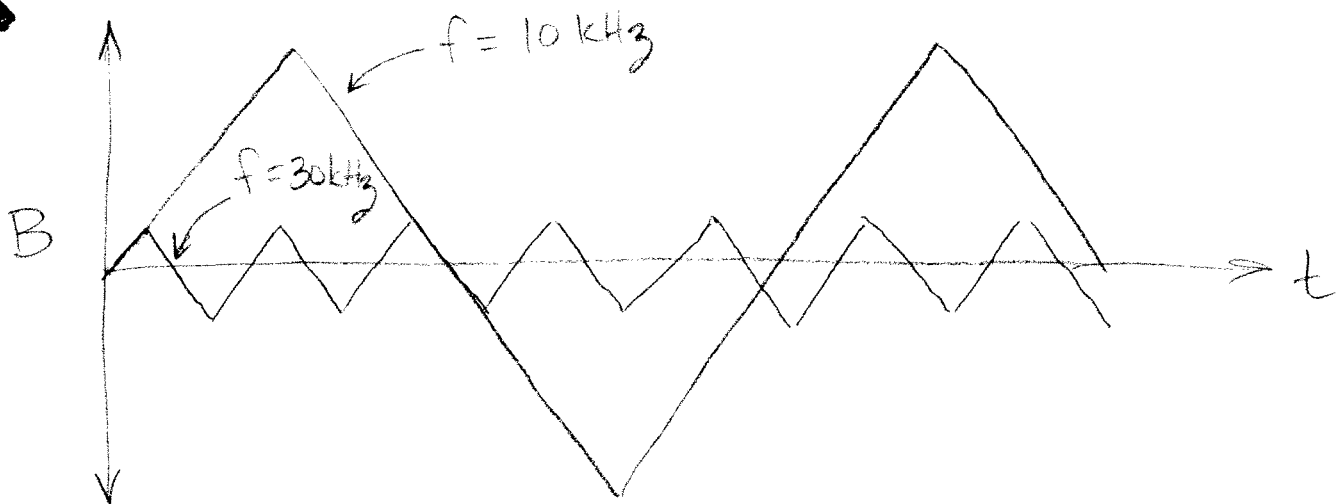
$$Z_{in}(w, C1) := \frac{\left[\frac{\frac{Zl(w, C1)}{j \cdot w(x) \cdot C2}}{Zl(w, C1) + \frac{1}{j \cdot w(x) \cdot C2}} + j \cdot w(x) \cdot L \right] \cdot \left(\frac{1}{j \cdot w(x) \cdot C1} + R \cdot \frac{Rm}{R + Rm} \right)}{\left[\frac{\frac{Zl(w, C1)}{j \cdot w(x) \cdot C2}}{Zl(w, C1) + \frac{1}{j \cdot w(x) \cdot C2}} + j \cdot w(x) \cdot L \right] + \left(\frac{1}{j \cdot w(x) \cdot C1} + R \cdot \frac{Rm}{R + Rm} \right)}$$



Overall the circuit's response is flat over the frequency of 450 KHz to 60 MHz under resistive and capacitive load at the output side. *The previous example in the book also meet the specification beyond 30 MHz*

17-1)

Lenz's law says that a changing flux at a given rate causes a corresponding voltage to appear at the inductor terminals. In a switching supply the desired output is fixed thus the rate of flux change is fixed. From the following diagram it can be seen that for a fixed rate of flux change the B fields get much larger in a slower switching supply.



Finally, to support the larger B fields without saturating the core, the transformer must have more magnetic core material and thus be physically larger. Typical switching frequencies vary from 20 kHz to 100 kHz.

Section 17

⑩ Work Problem 17.11

According to Faraday's Law: $V = -N \frac{d\Phi}{dt}$

if voltage variation is sinusoidal,
mag. flux also varies sinusoidally under
ideal conditions

$$\Phi = \Phi_m \sin \omega t$$

Now,

$$V = -N \omega \Phi_m \cos \omega t$$

Since $\Phi = B \cdot A$

$$V = -N \omega B \cdot A \cos \omega t = -N \underbrace{2\pi F}_{\uparrow} \underbrace{B A}_{\downarrow} \cos \omega t$$

V - want same

N - same

F - increases



Area ~~or~~ needs to be decreased
when Frequency is increased
to get same output voltage

If Frequency is increased 10x,
Area can be reduced by 10x

Typical Frequencies of switching power supplies:

10kHz to 1MHz [Nave pg. 2]

SECTION 26 PROBLEM 8

The non linear nature of the mic is being stressed because when radiation is coupled through nonlinear objects then the radiation is comprised of multiple frequencies due to clipping of the wave. When a sinusoidal of one frequency  IS CLIPPED 

The resultant waveform is NOT sinusoidal and thus it is comprised of many frequencies

Since the Load is $600\Omega =$ Low impedance an inductor can be used in series to create a low pass filter. CB frequencies range between 27MHz and 29MHz . The frequencies found on a telephone channel are between 300Hz to 3400Hz with the majority of the voice signals in the kilohertz range.

Any reasonable size inductor can be used since the frequencies of interest are low compared to the impedance of CB's

Frequency of interest 300 to $3400\text{kHz} = 1885$ to $21,362.8$ ¹⁻²/₃₄

Frequency to be filtered 27MHz to $29\text{MHz} = 169.6 \times 10^6$ to 182.21×10^6 ¹⁻²/₃₄

Impedance $= j\omega L$ we want this to be $>> 600\Omega$

so PICK INDUCTOR IN MILLIHENRY REGION

$Z = j\omega L$ at $\omega = 2\pi \cdot 28 \times 10^6 \text{ Hz}$ $L = 1 \times 10^{-3}$ Henries. $Z = 175,929\Omega$
 $L = 14/10^{-4}$ Henries $Z = 17,592.9\text{k}\Omega$
 $L = 1 \times 10^{-5}$ Henries $Z = 1,759.29\text{k}\Omega$

COULD AS PICK INDUCTOR IN OIL MILLIHENRY REGION.
OR EVEN IN 10MH REGION.

$I = 100\text{MH}$ DOWN TO 10MH

Rectifier Conductive Emission

Nonlinearities

$$I_0 + c_1[A \cdot (1 + M \cdot \cos(\Omega \cdot t)) \cdot \cos(\omega \cdot t)] + c_2[A \cdot (1 + M \cdot \cos(\Omega \cdot t))]^2 + c_3[A \cdot (1 + M \cdot \cos(\Omega \cdot t)) \cdot \cos(\omega \cdot t)]^3$$

$$I_0 + c_1 A \cdot \cos(\omega \cdot t) + c_1 A \cdot \cos(\omega \cdot t) \cdot M \cdot \cos(\Omega \cdot t) + c_2 A^2 \cdot \cos^2(\omega \cdot t) + 2 \cdot c_2 A^2 \cdot \cos(\omega \cdot t) \cdot M \cdot \cos(\Omega \cdot t) + c_2 A^2 \cdot \cos^2(\omega \cdot t) \cdot M^2 \cdot \cos^2(\Omega \cdot t) + c_3 A^3 \cdot \cos^3(\omega \cdot t)$$

$$I_0 + c_1 A \cdot \cos(\omega \cdot t) + c_1 A \cdot \cos(\omega \cdot t) \cdot M \cdot \cos(\Omega \cdot t) + c_2 A^2 \cdot \frac{1 + \cos(2 \cdot \omega \cdot t)}{2} + 2 \cdot c_2 A^2 \cdot \frac{1 + \cos(2 \cdot \omega \cdot t)}{2} \cdot M \cdot \cos(\Omega \cdot t) + c_2 A^2 \cdot \frac{1 + \cos(2 \cdot \omega \cdot t)}{2} \cdot M^2 \cdot \frac{1 + \cos(2 \cdot \Omega \cdot t)}{2}$$

$$I_0 + c_1 A \cdot \cos(\omega \cdot t) + c_1 A \cdot \cos(\omega \cdot t) \cdot M \cdot \cos(\Omega \cdot t) + \frac{1}{2} c_2 A^2 + \frac{1}{2} c_2 A^2 \cdot \cos(2 \cdot \omega \cdot t) + c_2 A^2 \cdot M \cdot \cos(\Omega \cdot t) + c_2 A^2 \cdot M \cdot \cos(\Omega \cdot t) \cdot \cos(2 \cdot \omega \cdot t) + \frac{1}{4} c_2 A^2 M^2 + \frac{1}{4} c_2 A^2 M^2 \cos(2 \cdot \Omega \cdot t) +$$

$$I_0 + \frac{1}{2} c_2 A^2 + \frac{1}{4} c_2 A^2 M^2 + c_2 A^2 M \cos(\Omega t) + \frac{1}{4} c_2 A^2 M^2 \cos(2 \Omega t)$$

$$+ A \left[c_1 M \cos(\Omega t) + \frac{3}{4} c_3 A^2 + \frac{2}{4} c_3 A^2 \cos(\Omega t) + c_1 \cos(\omega t) + \frac{2}{8} c_3 A^2 M^2 + \frac{2}{8} c_3 A^2 M^2 \cos(2 \Omega t) + \frac{2}{16} c_3 A^2 M^3 \cos(\Omega t) + \frac{2}{16} c_3 A^2 M^3 \cos(3 \Omega t) \right] \cos(\omega t)$$

$$3 + 3 \cdot c_3 \cdot A^3 \cdot \cos(\omega \cdot t) \cdot M \cdot \cos(\Omega \cdot t) + 3 \cdot c_3 \cdot A^3 \cdot \cos(\omega \cdot t)^3 \cdot M^2 \cdot \cos(\Omega \cdot t)^2 + c_3 \cdot A^3 \cdot \cos(\omega \cdot t) \cdot M^3 \cdot \cos(\Omega \cdot t)^3$$

$$\frac{\cdot \Omega \cdot t}{4} + c_3 \cdot A^3 \cdot \frac{3 \cdot 3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} + 3 \cdot c_3 \cdot A^3 \cdot \frac{3 \cdot 3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M \cdot \cos(\Omega \cdot t) + 3 \cdot c_3 \cdot A^3 \cdot \frac{3 \cdot 3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^2 \cdot \frac{1 + \cos(2 \cdot \Omega \cdot t)}{2} + c_3 \cdot A^3 \cdot \dots$$

$$\frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 \cdot \cos(2 \cdot \Omega \cdot t)$$

$$\frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 \cdot \cos(2 \cdot \omega \cdot t) + \frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 \cdot \cos(2 \cdot \omega \cdot t) \cdot \cos(2 \cdot \Omega \cdot t)$$

$$\frac{3}{4} \cdot c_3 \cdot A^3 \cdot \cos(\omega \cdot t) + \frac{3}{4} \cdot c_3 \cdot A^3 \cdot \cos(3 \cdot \omega \cdot t)$$

$$\frac{1}{4} \cdot c_3 \cdot A^3 \cdot \cos(3 \cdot \omega \cdot t) + \frac{9}{4} \cdot c_3 \cdot A^3 \cdot M \cdot \cos(\Omega \cdot t)$$

$$\frac{9}{4} \cdot c_3 \cdot A^3 \cdot M \cdot \cos(\Omega \cdot t)$$

Handwritten notes and scribbles:

- $\frac{1}{2} (2A)^2$
- $c_2 A^2 M \cos(\omega t)$
- $+ \frac{1}{4} c_2 A^2 M^2$
- $+ \frac{1}{4} c_2 A^2 M^2 \cos(2\omega t)$
- $+ \frac{1}{4} c_2 A^2 M^2 \cos(2\omega t)$
- $\cos(2\omega t)$
- $\cos(2\Omega t)$
- Large scribbles and a signature.

$$\frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^3 \cdot \frac{3 \cdot \cos(\Omega \cdot t) + \cos(3 \cdot \Omega \cdot t)}{4}$$

$$\frac{3}{4} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(\Omega \cdot t) \cdot \cos(3 \cdot \omega \cdot t) + \frac{9}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(\omega \cdot t) + \frac{9}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(\omega \cdot t) \cdot \cos(2 \cdot \Omega \cdot t) + \frac{3}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(3 \cdot \omega \cdot t) + \frac{3}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(3 \cdot \omega \cdot t) \cdot \cos(3 \cdot \Omega \cdot t)$$

$$\left[\frac{1}{4} c_3 A^3 + \frac{3}{8} c_3 A^3 M^2 + \frac{3}{8} c_3 A^3 M^2 \cos(2\omega t) + \frac{3}{16} c_3 A^3 M^3 \cos(\omega t) + \frac{3}{4} c_3 A^3 M \cos(\omega t) \right]$$

$$\cos(3\omega t)$$

$$+ \frac{1}{16} c_3 A^3 M^3 \cos(3\omega t)$$

$$is(\omega \cdot t) \cdot \cos(2 \cdot \Omega \cdot t) + \frac{9}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(w \cdot t) \cdot \cos(\Omega \cdot t) + \frac{3}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(w \cdot t) \cdot \cos(3 \cdot \Omega \cdot t) + \frac{3}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(3 \cdot \omega \cdot t) \cdot \cos(\Omega \cdot t) + \frac{1}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(3 \cdot \omega \cdot t) \cdot \cos(3 \cdot \Omega \cdot t)$$

$$\frac{3}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(w \cdot t) \cdot \cos(3 \cdot \Omega \cdot t) + \frac{3}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(3 \cdot \omega \cdot t) \cdot \cos(\Omega \cdot t)$$

$$\frac{3}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(w \cdot t) \cdot \cos(3 \cdot \Omega \cdot t)$$

$$\frac{9}{16} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \cos(w \cdot t) \cdot \cos(\Omega \cdot t)$$



Ken Kaiser



$$I_0 + c_1[A(1 + M \sin(\Omega t)) \cos(\omega t)] + c_2[A(1 + M \sin(\Omega t)) \cos(\omega t)]^2 + c_3[A(1 + M \sin(\Omega t)) \cos(\omega t)]^3$$

$$I_0 + c_1 A \cos(\omega t) + c_1 A \cos(\omega t) M \sin(\Omega t) + c_2 A^2 \cos^2(\omega t) + 2 c_2 A^2 \cos(\omega t)^2 M \sin(\Omega t) + c_2 A^2 \cos(\omega t)^2 M^2 - c_2 A^2 \cos(\omega t)^2 M^2 \cos(\Omega t) + c_3 A^3 \cos(\omega t)^3 + 3 c_3 A^3 \cos(\omega t)^3 M \sin(\Omega t)$$

$$I_0 + c_1 A \cos(\omega t) + c_1 A \cos(\omega t) M \sin(\Omega t) + c_2 A^2 \frac{1 + \cos(2\omega t)}{2} + 2 c_2 A^2 \frac{1 + \cos(2\omega t)}{2} M \sin(\Omega t) + c_2 A^2 \frac{1 + \cos(2\omega t)}{2} M^2 - c_2 A^2 \frac{1 + \cos(2\omega t)}{2} M^2 \cos(\Omega t) + c_3 A^3 \frac{3 \cos(\omega t)}{2} + c_3 A^3 \frac{3 \cos(\omega t)}{2} M \sin(\Omega t)$$

$$\boxed{I_0} + \frac{1}{4} c_3 A^3 \cos(3\omega t) + \frac{3}{4} c_3 A^3 \cos(\omega t) + \frac{1}{2} c_2 A^2 \cos(2\omega t) + \boxed{c_1 A \cos(\omega t)} - \frac{3}{8} c_3 A^3 M^3 \sin(\Omega t) \cos(\omega t) \cos(2\Omega t) - \frac{1}{8} c_3 A^3 M^3 \sin(\Omega t) \cos(3\omega t) \cos(2\Omega t) - \frac{9}{8} c_3 A^3 M^2 \cos(\omega t) \cos(\Omega t)$$

$$I_0 + \frac{1}{4} c_3 A^3 M^2 + \frac{1}{2} c_2 A^2 + c_2 A^2 M \sin(\Omega t)$$

$$- \frac{1}{4} c_2 A^2 M^2 \cos(2\Omega t)$$

$$\left[\begin{array}{l} \frac{3}{4} c_3 A^3 \\ \frac{3}{8} c_3 A^3 M \end{array} \right]$$

$$(\omega \cdot t)^3 \cdot M \cdot \sin(\Omega \cdot t) + 3 \cdot c_3 \cdot A^3 \cdot \cos(\omega \cdot t)^3 \cdot M^2 - 3 \cdot c_3 \cdot A^3 \cdot \cos(\omega \cdot t)^3 \cdot M^2 \cdot \cos(\Omega \cdot t)^2 + c_3 \cdot A^3 \cdot \cos(\omega \cdot t)^3 \cdot M^3 \cdot \sin(\Omega \cdot t) - c_3 \cdot A^3 \cdot \cos(\omega \cdot t)^3 \cdot M^3 \cdot \sin(\Omega \cdot t) \cdot \cos(\Omega \cdot t)^2$$

$$3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} + 3 \cdot c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M \cdot \sin(\Omega \cdot t) + 3 \cdot c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^2 - 3 \cdot c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^2 \cdot \frac{1 + \cos(2 \cdot \Omega \cdot t)}{2} + c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^3 \cdot \sin(\Omega \cdot t) - c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^3 \cdot \sin(\Omega \cdot t) \cdot \cos(2 \cdot \Omega \cdot t) - \frac{3}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(3 \cdot \omega \cdot t) \cdot \cos(2 \cdot \Omega \cdot t) + \frac{3}{8} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \sin(\Omega \cdot t) \cdot \cos(\omega \cdot t) + \frac{1}{8} \cdot c_3 \cdot A^3 \cdot M^3 \cdot \sin(\Omega \cdot t) \cdot \cos(3 \cdot \omega \cdot t) + \frac{9}{4} \cdot c_3 \cdot A^3 \cdot M \cdot \sin(\Omega \cdot t) \cdot \cos(\omega \cdot t) + \frac{3}{4} \cdot c_3 \cdot A^3 \cdot M \cdot \sin(\Omega \cdot t) \cdot \cos(3 \cdot \omega \cdot t) + \frac{9}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(\omega \cdot t)$$

$2 + C_1 A$
 $+ \frac{9}{8} C_3 A^2 M^2$
 $+ C_1 A M \sin(\omega t)$
 $\times \cos(\omega t)$
 $(3 A^3 M^3 \sin(\omega t) + \frac{9}{4} C_3 A M \sin(\omega t))$

$$\begin{aligned}
 & \frac{1 + \cos(2 \cdot \Omega \cdot t)}{2} + c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^3 \cdot \sin(\Omega \cdot t) - c_3 \cdot A^3 \cdot \frac{3 \cdot \cos(\omega \cdot t) + \cos(3 \cdot \omega \cdot t)}{4} \cdot M^3 \cdot \sin(\Omega \cdot t) \cdot \frac{1 + \cos(2 \cdot \Omega \cdot t)}{2} \\
 & \frac{3}{4} \cdot c_3 \cdot A^3 \cdot M \cdot \sin(\Omega \cdot t) \cdot \cos(3 \cdot \omega \cdot t) + \frac{9}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(\omega \cdot t) + \frac{3}{8} \cdot c_3 \cdot A^3 \cdot M^2 \cdot \cos(3 \cdot \omega \cdot t) - \frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 \cdot \cos(2 \cdot \omega \cdot t) \cdot \cos(2 \cdot \Omega \cdot t) - \frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 \cdot \cos(2 \cdot \Omega \cdot t) + \frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 + \frac{1}{4} \cdot c_2 \cdot A^2 \cdot M^2 \cdot \cos(2 \cdot \omega \cdot t) + c_2 \cdot A^2 \cdot M \cdot \sin(\Omega \cdot t) + c_2 \cdot A^2 \cdot M \cdot \sin
 \end{aligned}$$

$$2c_2 A^2 M^2 \cos(2\Omega t) + \frac{1}{4} c_2 A^2 M^2 + \frac{1}{4} c_2 A^2 M^2 \cos(2\omega t) + c_2 A^2 M \sin(\Omega t) + c_2 A^2 M \sin(\Omega t) \cos(2\omega t) + \frac{1}{2} c_2 A^2 + c_1 A \cos(\omega t) M \sin(\Omega t)$$