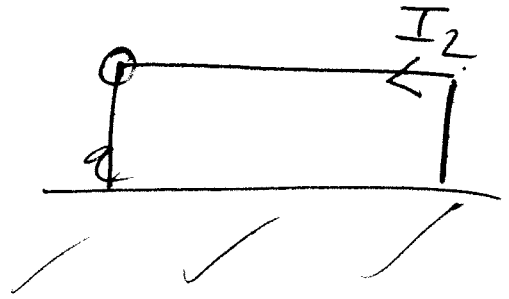


Compare to DeGauyn



p. 317 table

~~short and op~~

let $z = l$

$$\frac{2hEz}{Z_0} B^2 \left(\frac{l}{2} - l\right) l - \frac{2jExh}{Z_0} Bl$$

Need to be negative to agree with me
current direction - or their phase reference
is different

$$-\frac{2hEz}{Z_0} B^2 \left(-\frac{l}{2}\right) l + \frac{2jExh}{Z_0} Bl$$

e.g.
 $E_{z0}(1 + jBl)$

$$\frac{hEz B^2 l^2}{Z_0} + \frac{jExh B l^2}{Z_0}$$

My Method yields ^{Ken Kaiser} \cos of eqn in text

$$E_z \left(\frac{d l^2 B^2}{2Z_0} \right) + j E_x \left(\frac{l B d}{Z_0} \right)$$

$$\begin{array}{c} 0 \\ h \end{array} \Bigg| Z_c = \frac{2A}{60 \sqrt{\mu} \sqrt{\epsilon}} \quad \text{or} \quad \text{my} \quad \begin{array}{c} 0 \\ 0 \end{array} \Bigg| d \quad Z_0 = 120 \sqrt{\mu} \sqrt{\epsilon}$$

If $2h = d$ (images)

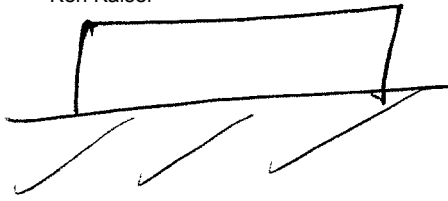
$$\frac{\frac{d}{2} E_z B^2 l^2}{Z_c} + j \frac{E_x \frac{d}{2} B l^2}{Z_c}$$

$$\boxed{\frac{E_z d B^2 l^2}{2 Z_c} + j \frac{E_x d B l^2}{Z_c}}$$

$$\frac{E_z d B^2 l^2}{120 \sqrt{\mu} \sqrt{\epsilon}} + j \frac{E_x d B l^2}{60 \sqrt{\mu} \sqrt{\epsilon}} ?$$

seems reasonable
but probably
needs to be
studied
again

• Net Charge



$$\frac{\sigma d}{-\sigma z_0} E_{z_0} \quad \text{or } E \quad M_z \text{ Math head}$$

$$-\frac{\sigma d E_{x_0}}{-\sigma z_0}$$

$$\frac{d}{z_0} (E_{x_0} - E_{z_0}) + \frac{3j d E_{x_0} B l}{-\sigma z_0}$$

$$\frac{d}{z_0} (E_{x_0} - E_{z_0}) + \frac{-j E_{x_0} d B l}{2z_0}$$

$$\frac{2h E_z}{z_c} - \frac{2j E_x h B (L)}{z_c}$$

$$\frac{2h E_z}{z_c} - \frac{j E_x h l B}{z_c}$$

• IF $2h=d$

$$\frac{d E_z}{z_c} - \frac{j E_x d l B}{2z_c}$$

see MZ
 NPW
 $E_x(x_0) = E_{x_0}^{inc}$
 $E_x(x_l) = E_{x_0}^{ref}$
 Same as my but I have one additional term
 those don't show are the same



$$|y_{ga}(x)| = \frac{w(x)}{w(a)}$$

$$|y_{ro}(x)| = \frac{w(x)}{w(r)}$$

my

$$\frac{b}{a} = h$$

$$b = 2h$$

$$Z_0 = 2 \int_0^a y |y_{ro}(y)| dy = 2 \int_0^a y \left(\frac{y}{a} \right) dy$$

$$= 2 \int_0^a \frac{y^2}{a} dy$$

$$= 2 \left(\frac{1}{3} \right) \frac{a^3}{a}$$

$$= \frac{2}{3} a^2$$

$$= \frac{2}{3} \left(\frac{2h}{a} \right) \frac{a^3}{a}$$

$$= \frac{2}{3} (2a^2) = \frac{4}{3} a^2$$

my

Apply Z for my

Smith

But also

$$\frac{Z_1}{\alpha} = Z_1$$

and

$$\frac{Z_2}{\alpha} = Z_2$$

$$Z_1 = \alpha Z_1$$

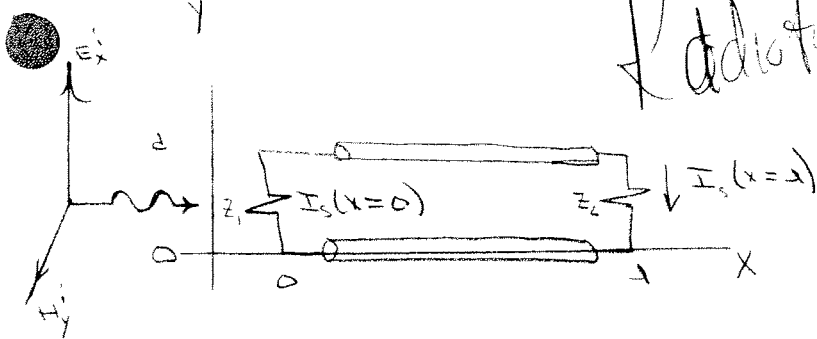
$$Z_2 = \alpha Z_2$$

$$D = (2Z_{om})(\alpha Z_1) \cos \beta_s + j(4Z_{om}^2 + 4Z_1 Z_2) \sin \beta$$

$$= 4(Z_{om} Z_1 \cos \beta_s + j(Z_{om}^2 + Z_1 Z_2) \sin \beta)$$

plug in
 $\beta = 0$
 (no β Smith in mu)
 result obtained

0-29



Reductive

$$I_s(x=d) = \frac{1}{D} \int_0^d K_s(x) [Z_0 \cos \beta x + j Z_1 \sin \beta x] dx - \frac{1}{D} [Z_0 \cos \beta d + j Z_1 \sin \beta d] \int_0^d E_{y_s}^{inc}(d,y) dy + \frac{1}{D} \int_0^d E_{y_s}^{ref}(d,y) dy$$

E_y IS CONSTANT SO $\int_0^d E_y^{inc}(d,y) dy = d E_y^{inc}(d)$

$K_s(x) = E_x^{inc}(x,d) - E_x^{ref}(x,d)$ IS ZERO SINCE \vec{E} IS IN y -DIRECTION
SO WHOLE 1ST TERM DROPS OUT

$$I_s(x=d) = -\frac{1}{D} [Z_0 \cos \beta d + j Z_1 \sin \beta d] d E_y^{inc}(d) + \frac{Z_0 d}{2} E_y^{inc}(d)$$

$$I_s(x=d) = \frac{Z_0 d E_y^{inc}(d) - d E_y^{inc}(d) [Z_0 \cos \beta d + j Z_1 \sin \beta d]}{D}$$

$$I_s(x=d) = \frac{Z_0 d E_y^{inc}(d) - d E_y^{inc}(d) Z_0 \cos \beta d - j Z_1 \sin \beta d d E_y^{inc}(d)}{D}$$

$$E_y^{inc}(d,d) = E_y^{inc}$$

$$E_y^{inc}(d,d) = E_y^i e^{-j\beta d \sin \theta}$$

$$H = \frac{bE_x}{2D} \left[z_0 - z_1 \cos[\beta s(1 + \sin \phi)] - (z_0 - z_1) \cos[\beta s(1 - \sin \phi)] \right]$$

$$+ j(z_0 - z_1) \sin[\beta s(1 + \sin \phi)] - j(z_0 - z_1) \sin[\beta s(1 - \sin \phi)]$$

$$\phi = 90$$

$$\sin \phi = 1$$

$$\sin 0 = 0$$

$$H = \frac{bE_x}{2D} (z_0 - z_1) [1 - \cos 2\beta s, \sin 2\beta s]$$

$$\frac{H}{E_x} = \frac{b(z_0 - z_1)}{2D} [1 - \cos 2\beta s, \sin 2\beta s]$$

$$j = \sqrt{-1}$$

Ken Kaiser

$$Z_0 := 150$$

$$Z_1 := 30$$

$$Z_2 := 3000$$

$$d := 1$$

$$C := 200 \cdot 10^{-12}$$

$$L := 0.5 \cdot 10^{-6}$$

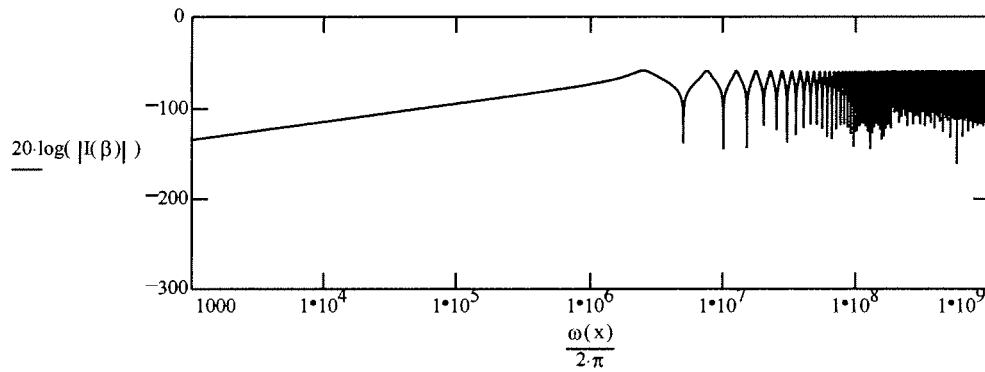
$$x := 0, 0.001 \dots 100$$

$$\omega(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$\beta(\omega) := \omega(x) \cdot \sqrt{L \cdot C}$$

$$D(\beta) := (Z_0 \cdot Z_1 + Z_0 \cdot Z_2) \cdot \cos(\beta(\omega) \cdot 10) + j \cdot (Z_0^2 + Z_1 \cdot Z_2) \cdot \sin(\beta(\omega) \cdot 10)$$

$$I(\beta) := \left[\frac{d \cdot (Z_0 - Z_1)}{2 \cdot D(\beta)} \right] \cdot (1 - \cos(2 \cdot \beta(\omega) \cdot 10) + j \cdot \sin(2 \cdot \beta(\omega) \cdot 10))$$



$$j = \sqrt{-1}$$

Ken Kaiser

$$Z_0 = 150$$

$$Z_1 = 300$$

$$Z_2 = 300$$

$$d = 1$$

$$C = 200 \cdot 10^{-12}$$

$$L = 0.5 \cdot 10^{-6}$$

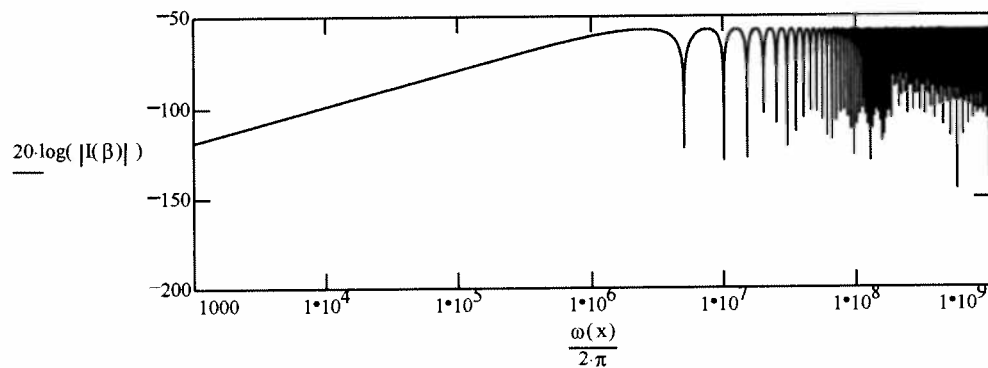
$$x = 0, 0.001 \dots 100$$

$$\omega(x) = \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$\beta(\omega) = \omega(x) \cdot \sqrt{L \cdot C}$$

$$D(\beta) = (Z_0 \cdot Z_1 + Z_0 \cdot Z_2) \cdot \cos(\beta(\omega) \cdot 10) + j \cdot (Z_0^2 + Z_1 \cdot Z_2) \cdot \sin(\beta(\omega) \cdot 10)$$

$$K(\beta) := \left[\frac{d \cdot (Z_0 - Z_1)}{2 \cdot D(\beta)} \right] \cdot (1 - \cos(2 \cdot \beta(\omega) \cdot 10) + j \cdot \sin(2 \cdot \beta(\omega) \cdot 10))$$



Radiated

PROBLEM # 137

$$\text{lgth} := 10 \quad E_{\text{inc}} := 1 \quad j := \sqrt{-1} \quad d := .01 \quad x := 50, 50.005..90$$

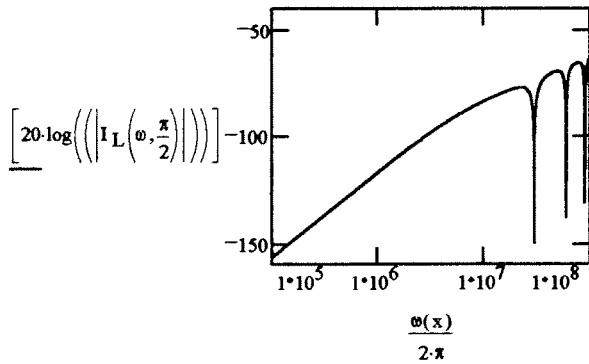
$$\omega(x) := \left[x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right]$$

$$\beta(\omega) := \frac{\omega(x)}{3 \cdot 10^8} \quad Z_0 := 150 \quad Z_s := 300 \quad Z_L := 300$$

$$D(\omega) := (Z_0 \cdot Z_s + Z_0 \cdot Z_L) \cdot \cos(\beta(\omega) \cdot \text{lgth}) + j \cdot (Z_0^2 + Z_s \cdot Z_L) \cdot \sin(\beta(\omega) \cdot \text{lgth})$$

$$K_s(\theta) := (E_{\text{inc}} \cdot e^{(\beta(\omega) \cdot d) - j \cdot \sin(\theta)}) - E_{\text{inc}}$$

$$I_L(\omega, \theta) := \frac{\text{lgth}}{D(\omega)} \cdot (K_s(\theta)) \cdot [Z_0 \cdot \cos(\beta(\omega) \cdot \text{lgth}) + j \cdot Z_s \cdot \sin(\beta(\omega) \cdot \text{lgth})] - \frac{Z_0 \cdot \text{lgth}}{D(\omega)} \cdot K_s(\theta)$$



When theta is 0 (broadside) IL = 0 When x = 0 ,ls = 0

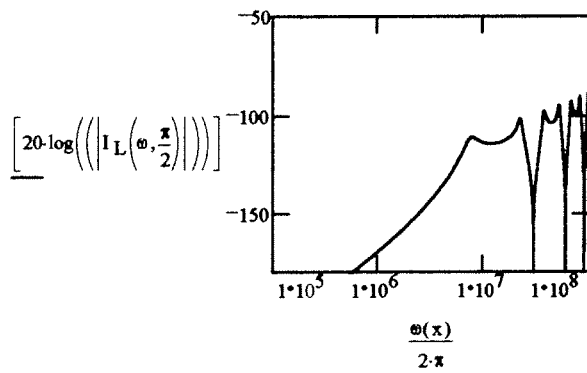
PROBLEM # 137

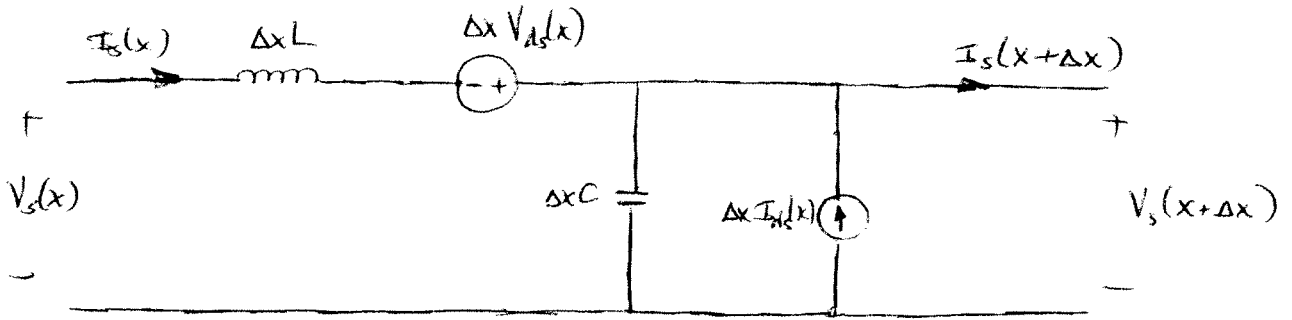
$$Z_o := 150 \quad Z_s := 30 \quad Z_L := 30000$$

$$D(\omega) := (Z_o \cdot Z_s + Z_o \cdot Z_L) \cdot \cos(\beta(\omega) \cdot \text{lgth}) + j \cdot (Z_o^2 + Z_s \cdot Z_L) \cdot \sin(\beta(\omega) \cdot \text{lgth})$$

$$K_s(\theta) := (E_{\text{inc}} \cdot e^{(\beta(\omega) \cdot d) - j \cdot \sin(\theta)}) - E_{\text{inc}}$$

$$I_L(\omega, \theta) := \frac{\text{lgth}}{D(\omega)} \cdot (K_s(\theta)) \cdot [Z_o \cdot \cos(\beta(\omega) \cdot \text{lgth}) + j \cdot Z_s \cdot \sin(\beta(\omega) \cdot \text{lgth})] - \frac{Z_o \cdot \text{lgth}}{D(\omega)} \cdot K_s(\theta)$$





KVL: $V_s(x+\Delta x) - V_s(x) = -j\omega L \Delta x I_s(x) + V_{dc}(x) \Delta x$

divide by Δx : $\frac{V_s(x+\Delta x) - V_s(x)}{\Delta x} = -j\omega L I_s(x) + \frac{V_{dc}(x)}{\Delta x}$

taking the limit as $\Delta x \rightarrow 0$

$$\frac{dV_s(x)}{dx} + j\omega L I_s(x) = V_{dc}(x) = j\omega\mu_0 \int_0^d H_{zs}^{inc}(x,y) dy$$

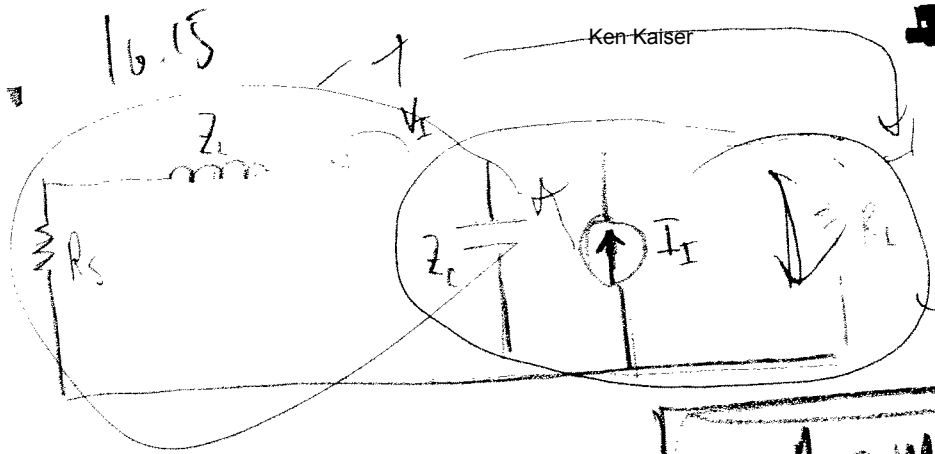
KCL: $I_s(x+\Delta x) - I_s(x) = -j\omega C \Delta x V_s(x+\Delta x) + I_{dc}(x) \Delta x$

divide by Δx : $\frac{I_s(x+\Delta x) - I_s(x)}{\Delta x} = -j\omega C V_s(x+\Delta x) + I_{dc}(x)$

~~$\frac{dI_s(x)}{dx} - j\omega C V_s(x) = I_{dc}(x)$~~

taking limit $\Delta x \rightarrow 0$

$$\frac{dI_s(x)}{dx} + j\omega C V_s(x) - I_{dc}(x) = -j\omega C \int_0^d E_{ys}^{inc}(x,y) dy$$



$V_{INDUCED} = j\omega M I_{STK}$

$I_{INDUCED} = j\omega I_{STK} E_1$

ASSUME
 $100 Z_L < R_L \neq R_S < 0.01 Z_C$

WORST CASE $100 Z_L = R_L = R_S = 0.01 Z_C$

$$V_L = \frac{Z_C R_L}{Z_C + R_L} \cdot \frac{Z_C R_L}{Z_C + R_L + R_S + Z_L} V_I$$

$$V_I \left[\frac{\left(\frac{R_L Z_C}{Z_C + R_L} \right) (R_S + Z_L)}{\frac{R_L Z_C}{Z_C + R_L} + R_S + Z_L} \right] I_I$$

$$\frac{(R_L)(R_S + Z_L)}{(R_L) + (R_S + Z_L)(Z_C + R_L)} Z_C$$

$$\frac{R_S R_L}{R_S R_L} \frac{V_I}{1 + \frac{R_S + Z_L}{Z_C R_L} V_I}$$

$$= \frac{-1}{1 + \frac{R_S}{R_L} + \frac{Z_L}{R_L} + \frac{R_S}{Z_C} + \frac{Z_L}{R_C}} V_I$$

$$= \frac{-1}{1 + \frac{R_S}{R_L} + 0.01 \frac{R_S}{R_L} + 0.01 \frac{R_S}{R_L} + 0.0001 \frac{R_S}{R_L}} V_I$$

$$= \frac{-1}{1 + 1.0201 \frac{R_S}{R_L}} V_I$$

$$\approx \frac{-1}{1 + \frac{R_S}{R_L}} V_I$$

$$\frac{R_L R_S + R_L Z_L}{R_L + R_S + \frac{R_S R_L}{Z_C} + Z_L + \frac{Z_L R_L}{Z_C}}$$

$$\frac{R_L R_S + 0.01 R_L R_S}{R_L + R_S + 0.01 R_S + 0.01 R_L + 0.0001 R_L}$$

$$+ \frac{R_L R_S}{R_L + R_S} \frac{I}{I}$$

WITH THESE ASSUMPTIONS
THIS REALIZES THE SAME FUNCTION POUL GAVE WHEN
NEGLECTING INDUCTANCE AND CAPACITANCE

$$V_L = \frac{-R_L}{R_L + R_S} V_I - \frac{R_S R_C}{R_L + R_S} I_I$$

No 16

WHAT FACTORS DECREASE THE SUSCEPTIBILITY OF AN ELECTRICALLY-SHORT TWIN-LEAD LINE TO FAR-FIELD RADIATION?

$$\frac{dV_s(z)}{dz} = j\omega L_s(z) = j\omega \mu_0 \int_0^d \frac{1}{r_s} I_s(x, y) dy$$

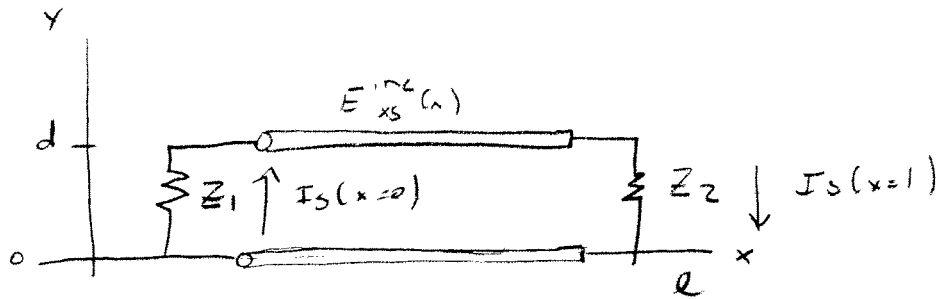
- LESSER INDUCTIVE & CAPACITIVE VALUES
- DECREASED FREQUENCY
- INCREASED DISTANCE TO FIELD
- DECREASED VOLTAGE & CURRENT
- DECREASED LINE LENGTH
- DECREASED PERMEABILITY
- INCREASED WIRE RADIUS
- INCIDENCE ANGLE TO \vec{E} AND \vec{H}
- PRESENCE OF COMMON MODE CURRENTS
- SHIELDING OF THE WIRES

SECTION 16 PROBLEM 31

ELECTRIC FIELD POLARIZED IN X DIRECTION. phase reference is at $y = d/2$

$$E_{xs}^{inc}(x, d) = E_x^{inc} e^{-j(\frac{\beta d}{2}) \sin \theta}$$

$$E_{xs}^{inc}(x, 0) = E_x^{inc} e^{+j(\frac{\beta d}{2}) \sin \theta}$$



$$I_s(x=l) = \frac{1}{D} \int_0^l k_s(x) [Z_0 \cos \beta x + j Z_1 \sin \beta x] dx + \frac{1}{D} [Z_0 \cos \beta l + j Z_1 \sin \beta l] \cdot \int_0^d E_{ys}^{inc}(l, y) dy + \frac{Z_0}{D} \int_0^d E_{ys}^{inc}(0, y) dy$$

$$0 = Z_0 Z_1 + Z_0 Z_2 \cos \beta l + j(Z_1^2 - Z_1 Z_2 \sin \beta l)$$

Z_0 = characteristic impedance.

$$k_s(x) = E_{xs}^{inc}(x, d) - E_{xs}^{inc}(x, 0)$$

For E polarized in x direction Equation reduces to the following

BECAUSE $E_{ys}^{inc} \text{ is } = 0$

$$I_s(x=l) = \frac{k_s(\omega)}{D} \int_0^l [Z_0 \cos \beta x + j Z_1 \sin \beta x] dx$$

$$I_s(x=l) = \frac{k_s(\omega)}{D} \cdot \left[\frac{1}{\beta} Z_0 \sin \beta x \Big|_0^l - \frac{1}{\beta} j Z_1 (\cos \beta x) \Big|_0^l \right]$$

$$= \frac{k_s(\omega)}{\beta D} [Z_0 \sin \beta l - \sin \beta \cdot 0] - j Z_1 (\cos \beta l - \cos 0)$$

$$= \frac{k_s(\omega)}{\beta D} [Z_0 \sin \beta l - 0] - j Z_1 (\cos \beta l - 1)$$

$$I_s(x=l) = \frac{k_s(\omega)}{\beta D} [Z_0 \sin \beta l + j Z_1 (1 - \cos \beta l)]$$

$$k(\omega) = E_{xs}^{inc} e^{-j(\beta d/2) \sin \theta} - E_{xs}^{inc} e^{+j(\beta d/2) \sin \theta}$$

using identity $e^{j\omega} = \cos \omega + j \sin \omega$

$$k(\omega) = E_{xs}^{inc} \left[-j Z_1 \sin \left(\frac{\beta d}{2} \sin \theta \right) \right]$$

Included steps

$$I_s(x=l) = E_{x5}^{inc} \frac{[-jZ_0 \sin(\beta d/2 \sin \theta)]}{\beta D} \cdot [Z_0 \sin \beta l + jZ_1(1 - \cos \beta l)]$$

$$f\lambda = v \quad \lambda = \frac{v}{f}$$

$$\beta = 2\pi/\lambda = \text{phase constant over line.}$$

$$\beta = \omega \sqrt{LC}$$

For 28 gauge wire: $L = 0.75 \mu\text{H}/\text{m}$
SEPARATION OF 50 MILS $C = 14.82 \text{ pF}/\text{m}$

$$\frac{I_s(x=l)}{E_{x5}^{inc}} = - \frac{jZ_0 \sin \beta d/2}{\beta D} [Z_0 \sin \beta l + jZ_1(1 - \cos \beta l)]$$

For 10 meter long line $L = 7.5 \mu\text{H}$ $C = 0.1482 \text{ nF}$

- ARE THE CURRENTS EQUAL TO ZERO WHEN THE SOURCE AND LOAD ARE MATCHED TO THE LINE?

- FROM MATHCAD ANALYSIS

AS can be seen from the graph the currents are not zero and vary with frequency when the source and loads are matched to the line.

- WHAT ARE THE CURRENTS IF THE ELECTRIC FIELD IS THE SAME (IN PHASE AND MAGNITUDE) EVERYWHERE ALONG THE LINE?

THROUGH THIS QUESTION ASSUME THE LINE IS ELECTRICALLY SHORT. ON AN ELECTRICALLY SHORT LINE THE WAVELIKE CHARACTERISTICS OF THE E FIELD ARE NOT NOTICEABLE. \vec{E} THE CURRENTS ALONG THE LINE ARE THE SAME BUT THEY STILL VARY WITH FREQUENCY UNTIL THE LINE BECOMES ELECTRICALLY LONG.

$$Z_0 = 150 \quad Z_1 = 300 \quad Z_2 = 300 \quad Z_3 = 300 \quad Z_4 = 3000 \quad Z_5 = 150 \quad Z_6 = 150$$

$$x = 30, 30.01, \dots, 90 \quad j = \sqrt{-1} \quad d = 1.27 \cdot 10^{-3} \quad l = 10$$

$$\omega(x) = \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$L = 7.5 \cdot 10^{-6} \quad C = .1482 \cdot 10^{-9}$$

$$\beta(\omega) = \omega(x) \cdot \sqrt{L \cdot C}$$

$$D_1(\omega) = (Z_0 \cdot Z_1 + Z_0 \cdot Z_2) \cdot \cos(\beta(\omega) \cdot l) + j \cdot (Z_0^2 + Z_1 \cdot Z_2) \cdot \sin(\beta(\omega) \cdot l)$$

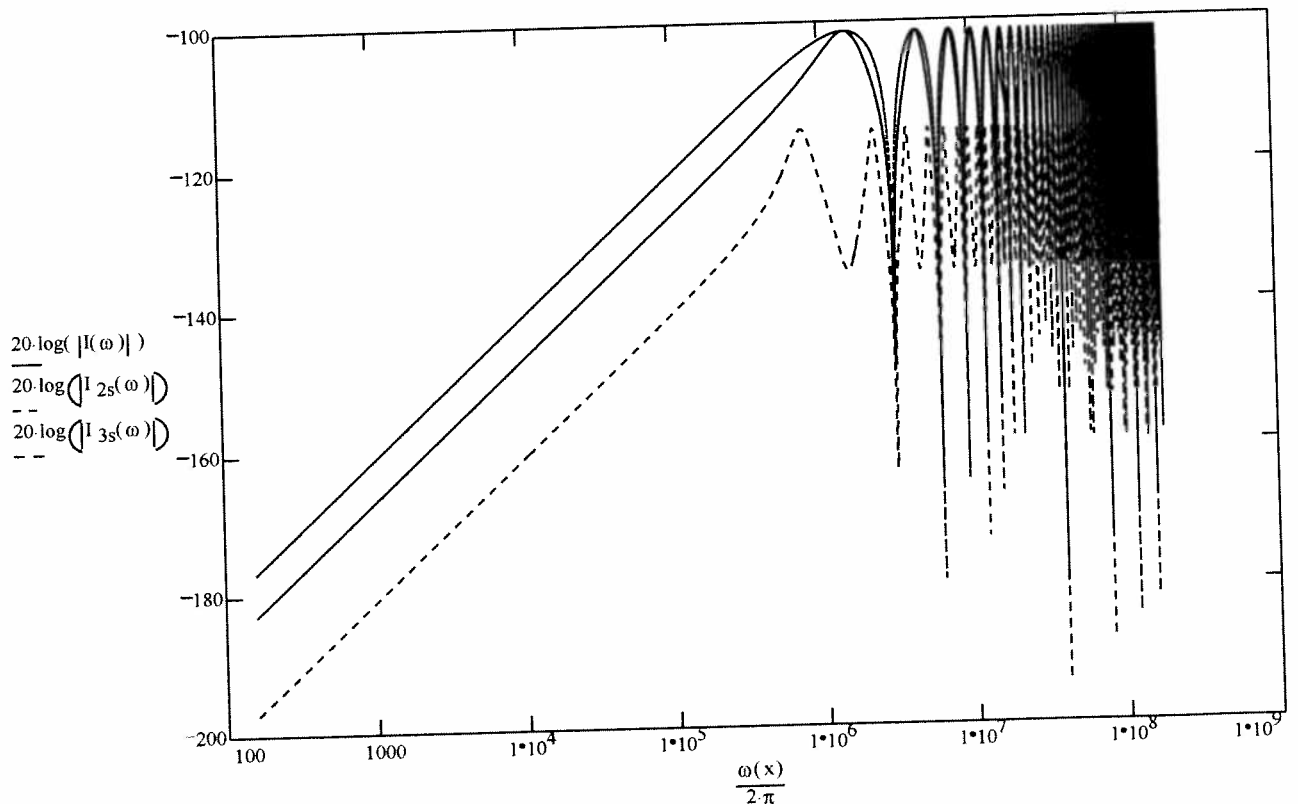
$$D_2(\omega) = (Z_0 \cdot Z_3 + Z_0 \cdot Z_4) \cdot \cos(\beta(\omega) \cdot l) + j \cdot (Z_0^2 + Z_3 \cdot Z_4) \cdot \sin(\beta(\omega) \cdot l)$$

$$D_3(\omega) = (Z_0 \cdot Z_5 + Z_0 \cdot Z_6) \cdot \cos(\beta(\omega) \cdot l) + j \cdot (Z_0^2 + Z_5 \cdot Z_6) \cdot \sin(\beta(\omega) \cdot l)$$

$$I(\omega) = \frac{-j \cdot 2 \cdot \sin\left(\beta(\omega) \cdot \frac{d}{2}\right)}{\beta(\omega) \cdot D_1(\omega)} \cdot [Z_0 \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_1 \cdot (1 - \cos(\beta(\omega) \cdot l))]$$

$$I_{2s}(\omega) = \frac{-j \cdot 2 \cdot \sin\left(\beta(\omega) \cdot \frac{d}{2}\right)}{\beta(\omega) \cdot D_2(\omega)} \cdot [Z_0 \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_3 \cdot (1 - \cos(\beta(\omega) \cdot l))]$$

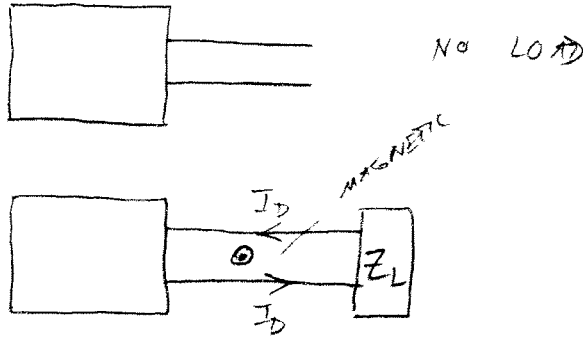
$$I_{3s}(\omega) = \frac{-j \cdot 2 \cdot \sin\left(\beta(\omega) \cdot \frac{d}{2}\right)}{\beta(\omega) \cdot D_3(\omega)} \cdot [Z_0 \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_5 \cdot (1 - \cos(\beta(\omega) \cdot l))]$$



00mHz = 1m

16.4

To determine whether an interfering signal was common-mode or differential mode signal, the load was removed. Noise level remained constant. What can be concluded?



- 1) NOISE SOURCE MAY NOT BE ASSOCIATED WITH LOAD
- 2) IF ASSOCIATED WITH CONNECTIONS TO LOAD, NOT DIFFERENTIAL MODE CAUSED BY MAGNETIC FIELDS THROUGH LOOP.
- 3) COULD BE COMMON-MODE CURRENTS DUE TO PROXIMITY EFFECTS TO GROUND, POWER, OTHER SIGNALS

4-164
4-280
4-281
4-282
4-283
4-284
4-285
4-286
4-287
4-288
4-289
4-290
4-291
4-292
4-293
4-294
4-295
4-296
4-297
4-298
4-299
4-300
4-301
4-302
4-303
4-304
4-305
4-306
4-307
4-308
4-309
4-310
4-311
4-312
4-313
4-314
4-315
4-316
4-317
4-318
4-319
4-320
4-321
4-322
4-323
4-324
4-325
4-326
4-327
4-328
4-329
4-330
4-331
4-332
4-333
4-334
4-335
4-336
4-337
4-338
4-339
4-340
4-341
4-342
4-343
4-344
4-345
4-346
4-347
4-348
4-349
4-350
4-351
4-352
4-353
4-354
4-355
4-356
4-357
4-358
4-359
4-360
4-361
4-362
4-363
4-364
4-365
4-366
4-367
4-368
4-369
4-370
4-371
4-372
4-373
4-374
4-375
4-376
4-377
4-378
4-379
4-380
4-381
4-382
4-383
4-384
4-385
4-386
4-387
4-388
4-389
4-390
4-391
4-392
4-393
4-394
4-395
4-396
4-397
4-398
4-399
4-400
4-401
4-402
4-403
4-404
4-405
4-406
4-407
4-408
4-409
4-410
4-411
4-412
4-413
4-414
4-415
4-416
4-417
4-418
4-419
4-420
4-421
4-422
4-423
4-424
4-425
4-426
4-427
4-428
4-429
4-430
4-431
4-432
4-433
4-434
4-435
4-436
4-437
4-438
4-439
4-440
4-441
4-442
4-443
4-444
4-445
4-446
4-447
4-448
4-449
4-450
4-451
4-452
4-453
4-454
4-455
4-456
4-457
4-458
4-459
4-460
4-461
4-462
4-463
4-464
4-465
4-466
4-467
4-468
4-469
4-470
4-471
4-472
4-473
4-474
4-475
4-476
4-477
4-478
4-479
4-480
4-481
4-482
4-483
4-484
4-485
4-486
4-487
4-488
4-489
4-490
4-491
4-492
4-493
4-494
4-495
4-496
4-497
4-498
4-499
4-500
4-501
4-502
4-503
4-504
4-505
4-506
4-507
4-508
4-509
4-510
4-511
4-512
4-513
4-514
4-515
4-516
4-517
4-518
4-519
4-520
4-521
4-522
4-523
4-524
4-525
4-526
4-527
4-528
4-529
4-530
4-531
4-532
4-533
4-534
4-535
4-536
4-537
4-538
4-539
4-540
4-541
4-542
4-543
4-544
4-545
4-546
4-547
4-548
4-549
4-550
4-551
4-552
4-553
4-554
4-555
4-556
4-557
4-558
4-559
4-560
4-561
4-562
4-563
4-564
4-565
4-566
4-567
4-568
4-569
4-570
4-571
4-572
4-573
4-574
4-575
4-576
4-577
4-578
4-579
4-580
4-581
4-582
4-583
4-584
4-585
4-586
4-587
4-588
4-589
4-590
4-591
4-592
4-593
4-594
4-595
4-596
4-597
4-598
4-599
4-600
4-601
4-602
4-603
4-604
4-605
4-606
4-607
4-608
4-609
4-610
4-611
4-612
4-613
4-614
4-615
4-616
4-617
4-618
4-619
4-620
4-621
4-622
4-623
4-624
4-625
4-626
4-627
4-628
4-629
4-630
4-631
4-632
4-633
4-634
4-635
4-636
4-637
4-638
4-639
4-640
4-641
4-642
4-643
4-644
4-645
4-646
4-647
4-648
4-649
4-650
4-651
4-652
4-653
4-654
4-655
4-656
4-657
4-658
4-659
4-660
4-661
4-662
4-663
4-664
4-665
4-666
4-667
4-668
4-669
4-670
4-671
4-672
4-673
4-674
4-675
4-676
4-677
4-678
4-679
4-680
4-681
4-682
4-683
4-684
4-685
4-686
4-687
4-688
4-689
4-690
4-691
4-692
4-693
4-694
4-695
4-696
4-697
4-698
4-699
4-700
4-701
4-702
4-703
4-704
4-705
4-706
4-707
4-708
4-709
4-710
4-711
4-712
4-713
4-714
4-715
4-716
4-717
4-718
4-719
4-720
4-721
4-722
4-723
4-724
4-725
4-726
4-727
4-728
4-729
4-730
4-731
4-732
4-733
4-734
4-735
4-736
4-737
4-738
4-739
4-740
4-741
4-742
4-743
4-744
4-745
4-746
4-747
4-748
4-749
4-750
4-751
4-752
4-753
4-754
4-755
4-756
4-757
4-758
4-759
4-760
4-761
4-762
4-763
4-764
4-765
4-766
4-767
4-768
4-769
4-770
4-771
4-772
4-773
4-774
4-775
4-776
4-777
4-778
4-779
4-780
4-781
4-782
4-783
4-784
4-785
4-786
4-787
4-788
4-789
4-790
4-791
4-792
4-793
4-794
4-795
4-796
4-797
4-798
4-799
4-800
4-801
4-802
4-803
4-804
4-805
4-806
4-807
4-808
4-809
4-810
4-811
4-812
4-813
4-814
4-815
4-816
4-817
4-818
4-819
4-820
4-821
4-822
4-823
4-824
4-825
4-826
4-827
4-828
4-829
4-830
4-831
4-832
4-833
4-834
4-835
4-836
4-837
4-838
4-839
4-840
4-841
4-842
4-843
4-844
4-845
4-846
4-847
4-848
4-849
4-850
4-851
4-852
4-853
4-854
4-855
4-856
4-857
4-858
4-859
4-860
4-861
4-862
4-863
4-864
4-865
4-866
4-867
4-868
4-869
4-870
4-871
4-872
4-873
4-874
4-875
4-876
4-877
4-878
4-879
4-880
4-881
4-882
4-883
4-884
4-885
4-886
4-887
4-888
4-889
4-890
4-891
4-892
4-893
4-894
4-895
4-896
4-897
4-898
4-899
4-900
4-901
4-902
4-903
4-904
4-905
4-906
4-907
4-908
4-909
4-910
4-911
4-912
4-913
4-914
4-915
4-916
4-917
4-918
4-919
4-920
4-921
4-922
4-923
4-924
4-925
4-926
4-927
4-928
4-929
4-930
4-931
4-932
4-933
4-934
4-935
4-936
4-937
4-938
4-939
4-940
4-941
4-942
4-943
4-944
4-945
4-946
4-947
4-948
4-949
4-950
4-951
4-952
4-953
4-954
4-955
4-956
4-957
4-958
4-959
4-960
4-961
4-962
4-963
4-964
4-965
4-966
4-967
4-968
4-969
4-970
4-971
4-972
4-973
4-974
4-975
4-976
4-977
4-978
4-979
4-980
4-981
4-982
4-983
4-984
4-985
4-986
4-987
4-988
4-989
4-990
4-991
4-992
4-993
4-994
4-995
4-996
4-997
4-998
4-999
5-000



15.12

15.12

$$\text{lgth} := 10 \quad E_{\text{inc}} := 1 \quad j := \sqrt{-1} \quad d := .005$$

$$x := 50, 50.005 .. 90$$

$$\omega(x) := \left[x + 1 - 10 \cdot \text{floor} \left(\left(\frac{x}{10} \right) \right) \right] \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

$$\beta(\omega) := \frac{\omega(x)}{3 \cdot 10^8}$$

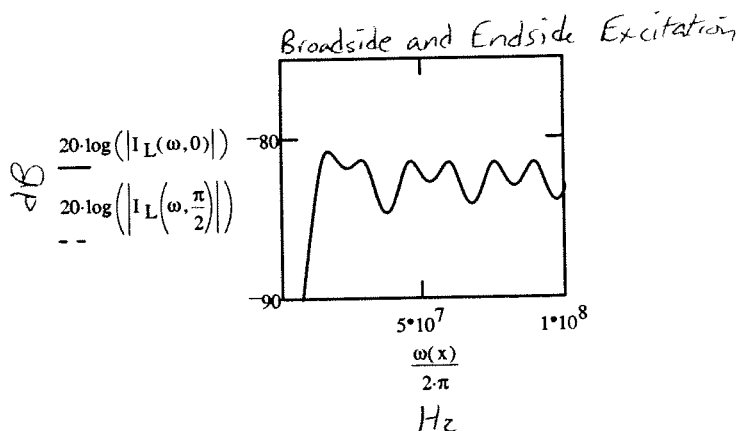
$$Z_0 := 150 \quad Z_s := 300 \quad Z_1 := 300$$

$$D(\omega) := (Z_0 \cdot Z_s + Z_0 \cdot Z_1) \cdot \cos(\beta(\omega) \cdot \text{lgth}) + j \cdot (Z_0^2 + Z_s \cdot Z_1) \cdot \sin(\beta(\omega) \cdot \text{lgth})$$

$$E_{\text{xs}}(d, \theta) := E_{\text{inc}} \cdot e^{-j \frac{\beta(\omega) \cdot d \cdot \sin(\theta)}{2}} \quad E_{\text{xs}}(d, \theta) := E_{\text{inc}} \cdot e^{\frac{j \beta(\omega) \cdot d \cdot \sin(\theta)}{2}}$$

$$K_s(x) := E_{\text{xs}}(x, d) - E_{\text{xs}}(x, 0)$$

$$I_L(\omega, \theta) := \frac{1}{D(\omega)} \int_0^{\text{lgth}} [K_s(x) \cdot [Z_0 \cdot (\cos(\beta(\omega) \cdot (\text{lgth} - x))) + j \cdot Z_1 \cdot \sin(\beta(\omega) \cdot (\text{lgth} - x))]] dx$$



Wave polarized in x-direction

Graph appears to be independent of θ

15.12

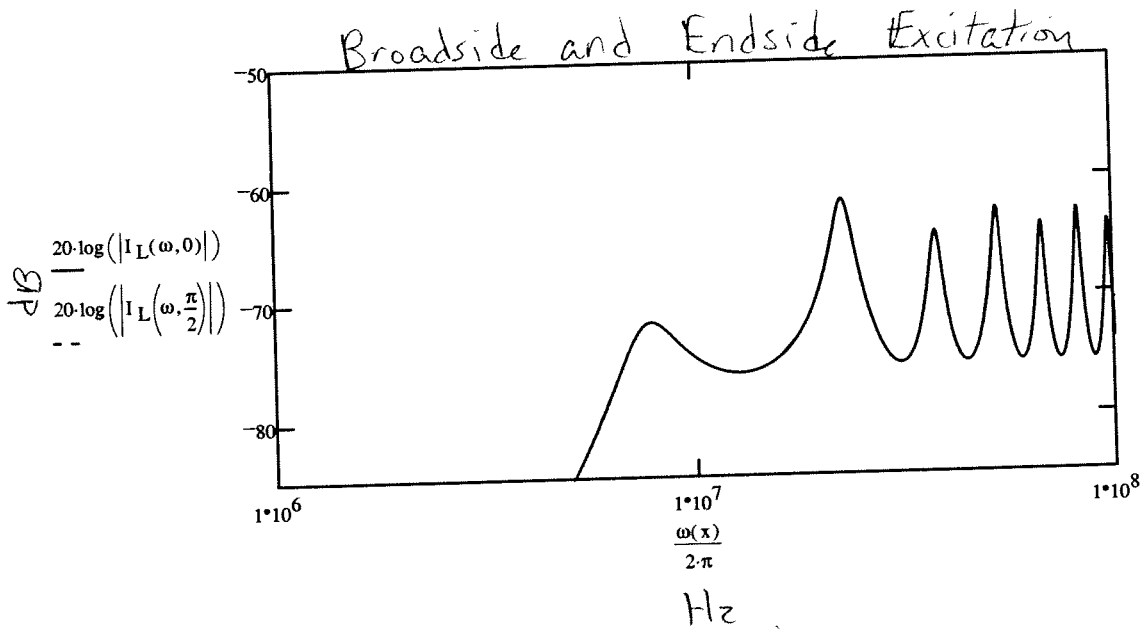
$$Z_0 := 150 \quad Z_s := 30 \quad Z_1 := 3000$$

$$D(\omega) := (Z_0 \cdot Z_s + Z_0 \cdot Z_1) \cdot \cos(\beta(\omega) \cdot \text{lgth}) + j \cdot (Z_0^2 + Z_s \cdot Z_1) \cdot \sin(\beta(\omega) \cdot \text{lgth})$$

$$E_{XS}(d, \theta) := E_{inc} \cdot e^{-j \frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)} \quad E_{XS}(d, \theta) := E_{inc} \cdot e^{j \frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)}$$

$$K_s(x) := E_{XS}(x, d) - E_{XS}(x, 0)$$

$$I_L(\omega, \theta) := \frac{1}{D(\omega)} \int_0^{\text{lgth}} [K_s(x) \cdot [Z_0 \cdot (\cos(\beta(\omega) \cdot (\text{lgth} - x))) + j \cdot Z_1 \cdot \sin(\beta(\omega) \cdot (\text{lgth} - x))]] dx$$



Wave polarized in the x-direction

Graph appears to be independent of θ

Section 16

(*21)

$$I_s(x=0) = \frac{1}{D} \int_0^l k_s(x) [z_0 \cos \beta(l-x) + j z_L \sin \beta(l-x)] dx + \frac{1}{D} [z_0 \cos \beta l + j z_L \sin \beta l] \int_0^d E_{ys}^{inc}(y) dy - \frac{z_0}{D} \int_0^d E_{ys}^{inc}(l, y) dy$$

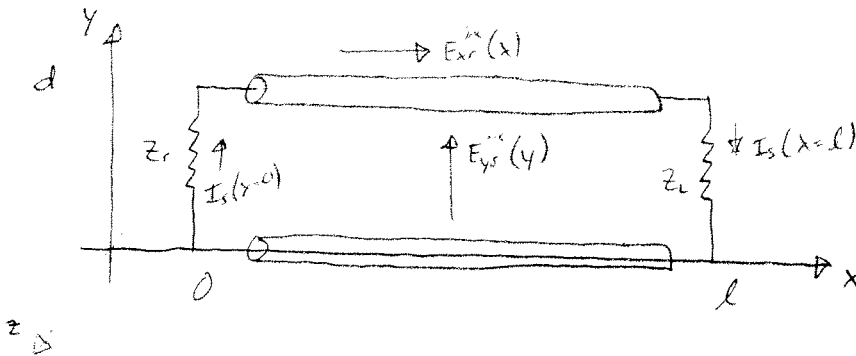
$$I_s(x=l) = \frac{1}{D} \int_0^l k_s(x) [z_0 \cos \beta x + j z_L \sin \beta x] dx + -\frac{1}{D} [z_0 \cos \beta l + j z_L \sin \beta l] \int_0^d E_{ys}^{inc}(l, y) dy + \frac{z_0}{D} \int_0^d E_{ys}^{inc}(0, y) dy$$

$$D = (z_0 z_s + z_0 z_L) \cos \beta l + j (z_0^2 + z_0 z_L) \sin \beta l$$

$$k_s(x) = E_{xs}^{inc}(x, d) - E_{xs}^{inc}(x, 0)$$

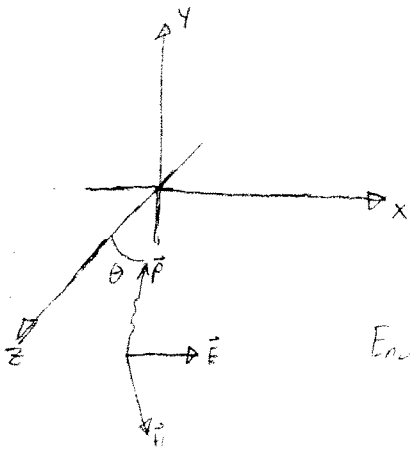
$E_{xs}^{inc}(x, d)$ = incident elec. field in x-dir. along upper conductor

$E_{xs}^{inc}(x, 0)$ = incident elec. field in x-dir. along lower conductor



Electric Field polarized in x-direction (parallel w/ transmission line) + phase ref. at $y = d/2$

$$E_{xs}^{inc}(x, d) = E_x^{inc} e^{-j(\frac{\beta d}{2}) \sin \theta} \quad E_{xs}^{inc}(x, 0) = E_x^{inc} e^{+j(\frac{\beta d}{2}) \sin \theta}$$



Broadside excitation: $\theta = 0^\circ$ - z travel

$$E_{xs}^{inc}(x, d) = E_x^{inc} e^0 \quad E_{xs}^{inc}(x, 0) = E_x^{inc} e^0$$

Volt. induced along line

$$V = - \int_0^l \vec{E} \cdot d\vec{l} \approx -l E_x^{inc}$$

Endfire excitation: $\theta = 90^\circ$

can't have endfire w/ E-field polarized in x-direction.

$E_{ys} = 0$ + E_{zs} is uniform over each conductor
 4 terms go to zero, left only w/ x

$$I_s(x=0) = \frac{1}{D} \int_0^l k_s(x) [Z_0 \cos \beta(l-x) + j \sin \beta(l-x)] dx$$

$$I_s(x=l) = \frac{1}{D} \int_0^l k_s(x) [Z_0 \cos \beta x + j Z_s \sin \beta x] dx$$

$$k_s(x) = \frac{E_x^{inc}}{E_x^{inc}} \left[e^{-j \frac{\beta D}{2} \sin \theta} - e^{+j \frac{\beta D}{2} \sin \theta} \right] \quad e^{+j\theta} = \cos \theta + j \sin \theta$$

$$= \frac{E_x^{inc}}{E_x^{inc}} \left[-j 2 \sin \left(\frac{\beta D}{2} \sin \theta \right) \right]$$

$$I_s(x=0) = \frac{k_s(x)}{D} \int_0^l [Z_0 \cos \beta(l-x) + j \sin \beta(l-x)] dx = \frac{k_s(x)}{D} [j - (-Z_0 \sin \beta l + j \cos \beta l)]$$

$$I_s(x=l) = \frac{k_s(x)}{D} \int_0^l [Z_0 \cos \beta x + j Z_s \sin \beta x] dx = \frac{k_s(x)}{D} [Z_0 \sin \beta l + j Z_s (1 - \cos \beta l)]$$

↑ obtained from Smith's book

electrically long line

$$Z_0 = 150$$

$$1) Z_s = Z_L = 300$$

$$2) Z_s = 30 \quad Z_L = 3000$$

$$\text{let } l = 10m$$

lossless line

$$\lambda = \frac{v}{f}$$

freq corresponding to 10m λ

$$f = 30MHz$$

$f \leq 30MHz$ for l to be electrically short

Section 16, number #21:

Determine value for current at source end:

$$j = \sqrt{-1}$$

$$I_{s0} = \frac{K_s(x)}{D} \int_0^1 [Z_o \cdot \cos(\beta \cdot (1-x)) + j \cdot \sin(\beta \cdot (1-x))] dx$$

$$I_{s0} = \frac{K_s(x)}{D} \left[\frac{j}{\beta} \frac{(-\sin(\beta \cdot 1) \cdot Z_o + \cos(\beta \cdot 1) \cdot j)}{\beta} \right]$$

Plot the load current divided by the incident electric field:

$$l = 10 \quad E_{inc} = 1 \quad d = .01 \quad x = 50, 50.005 \dots 90$$

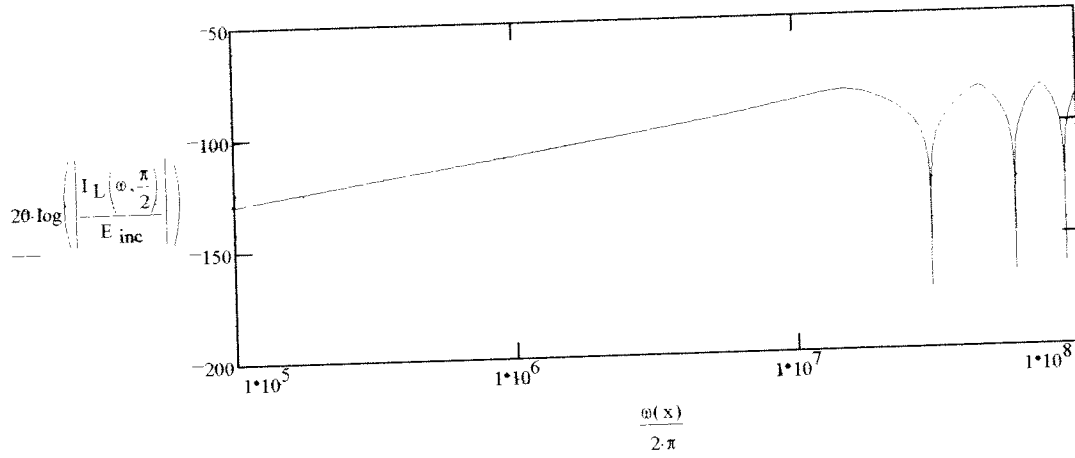
$$\omega(x) = \left(x + 1 - 10 \cdot \text{floor}\left(\frac{x}{10}\right) \right) \cdot 10^{\text{floor}\left(\frac{x}{10}\right)} \quad \beta(\omega) := \frac{\omega(x)}{3 \cdot 10^8}$$

Case 1: $Z_o = 150$, $Z_s = Z_L = 300$

$$Z_o := 150 \quad Z_s := 300 \quad Z_L := 300$$

$$D(\omega) := (Z_o \cdot Z_s + Z_o \cdot Z_L) \cdot \cos(\beta(\omega) \cdot 1) + j \cdot (Z_o^2 + Z_s \cdot Z_L) \cdot \sin(\beta(\omega) \cdot 1)$$

$$I_L(\omega, \theta) = \frac{-j \cdot 2 \cdot \sin\left(\frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)\right)}{\beta(\omega) \cdot D(\omega)} \cdot [Z_o \cdot \sin(\beta(\omega) \cdot 1) + j \cdot Z_s \cdot (1 - \cos(\beta(\omega) \cdot 1))]$$

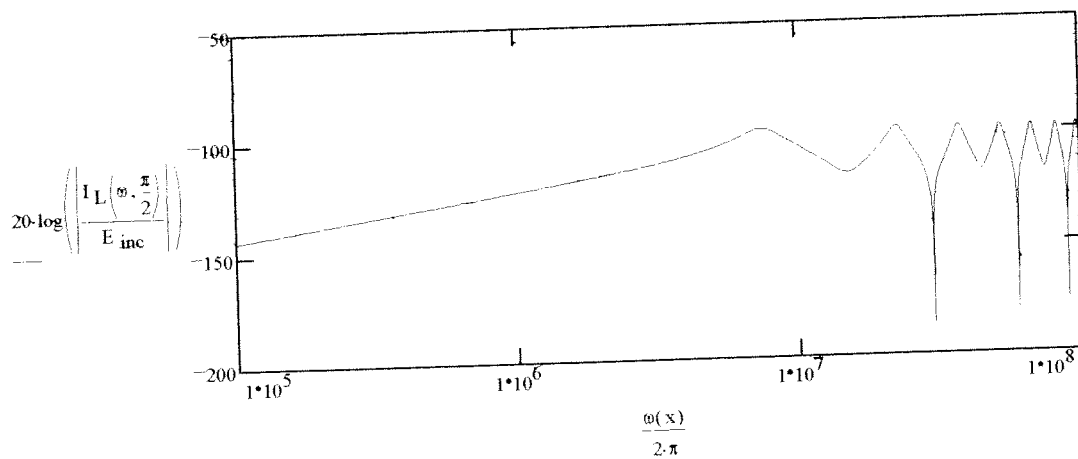


Case 2: $Z_o=150, Z_s=30, Z_L=3000$

$$Z_o = 150 \quad Z_s = 30 \quad Z_L = 3000$$

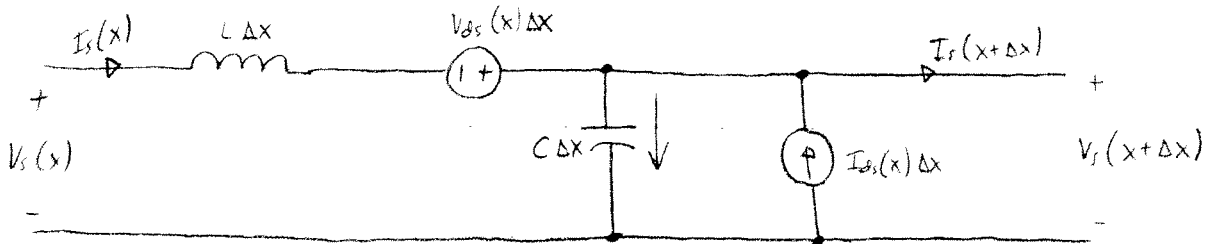
$$D(\omega) := (Z_o \cdot Z_s + Z_o \cdot Z_L) \cdot \cos(\beta(\omega) \cdot l) + j \cdot (Z_o^2 + Z_s \cdot Z_L) \cdot \sin(\beta(\omega) \cdot l)$$

$$I_L(\omega, \theta) := \frac{-j \cdot 2 \cdot \sin\left(\frac{\beta(\omega) \cdot d}{2} \cdot \sin(\theta)\right)}{\beta(\omega) \cdot D(\omega)} \cdot [Z_o \cdot \sin(\beta(\omega) \cdot l) + j \cdot Z_s \cdot (1 - \cos(\beta(\omega) \cdot l))]$$



Section 16

① reference Problem 16.9



KVL: $-V_s(x) + j\omega L \Delta x I_s(x) - V_{os}(x) \Delta x + V_s(x + \Delta x) = 0$

$$V_s(x) = j\omega L \Delta x I_s(x) - V_{os}(x) \Delta x + V_s(x + \Delta x)$$

Divide through by Δx

$$\frac{V_s(x)}{\Delta x} = j\omega L I_s(x) - V_{os}(x) + \frac{V_s(x + \Delta x)}{\Delta x}$$

$$V_{os}(x) = j\omega L I_s(x) + \frac{V_s(x + \Delta x)}{\Delta x} - \frac{V_s(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$V_{os}(x) = \frac{1}{\Delta x} (V_s(x + \Delta x) - V_s(x)) + j\omega L I_s(x)$$

$$\frac{\partial V_s(x)}{\partial x}$$

$$V_{os}(x) = \frac{\partial V_s(x)}{\partial x} + j\omega L I_s(x)$$

KCL:
(in +)

$$I_s(x) - j\omega C \Delta x V_s(x + \Delta x) + I_{os}(x) \Delta x - I_s(x + \Delta x) = 0$$

$$I_{os}(x) \Delta x = I_s(x + \Delta x) - I_s(x) + j\omega C \Delta x V_s(x + \Delta x)$$

- divide through by Δx -

$$I_{os}(x) = \frac{I_s(x + \Delta x) - I_s(x)}{\Delta x} + j\omega C \frac{\Delta x}{\Delta x} V_s(x + \Delta x)$$

$$I_{os}(x) = \frac{\partial I_s(x)}{\partial x} + j\omega C V_s(x + \Delta x)$$

$\Delta x \rightarrow 0$

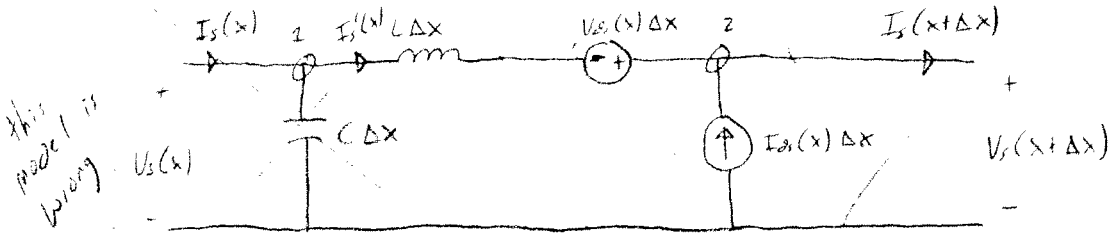
$$\frac{V_s(x + \Delta x)}{j\omega C \Delta x}$$

Not the same

Unless $\Delta x \ll x \Rightarrow V_s(x) \approx V_s(x + \Delta x)$

unless $\Delta x \ll x + V_s(x + \Delta x) \approx V_s(x)$

But, if $C \Delta x$ is moved to parallel w/ source -
would get exact answer



Now, Node 1: $I_s(x) - j\omega C \Delta x V_s(x) - I_s'(x) = 0$

$$I_s'(x) = I_s(x) - j\omega C \Delta x V_s(x)$$

Node 2: $I_s'(x) + I_{sr}(x) \Delta x - I_s(x + \Delta x) = 0$

$$I_{sr}(x) \Delta x = I_s(x + \Delta x) - I_s'(x)$$

$$I_{sr}(x) \Delta x = I_s(x + \Delta x) - I_s(x) + j\omega C \Delta x V_s(x)$$

$$I_{sr}(x) = \frac{\partial I_s(x)}{\partial x} + j\omega C V_s(x)$$

~~KVL is not satisfied, $V_s(x)$~~

But - KVL does not yield same result. (Different current $I_s(x)$ through inductor)

Conclusion

must assume that for the original circuit,
 $V_s(x + \Delta x) \approx V_s(x)$ (line is electrically short)

$$V_s(x) = \frac{\partial V_s(x)}{\partial x} + j\omega L I_s(x)$$

$$I_{sr}(x) = \frac{\partial I_s(x)}{\partial x} + j\omega C V_s(x)$$

DELPHIEnergy & Engine
Management Systems**Problem 16.31****ECE-640**

For E-Field polarized in the x-direction:

$$I_s(x=l) = \frac{1}{D} \int_0^l E_x^{inc} \begin{bmatrix} e^{-j(\frac{\beta l}{2}) \sin \theta} & e^{j(\frac{\beta l}{2}) \sin \theta} \\ & -e^{j(\frac{\beta l}{2}) \sin \theta} \end{bmatrix} \begin{bmatrix} Z_0 \cos \beta x + j Z_1 \sin \beta x \end{bmatrix} dx$$

$$Z_0 = 150 \Omega \quad D = (Z_0 Z_1 + Z_0 Z_2) \cos \beta l + j (Z_0^2 + Z_1 Z_2) \sin \beta l$$

$$\frac{I_s(x=l)}{E_x^{inc}} = \frac{1}{D} \begin{bmatrix} e^{-j(\frac{\beta l}{2}) \sin \theta} & e^{j(\frac{\beta l}{2}) \sin \theta} \\ & -e^{j(\frac{\beta l}{2}) \sin \theta} \end{bmatrix} \int_0^l [Z_0 \cos \beta x + j Z_1 \sin \beta x] dx$$

$$= \frac{1}{\beta D} \begin{bmatrix} e^{-j(\frac{\beta l}{2}) \sin \theta} & e^{j(\frac{\beta l}{2}) \sin \theta} \\ & -e^{j(\frac{\beta l}{2}) \sin \theta} \end{bmatrix} \begin{bmatrix} Z_0 \sin \beta x - j Z_1 \cos \beta x \end{bmatrix}_0^l$$

$$= \frac{1}{\beta D} \begin{bmatrix} e^{-j(\frac{\beta l}{2}) \sin \theta} & e^{j(\frac{\beta l}{2}) \sin \theta} \\ & -e^{j(\frac{\beta l}{2}) \sin \theta} \end{bmatrix} [Z_0 \sin \beta l - j Z_1 \cos \beta l + j Z_1]$$

MATHEMATICAL PLOTS AND ANALYSIS FOLLOW :

Problem 16.31

ECE-640

$Z_0 := 300 \quad Z_1 := 300 \quad Z_2 := 300 \quad l := 5 \quad \beta(l) := \frac{2 \cdot \pi \cdot f}{3 \cdot 10^8} \quad \theta := \frac{\pi}{2} \quad d := 2 \cdot 10^{-3} \quad j := \sqrt{-1}$

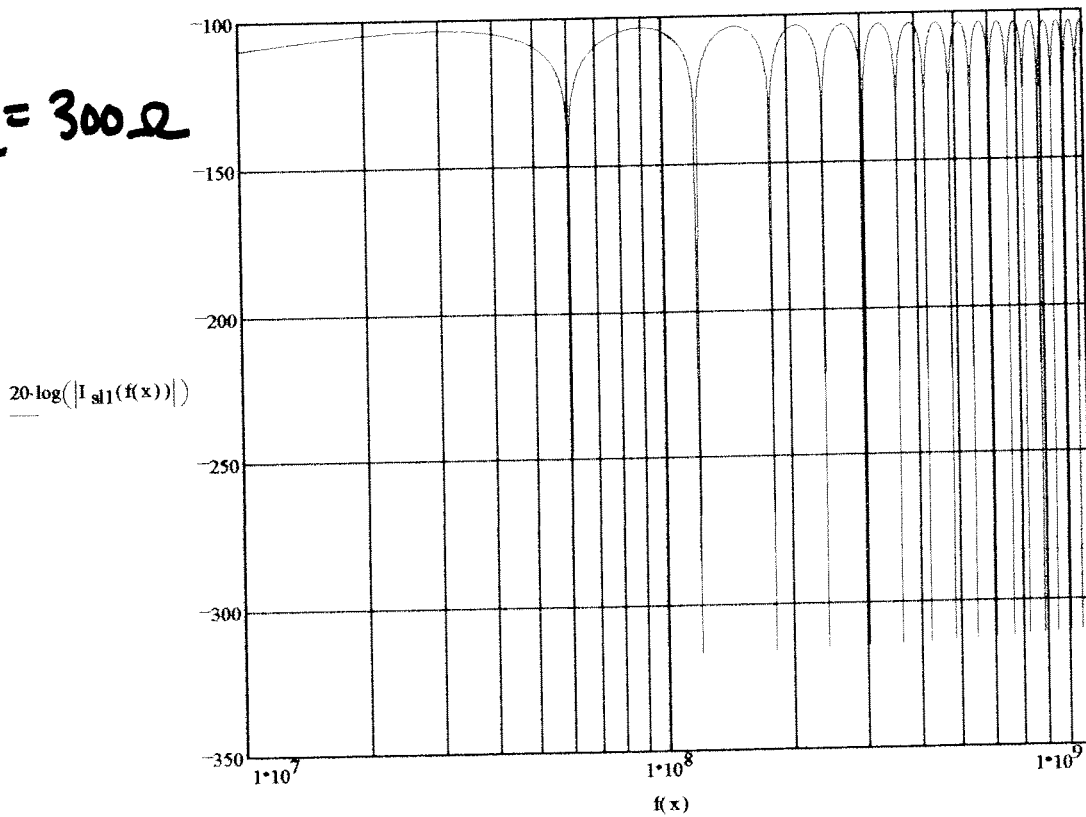
$x := 1, 1.01, \dots, 110$

$f(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$

$D_1(x) := (Z_0 \cdot Z_1 + Z_0 \cdot Z_2) \cdot \cos(\beta(x) \cdot l) + j \cdot (Z_0^2 + Z_1 \cdot Z_2) \cdot \sin(\beta(x) \cdot l)$

$I_{sl1}(x) := \frac{1}{D_1(x) \cdot \beta(x)} \cdot \left[e^{-j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} - e^{j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} \right] \cdot (Z_0 \cdot \sin(\beta(x) \cdot l) - j \cdot Z_1 \cdot \cos(\beta(x) \cdot l) + j \cdot Z_2)$

$Z_0 = Z_1 = Z_2 = 300 \Omega$



Problem 16.31

ECE-640

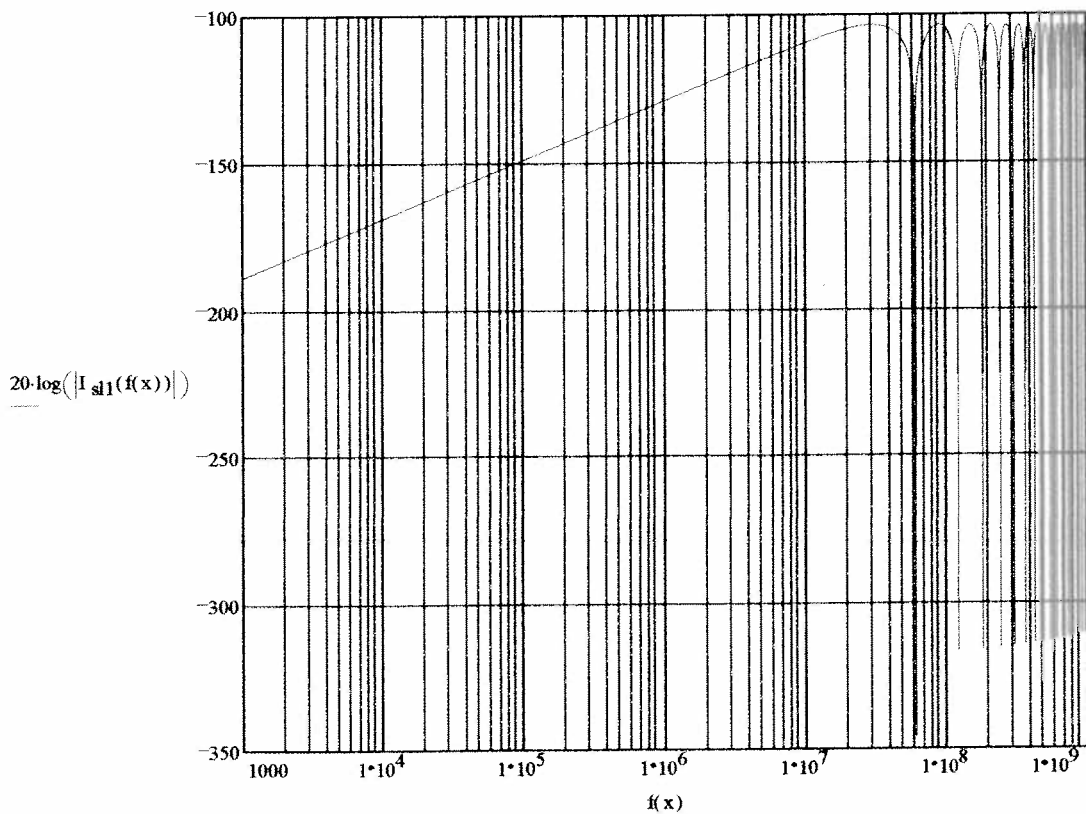
$$Z_0 := 300 \quad Z_1 := 300 \quad Z_2 := 300 \quad l := 5 \quad \beta(f) := \frac{2 \cdot \pi \cdot f}{3 \cdot 10^8} \quad \theta := \frac{\pi}{2} \quad d := 2 \cdot 10^{-3} \quad j := \sqrt{-1}$$

$$x := 1, 1.01 \dots 110$$

$$f(x) := \left(x + 1 - 10 \cdot \text{floor} \left(\frac{x}{10} \right) \right) \cdot 10^{\text{floor} \left(\frac{x}{10} \right)}$$

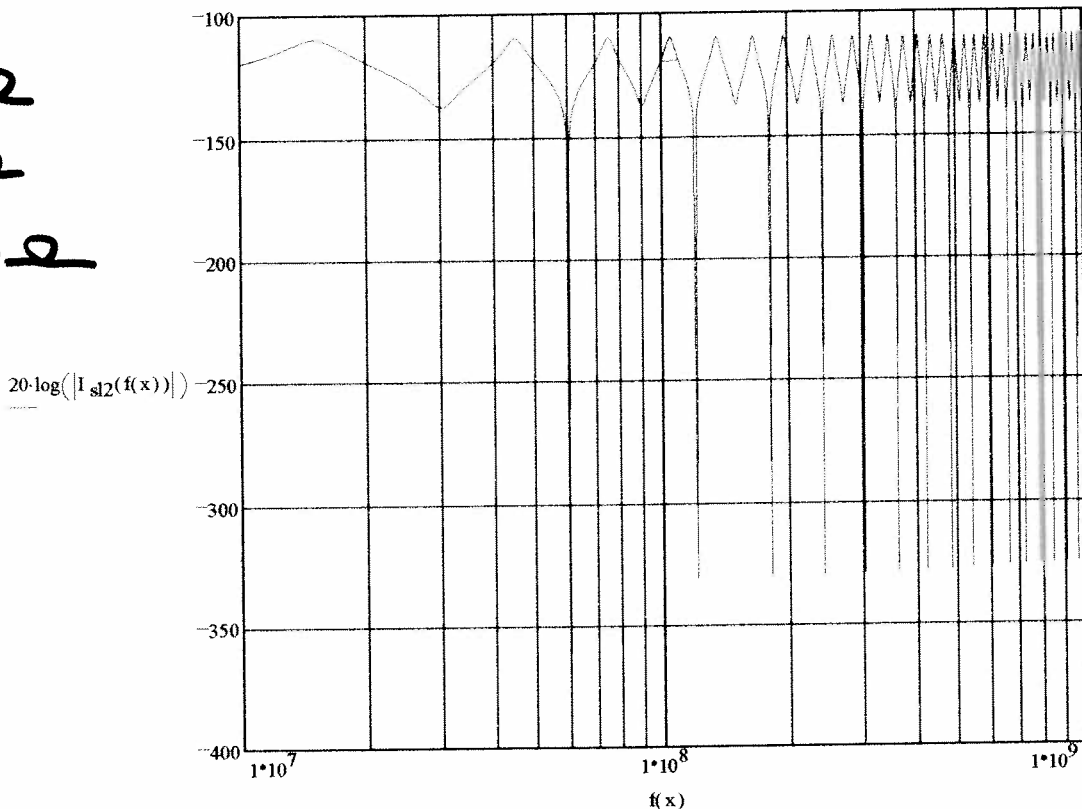
$$D_1(x) := (Z_0 \cdot Z_1 + Z_0 \cdot Z_2) \cdot \cos(\beta(x) \cdot l) + j \cdot (Z_0^2 + Z_1 \cdot Z_2) \cdot \sin(\beta(x) \cdot l)$$

$$I_{sl1}(x) := \frac{1}{D_1(x) \cdot \beta(x)} \cdot \left[e^{-j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} - e^{j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} \right] \cdot (Z_0 \cdot \sin(\beta(x) \cdot l) - j \cdot Z_1 \cdot \cos(\beta(x) \cdot l) + j \cdot Z_2)$$



$$D_2(x) := \left(Z_0 \frac{Z_1}{10} + Z_0 \cdot Z_2 \cdot 10 \right) \cdot \cos(\beta(x) \cdot l) + j \cdot \left(Z_0^2 + \frac{Z_1}{10} \cdot Z_2 \cdot 10 \right) \cdot \sin(\beta(x) \cdot l)$$

$$I_{sl2}(x) = \frac{1}{D_2(x) \cdot \beta(x)} \left[e^{-j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} - e^{j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} \right] \cdot \left(Z_0 \cdot \sin(\beta(x) \cdot l) - j \cdot \frac{Z_1}{10} \cdot \cos(\beta(x) \cdot l) + j \cdot \frac{Z_1}{10} \right)$$



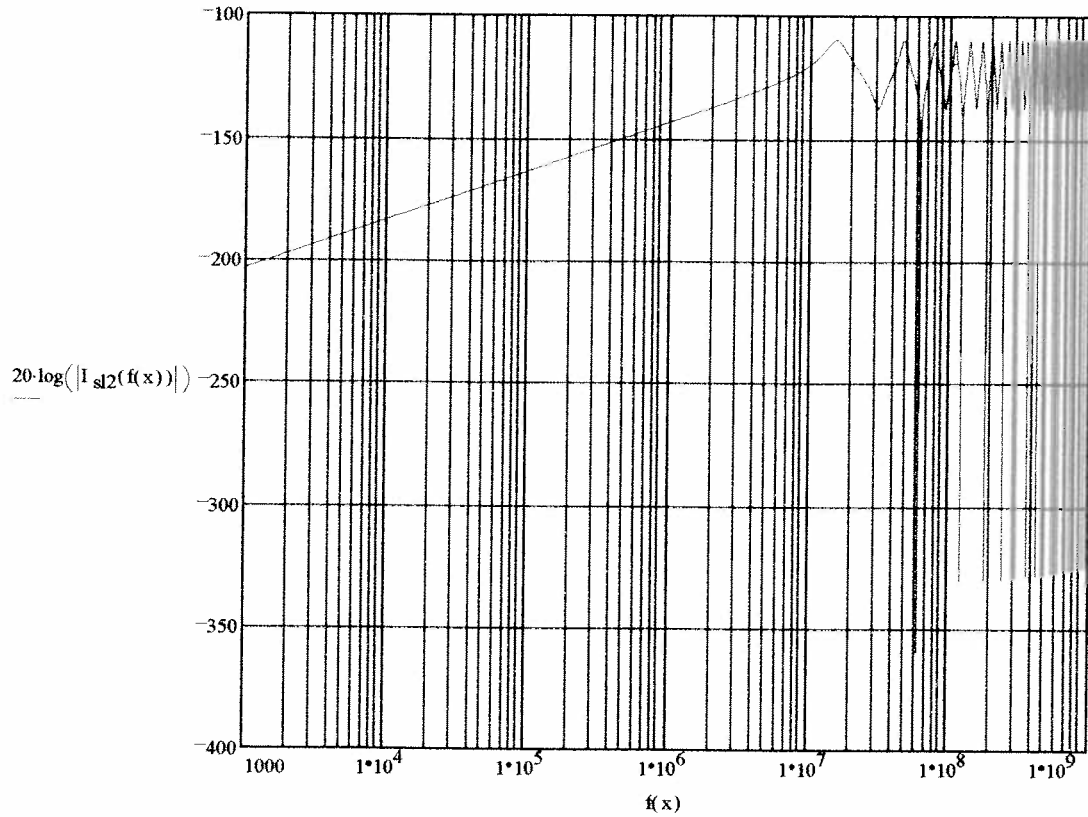
1. Resonance points are the same for both matched and unmatched cases.

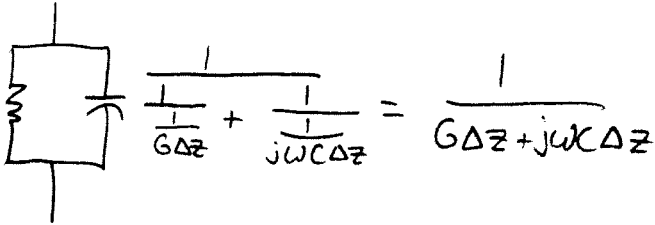
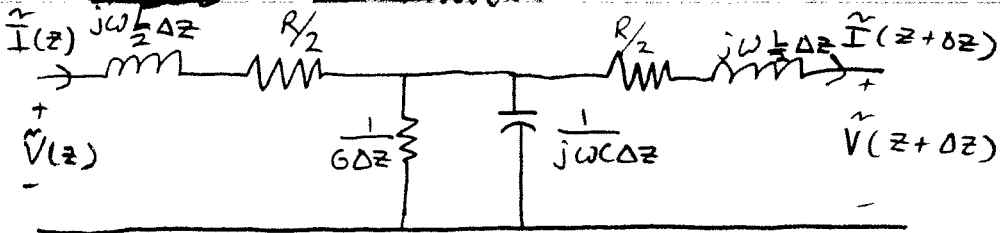
In the unmatched case the Load current varies more w/ frequency than the matched case.

2. If the x-polarized E-Field is the same in magnitude and phase everywhere along the Line then no potential will exist across the Load thus the Load current = 0.

$$D_2(x) := \left(Z_0 \cdot \frac{Z_1}{10} + Z_0 \cdot Z_2 \cdot 10 \right) \cdot \cos(\beta(x) \cdot 1) + j \cdot \left(Z_0^2 + \frac{Z_1}{10} \cdot Z_2 \cdot 10 \right) \cdot \sin(\beta(x) \cdot 1)$$

$$I_{sl2}(x) := \frac{1}{D_2(x) \cdot \beta(x)} \cdot \left[e^{-j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} - e^{j \cdot \left(\frac{\beta(x) \cdot d}{2} \right) \cdot \sin(\theta)} \right] \cdot \left(Z_0 \cdot \sin(\beta(x) \cdot 1) - j \cdot \frac{Z_1}{10} \cdot \cos(\beta(x) \cdot 1) + j \cdot \frac{Z_1}{10} \right)$$





$$\text{KVL: } -\tilde{V}(z) + \tilde{I}(z)[R/2 + j\omega L/2]\Delta z + \tilde{I}(z+\Delta z)[R/2 + j\omega L/2]\Delta z + \tilde{V}(z+\Delta z) = 0$$

$$\tilde{V}(z+\Delta z) - \tilde{V}(z) + (\tilde{I}(z) + \tilde{I}(z+\Delta z))[R/2 + j\omega L/2]\Delta z = 0$$

$$\frac{\tilde{V}(z+\Delta z) - \tilde{V}(z)}{\Delta z} + [\tilde{I}(z) + \tilde{I}(z+\Delta z)](R/2 + j\omega L/2) = 0$$

$$\frac{d\tilde{V}(z)}{dz} + \tilde{I}(z)(R + j\omega L) = 0$$

$$\text{KCL: } -\tilde{I}(z) + \tilde{I}(z+\Delta z) + \frac{[\tilde{V}(z) - \tilde{I}(z)[R/2 + j\omega L/2]\Delta z]}{G\Delta z + j\omega C\Delta z} = 0$$

$$\frac{\tilde{I}(z+\Delta z) - \tilde{I}(z)}{\Delta z} + [\tilde{V}(z) - \tilde{I}(z)[R/2 + j\omega L/2]\Delta z](G + j\omega C) = 0$$

$$\frac{d\tilde{I}(z)}{dz} + \tilde{V}(z)(G + j\omega C) = 0$$

$$\frac{d\tilde{I}(z)}{dz} = -\tilde{V}(z)(G + j\omega C)$$

$$\frac{d^2\tilde{V}(z)}{dz^2} + \frac{d\tilde{I}(z)}{dz}(R + j\omega L) = 0$$

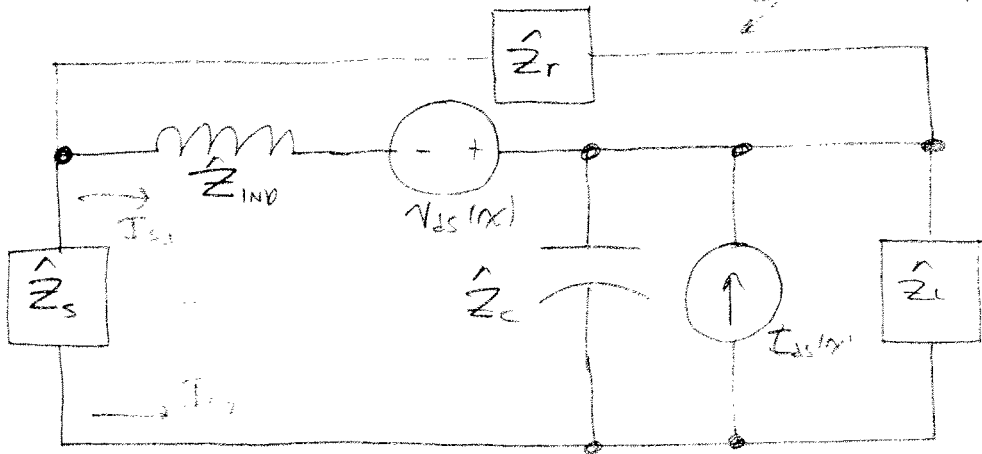
$$\frac{d^2\tilde{V}(z)}{dz^2} - (G + j\omega C)(R + j\omega L)\tilde{V}(z) = 0$$

$$\gamma = \sqrt{(G + j\omega C)(R + j\omega L)}$$

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0$$

Problem 16-12 Comm. Path Impedance

The model as given is adequate for present purposes. However, it is noted that the model is a "lumped" model. It is usually leakage capacitance due to the lines.



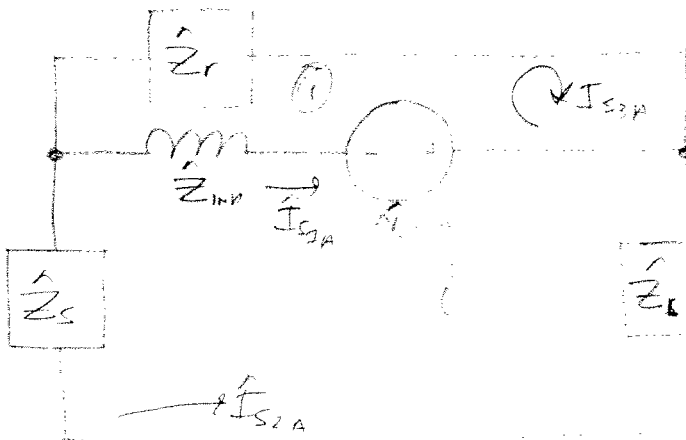
$$\hat{Z}_{IND} = j\omega(\Delta X)L$$

$$\hat{Z}_c = \frac{1}{j\omega\Delta X}$$

$$\hat{Z}_L = R_L + jX_L \quad \hat{Z}_r = R_r + jX_r \quad \hat{Z}_s = R_s + jX_s$$

As we assumed sinusoidal plane wave in the entire derivation of the model, we can model $N_{DS}(10X)$ as a phase-type source in uniform plane wave or entire cell.

Part A. ZERE I_{S2A} (with T_{S2A} and S_{S2A})



$$0 = \hat{I}_{S2B} (\hat{Z}_s + \hat{Z}_L/\hat{Z}_c) + \hat{N}_{DS}(10X) + \hat{Z}_{IND} (I_{S2A} + I_{S2B})$$

$$0 = \hat{I}_{S2B} \left(\hat{Z}_s + \frac{\hat{Z}_L}{\hat{Z}_c} \right) + \hat{N}_{DS}(10X) + \hat{Z}_{IND} I_{S2A} + (\hat{Z}_r) I_{S2B}$$

$$\hat{I}_{S2B} (\hat{Z}_r + \hat{Z}_{IND}) = -\hat{N}_{DS}(10X) - \hat{Z}_{IND} I_{S2A}$$

$$\hat{I}_{S2B} = \frac{-\hat{N}_{DS}(10X)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_{IND} I_{S2A}}{\hat{Z}_r + \hat{Z}_{IND}}$$

$$0 = \hat{I}_{S2A} (\hat{Z}_S + \hat{Z}_L // \hat{Z}_C) + N_{DS}(\alpha) - \hat{Z}_R (\hat{I}_{S2A} - \frac{N_{DS}(\alpha)}{\hat{Z}_R + \hat{Z}_{IND}} - \frac{\hat{Z}_{IND}}{\hat{Z}_{IND} + \hat{Z}_R} \hat{I}_{S2A})$$

$$0 = \hat{I}_{S2A} \left([\hat{Z}_S + \hat{Z}_L // \hat{Z}_C] + \hat{Z}_{IND} - \frac{\hat{Z}_{IND}^2}{\hat{Z}_{IND} + \hat{Z}_R} \right) + N_{DS}(\alpha) \left[1 - \frac{1}{\hat{Z}_R + \hat{Z}_{IND}} \right]$$

$$\hat{I}_{S2A} = \frac{N_{DS}(\alpha) \left[\frac{1}{\hat{Z}_R + \hat{Z}_{IND}} \right]}{[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C] + \hat{Z}_{IND} - \frac{\hat{Z}_{IND}^2}{\hat{Z}_{IND} + \hat{Z}_R}}$$

$$\hat{I}_{S2A} = \frac{N_{DS}(\alpha) [1 - \hat{Z}_R - \hat{Z}_{IND}]}{[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C] [\hat{Z}_{IND} + \hat{Z}_R] + \hat{Z}_{IND} \hat{Z}_R}$$

$$\hat{I}_{S3A} = - \frac{N_{DS}(\alpha)}{\hat{Z}_R + \hat{Z}_{IND}} - \left(\frac{\hat{Z}_{IND}}{\hat{Z}_{IND} + \hat{Z}_R} \right) \frac{N_{DS}(\alpha) [1 - \hat{Z}_R - \hat{Z}_{IND}]}{[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C] [\hat{Z}_{IND} + \hat{Z}_R] + \hat{Z}_{IND} \hat{Z}_R}$$

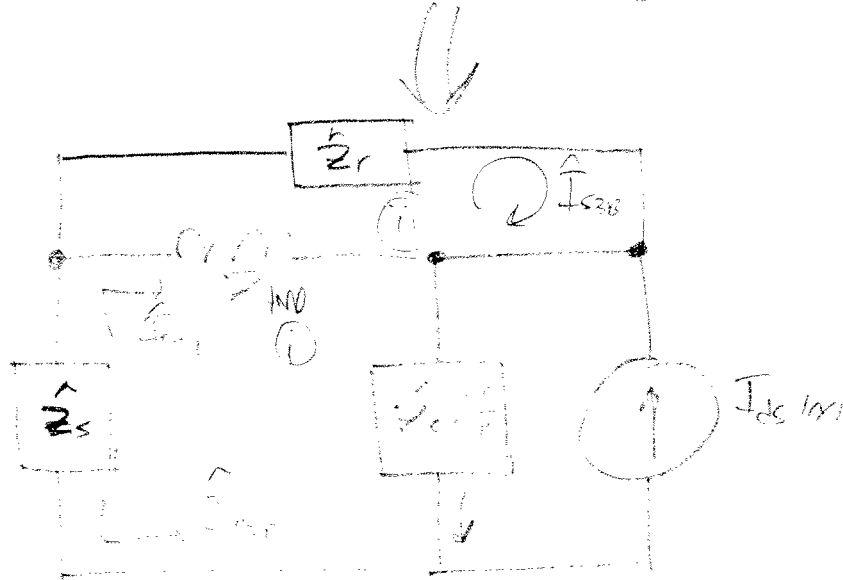
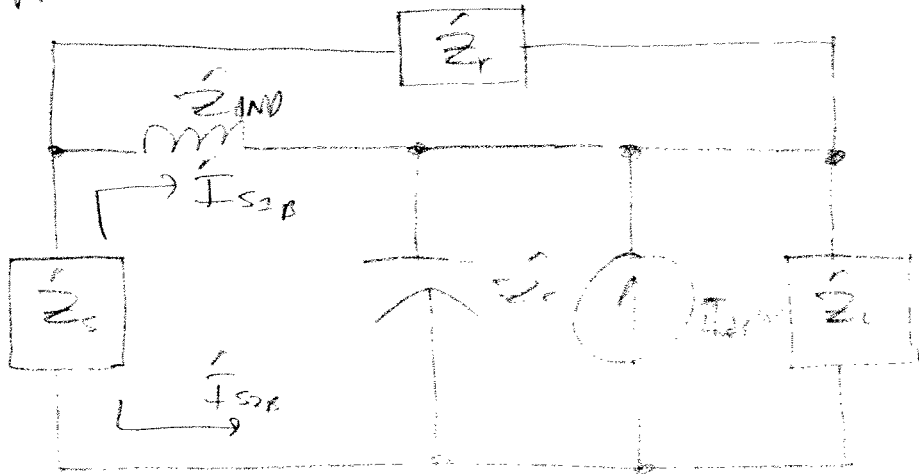
$$\hat{I}_{S1A} = -(\hat{I}_{S2A} + \hat{I}_{S3A})$$

$$= \frac{\hat{N}_{DS}(\alpha)}{\hat{Z}_R + \hat{Z}_{IND}} - \frac{\hat{Z}_R}{\hat{Z}_{IND} + \hat{Z}_R} \left[\frac{N_{DS}(\alpha) [1 - \hat{Z}_R - \hat{Z}_{IND}]}{[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C] [\hat{Z}_{IND} + \hat{Z}_R] + \hat{Z}_{IND} \hat{Z}_R} \right]$$

$$= \frac{\hat{N}_{DS}(\alpha)}{\hat{Z}_R + \hat{Z}_{IND}} \left[1 - \frac{\hat{Z}_R [1 - \hat{Z}_R - \hat{Z}_{IND}]}{[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C] [\hat{Z}_{IND} + \hat{Z}_R] + \hat{Z}_{IND} \hat{Z}_R} \right]$$

$$= \frac{N_{DS}(\alpha)}{\hat{Z}_R + \hat{Z}_{IND}} - \frac{\hat{Z}_R}{\hat{Z}_{IND} + \hat{Z}_R} \hat{I}_{S2A}$$

Part B: Zero current (S.C.)



$$\textcircled{1} 0 = \hat{I}_{sc0}(\hat{Z}_s) + (\hat{I}_{sc0} - I_{sc0})\hat{Z}_c // \hat{Z}_L + (\hat{I}_{sc0} + \beta \hat{I}_{sc0})\hat{Z}_{IND}$$

$$\textcircled{2} 0 = \hat{I}_{sc0}(\hat{Z}_r) + [\hat{I}_{sc0} + (\beta \hat{I}_{sc0})]\hat{Z}_{IND}$$

$$\text{from } \hat{I}_{sc0} = - \hat{I}_{sc0} \left[\frac{\hat{Z}_c}{\hat{Z}_{IND} + \hat{Z}_r} \right] \textcircled{2}$$

$$\hat{I}_{sc0} \left[\frac{\hat{Z}_c}{\hat{Z}_{IND} + \hat{Z}_r} \right] = \hat{I}_{sc0} \left[\frac{\hat{Z}_c}{\hat{Z}_{IND} + \hat{Z}_r} \right]$$

$$0 = \hat{I}_{sc0} \left[\frac{\hat{Z}_c}{\hat{Z}_{IND} + \hat{Z}_r} \right] - \hat{I}_{sc0} \left[\frac{\hat{Z}_c}{\hat{Z}_{IND} + \hat{Z}_r} \right]$$

$$= \hat{I}_{sc0} \left[\frac{\hat{Z}_c}{\hat{Z}_{IND} + \hat{Z}_r} \right]$$

$$0 = \hat{I}_{S2B} \left[\hat{Z}_S + Z_C // Z_L + \frac{\hat{Z}_{IND} \hat{Z}_R}{\hat{Z}_{IND} + \hat{Z}_R} \right] - \left[\hat{I}_{S1} (N) \right] \hat{Z}_C // \hat{Z}_L$$

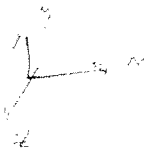
$$\begin{aligned} \hat{I}_{S2B} &= \frac{\left[\hat{I}_{S1} (N) \right] \hat{Z}_C // \hat{Z}_L}{\hat{Z}_S + Z_C // Z_L + \frac{\hat{Z}_{IND} \hat{Z}_R}{\hat{Z}_{IND} + \hat{Z}_R}} \\ &= \frac{\left[\hat{I}_{S1} (N) \right] (\hat{Z}_C // \hat{Z}_L) (\hat{Z}_{IND} + \hat{Z}_R)}{(\hat{Z}_S + \hat{Z}_C // \hat{Z}_L) (\hat{Z}_{IND} + \hat{Z}_R) + \hat{Z}_{IND} \hat{Z}_R} \end{aligned}$$

$$\begin{aligned} \hat{I}_{S2} &= - (\hat{I}_{S1} + \hat{I}_{S2B}) \\ &= - \frac{\hat{Z}_R}{\hat{Z}_{IND} + \hat{Z}_R} \hat{I}_{S1} \end{aligned}$$

TOTAL CURRENTS:

$$\begin{aligned} \boxed{F1} \quad \hat{I}_{S2} &= \hat{I}_{S2A} + \hat{I}_{S2B} \\ &= \frac{N_{S1} I_1}{\left[\hat{Z}_S + \hat{Z}_L // \hat{Z}_C \right] \left[\hat{Z}_{IND} + \hat{Z}_R \right] + \hat{Z}_{IND} \hat{Z}_R} \left[1 - \frac{\hat{Z}_R}{\hat{Z}_{IND} + \hat{Z}_R} \right] + \hat{I}_{S1} (N) \frac{\hat{Z}_C // \hat{Z}_L}{\left[\hat{Z}_{IND} + \hat{Z}_R \right] + \hat{Z}_{IND} \hat{Z}_R} \end{aligned}$$

$$\begin{aligned} \boxed{F2} \quad \hat{I}_{S2} &= \hat{I}_{S2A} + \hat{I}_{S2B} \\ &= \frac{N_{S1} I_1}{Z_R + Z_{IND}} - \frac{\hat{Z}_R}{\hat{Z}_R + \hat{Z}_{IND}} \hat{I}_{S2A} - \frac{\hat{Z}_R}{\hat{Z}_R + \hat{Z}_{IND}} \hat{I}_{S2B} \\ &= \frac{N_{S1} I_1}{Z_R + Z_{IND}} - \frac{\hat{Z}_R}{\hat{Z}_R + \hat{Z}_{IND}} \left[\hat{I}_{S2A} + \hat{I}_{S2B} \right] \\ &= \frac{N_{S1} I_1}{Z_R + Z_{IND}} - \hat{Z}_R (\hat{I}_{S2}) \end{aligned}$$



$$I_C = \frac{I_{S1} + I_{S2}}{2}$$

$$= \frac{1}{2} \left[I_{S1} + \frac{N_{ds}(w)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}} \hat{I}_{S2} \right]$$

usually $\hat{Z}_r \gg \hat{Z}_{IND}$
 VERY SMALL LOSS DUE TO LEAKAGE CAPACITANCE

F3

$$I_C = \frac{N_{ds}(w)}{2(\hat{Z}_r + \hat{Z}_{IND})} + \frac{\hat{Z}_{IND}}{\hat{Z}_r + \hat{Z}_{IND}} \hat{I}_{S2}$$

usually very small

$$I_D = \frac{|I_{S1} - I_{S2}|}{2}$$

$$= \frac{\left| \frac{N_{ds}(w)}{\hat{Z}_r + \hat{Z}_{IND}} - \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}} \hat{I}_{S2} - \hat{I}_{S1} \right|}{2}$$

$$= \frac{1}{2} \left| \hat{I}_{S2} \left(1 + \frac{\hat{Z}_r}{\hat{Z}_r + \hat{Z}_{IND}} \right) - \frac{N_{ds}(w)}{\hat{Z}_r + \hat{Z}_{IND}} \right|$$

$$I_{ds}(w) = -j\omega C \int_0^d \hat{E}_{ys}^{INC}(w, y) dy$$

$$V_d(w) = j\omega \mu \int_0^d \hat{H}_{zs}^{INC}(w, y) dy$$

for a planar TEM wave propagation in the xz plane \hat{E}_y and \hat{H}_z are the only non-zero components

$\therefore I_{ds} \neq V_d$ since they are spatially averaged

if \hat{E}_y and \hat{H}_z are uniform over the gap $0 < y < d$

then $I_{ds}(w) = -j\omega C V_d(w)$ and $V_d(w) = j\omega \mu I_{ds}(w) d$

if \hat{E}_y and \hat{H}_z are not uniform over the gap $0 < y < d$ OTHERWISE $I_D \gg I_C$