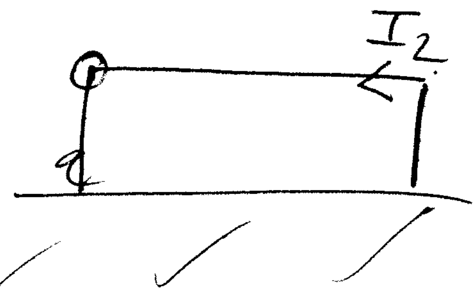
Compare to DeGauque



p. 311 table

~~short and op~~

let $z = l$

$$\frac{2hE_z}{Z_0}\beta^2\left(\frac{l}{2}-l\right)l - \frac{2jE_xh}{Z_C}\beta l$$

Need to negative to agree with me current direction — or their phase reference is different

e.g. $E_{z0}(1 \pm j\beta l_{eq})$

$$-\frac{2hE_z}{Z_0}\beta^2\left(-\frac{l}{2}\right)l + \frac{2jE_xh}{Z_0}\beta l$$

$$\frac{hE_z\beta^2l^2}{Z_0} + j\frac{E_xh\beta l 2}{Z_0}$$

My Mathcad yield (using my eqn in text)

$$E_z\left(\frac{d l^2 \beta^2}{2Z_0}\right) + jE_x\left(\frac{l\beta d}{Z_0}\right)$$

$h \,|\, Z_c = 60\ln\frac{2h}{r_w}$ my

$d \,|\, Z_0 = 120\ln\frac{d}{r_w}$

If $2h = d$ (images)

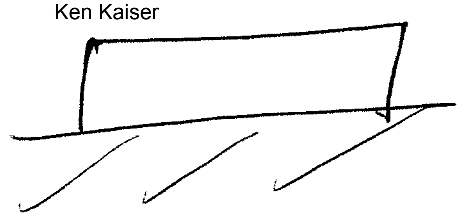
$$\frac{\frac{d}{2}E_z\beta^2 l^2}{Z_c} + j\frac{E_x\frac{d}{2}\beta l^2}{Z_c}$$

$$\boxed{\frac{E_z d\beta^2 l^1}{2Z_c} + j\frac{E_x d\beta l}{Z_c}}$$

seems reasonable but probably needs to be studied again

$$\frac{E_z d\beta^2 l^1}{120\ln\frac{d}{r_w}} + j\frac{E_x d\beta l}{60\ln\frac{d}{r_w}} \;?$$

Ken Kaiser



DeGauque

$$\frac{\sigma d}{-\sigma Z_0} E_{z0} \quad \cancel{E} \quad \frac{-\sigma d E_{x0}}{-\sigma Z_0}$$

My Mathcad

$$\frac{d}{Z_0}\left(E_{x0} - E_{z0}\right) + \frac{3 j d E_{x0} \beta l}{-\sigma Z_0}$$

$$\frac{d}{Z_0}\left(E_{x0} - E_{z0}\right) + \frac{-j E_{x0} d \beta l}{2 Z_0}$$

$$\frac{2hE_z}{Z_c} - \frac{2jE_x h}{Z_c}\beta\left(\frac{L}{2}\right)$$

$$\frac{2hE_z}{Z_c} - \frac{jE_x h l \beta}{Z_c}$$

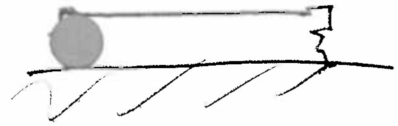
If $2h = d$

$$\frac{dE_z}{Z_c} - \frac{jE_x d l \beta}{2Z_c}$$

See my new $E_{xs}(x,0) = E_{x0}^{inc}$ $E_{xs}(x,l) = E_{x0}^{inc}$ no phase diff. then they are the same

Same as my but I have one additional term

Smith p. 11



$$Z_0 = 276 \log_{10}\left(\frac{2b}{a}\right)$$

$$= 120 \ln\left(\frac{2b}{a}\right)$$

$$= 120 \ln \frac{2b}{2r_w}$$

$$= 120 \ln \frac{b}{r_w}$$

$$= 120 \ln \frac{2h}{r_w}$$

$$= 2\left(60 \ln\left(\frac{2h}{r_w}\right)\right) = 2 Z_{om}$$

true Z_0 for situation

mr

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)}$$

mr

$$\frac{b}{2} = h$$

$$b = 2h$$

But also $\frac{Z_1}{2} = Z_s$ and $\frac{Z_2}{2} = Z_L$

$Z_1 = 2Z_s \qquad Z_2 = 2Z_L$

$$D = (2Z_{om})(2Z_s)\cos\beta s + j(4Z_{om}^2 + 4Z_sZ_L)\sin\beta s$$

$$= 4\left(Z_{om}Z_s\cos\beta s + j(Z_{om}^2 + Z_sZ_L)\sin\beta s\right)$$

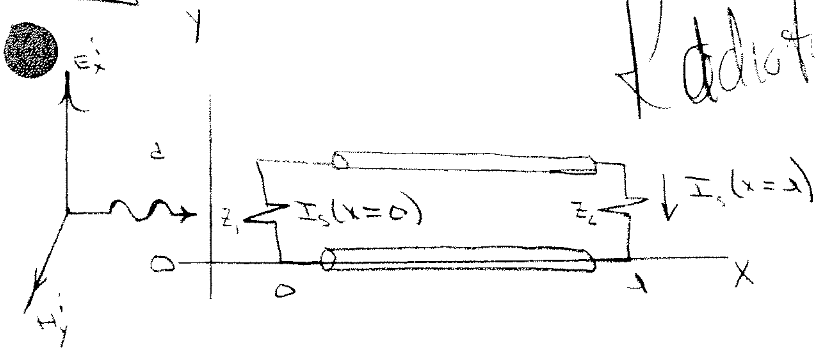
plugs the
into
1-8, 1-9
of Smith
results in the
text obtained

[redacted]

Ken Kaiser

6-29

Radiation



$$I_s(x=l) = \frac{1}{D}\int_0^l K_s(x)\left[Z_0\cos\beta x + jZ_1\sin\beta x\right]dx - \frac{1}{D}\left[Z_0\cos\beta l + jZ_1\sin\beta l\right]\int_0^d E_{ys}^{inc}(l,y) + \frac{Z_0}{D}\int_0^d E_{ys}^{inc}(0,y)\,dy$$

E_y IS CONSTANT SO $\int_0^d E_y^{inc}(0,y) = dE_y^{inc}(0,y)$

$K_s(x) = E_x^{inc}(x,d) - E_{xs}^{inc}(x,0)$ IS ZERO SINCE $\bar{E}$ IS IN Y-DIRECTION

SO WHOLE 1st TERM DROPS OUT

$$I_s(x=l) = -\frac{1}{D}\left[Z_0\cos\beta l + jZ_1\sin\beta l\right] dE_{ys}^{inc}(l,y) + \frac{Z_0}{D} dE_y^{inc}(0,y)$$

$$I_s(\cancel{x}=l) = \frac{Z_0 dE_y^{inc}(0,y) - dE_{ys}^{inc}(l,y)\left[Z_0\cos\beta l + jZ_1\sin\beta l\right]}{D}$$

$$I_s(x=l) = \frac{Z_0 dE_y^{inc}(0,y) - dE_{ys}^{inc}(l,y) Z_0\cos\beta l - jZ_1\sin\beta l\, dE_{ys}^{inc}(l,y)}{D}$$

$E_y^{inc}(0,l) = E_y^{inc}$

$E_y^{inc}(l, \ ) = E_y^i e^{-j\beta\sin\phi}$

Ken Kaiser

$$I = \frac{bE_y^i}{2D}\left\{2Z_0 - (Z_0 - Z_1)\cos\left[\beta s(1+\cancel{\sin\phi})\right] - (Z_0 - Z_1)\cos\left[\beta s(1-\cancel{\sin\phi})\right]\right.$$

$$\left. + j\cancel{(Z_0 - Z_1)\sin\left[\beta s(1+\sin\phi)\right]} - j\cancel{(Z_0 - Z_1)\sin\left[\beta s(1-\sin\phi)\right]}\right]$$

$\phi = 90$

$\sin\phi = 1$

$\sin 0 = 0$

$$I = \frac{bE_x^i}{2D}(Z_0 - Z_1)\left[1 - \cos 2\beta s \quad \cancel{\pm j \sin 2\beta s}\right]$$

$$\frac{I}{E_x^i} = \frac{b(Z_0 - Z_1)}{2D}\left[1 - \cos 2\beta s \quad \cancel{\pm j \sin 2\beta s}\right]$$

$j := \sqrt{-1}$

Ken Kaiser

$Zo := 150$

$Z1 := 30$

$Z2 := 3000$ $\qquad d := 1$

$C := 200 \cdot 10^{-12}$

$L := 0.5 \cdot 10^{-6}$

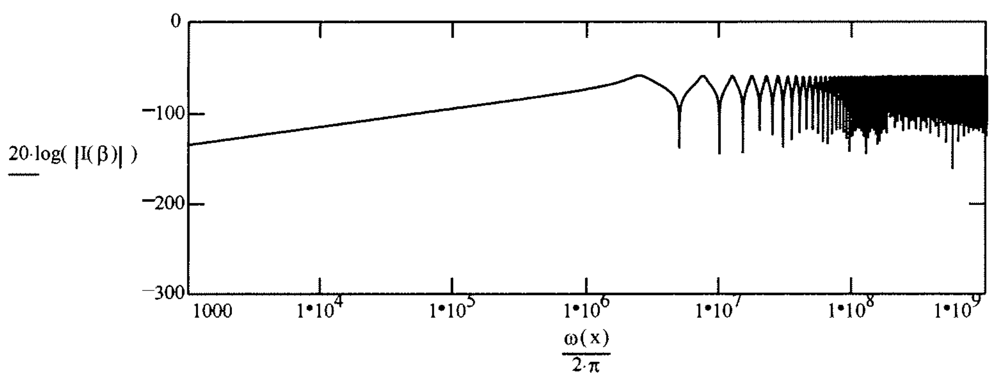
$x := 0, 0.001 .. 100$

$$\omega(x) := \left(x + 1 - 10 \cdot \mathrm{floor}\left(\frac{x}{10}\right)\right) \cdot 10^{\mathrm{floor}\left(\frac{x}{10}\right)}$$

$$\beta(\omega) := \omega(x) \cdot \sqrt{L \cdot C}$$

$$D(\beta) := (Zo \cdot Z1 + Zo \cdot Z2) \cdot \cos(\beta(\omega) \cdot 10) + j \cdot \left(Zo^2 + Z1 \cdot Z2\right) \cdot \sin(\beta(\omega) \cdot 10)$$

$$I(\beta) := \left[\frac{d \cdot (Zo - Z1)}{2 \cdot D(\beta)}\right] \cdot (1 - \cos(2 \cdot \beta(\omega) \cdot 10) + j \cdot \sin(2 \cdot \beta(\omega) \cdot 10))$$



6-29

Ken Kaiser

$j := \sqrt{-1}$

$Zo := 150$

$Z1 := 300$

$Z2 := 300$ $\quad d := 1$

$C := 200 \cdot 10^{-12}$

$L := 0.5 \cdot 10^{-6}$

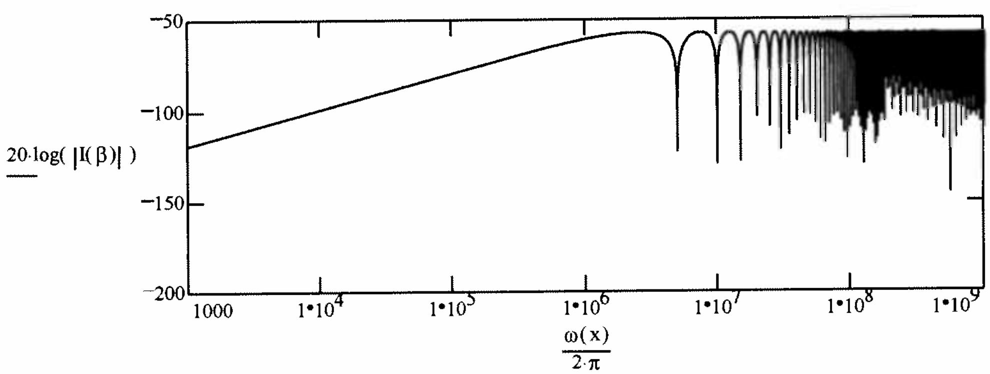
$x := 0, 0.001 .. 100$

$$\omega(x) := \left(x + 1 - 10 \cdot \mathrm{floor}\left(\frac{x}{10}\right)\right) \cdot 10^{\mathrm{floor}\left(\frac{x}{10}\right)}$$

$$\beta(\omega) := \omega(x) \cdot \sqrt{L \cdot C}$$

$$D(\beta) := (Zo \cdot Z1 + Zo \cdot Z2) \cdot \cos(\beta(\omega) \cdot 10) + j \cdot \left(Zo^2 + Z1 \cdot Z2\right) \cdot \sin(\beta(\omega) \cdot 10)$$

$$I(\beta) := \left[\frac{d \cdot (Zo - Z1)}{2 \cdot D(\beta)}\right] \cdot (1 - \cos(2 \cdot \beta(\omega) \cdot 10) + j \cdot \sin(2 \cdot \beta(\omega) \cdot 10))$$



Radiated

PROBLEM # 137

$lgth := 10 \quad E_{inc} := 1 \quad j := \sqrt{-1} \quad d := .01 \quad x := 50, 50.005 .. 90$

$\omega(x) := \left[x + 1 - 10\cdot \text{floor}\left(\left(\frac{x}{10}\right)\right)\right]\cdot 10^{\text{floor}\left(\frac{x}{10}\right)}$

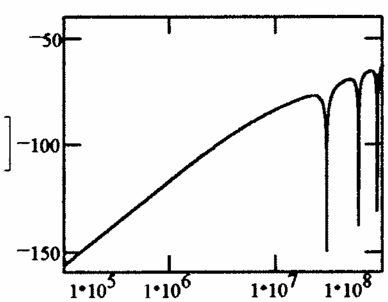
$\beta(\omega) := \frac{\omega(x)}{3\cdot 10^{8}} \quad Z_o := 150 \quad Z_s := 300 \quad Z_L := 300$

$D(\omega) := \left(Z_o\cdot Z_s + Z_o\cdot Z_L\right)\cdot\cos(\beta(\omega)\cdot lgth) + j\cdot\left(Z_o^{\,2} + Z_s\cdot Z_L\right)\cdot\sin(\beta(\omega)\cdot lgth)$

$K_s(\theta) := \left(E_{inc}\cdot e\right)^{(\beta(\omega)\cdot d)\cdot -j\cdot\sin(\theta)} - E_{inc}$

$I_L(\omega,\theta) := \frac{lgth}{D(\omega)}\cdot\left(K_s(\theta)\right)\cdot\left[Z_o\cdot\cos((\beta(\omega))\cdot lgth) + j\cdot Z_s\cdot\sin(\beta(\omega)\cdot lgth)\right] - \frac{Z_o\cdot lgth}{D(\omega)}\cdot K_s(\theta)$

$\left[20\cdot\log\left(\left(\left|I_L\left(\omega,\frac{\pi}{2}\right)\right|\right)\right)\right]$



$\frac{\omega(x)}{2\cdot\pi}$

When theda is 0 (broadside) IL = 0 When x = 0 ,Is = 0

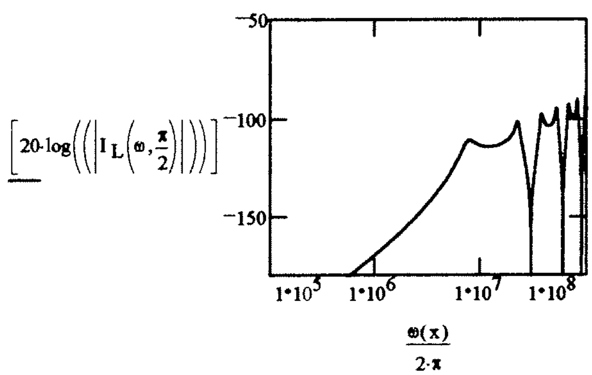
PROBLEM # 137

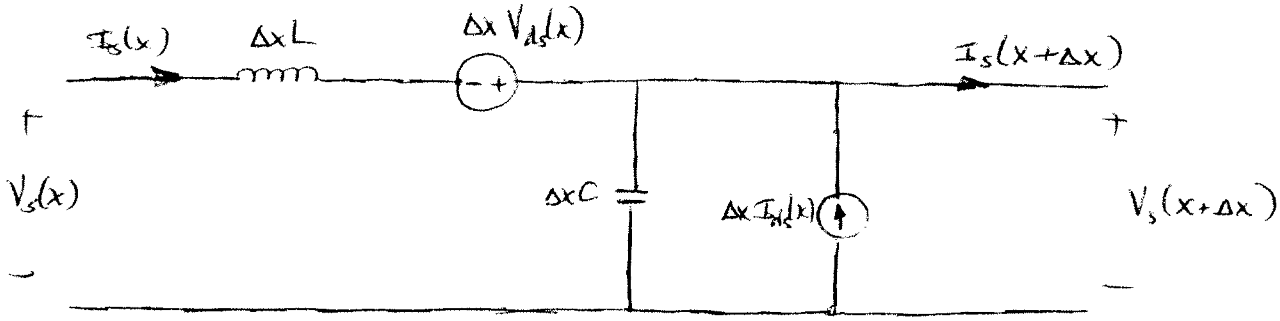
$Z_o := 150$ $\quad Z_s := 30$ $\quad Z_L := 30000$

$$D(\omega) := (Z_o \cdot Z_s + Z_o \cdot Z_L) \cdot \cos(\beta(\omega) \cdot lgth) + j \cdot (Z_o^{\,2} + Z_s \cdot Z_L) \cdot \sin(\beta(\omega) \cdot lgth)$$

$$K_s(\theta) := (E_{inc} \cdot e)^{(\beta(\omega) \cdot d) \cdot -j \cdot \sin(\theta)} - E_{inc}$$

$$I_L(\omega, \theta) := \frac{lgth}{D(\omega)} \cdot (K_s(\theta)) \cdot \left[Z_o \cdot \cos((\beta(\omega)) \cdot lgth) + j \cdot Z_s \cdot \sin(\beta(\omega) \cdot lgth) \right] - \frac{Z_o \cdot lgth}{D(\omega)} \cdot K_s(\theta)$$





KVL: $V_s(x+\Delta x) - V_s(x) = -j\omega L \Delta x\, I_s(x) + V_{ds}(x)\,\Delta x$

divide by Δx: $$\frac{V_s(x+\Delta x) - V_s(x)}{\Delta x} = -j\omega L\, I_s(x) + V_{ds}(x)$$

taking the lim as $\Delta x \to 0$

$$\frac{dV_s(x)}{dx} + j\omega L\, I_s(x) = V_{ds}(x) = j\omega\mu_0 \int_0^d H^{inc}_{zs}(x,y)\,dy$$

KCL: $I_s(x+\Delta x) - I_s(x) = -j\omega C \Delta x\, V_s(x+\Delta x) + I_{ds}(x)\,\Delta x$

divide by Δx: $$\frac{I_s(x+\Delta x) - I_s(x)}{\Delta x} = -j\omega C\, V_s(x+\Delta x) + I_{ds}(x)$$

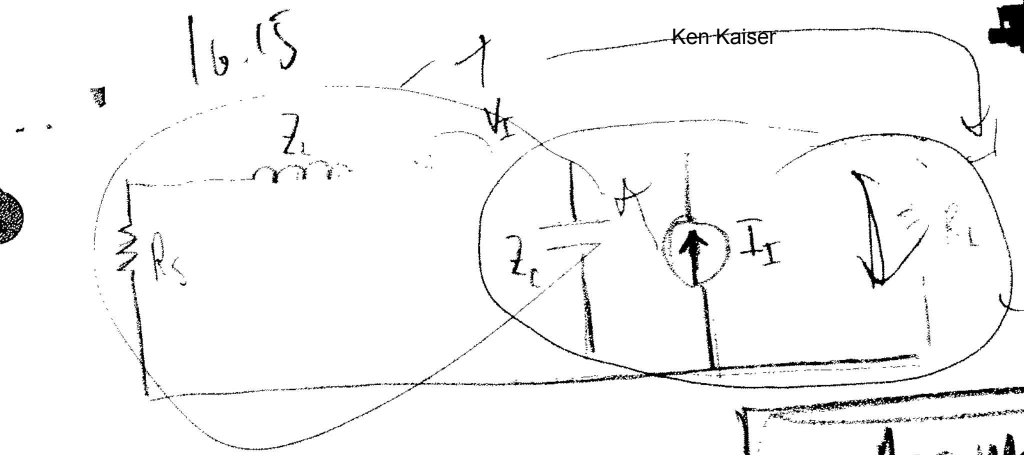
$$\cancel{\frac{dI_s(x)}{dx} + j\omega C\, V_s(x+\Delta x) = I_{ds}(x)}$$

taking $\lim_{\Delta x \to 0}$ $$\frac{dI_s(x)}{dx} + j\omega C\, V_s(x) = I_{ds}(x) = -j\omega C \int_0^d E^{inc}_{ys}(x,y)\,dy$$

[redacted]

16.15

Ken Kaiser



$V_{INDUCED} = j\omega\mu_0 \oint s H_n^i$

$I_{INDUCED} = j\omega\varepsilon \oint s E_t^i$

ASSUME

$100 Z_L < R_L, R_S < 0.01 Z_C$

WORST CASE $100 Z_L = R_L = R_S = 0.01 Z_C$

$$V_L = \frac{-\dfrac{Z_C R_L}{Z_C + R_L}}{\dfrac{Z_C R_L}{Z_C + R_L} + R_S + Z_L} V_I + \frac{\left(\dfrac{R_L Z_C}{Z_C + R_L}\right)\left(R_S + Z_L\right)}{\dfrac{R_L Z_C}{Z_C + R_L} + R_S + Z_L} I_I$$

$$\frac{-R_J\ V_I}{R_L + R_S}$$

$$\frac{-1}{1 + \dfrac{(R_S + Z_L)(Z_C + R_L)}{Z_C R_L}} V_I \qquad \frac{(R_L)(R_S + Z_L)}{(R_L) + \dfrac{(R_S + Z_L)(Z_C + R_L)}{Z_C}}$$

$$= \frac{-1}{1 + \dfrac{R_S}{R_L} + \dfrac{Z_L}{R_L} + \dfrac{R_S}{Z_C} + \dfrac{Z_L}{R_C}} V_I \qquad \frac{R_L R_S + R_L Z_L}{R_L + R_S + \dfrac{R_S R_L}{Z_C} + Z_L + \dfrac{Z_L R_L}{Z_C}}$$

$$= \frac{-1}{1 + \dfrac{R_S}{R_L} + 0.01\dfrac{R_S}{R_L} + 0.01\dfrac{R_S}{R_L} + 0.0001\dfrac{R_S}{R_L}} V_I \qquad \frac{R_L R_S + 0.01 R_L R_S}{R_L + R_S + 0.01 R_S + 0.01 R_L + 0.0001 R}$$

$$= \frac{-1}{1 + 1.0201\dfrac{R_S}{R_L}}$$

$$\approx \frac{-1}{1 + \dfrac{R_S}{R_L}} V_I + \frac{R_L R_S}{R_L + R_S} I_I$$

WITH THESE ASSUMPTIONS

THIS REALIZES THE SAME FUNCTION PAUL GAVE WHEN NEGLECTING INDUCTANCE AND CAPACITANCE

$$\boxed{V_L = \frac{-R_L}{R_L + R_S} V_I - \frac{R_S R_L}{R_L + R_S} I_I}$$

L-599
T_{11}
[redacted]

16-16

WHAT FACTORS DECREASE THE SUSCEPTIBILITY OF AN ELECTRICALLY-SHORT TWIN-LEAD LINE TO FAR-FIELD RADIATION?

$$\frac{dV_S(x)}{dx} + j\omega L I_S(x) = j\omega\mu_0 \int_0^d H^{inc}_{zs}(x,y)\,dy$$

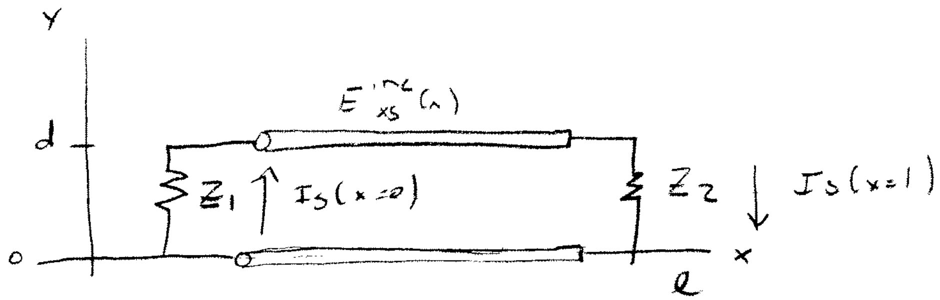
- LESSER INDUCTIVE & CAPACITIVE VALUES
- DECREASED FREQUENCY
- INCREASED DISTANCE TO FIELD
- DECREASED VOLTAGE + CURRENT
- DECREASED LINE LENGTH
- DECREASED PERMEABILITY
- INCREASED WIRE RADIUS
- INCIDENCE ANGLE TO $\hat{E}$ and $\hat{H}$
- CONTROL OF COMMON MODE CURRENTS
- SHIELDING OF THE WIRE

 [redacted]

SECTION 16 PROBLEM 31

ELECTRIC FIELD POLARIZED IN X DIRECTION. phase reference is at $y = d/2$

$$E^{inc}_{xs}(x,d) = E^{inc}_{x} e^{-j(\beta d/2)\sin\theta} \qquad E^{inc}_{xs}(x,0) = E^{inc}_{x} e^{+j(\beta d/2)\sin\theta}$$



$$I_S(x=l) = \frac{1}{D}\int_0^l K_S(x)\left[Z_0\cos\beta x + jZ_1\sin\beta x\right]dx - \frac{1}{D}\left[Z_0\cos\beta l + jZ_1\sin\beta l\right]\cdot \int_0^d E^{inc}_{ys}(l,y)\,dy + \frac{Z_0}{D}\int_0^d E^{inc}_{ys}(0,y)\,dy$$

$$D = Z_0Z_1 + Z_0Z_2\cos\beta l + j(Z_0^2 - Z_1Z_2)\sin\beta l$$

Z_0 = characteristic impedance.

$K_S(x) = E^{inc}_{xs}(x,d) - E^{inc}_{xs}(x,0)$

FOR E polarized in x direction EQUATION Reduces to the following BECAUSE E^{inc}_{ys} IS $= 0$.

INCLUDE STEPS

$$I_S(x=l) = \frac{K_S(\omega)}{D}\int_0^l Z_0\cos\beta x + jZ_1\sin\beta x\,]\,dx$$

$$I_S(x=l) = \frac{K_S(\omega)}{D}\cdot\left[\frac{1}{\beta}Z_0\sin\beta x\Big|_0^l - \frac{1}{\beta}jZ_1\cos\beta x\Big|_0^l\right]$$

$$= \frac{K_S(\omega)}{\beta D}\left[Z_0(\sin\beta l - \sin\beta 0) - jZ_1(\cos\beta l - \cos 0)\right]$$

$$= \frac{K_S(\omega)}{\beta D}\left[Z_0\sin\beta l - 0) - jZ_1((\cos\beta l - 1)\right]$$

$$I_S(x=l) = \frac{K_S(\omega)}{\beta D}\left[Z_0\sin\beta l + jZ_1(1 - \cos\beta l)\right]$$

$$K(\omega) = E^{inc}_{xs}e^{-j(\beta d/2)\sin\theta} - E^{inc}_{xs}e^{j(\beta d/2)\sin\theta}$$

using identity $e^{\pm jw} = \cos w \pm j\sin w$

$$K(\omega) = E^{inc}_{xs}\left[-j2\sin\left(\frac{\beta d}{2}\sin\theta\right)\right]$$

$$I_s(x=\ell) = E_{xs}^{inc}\frac{\left[-j2\sin\left(\beta d/2\sin\theta\right)\right]}{\beta D}\cdot\left[Z_0\sin\beta\ell + jZ_1(1-\cos\beta\ell)\right]$$

$f\lambda = v \quad \lambda = v/f$

$\beta = 2\pi/\lambda =$ phase constant over line.

$\beta = \omega\sqrt{LC}$ FOR 28 guage WIRE SEPARATION OF 50 MILS $L = 0.75\mu H/m$, $C = 14.82\,pF/m$

$$\frac{I_s(x=\ell)}{E_{xs}^{inc}} = -\frac{j2\sin\beta d/2}{\beta D}\left[Z_0\sin\beta\ell + jZ_1(1-\cos\beta\ell)\right]$$

For 10 meter long line $L = 7.5\mu H$ $C = 0.1482\,nF$

- ARE THE CURRENTS EQUAL TO ZERO WHEN THE SOURCE AND LOAD ARE MATCHED TO THE LINE?

- FROM MATHCAD ANALYSIS
  As can be seen from the graph the currents are not zero and vary with frequency when the source and loads are matched to the line.

- WHAT ARE THE CURRENTS IF THE ELECTRIC FIELD IS THE SAME (IN PHASE AND MAGNITUDE) EVERYWHERE ALONG THE LINE?

THROUGH THIS QUESTION ASSUME THE LINE IS ELECTRICALLY SHORT. ON AN ELECTRICALLY SHORT LINE THE WAVELIKE CHARACTERISTICS OF THE E FIELD ARE NOT NOTICEABLE. ∴ THE CURRENTS ALONG THE LINE ARE THE SAME BUT THEY STILL VARY WITH FREQUENCY UNTIL THE LINE BECOMES ELECTRICALLY LONG.

$Z_o := 150$ $\quad Z_1 := 300$ $\quad Z_2 := 300$ $\quad Z_3 := 30$ Ken Kaiser $\quad Z_4 := 3000$ $\quad Z_5 := 150$ $\quad Z_6 := 150$

$x := 30, 30.01 .. 90$ $\quad j := \sqrt{-1}$ $\quad d := 1.27 \cdot 10^{-3}$ $\quad l := 10$

$$\omega(x) := \left(x + 1 - 10 \cdot \mathrm{floor}\left(\frac{x}{10}\right)\right) \cdot 10^{\mathrm{floor}\left(\frac{x}{10}\right)}$$

$L := 7.5 \cdot 10^{-6}$ $\quad C := .1482 \cdot 10^{-9}$

$$\beta(\omega) := \omega(x) \cdot \sqrt{L \cdot C}$$

$$D_1(\omega) := \left(Z_o \cdot Z_1 + Z_o \cdot Z_2\right) \cdot \cos(\beta(\omega) \cdot l) + j \cdot \left(Z_o^{\,2} + Z_1 \cdot Z_2\right) \cdot \sin(\beta(\omega) \cdot l)$$

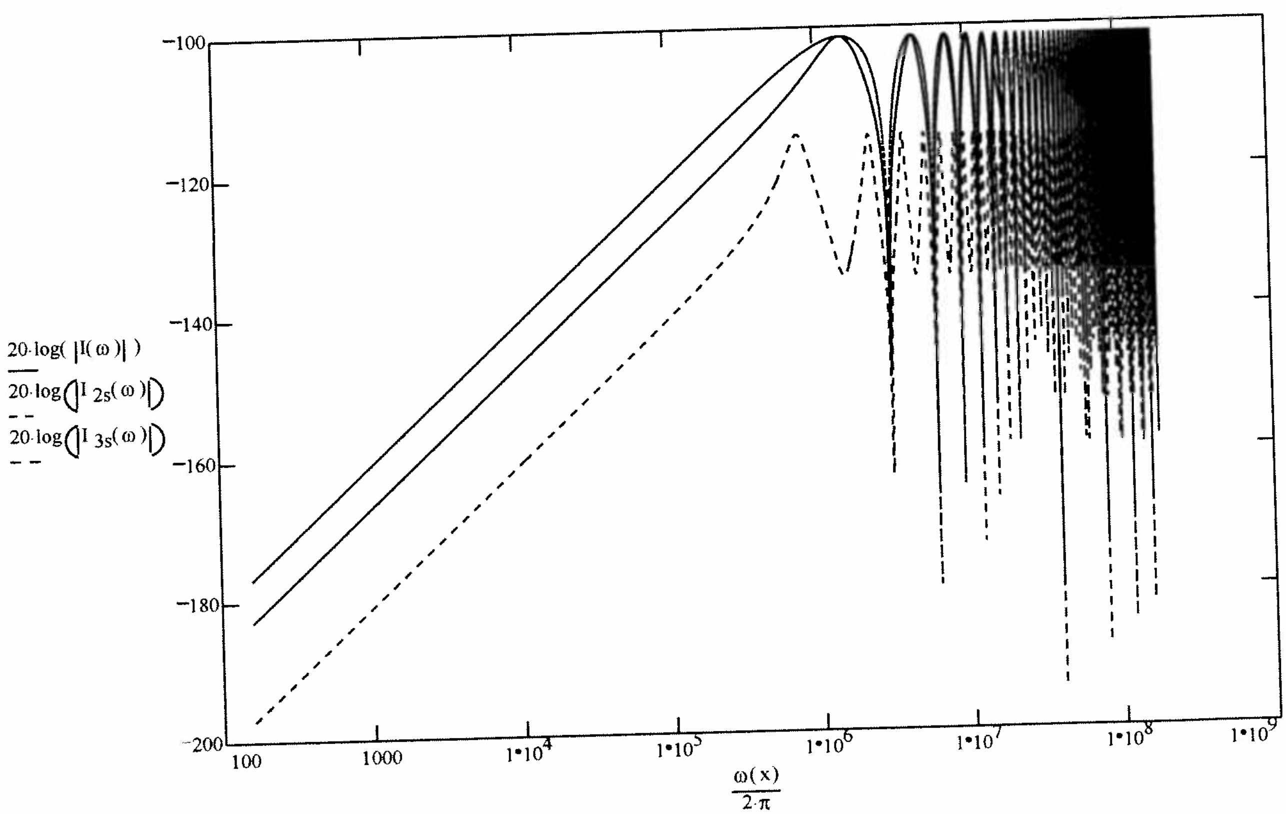
$$D_2(\omega) := \left(Z_o \cdot Z_3 + Z_o \cdot Z_4\right) \cdot \cos(\beta(\omega) \cdot l) + j \cdot \left(Z_o^{\,2} + Z_3 \cdot Z_4\right) \cdot \sin(\beta(\omega) \cdot l)$$

$$D_3(\omega) := \left(Z_o \cdot Z_5 + Z_o \cdot Z_6\right) \cdot \cos(\beta(\omega) \cdot l) + j \cdot \left(Z_o^{\,2} + Z_5 \cdot Z_6\right) \cdot \sin(\beta(\omega) \cdot l)$$

$$I(\omega) := \frac{-j \cdot 2 \cdot \sin\left(\beta(\omega) \cdot \frac{d}{2}\right)}{\beta(\omega) \cdot D_1(\omega)} \cdot \left[Z_o \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_1 \cdot (1 - \cos(\beta(\omega) \cdot l))\right]$$

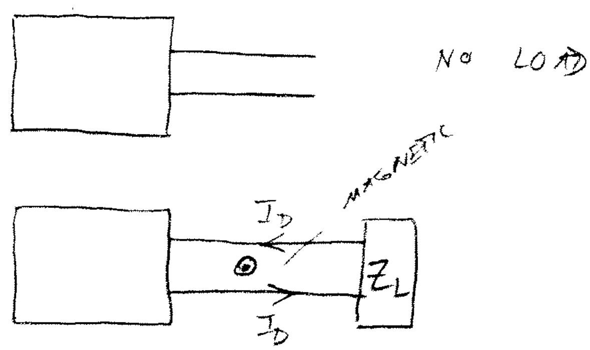
$$I_{2s}(\omega) := \frac{-j \cdot 2 \cdot \sin\left(\beta(\omega) \cdot \frac{d}{2}\right)}{\beta(\omega) \cdot D_2(\omega)} \cdot \left[Z_o \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_3 \cdot (1 - \cos(\beta(\omega) \cdot l))\right]$$

$$I_{3s}(\omega) := \frac{-j \cdot 2 \cdot \sin\left(\beta(\omega) \cdot \frac{d}{2}\right)}{\beta(\omega) \cdot D_3(\omega)} \cdot \left[Z_o \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_5 \cdot (1 - \cos(\beta(\omega) \cdot l))\right]$$



oomtte = 1m

16.4 To determine whether an interfering signal was common-mode or differential mode signal, the load was removed. Noise level remained constant. What can be concluded?



1) NOISE SOURCE MAY NOT BE ASSOCIATED WITH LOAD

2) IF ASSOCIATED WITH CONNECTIONS TO LOAD, NOT DIFFERENTIAL MODE CAUSED BY MAGNETIC FIELDS THROUGH LOOP.

3) COULD BE COMMON-MODE CURRENTS DUE TO PROXIMITY EFFECTS TO GROUND, POWER, OTHER SIGNALS

Ken Kaiser

15.12

15.12

$lgth := 10 \qquad E_{inc} := 1 \qquad j := \sqrt{-1} \qquad d := .005$

$x := 50, 50.005 .. 90$

$$\omega(x) := \left[x + 1 - 10 \cdot \text{floor}\left(\left(\frac{x}{10}\right)\right)\right] \cdot 10^{\text{floor}\left(\frac{x}{10}\right)}$$

$$\beta(\omega) := \frac{\omega(x)}{3 \cdot 10^8}$$

$Z_o := 150 \qquad Z_s := 300 \qquad Z_1 := 300$

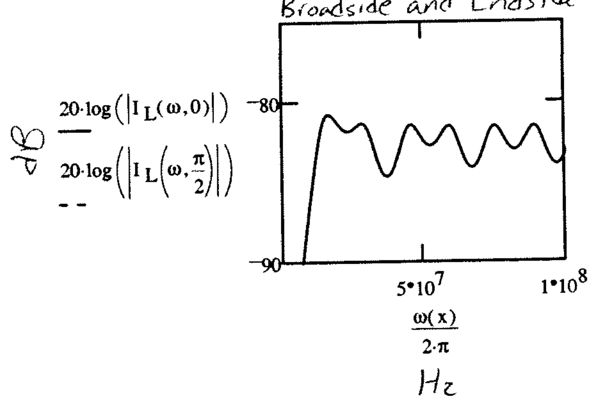
$$D(\omega) := (Z_o \cdot Z_s + Z_o \cdot Z_1) \cdot \cos(\beta(\omega) \cdot lgth) + j \cdot (Z_o{}^2 + Z_s \cdot Z_1) \cdot \sin(\beta(\omega) \cdot lgth)$$

$$E_{xs}(d,\theta) := E_{inc} \cdot e^{-j \cdot \frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)} \qquad E_{xs}(d,\theta) := E_{inc} \cdot e^{\frac{j \cdot \beta(\omega) \cdot d}{2} \cdot \sin(\theta)}$$

$$K_s(x) := E_{xs}(x,d) - E_{xs}(x,0)$$

$$I_L(\omega,\theta) := \frac{1}{D(\omega)} \cdot \int_0^{lgth} \left[K_s(x) \cdot \left[Z_o \cdot (\cos(\beta(\omega) \cdot (lgth - x))) + j \cdot Z_1 \cdot \sin(\beta(\omega) \cdot (lgth - x))\right]\right] dx$$

Broadside and Endside Excitation



Wave polarized in x − direction

Graph appears to be independent of θ

15.12

$Z_o := 150 \quad Z_s := 30 \quad Z_l := 3000$

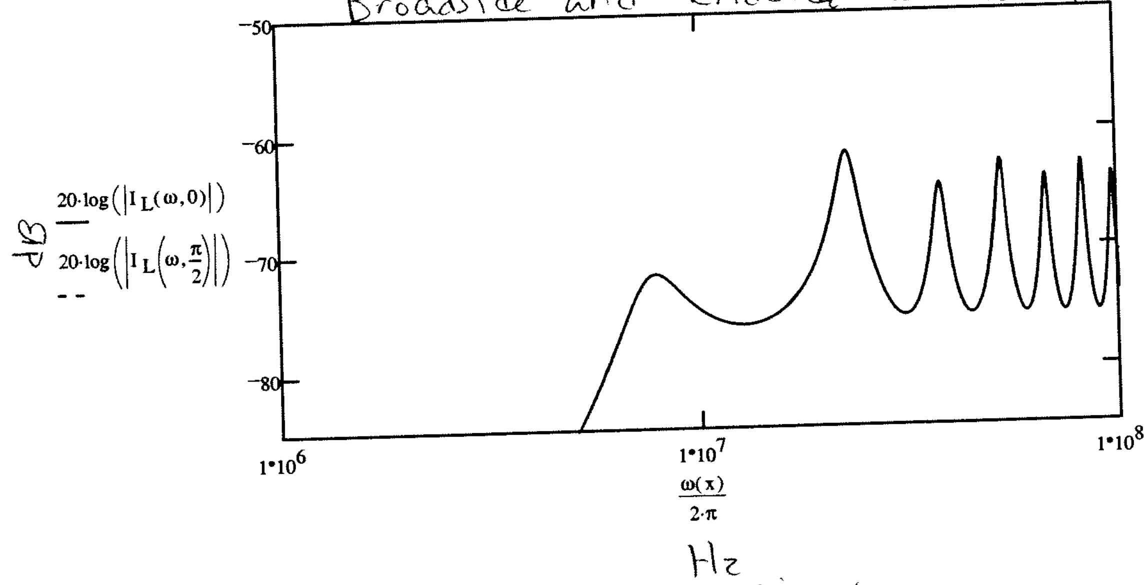
$$D(\omega) := \left(Z_o \cdot Z_s + Z_o \cdot Z_l\right) \cdot \cos(\beta(\omega) \cdot lgth) + j \cdot \left(Z_o^2 + Z_s \cdot Z_l\right) \cdot \sin(\beta(\omega) \cdot lgth)$$

$$E_{xs}(d,\theta) := E_{inc} \cdot e^{-j \cdot \frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)} \qquad E_{xs}(d,\theta) := E_{inc} \cdot e^{\frac{j \cdot \beta(\omega) \cdot d}{2} \cdot \sin(\theta)}$$

$$K_s(x) := E_{xs}(x,d) - E_{xs}(x,0)$$

$$I_L(\omega,\theta) := \frac{1}{D(\omega)} \cdot \int_0^{lgth} \left[K_s(x) \cdot \left[Z_o \cdot (\cos(\beta(\omega) \cdot (lgth - x))) + j \cdot Z_l \cdot \sin(\beta(\omega) \cdot (lgth - x))\right]\right] dx$$

Broadside and Endside Excitation



Wave polarized in the x-direction

Graph appears to be independent of θ

Section 16

(* 21)

$$I_s(x=0) = \frac{1}{D}\int_0^{\ell} k_s(x)\left[Z_0\cos\beta(\ell-x) + jZ_L\sin\beta(\ell-x)\right]dx + \frac{1}{D}\left[Z_0\cos\beta\ell + jZ_L\sin\beta\ell\right]\int_0^{d} E_{ys}^{inc}(0,y)dy - \frac{Z_0}{D}\int_0^{d} E_{ys}^{inc}(\ell,y)dy$$

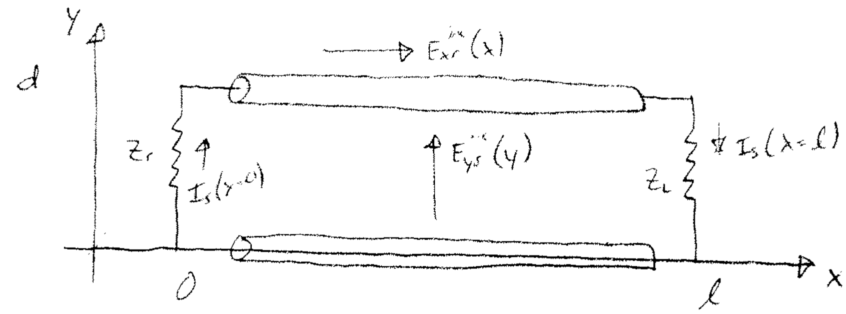
$$I_s(x=\ell) = \frac{1}{D}\int_0^{\ell} k_s(x)\left[Z_0\cos\beta x + jZ_s\sin\beta x\right]dx + -\frac{1}{D}\left[Z_0\cos\beta\ell + jZ_s\sin\beta\ell\right]\int_0^{d} E_{ys}^{inc}(\ell,y)dy + \frac{Z_0}{D}\int_0^{d} E_{ys}^{inc}(0,y)dy$$

$$D = (Z_0Z_s + Z_0Z_L)\cos\beta\ell + j(Z_0^2 + Z_sZ_L)\sin\beta\ell$$

$$k_s(x) = E_{xs}^{inc}(x,d) - E_{xs}^{inc}(x,0)$$

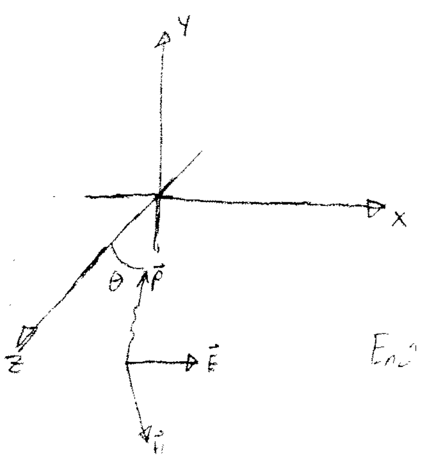
$E_{xs}^{inc}(x,d)$ = incident elec. field in x-dir. along upper conductor

$E_{xs}^{inc}(x,0)$ = incident elec field in x-dir along lower conductor



Electric Field polarized in x-direction (parallel w/ transmission line) + phase ref. at $y = d/2$

$$E_{xs}^{inc}(x,d) = E_x^{inc}e^{-j\left(\frac{\beta d}{2}\right)\sin\theta} \qquad E_{xs}^{inc}(x,0) = E_x^{inc}e^{+j\left(\frac{\beta d}{2}\right)\sin\theta}$$



Broadside excitation $\theta = 0°$ −z travel

$$E_{xs}^{inc}(x,d) = E_x^{inc}e^0 \qquad E_{xs}^{inc}(x,0) = E_x^{inc}e^0$$

Volt. induced along line

$$V = -\int_0^{\ell}\vec{E}\cdot d\vec{l} \approx -\ell E_x^{inc}$$

Endfire excitation: $\cancel{\theta = 90°}$

can't have endfire w/ E-field polarized in x-direction.

[redacted]

$E_{ys}^{i} = 0$ + E_{xs}^{i} is uniform over each conductor

y- terms go to zero, left only w/ x

$$I_s(x=0) = \frac{1}{D}\int_0^{\ell} k_s(x)\left[Z_0\cos\beta(\ell-x) + j\sin\beta(\ell-x)\right]dx$$

$$I_s(x=\ell) = \frac{1}{D}\int_0^{\ell} k_s(x)\left[Z_0\cos\beta x + jZ_s\sin\beta x\right]dx$$

$$k_s(x) = E_x^{inc}\left[e^{-j\frac{\beta d}{2}\sin\theta} - e^{+j\frac{\beta d}{2}\sin\theta}\right] \qquad e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$= E_x^{inc}\left[-j2\sin\left(\frac{\beta d}{2}\sin\theta\right)\right]$$

using Mathcad

$$I_s(x=0) = \frac{K_s(x)}{D}\int_0^{\ell}\left[Z_0\cos\beta(\ell-x) + j\sin\beta(\ell-x)\right]dx = \frac{k_s(x)}{\beta D}\left[j - (-Z_0\sin\beta\ell + j\cos\beta\ell)\right]$$

$$I_s(x=\ell) = \frac{k_s(x)}{D}\int_0^{\ell}\left[Z_0\cos\beta x + jZ_s\sin\beta x\right]dx = \frac{k_s(x)}{\beta D}\left[Z_0\sin\beta\ell + jZ_s(1-\cos\beta\ell)\right]$$

obtained from Smith's book

electrically long line

$Z_0 = 150$

1) $Z_s = Z_L = 300$

2) $Z_s = 30$ $Z_L = 3000$

let $\ell = 10m$

lossless line

$$\lambda = \frac{v}{f}$$

freq corresponding to 10m λ

$f = 30MHz$

$f \leq 3MHz$ for line to be electrically short

Section 16, number *21:

Determine value for current at source end:

$$j := \sqrt{-1}$$

$$I_{s0} = \frac{K_s(x)}{D} \cdot \int_0^1 \left[Z_o \cdot \cos(\beta \cdot (1 - x)) + j \cdot \sin(\beta \cdot (1 - x))\right] dx$$

$$I_{s0} = \frac{K_s(x)}{D} \cdot \left[\frac{j}{\beta} - \frac{(-\sin(\beta \cdot 1) \cdot Z_o + \cos(\beta \cdot 1) \cdot j)}{\beta}\right]$$

Plot the load current divided by the incident electric field:

$$l := 10 \qquad E_{inc} := 1 \qquad d := .01 \qquad x := 50, 50.005 .. 90$$

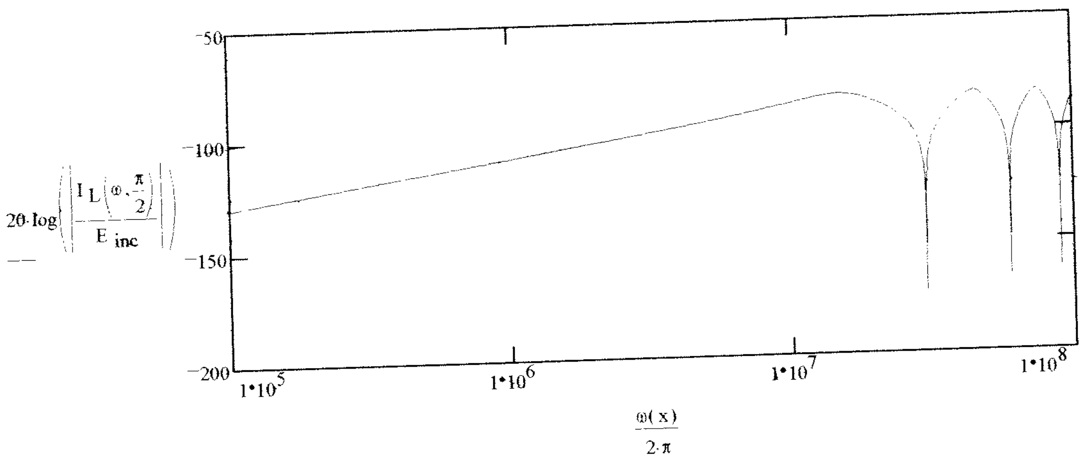
$$\omega(x) := \left(x + 1 - 10 \cdot \mathrm{floor}\left(\frac{x}{10}\right)\right) \cdot 10^{\mathrm{floor}\left(\frac{x}{10}\right)} \qquad \beta(\omega) := \frac{\omega(x)}{3 \cdot 10^8}$$

Case 1: Z_O=150, Z_S=Z_L=300

$$Z_o := 150 \qquad Z_s := 300 \qquad Z_L := 300$$

$$D(\omega) := (Z_o \cdot Z_s + Z_o \cdot Z_L) \cdot \cos(\beta(\omega) \cdot l) + j \cdot (Z_o^2 + Z_s \cdot Z_L) \cdot \sin(\beta(\omega) \cdot l)$$

$$I_L(\omega, \theta) := \frac{-j \cdot 2 \cdot \sin\left(\frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)\right)}{\beta(\omega) \cdot D(\omega)} \cdot \left[Z_o \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_s \cdot (1 - \cos(\beta(\omega) \cdot l))\right]$$

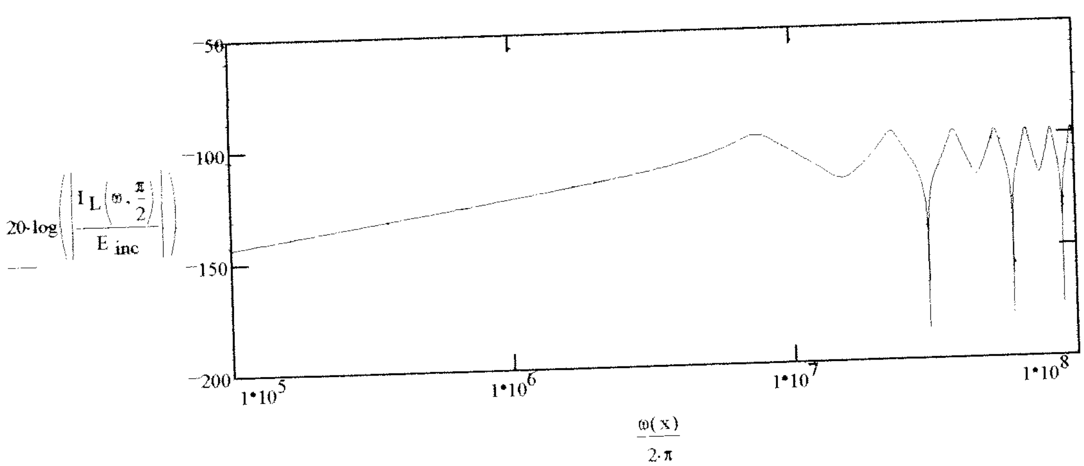


Case 2: Z_O=150, Z_S=30, Z_L=3000

$Z_o = 150 \qquad Z_s = 30 \qquad Z_L = 3000$

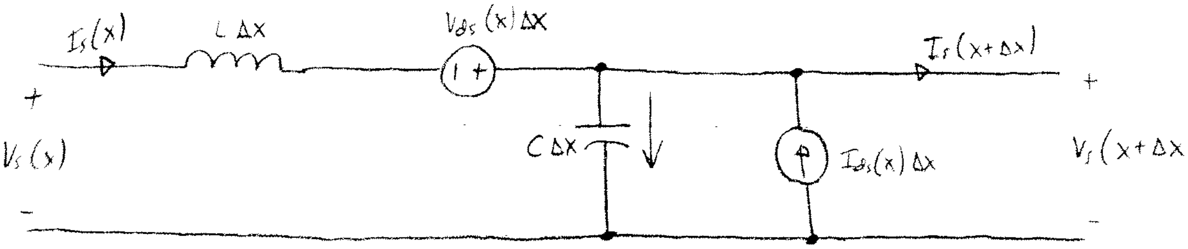
$$D(\omega) := \left(Z_o\cdot Z_s + Z_o\cdot Z_L\right)\cdot\cos(\beta(\omega)\cdot l) + j\cdot\left(Z_o^{\;2} + Z_s\cdot Z_L\right)\cdot\sin(\beta(\omega)\cdot l)$$

$$I_L(\omega,\theta) := \frac{-j\cdot 2\cdot\sin\left(\frac{\beta(\omega)\cdot d}{2}\cdot\sin(\theta)\right)}{\beta(\omega)\cdot D(\omega)}\cdot\left[Z_o\cdot\sin(\beta(\omega)\cdot l) + j\cdot Z_s\cdot(1-\cos(\beta(\omega)\cdot l))\right]$$



Section 16

(1) Reference Problem 16.9



KVL: $-V_s(x) + j\omega L\,\Delta x\, I_s(x) - V_{\partial s}(x)\,\Delta x + V_s(x+\Delta x) = 0$

$$V_s(x) = j\omega L\,\Delta x\, I_s(x) - V_{\partial s}(x)\,\Delta x + V_s(x+\Delta x)$$

divide through by Δx

$$\frac{V_s(x)}{\Delta x} = j\omega L\, I_s(x) - V_{\partial s}(x) + \frac{V_s(x+\Delta x)}{\Delta x}$$

$$V_{\partial s}(x) = j\omega L\, I_s(x) + \frac{V_s(x+\Delta x)}{\Delta x} - \frac{V_s(x)}{\Delta x}$$

$f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$V_{\partial s}(x) = \underbrace{\frac{1}{\Delta x}\left(V_s(x+\Delta x) - V_s(x)\right)}_{\frac{\partial V_s(x)}{\partial x}} + j\omega L\, I_s(x)$$

$$V_{\partial s}(x) = \frac{\partial V_s(x)}{\partial x} + j\omega L\, I_s(x)$$

✓ ✓ ✓ ✓

KCL: (in +) $I_s(x) - j\omega C\,\Delta x\, V_s(x+\Delta x) + I_{\partial s}(x)\,\Delta x - I_s(x+\Delta x) = 0$

$$I_{\partial s}(x)\,\Delta x = I_s(x+\Delta x) - I_s(x) + j\omega C\,\Delta x\, V_s(x+\Delta x)$$

– divide through by Δx –

$$I_{\partial s}(x) = \frac{I_s(x+\Delta x) - I_s(x)}{\Delta x} + j\omega C\,\frac{\Delta x}{\Delta x}\, V_s(x+\Delta x)$$

$$I_{\partial s}(x) = \frac{\partial I_s(x)}{\partial x} + j\omega C\, V_s(x+\Delta x)$$

$\Delta x \to 0$

$\dfrac{V_s(x+\Delta x)}{\frac{1}{j\omega C\Delta x}}$

Unless ~~$\Delta x \ll x$ + $V_s(x+\Delta x) \approx V_s(x)$~~

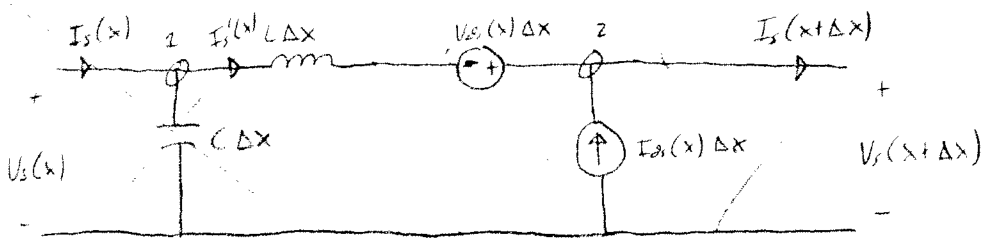
Not the same

Unless $\Delta x \ll x \Rightarrow V_s(x) \approx V_s(x+\Delta x)$

[redacted]

But, if $C\Delta x$ is moved to parallel w/ source – would get exact answer

this model is wrong



Now, node 1 :

$$I_s(x) - j\omega C\Delta x\, V_s(x) - I_s'(x) = 0$$

$$I_s'(x) = I_s(x) - j\omega C\Delta x\, V_s(x)$$

Node 2 :

$$I_s'(x) + I_{ds}(x)\Delta x - I_s(x+\Delta x) = 0$$

$$I_{ds}(x)\Delta x = I_s(x+\Delta x) - I_s'(x)$$

$$I_{ds}(x)\Delta x = I_s(x+\Delta x) - I_s(x) + j\omega C\Delta x\, V_s(x)$$

$$I_{ds}(x) = \frac{\partial I_s(x)}{\partial x} + j\omega C V_s(x)$$

But – KVL does not yield same result. (Different current, $I_s'(x)$ through inductor)

Conclusion:

must assume that for the original circuit, $V_s(x+\Delta x) \approx V_s(x)$ (line is electrically short)

$\therefore$

$$\boxed{\begin{aligned} V_{ds}(x) &= \frac{\partial V_s(x)}{\partial x} + j\omega L I_s(x) \\ I_{ds}(x) &= \frac{\partial I_s(x)}{\partial x} + j\omega C V_s(x) \end{aligned}}$$

Problem 16.31 ECE-640 [redacted]

For E-Field polarized in the x-direction:

$$I_s(x=l) = \frac{1}{D}\int_0^l E_x^{inc}\left[e^{-j\left(\frac{\beta d}{2}\right)\sin\theta} - e^{j\left(\frac{\beta d}{2}\right)\sin\theta}\right]\left[Z_0\cos\beta x + jZ_1\sin\beta x\right]dx$$

$Z_0 = 150\,\Omega$ $\quad D = \left(Z_0 Z_1 + Z_0 Z_2\right)\cos\beta l + j\left(Z_0^2 + Z_1 Z_2\right)\sin\beta l$

$$\frac{I_s(x=l)}{E_x^{inc}} = \frac{1}{D}\left[e^{-j\left(\frac{\beta d}{2}\right)\sin\theta} - e^{j\left(\frac{\beta d}{2}\right)\sin\theta}\right]\int_0^l\left[Z_0\cos\beta x + jZ_1\sin\beta x\right]dx$$

$$= \frac{1}{\beta D}\left[e^{-j\left(\frac{\beta d}{2}\right)\sin\theta} - e^{j\left(\frac{\beta d}{2}\right)\sin\theta}\right]\left[Z_0\sin\beta x - jZ_1\cos\beta x\right]_0^l$$

$$= \frac{1}{\beta D}\left[e^{-j\left(\frac{\beta d}{2}\right)\sin\theta} - e^{j\left(\frac{\beta d}{2}\right)\sin\theta}\right]\left[Z_0\sin\beta l - jZ_1\cos\beta l + jZ_1\right]$$

MATHCAD PLOTS AND ANALYSIS FOLLOW:

Problem 16.31 ECE-640 [redacted]

$Z_o := 300 \quad Z_1 := 300 \quad Z_2 := 300 \quad l := 5 \quad \beta(f) := \dfrac{2\cdot\pi\cdot f}{3\cdot 10^8} \quad \theta := \dfrac{\pi}{2} \quad d := 2\cdot 10^{-3} \quad j := \sqrt{-1}$

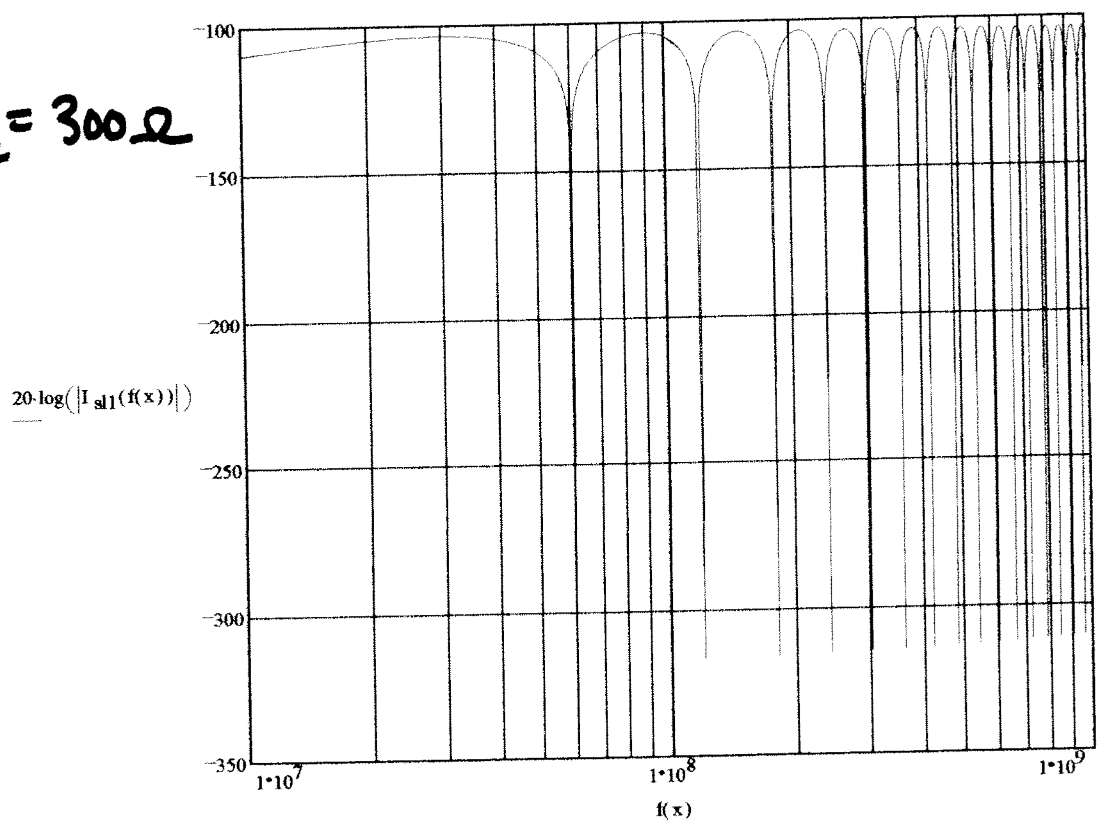
$x := 1, 1.01 .. 110$

$$f(x) := \left(x + 1 - 10\cdot\mathrm{floor}\left(\frac{x}{10}\right)\right)\cdot 10^{\mathrm{floor}\left(\frac{x}{10}\right)}$$

$$D_1(x) := \left(Z_o\cdot Z_1 + Z_o\cdot Z_2\right)\cdot\cos(\beta(x)\cdot l) + j\cdot\left(Z_o^2 + Z_1\cdot Z_2\right)\cdot\sin(\beta(x)\cdot l)$$

$$I_{sl1}(x) := \frac{1}{D_1(x)\cdot\beta(x)}\cdot\left[e^{-j\cdot\left(\frac{\beta(x)\cdot d}{2}\right)\cdot\sin(\theta)} - e^{j\cdot\left(\frac{\beta(x)\cdot d}{2}\right)\cdot\sin(\theta)}\right]\cdot\left(Z_o\cdot\sin(\beta(x)\cdot l) - j\cdot Z_1\cdot\cos(\beta(x)\cdot l) + j\cdot Z_1\right)$$

$Z_o = Z_1 = Z_2 = 300\,\Omega$



Problem 16.31 **ECE-640** [redacted]

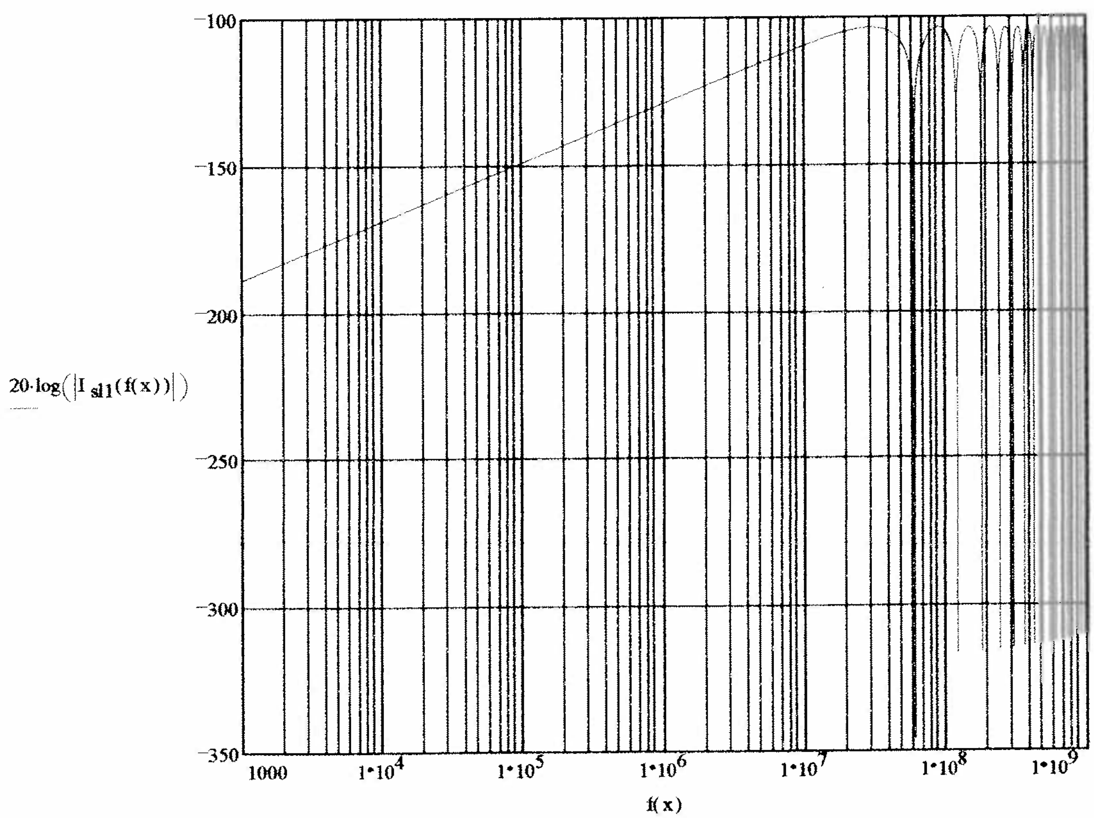
$Z_o := 300 \quad Z_1 := 300 \quad Z_2 := 300 \quad l := 5 \quad \beta(f) := \dfrac{2\cdot\pi\cdot f}{3\cdot 10^8} \quad \theta := \dfrac{\pi}{2} \quad d := 2\cdot 10^{-3} \quad j := \sqrt{-1}$

$x := 1, 1.01 .. 110$

$$f(x) := \left(x + 1 - 10\cdot\mathrm{floor}\left(\frac{x}{10}\right)\right)\cdot 10^{\mathrm{floor}\left(\frac{x}{10}\right)}$$

$$D_1(x) := \left(Z_o\cdot Z_1 + Z_o\cdot Z_2\right)\cdot\cos(\beta(x)\cdot l) + j\cdot\left(Z_o^{\,2} + Z_1\cdot Z_2\right)\cdot\sin(\beta(x)\cdot l)$$

$$I_{sl1}(x) := \frac{1}{D_1(x)\cdot\beta(x)}\cdot\left[e^{-j\cdot\left(\frac{\beta(x)\cdot d}{2}\right)\cdot\sin(\theta)} - e^{j\cdot\left(\frac{\beta(x)\cdot d}{2}\right)\cdot\sin(\theta)}\right]\cdot\left(Z_o\cdot\sin(\beta(x)\cdot l) - j\cdot Z_1\cdot\cos(\beta(x)\cdot l) + j\cdot Z_1\right)$$



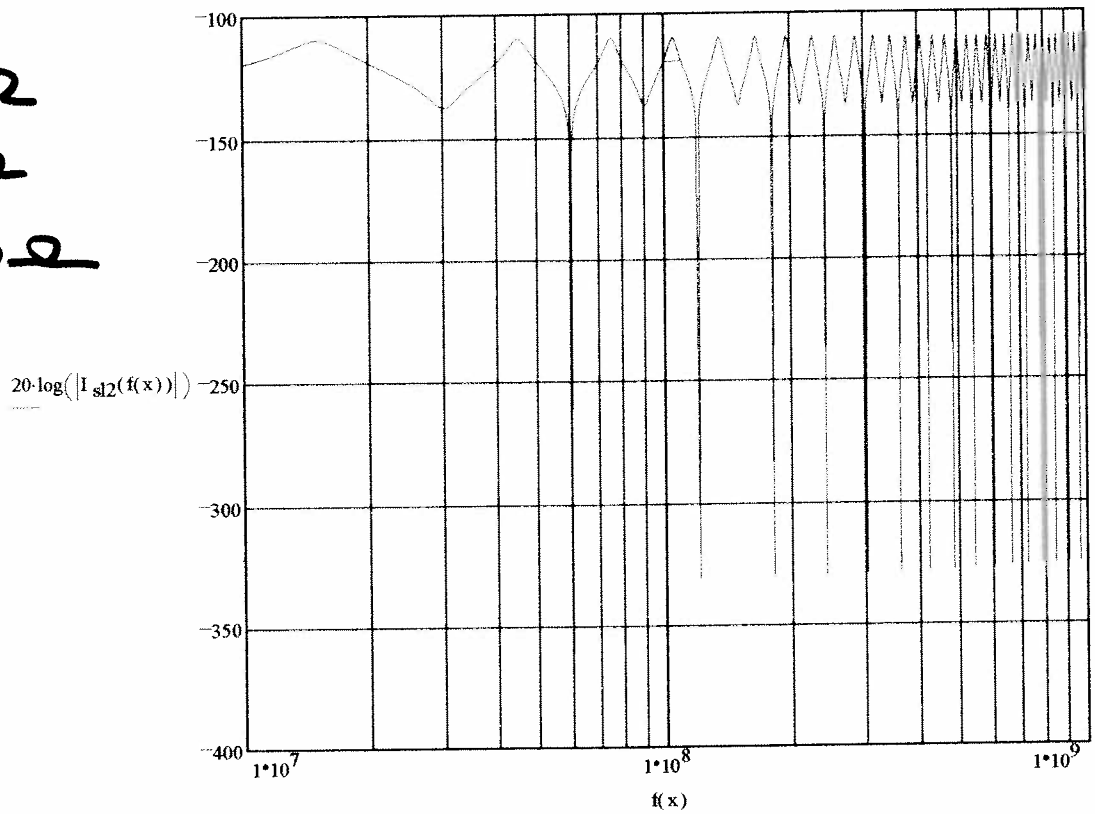
$$D_2(x) := \left(Z_o \cdot \frac{Z_1}{10} + Z_o \cdot Z_2 \cdot 10\right)\cdot \cos(\beta(x)\cdot l) + j\cdot\left(Z_o^{2} + \frac{Z_1}{10}\cdot Z_2\cdot 10\right)\cdot\sin(\beta(x)\cdot l)$$

$$I_{sl2}(x) := \frac{1}{D_2(x)\cdot\beta(x)}\cdot\left[e^{-j\cdot\left(\frac{\beta(x)\cdot d}{2}\right)\cdot\sin(\theta)} - e^{j\cdot\left(\frac{\beta(x)\cdot d}{2}\right)\cdot\sin(\theta)}\right]\cdot\left(Z_o\cdot\sin(\beta(x)\cdot l) - j\cdot\frac{Z_1}{10}\cdot\cos(\beta(x)\cdot l) + j\cdot\frac{Z_1}{10}\right)$$

$Z_0 = 300\,\Omega$

$Z_1 = 30\,\Omega$

$Z_2 = 3000\,\Omega$



1. Resonance points are the same for both matched and unmatched cases.

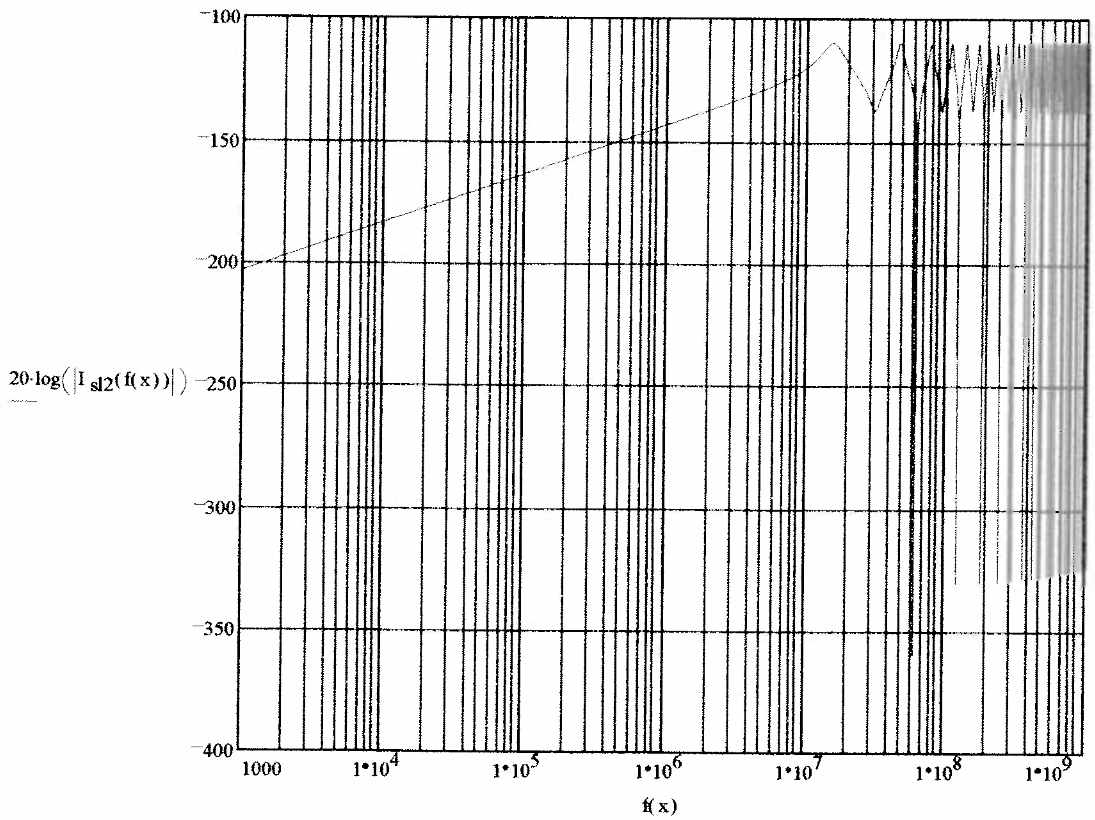
   In the unmatched case the Load current varies more w/ frequency than the matched case.

---

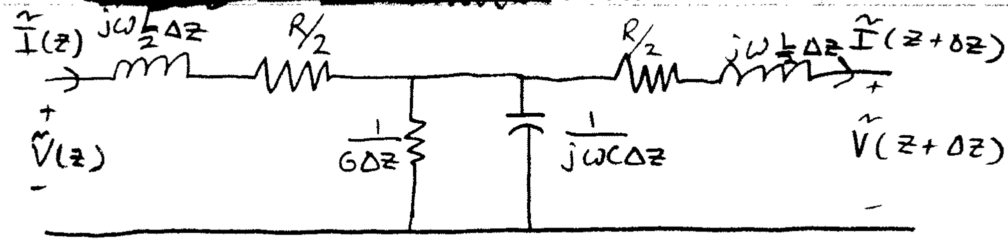
2. If the x-polarized E-Field is the same in magnitude and phase everywhere along the Line then no potential will exist across the Load thus the Load current $= 0$.

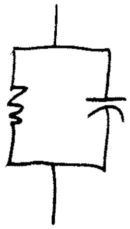
$$D_2(x) := \left(Z_o \cdot \frac{Z_1}{10} + Z_o \cdot Z_2 \cdot 10\right) \cdot \cos(\beta(x) \cdot 1) + j \cdot \left(Z_o^{\,2} + \frac{Z_1}{10} \cdot Z_2 \cdot 10\right) \cdot \sin(\beta(x) \cdot 1)$$

$$I_{sl2}(x) := \frac{1}{D_2(x) \cdot \beta(x)} \cdot \left[e^{-j \cdot \left(\frac{\beta(x) \cdot d}{2}\right) \cdot \sin(\theta)} - e^{j \cdot \left(\frac{\beta(x) \cdot d}{2}\right) \cdot \sin(\theta)}\right] \cdot \left(Z_o \cdot \sin(\beta(x) \cdot 1) - j \cdot \frac{Z_1}{10} \cdot \cos(\beta(x) \cdot 1) + j \cdot \frac{Z_1}{10}\right)$$



[redacted] Ken Kaiser





$$\frac{1}{\frac{1}{\frac{1}{G\Delta z}}+\frac{1}{\frac{1}{j\omega C\Delta z}}} = \frac{1}{G\Delta z + j\omega C\Delta z}$$

KVL: $$-\tilde{V}(z) + \tilde{I}(z)\left[R/2 + j\omega L/2\right]\Delta z + \tilde{I}(z+\Delta z)\left[R/2 + j\omega L/2\right]\Delta z + \tilde{V}(z+\Delta z) = 0$$

$$\tilde{V}(z+\Delta z) - \tilde{V}(z) + \left(\tilde{I}(z) + \tilde{I}(z+\Delta z)\right)\left[R/2 + j\omega L/2\right]\Delta z = 0$$

$$\frac{\tilde{V}(z+\Delta z) - \tilde{V}(z)}{\Delta z} + \left[\tilde{I}(z) + \tilde{I}(z+\Delta z)\right]\left(R/2 + j\omega L/2\right) = 0$$

$$\frac{d\tilde{V}(z)}{dz} + \tilde{I}(z)(R + j\omega L) = 0$$

KCL: $$-\tilde{I}(z) + \tilde{I}(z+\Delta z) + \frac{\left[\tilde{V}(z) - \tilde{I}(z)\left[R/2 + j\omega L/2\right]\Delta z\right]}{\frac{1}{G\Delta z + j\omega C\Delta z}} = 0$$

$$\frac{\tilde{I}(z+\Delta z) - \tilde{I}(z)}{\Delta z} + \left[\tilde{V}(z) - \tilde{I}(z)\left[R/2 + j\omega L/2\right]\Delta z\right](G + j\omega C) = 0$$

$$\frac{d\tilde{I}(z)}{dz} + \tilde{V}(z)(G + j\omega C) = 0$$

$$\frac{d\tilde{I}(z)}{dz} = -\tilde{V}(z)(G + j\omega C)$$

$$\frac{d^2\tilde{V}(z)}{dz^2} + \frac{d\tilde{I}(z)}{dz}(R + j\omega L) = 0$$

$$\frac{d^2\tilde{V}(z)}{dz^2} - (G + j\omega C)(R + j\omega L)\tilde{V}(z) = 0$$

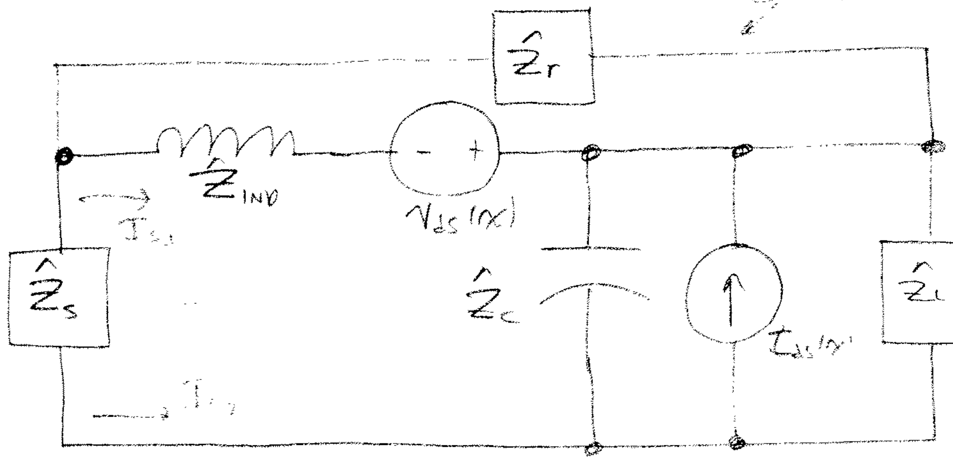
$$\gamma = \sqrt{(G + j\omega C)(R + j\omega L)}$$

$$\boxed{\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0}$$

Problem 16-12 Common Mode Impedance

- the model as given [illegible] Conducted [illegible] is adequate for [illegible] [illegible] [illegible] with [illegible]

goes here (usually leakage capacitance [illegible] [illegible] plane)



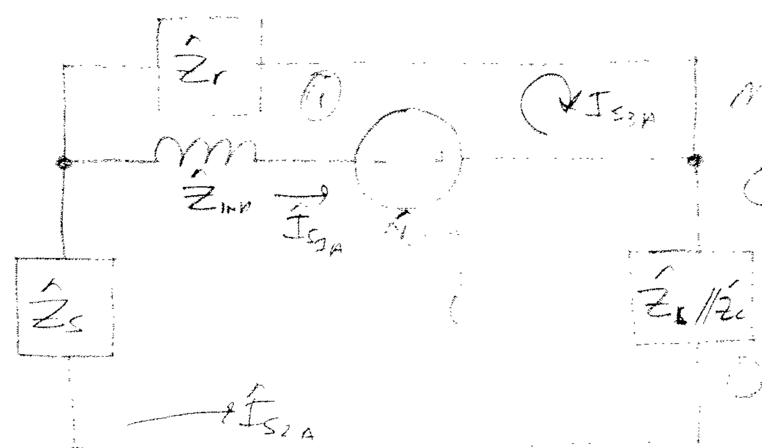
$\hat{Z}_{IND} = j\omega(\Delta x)L$

$\hat{Z}_C = \frac{1}{j\omega C\Delta x}$

$\hat{Z}_L = R_L + jX_L \quad \hat{Z}_r = R_r + jX_r \quad \hat{Z}_S = R_S + jX_S$

as we assumed sinusoidal plane [illegible] with the [illegible] derivation of the circuit given in the appendix, we can model $V_{ds}(x)$ & $I_{ds}(x)$ as phasor-type sources & perform phasor analysis on entire ckt

Part A. Zero $I_{ds}(x)$ (make $I_{ds}(x)$ a S.C.)



mesh (i)

$0 = \hat{I}_{S2A}(\hat{Z}_S + \hat{Z}_L // \hat{Z}_C) + \hat{V}_{ds}(x) + \hat{Z}_{IND}(I_{S2A} + I_{S3A})$

(ii)

$0 = (\hat{I}_{S2A} + \hat{I}_{S3A})\hat{Z}_{IND} + \hat{V}_{ds}(x) + (\hat{Z}_r)\hat{I}_{S3A}$

$\therefore \hat{I}_{S3A}(\hat{Z}_r + \hat{Z}_{IND}) = -\hat{V}_{ds}(x) - \hat{Z}_{IND}\hat{I}_{S2A}$

$\hat{I}_{S3A} = -\frac{\hat{V}_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_{IND}}{\hat{Z}_r + \hat{Z}_{IND}}\hat{I}_{S2A}$ (iii)

$$0 = \hat{I}_{S2A}\left(\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right) + N_{ds}(x) - \hat{Z}_{IND}\left(\hat{I}_{S2A} - \frac{N_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_{IND}}{\hat{Z}_{IND} + \hat{Z}_r}\hat{I}_{S2A}\right)$$

$$0 = \hat{I}_{S2A}\left(\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right] + \hat{Z}_{IND} - \frac{\hat{Z}_{IND}^2}{\hat{Z}_{IND} + \hat{Z}_r}\right)$$

$$+ N_{ds}(x)\left[1 - \frac{1}{\hat{Z}_r + \hat{Z}_{IND}}\right]$$

$$\hat{I}_{S2A} = \frac{N_{ds}(x)\left[\frac{1}{\hat{Z}_r + \hat{Z}_{IND}} - \right]}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right] + \hat{Z}_{IND} - \frac{\hat{Z}_{IND}^2}{Z_{IND} + \hat{Z}_r}}$$

$$\hat{I}_{S2A} = \frac{N_{ds}(x)\left[1 - \hat{Z}_r - \hat{Z}_{IND}\right]}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right]\left[\hat{Z}_{IND} + \hat{Z}_r\right] + \hat{Z}_{IND}\hat{Z}_r}$$

$$\hat{I}_{S3A} = -\frac{\hat{N}_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \left(\frac{\hat{Z}_{IND}}{\hat{Z}_{IND} + \hat{Z}_r}\right)\left[\frac{N_{ds}(x)\left[1 - \hat{Z}_r - \hat{Z}_{IND}\right]}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right]\left[\hat{Z}_{IND} + \hat{Z}_r\right] + \hat{Z}_{IND}\hat{Z}_r}\right]$$

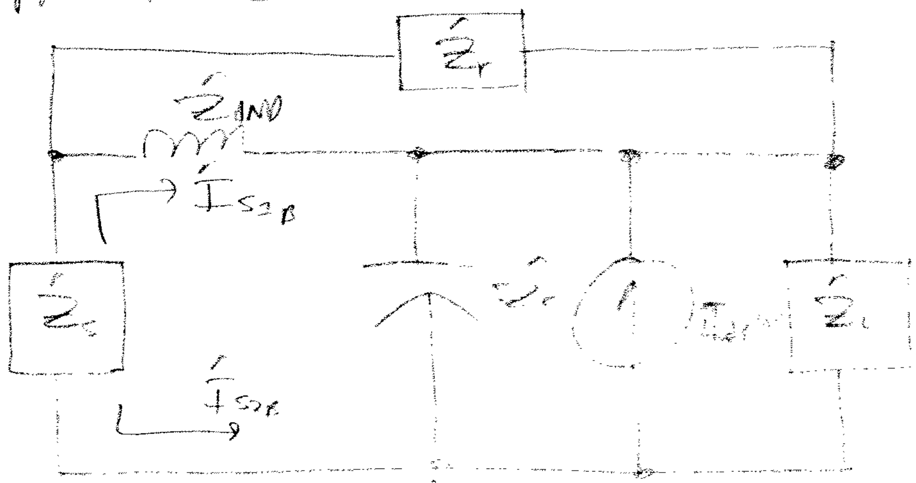
$$\hat{I}_{S1A} = -\left(\hat{I}_{S2A} + \hat{I}_{S3A}\right)$$

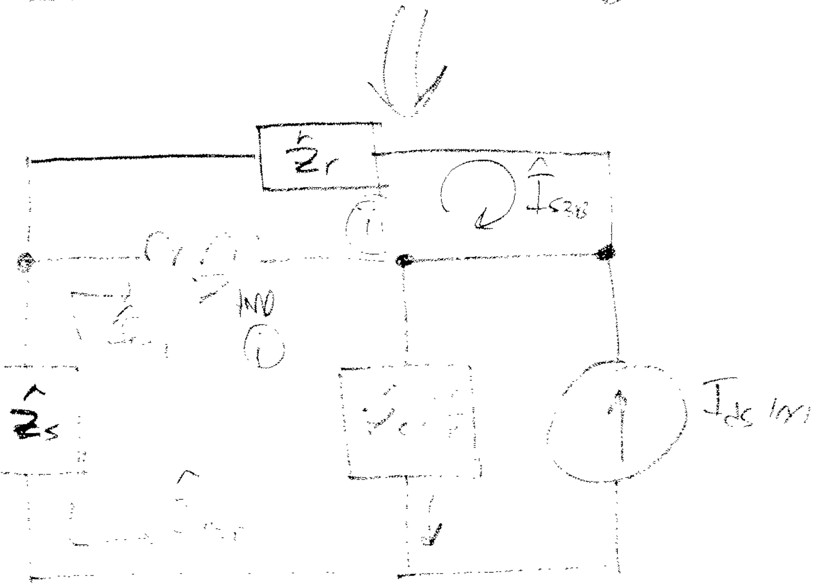
$$= \frac{\hat{N}_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_r}{\hat{Z}_{IND} + \hat{Z}_r}\left[\frac{N_{ds}(x)\left[1 - \hat{Z}_r - \hat{Z}_{IND}\right]}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right]\left[\hat{Z}_{IND} + \hat{Z}_r\right] + \hat{Z}_{IND}\hat{Z}_r}\right]$$

$$= \frac{\hat{N}_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}}\left[1 - \frac{\hat{Z}_r\left[1 - \hat{Z}_r - \hat{Z}_{IND}\right]}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right]\left[\hat{Z}_{IND} + \hat{Z}_r\right] + \hat{Z}_{IND}\hat{Z}_r}\right]$$

$$= \frac{\hat{N}_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_r}{\hat{Z}_{IND} + \hat{Z}_r}\hat{I}_{S2A}$$

Part B: $\hat{Z}_{s2}$ [illegible] (S.C.)





$$(1)\quad 0 = \hat{I}_{s2b}(\hat{Z}_s) + (\hat{I}_{s2b} - I_{dc\,in})\,\hat{Z}_c // \hat{Z}_L + (\hat{I}_{s2b} + \hat{I}_{s2x})\,\hat{Z}_{IND}$$

$$(2)\quad 0 = \hat{I}_{s2x}(\hat{Z}_r) + \left[\hat{I}_{s2x} + \hat{I}_{s2b}\right]\hat{Z}_{IND}$$

from (2): $$\hat{I}_{s2x} = -\hat{I}_{s2b}\left[\frac{\hat{Z}_L}{\hat{Z}_{IND}+\hat{Z}_r}\right]$$

[illegible] $\left[\frac{\hat{Z}_{IND}}{\hat{Z}_{IND}+\hat{Z}_r}\right]\Big)\,\hat{Z}_{IND}$

$$0 = \hat{I}_{s2b}\left[\hat{Z}_s + \hat{Z}_c // \hat{Z}_L\right] - I_{dc\,in}(\hat{Z}_c // \hat{Z}_L) + \hat{I}_{s2b}\left[\frac{\hat{Z}_{IND}\hat{Z}_r}{\hat{Z}_{IND}+\hat{Z}_r}\right]$$

$$0 = \hat{I}_{S2B}\left[\hat{Z}_S + Z_C // Z_L + \frac{\hat{Z}_{IND}\hat{Z}_r}{\hat{Z}_{IND} + \hat{Z}_r}\right] - \boxed{I_{ds}(x)}\,\hat{Z}_C // \hat{Z}_L$$

$$\therefore\ I_{S2B} = \frac{\boxed{I_{ds}(x)}\ \hat{Z}_C // \hat{Z}_L}{\hat{Z}_S + Z_C // Z_L + \frac{\hat{Z}_{IND}\hat{Z}_r}{\hat{Z}_{IND} + \hat{Z}_r}}$$

$$= \frac{\boxed{I_{ds}(x)}\left(\hat{Z}_C // \hat{Z}_L\right)\left(\hat{Z}_{IND} + \hat{Z}_r\right)}{\left(\hat{Z}_S + \hat{Z}_C // \hat{Z}_L\right)\left(\hat{Z}_{IND} + \hat{Z}_r\right) + \hat{Z}_{IND}\hat{Z}_r}$$

$$\hat{I}_{SC2} = -\left(\hat{I}_{S2} + \hat{I}_{SR}\right)$$

$$= -\frac{\hat{Z}_r}{\hat{Z}_{IND} + \hat{Z}_r}\hat{I}_{S2B}$$

TOTAL CURRENTS:

$\boxed{F1}$
$$\hat{I}_{S2} = \hat{I}_{S2A} + \hat{I}_{S2B}$$

TERM 1) TERM

$$= \frac{\hat{N}_{ds}(x)\left[1 - \hat{Z}_r - \hat{Z}_{IND}\right] + \hat{I}_{ds}(x)\left(\hat{Z}_C // \hat{Z}_L\right)\left(\hat{Z}_{IND} + \hat{Z}_r\right)}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C\right]\left[\hat{Z}_{IND} + \hat{Z}_r\right] + \hat{Z}_{IND}\hat{Z}_r}$$

$\boxed{F2}$
$$\hat{I}_{S1} = \hat{I}_{S2A} + \hat{I}_{S2B}$$

$$= \frac{N_{ds}(x)}{Z_r + Z_{IND}} - \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}}\hat{I}_{S2A} - \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}}\hat{I}_{S2B}$$

$$= \frac{N_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}}\left[\hat{I}_{S2A} + \hat{I}_{S2B}\right]$$

$$= \frac{N_{ds}(x) - \hat{Z}_r\left(\hat{I}_{S2}\right)}{\hat{Z}_r + \hat{Z}_{IND}}$$

$$I_C = \frac{I_{S1} + I_{S2}}{2}$$

$$= \frac{1}{2}\left[I_{S2} + \frac{V_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{Z_r}{\hat{Z}_r + Z_{IND}} \hat{I}_{S2} \right]$$

assume $\hat{Z}_r >> \hat{Z}_{IND}$

VERY [illegible] DUE TO LEAKAGE [illegible]

$$\boxed{F3} \quad I_C = \frac{V_{ds}(x)}{2(\hat{Z}_r + \hat{Z}_{IND})} + \frac{\hat{Z}_{IND}}{\hat{Z}_r + \hat{Z}_{IND}} \hat{I}_{S2}$$

[illegible]

$$I_D = \left| \frac{I_{S2} - I_{S1}}{2} \right|$$

$$= \frac{\left| \frac{V_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_r(\hat{I}_{S2})}{\hat{Z}_r + \hat{Z}_{IND}} - \hat{I}_{S2} \right|}{2}$$

$$= \frac{1}{2} \left| \hat{I}_{S2}\left(1 + \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}}\right) - \frac{V_{ds}(x)}{\hat{Z}_r + \hat{Z}_{IND}} \right|$$

$$I_{ds}(x) = -j\omega C \int_0^d \hat{E}^{inc}_{ys}(x,y)\,dy$$

$$V_{ds}(x) = j\omega\mu \int_0^d \hat{H}^{inc}_{zs}(x,y)\,dy$$

for a plane TEM wave [illegible]

[illegible]

I_{ds} & V_{ds} [illegible]

[illegible]

[illegible] $\approx$ [illegible]

OTHERWISE $I_D >> I_C$