26 - 2 [redacted]

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1 $j\cdot w\cdot L1\cdot(Ia)+j\cdot w\cdot L1\cdot(Ib)+(Ia-Ia1)\cdot(R1)-Vs$

2 $j\cdot w\cdot L1\cdot(Ia)+j\cdot w\cdot L1\cdot(Ib)+j\cdot w\cdot L2\cdot Ia1$

3 $j\cdot w\cdot L1\cdot(Ia)+j\cdot w\cdot L1\cdot(Ib)+(Ib-Ib1)\cdot(R2)$

4 $j\cdot w\cdot L1\cdot(Ia)+j\cdot w\cdot L1\cdot(Ib)+j\cdot w\cdot L2\cdot Ib1$

solve for Ia1 $\frac{-(j\cdot w\cdot L1\cdot Ia+j\cdot w\cdot L1\cdot Ib)}{(j\cdot(w\cdot L2))}$

solve for Ib1 $\frac{-(j\cdot w\cdot L1\cdot Ia+j\cdot w\cdot L1\cdot Ib)}{(j\cdot(w\cdot L2))}$

1 $j\cdot w\cdot L1\cdot(Ia)+j\cdot w\cdot L1\cdot(Ib)+\left[Ia-\frac{-(j\cdot w\cdot L1\cdot Ia+j\cdot w\cdot L1\cdot Ib)}{(j\cdot(w\cdot L2))}\right]\cdot(R1)-Vs$

$$\frac{(j\cdot w\cdot L1\cdot Ia\cdot L2+j\cdot w\cdot L1\cdot Ib\cdot L2+R1\cdot Ia\cdot L2+R1\cdot L1\cdot Ia+R1\cdot L1\cdot Ib-Vs\cdot L2)}{L2}$$

$$Ia=\frac{-(j\cdot w\cdot L1\cdot Ib\cdot L2+R1\cdot L1\cdot Ib-Vs\cdot L2)}{(j\cdot w\cdot L1\cdot L2+R1\cdot L2+R1\cdot L1)}$$

3 $j\cdot w\cdot L1\cdot(Ia)+j\cdot w\cdot L1\cdot(Ib)+\left[Ib-\frac{-(j\cdot w\cdot L1\cdot Ia+j\cdot w\cdot L1\cdot Ib)}{(j\cdot(w\cdot L2))}\right]\cdot(R2)$

$$\frac{(j\cdot w\cdot L1\cdot Ia\cdot L2+j\cdot w\cdot L1\cdot Ib\cdot L2+R2\cdot Ib\cdot L2+R2\cdot L1\cdot Ia+R2\cdot L1\cdot Ib)}{L2}$$

$$Ib=\frac{-(j\cdot w\cdot L1\cdot Ia\cdot L2+R2\cdot L1\cdot Ia)}{(j\cdot w\cdot L1\cdot L2+R2\cdot L2+R2\cdot L1)}$$

plug Ib into equation 1:

$$\frac{\left[j\cdot w\cdot L1\cdot Ia\cdot L2+j\cdot w\cdot L1\cdot\frac{-(j\cdot w\cdot L1\cdot Ia\cdot L2+R2\cdot L1\cdot Ia)}{(j\cdot w\cdot L1\cdot L2+R2\cdot L2+R2\cdot L1)}\cdot L2+R1\cdot Ia\cdot L2+R1\cdot L1\cdot Ia+R1\cdot L1\cdot\frac{-(j\cdot w\cdot L1\cdot Ia\cdot L2+R2\cdot L1\cdot Ia)}{(j\cdot w\cdot L1\cdot L2+R2\cdot L2+R2\cdot L1)}-Vs\cdot\right.}{L2}$$

$$\frac{-(-j\cdot w\cdot L1\cdot Ia\cdot L2\cdot R2-R1\cdot Ia\cdot L2\cdot j\cdot w\cdot L1-R1\cdot Ia\cdot L2\cdot R2-2\cdot R1\cdot Ia\cdot R2\cdot L1+Vs\cdot L2\cdot j\cdot w\cdot L1+Vs\cdot L2\cdot R2+Vs\cdot R2\cdot L1)}{(j\cdot w\cdot L1\cdot L2+R2\cdot L2+R2\cdot L1)}$$

$$Ia=\frac{-(-Vs\cdot L2\cdot j\cdot w\cdot L1-Vs\cdot L2\cdot R2-Vs\cdot R2\cdot L1)}{(j\cdot w\cdot L1\cdot L2\cdot R2+R1\cdot L2\cdot j\cdot w\cdot L1+R1\cdot L2\cdot R2+2\cdot R1\cdot R2\cdot L1)}$$

plug Ia into equation 3:

$$\frac{\left[j\cdot w\cdot L1\cdot\frac{-(j\cdot w\cdot L1\cdot Ib\cdot L2+R1\cdot L1\cdot Ib-Vs\cdot L2)}{(j\cdot w\cdot L1\cdot L2+R1\cdot L2+R1\cdot L1)}\cdot L2+j\cdot w\cdot L1\cdot Ib\cdot L2+R2\cdot Ib\cdot L2+R2\cdot L1\cdot\frac{-(j\cdot w\cdot L1\cdot Ib\cdot L2+R1\cdot L1\cdot Ib-Vs\cdot L2)}{(j\cdot w\cdot L1\cdot L2+R1\cdot L2+R1\cdot L1)}+R\right.}{L2}$$

$$\frac{(Vs\cdot L2\cdot j\cdot w\cdot L1+j\cdot w\cdot L1\cdot Ib\cdot L2\cdot R1+R2\cdot Ib\cdot L2\cdot j\cdot w\cdot L1+R2\cdot Ib\cdot L2\cdot R1+2\cdot R2\cdot Ib\cdot R1\cdot L1+Vs\cdot R2\cdot L1)}{(j\cdot w\cdot L1\cdot L2+R1\cdot L2+R1\cdot L1)}$$

$$Ib=\frac{-(Vs\cdot L2\cdot j\cdot w\cdot L1+Vs\cdot R2\cdot L1)}{(j\cdot w\cdot L1\cdot L2\cdot R2+R1\cdot L2\cdot j\cdot w\cdot L1+R1\cdot L2\cdot R2+2\cdot R1\cdot R2\cdot L1)}$$

$$Ia=\frac{-(-Vs\cdot L2\cdot j\cdot w\cdot L1-Vs\cdot L2\cdot R2-Vs\cdot R2\cdot L1)}{(j\cdot w\cdot L1\cdot L2\cdot R2+R1\cdot L2\cdot j\cdot w\cdot L1+R1\cdot L2\cdot R2+2\cdot R1\cdot R2\cdot L1)}$$

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$$I_a = \frac{V_s \cancel{R_2}(L_2 - L_1)}{\cancel{R_2} R_1 (L_2 + 2L_1)} \cdot \frac{\left(1 + \frac{j\omega}{\frac{R_2(L_2-L_1)}{L_2L_1}}\right)}{\left(1 + \frac{j\omega}{\frac{R_1R_2(L_2+2L_1)}{L_1L_2(R_2+R_1)}}\right)}$$

$$I_b = \frac{-L_1 V_s \cancel{R_2}}{\cancel{R_2} R_1 (L_2 + 2L_1)} \cdot \frac{\left(1 + \frac{j\omega}{\frac{R_2}{L_2}}\right)}{\left(1 + \frac{j\omega}{\frac{R_1R_2(L_2+2L_1)}{L_1L_2(R_2+R_1)}}\right)}$$

$$\omega_c = \frac{R_2}{L_2},\ \frac{R_2(L_2-L_1)}{L_2L_1},\ \frac{R_1R_2(L_2+2L_1)}{L_1L_2(R_2+R_1)}$$

if ω is well below ω_c

$$I_a = \frac{V_s(L_2-L_1)}{R_1(L_2+2L_1)} \qquad I_b = \frac{-V_s L_1}{R_1(L_2+2L_1)}$$

$$I_a \neq I_b$$

if ω is well above ω_c

$$I_a = \frac{V_s L_2 L_1}{(L_1L_2R_2 + R_1L_2L_1)} = \frac{V_s}{(R_2+R_1)}$$

$$I_b = \frac{-V_s L_2 L_1}{(L_1L_2R_2 + R_1L_2L_1} = \frac{-V_s}{R_2+R_1}$$

equal & opposite

$$Z_{in} = \frac{V_s}{I_1} = \frac{R_1(L_2+2L_1)}{(L_2-L_1)} \cdot \frac{\left(1 + \dfrac{j\omega}{\dfrac{R_1R_2(L_2+2L_1)}{L_1L_2(R_2+R_1)}}\right)}{\left(1 + \dfrac{j\omega}{\dfrac{R_2(L_2-L_1)}{L_2L_1}}\right)}$$

when frequency is low

$$Z_{in} = \frac{V_s}{I_1} = \frac{R_1(L_2+2L_1)}{(L_2-L_1)}$$

when frequency is high

$R_1 = R_2$

$$Z_{in} = \frac{V_s}{I_1} = \frac{V_s}{R_2+R_1} = \frac{V_s}{2R}$$

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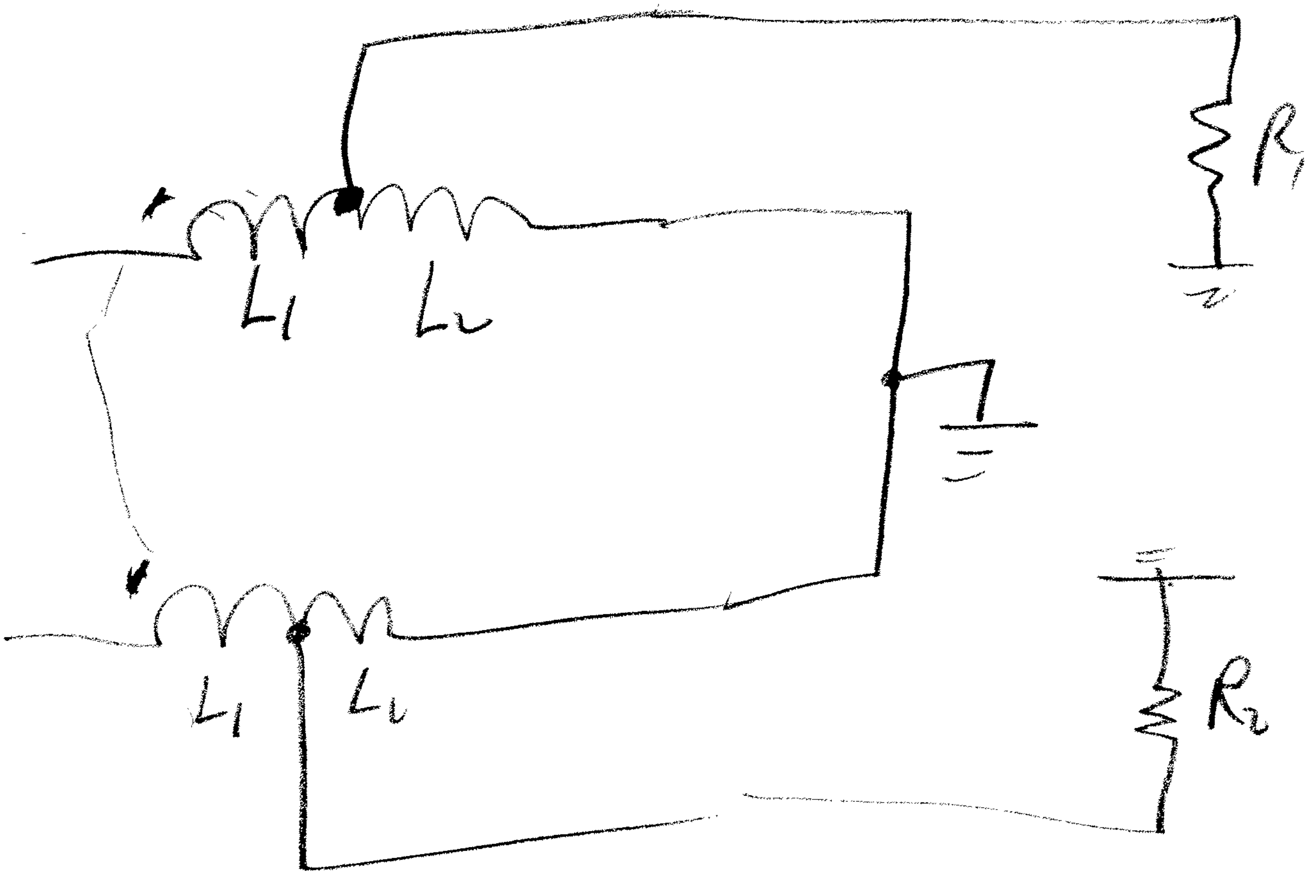

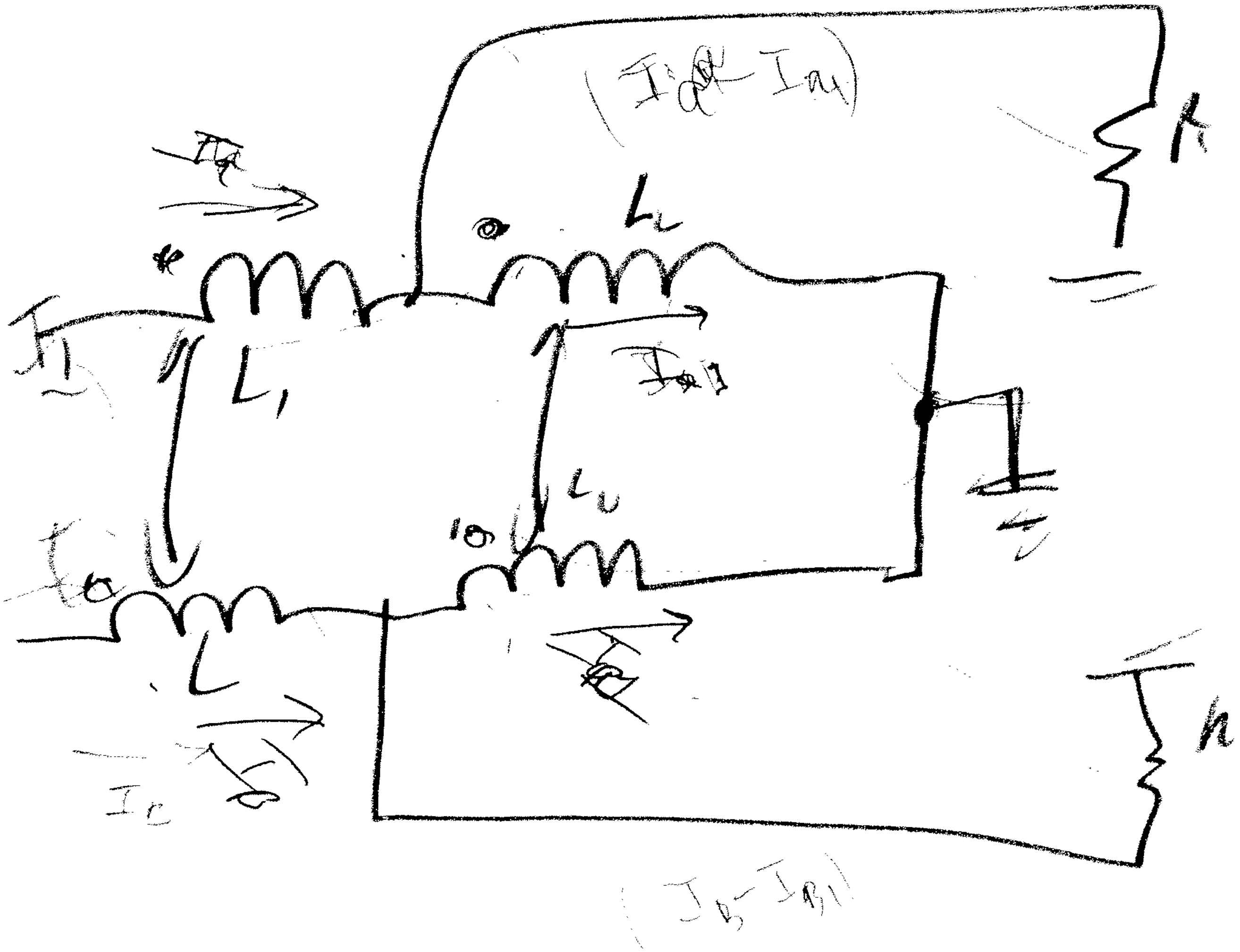

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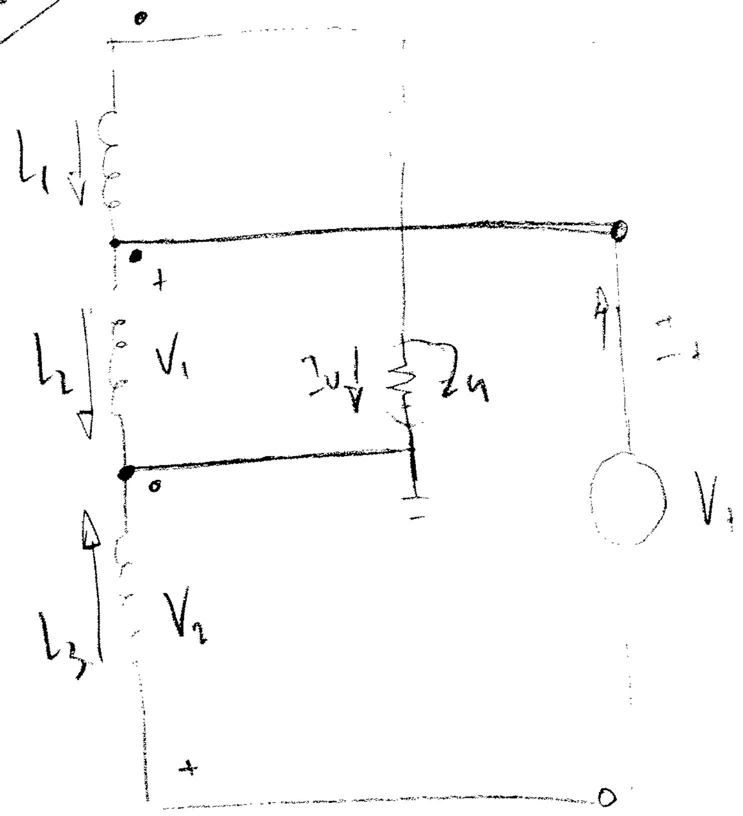


$$V_1 = L_2\frac{di_2}{dt} - M\frac{di_3}{dt} + M\frac{di_1}{dt} = L\left(\frac{di_2}{dt} - \frac{di_3}{dt} + \frac{di_1}{dt}\right)$$

$$V_2 = L_3\frac{di_3}{dt} - M\frac{di_2}{dt} - M\frac{di_1}{dt} = L\left(\frac{di_3}{dt} - \frac{di_2}{dt} - \frac{di_1}{dt}\right)$$

OR $\underline{V_1 = -V_2}$

$0 = -V_+ + j\omega L_2(I_+ - I_M) + j\omega M(I_+ - I_M) + j\omega L_3(I_+) + j\omega M(I_+ - I_+ - I$

$0 = Z_M I_M + j\omega L_2(I_M - I_+) + j\omega M(I_M - I_+) + j\omega L_4(I_M) + j\omega M(I_M - I_+ - I_+)$

$L_1 = L_2 = L_3 = M$

$V_+ = 4j\omega L I_+ - 4j\omega L I_M$

$I_M(Z_M + 4j\omega L) - I_+(4j\omega L)$

$$I_M = \frac{4j\omega L(I_+)}{4j\omega L + Z_M}$$

$$V_+ = 4j\omega L I_+ - 4j\omega L\left(\frac{4j\omega L(I_+)}{4j\omega L + Z_M}\right)$$

$$Z_D = \frac{V_+}{I_+} = 4j\omega L\left(1 - \frac{4j\omega L}{4j\omega L + Z_M}\right) = 4j\omega L\left(\frac{\cancel{4j\omega L} + Z_M - \cancel{4j\omega L}}{4j\omega L + Z_M}\right)$$

$$= \frac{4j\omega L Z_M}{4j\omega L + Z_M}$$

for $j\omega L >> Z_M$ $\quad Z_D = \dfrac{4j\omega L Z_M}{4j\omega L} = \underline{\underline{Z_M}}$

$1:1$

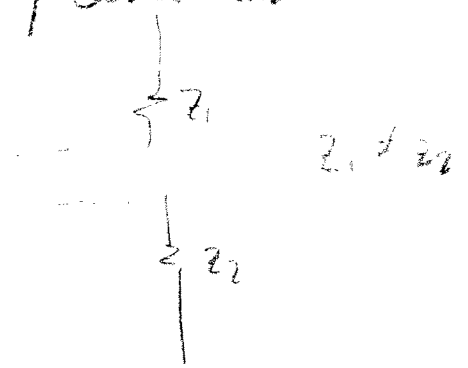
TWO DISADVANTAGES OF THIS BALUN.

assumption $j\omega L >> Z_u$ to have $Z_D = Z_u$

at lower frequencies, this will not be true and $Z_b = \dfrac{4j\omega L Z_u}{4j\omega L + Z_u}$

thus with $\underline{Z_b \neq Z_u}$ SWR will not equal 1, there will be standing waves and higher I^2R losses

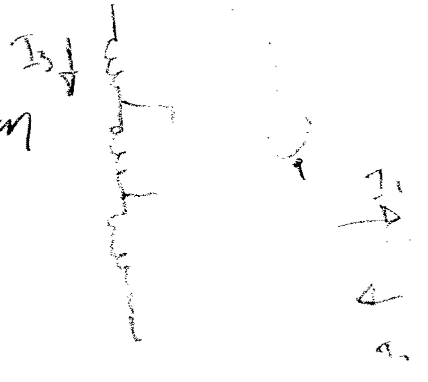
if balanced side is connected to an antenna that is not perfectly symmetrical



then unequal currents will flow through balun.

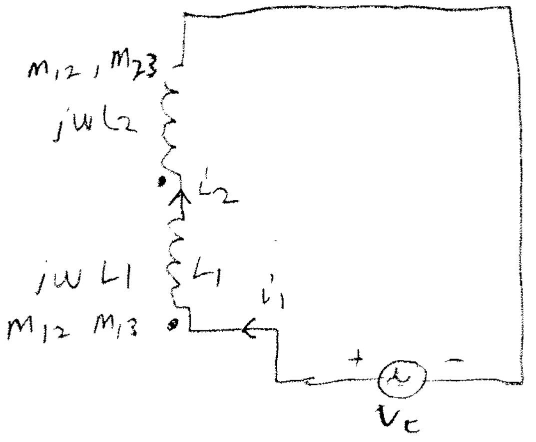
this results in a current drawn from the other side of the balun.

and the transmission line will radiate



if $I_1 \neq I_2$ then an I_3 will be introduced.

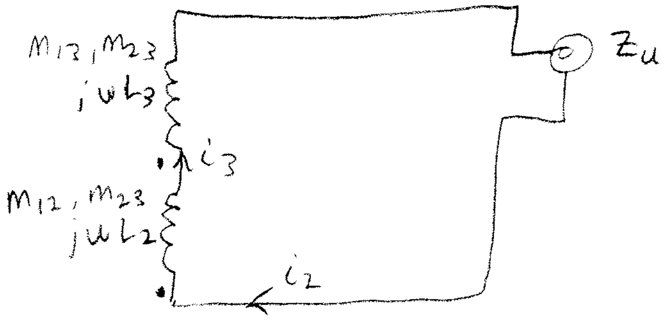
Two loops



$$-U_t + j\omega L_1 I_1 + j\omega M_{13} I_3 + j\omega M_{12} I_2 + j\omega L_2 I_2 + j\omega M_{23} I_3 + j\omega M_{12} I_1 = 0$$

and since K=1 & all L's are equal

$$-U_t + j\omega L I_1 + j\omega L I_3 + j\omega L I_2 + j\omega L I_2 + j\omega L I_3 + j\omega L I_1 = 0$$



$$Z_u I_3 + j\omega L_2 I_2 + j\omega M_{12} I_1 + j\omega M_{23} I_3 + j\omega L_3 I_3 + j\omega M_{13} I_1 + j\omega M_{23} I_2 = 0$$

$$Z_u I_3 + j\omega L I_2 + j\omega L I_1 + j\omega L I_3 + j\omega L I_3 + j\omega L I_1 + j\omega L I_2 = 0$$

⑥

Verification of the ratio $V_t : I_1$

Given

$$-V_t + 2\cdot j\cdot\omega\cdot L\cdot I_1 + 2\cdot j\cdot\omega\cdot L\cdot I_3 + 2\cdot j\cdot\omega\cdot L\cdot\left(I_1 + I_3\right) = 0$$

$$Z_u\cdot I_3 + 2\cdot j\cdot\omega\cdot L\cdot I_1 + 2\cdot j\cdot\omega\cdot L\cdot I_3 + 2\cdot j\cdot\omega\cdot L\cdot\left(I_1 + I_3\right) = 0$$

$$\mathrm{Find}\left(I_1, I_3\right) \rightarrow \begin{bmatrix} \frac{1}{4}\cdot V_t\cdot\frac{\left(Z_u + 4\cdot j\cdot\omega\cdot L\right)}{\left[Z_u\cdot\left(j\cdot(\omega\cdot L)\right)\right]} \\ \frac{-V_t}{Z_u} \end{bmatrix}$$

$$Z_b = \frac{V_t}{I_1} = \frac{V_t}{\frac{1}{4}\cdot V_t\cdot\frac{\left(Z_u + 4\cdot j\cdot\omega\cdot L\right)}{\left[Z_u\cdot\left(j\cdot(\omega\cdot L)\right)\right]}}$$

$Z_b = \frac{V_t}{I_1} = \frac{4}{\left(Z_u + 4\cdot j\cdot\omega\cdot L\right)}\cdot Z_u\cdot j\cdot\omega\cdot L$ which is the expression shown on p 628. ✓

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27-λ

Given

$$-V_S + R_s \cdot I_1 + \sqrt{-1}\left[w \cdot L \cdot (I_1 - I_2)\right] + \sqrt{-1}\left[w \cdot L \cdot (I_3 - I_2)\right] + \sqrt{-1}\left[w \cdot L \cdot (I_1 - I_2)\right] + -\left(\sqrt{-1} \cdot w \cdot L \cdot I_2\right) = 0$$

$$\sqrt{-1} \cdot w \cdot L \cdot I_2 + \left[\left[w \cdot \sqrt{-1} \cdot L \cdot (I_2 - I_1)\right] + R_1 \cdot I_2 + R_2 \cdot (I_2 - I_3) + \sqrt{-1} \cdot \left[w \cdot L \cdot (I_2 - I_3)\right] + \sqrt{-1} \cdot \left[w \cdot L \cdot (I_2 - I_1)\right] + \right.$$

$$\sqrt{-1}\left[w \cdot L \cdot (I_3 - I_2)\right] - \sqrt{-1}\left[w \cdot L \cdot (I_2 - I_1)\right] + R_2 \cdot (I_3 - I_2) = 0$$

$$\mathrm{Find}(I_1, I_2, I_3) \rightarrow \left[\begin{array}{l} i \cdot V_s \cdot \dfrac{(5 \cdot w \cdot L \cdot R_2 + L \cdot w \cdot R_1 + 4 \cdot i \cdot L^2 \cdot w^2 - i \cdot R_2 \cdot R_1)}{(-4 \cdot R_s \cdot w^2 \cdot L^2 + 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 - L^2 \cdot w^2 \cdot R_2 + i \cdot R_s \cdot w \cdot L \cdot R_1 + R_s \cdot R_2 \cdot R_1 -} \\ i \cdot V_s \cdot L \cdot \dfrac{w}{(-4 \cdot R_s \cdot w^2 \cdot L^2 + 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 - L^2 \cdot w^2 \cdot R_2 + i \cdot R_s \cdot w \cdot L \cdot R_1 + R_s \cdot R_2 \cdot R_1 - L^2 \cdot w^2 \cdot} \\ i \cdot V_s \cdot L \cdot w \cdot \dfrac{(3 \cdot R_2 - R_1)}{(-4 \cdot R_s \cdot w^2 \cdot L^2 + 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 - L^2 \cdot w^2 \cdot R_2 + i \cdot R_s \cdot w \cdot L \cdot R_1 + R_s \cdot R_2 \cdot R_1} \end{array}\right.$$

$L := 1 \cdot 10^{-3}$ $R_s := 0.1$ $V_s := 10$ $R_1 := 150$

$R_2 := R_1, 1.01 \cdot R_1 .. 1.5 \cdot R_1$

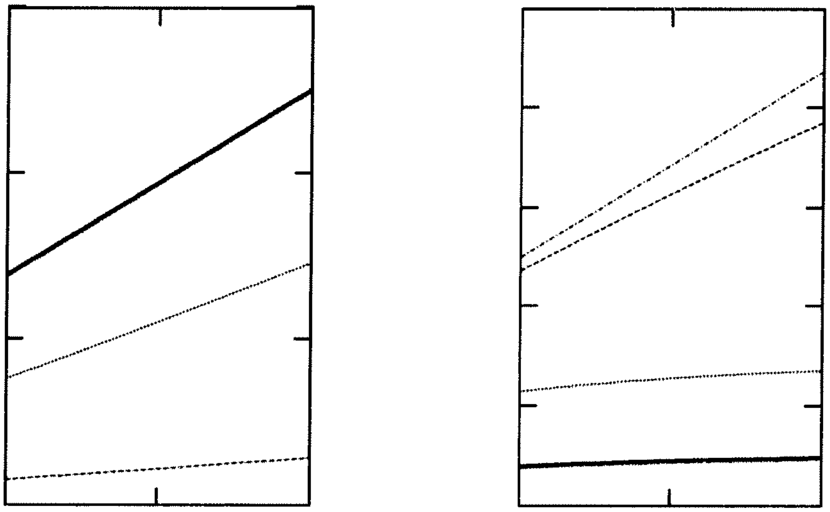
$$I_x(w, R_2) := V_s \cdot L \cdot \frac{w}{(4 \cdot R_s \cdot w^2 \cdot L^2 - 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 + w^2 \cdot L^2 \cdot R_2 - i \cdot R_s \cdot w \cdot L \cdot R_1 - R_s \cdot R_2 \cdot R_1 + w^2 \cdot L^2 \cdot R_1 - 2 \cdot i \cdot L \cdot v}$$

$$I_y(w, R_2) := V_s \cdot L \cdot w \cdot \frac{(2 \cdot L \cdot w - i \cdot R_1)}{(4 \cdot R_s \cdot w^2 \cdot L^2 - 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 + w^2 \cdot L^2 \cdot R_2 - i \cdot R_s \cdot w \cdot L \cdot R_1 - R_s \cdot R_2 \cdot R_1 + w^2 \cdot L^2 \cdot R_1 - 2 \cdot i \cdot}$$

$$Z_{in}(w,R_2) := \frac{\left(-4\cdot R_s\cdot w^2\cdot L^2 + 5\cdot i\cdot R_s\cdot w\cdot L\cdot R_2 - w^2\cdot L^2\cdot R_2 + i\cdot R_s\cdot w\cdot L\cdot R_1 + R_s\cdot R_2\cdot R_1 - w^2\cdot L^2\cdot R_1 + 2\cdot i\cdot L\right.}{\left(5\cdot i\cdot L\cdot w\cdot R_2 + i\cdot L\cdot w\cdot R_1 - 4\cdot w^2\cdot L^2 + R_2\cdot R_1\right)}$$

$$Z_L(R_2) := R_1 + R_2$$

$$w := 10^4\cdot 2\cdot \pi$$



$$V_s \cdot L \cdot \frac{w}{\left(4 \cdot R_s \cdot w^2 \cdot L^2 - 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 + w^2 \cdot L^2 \cdot R_2 - i \cdot R_s \cdot w \cdot L \cdot R_1 - R_s \cdot R_2 \cdot R_1 + w^2 \cdot L^2 \cdot R_1 - 2 \cdot i \cdot L \cdot w \cdot R_2 \cdot R_1\right.}$$

$$V_s \cdot L \cdot \frac{w}{\left(4 \cdot R_s \cdot w^2 \cdot L^2 - 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 + w^2 \cdot L^2 \cdot R_2 - i \cdot R_s \cdot w \cdot L \cdot R_1 - R_s \cdot R_2 \cdot R_1 + w^2 \cdot L^2 \cdot R_1 - 2 \cdot i \cdot L \cdot w \cdot R_2 \cdot R_1\right.}$$

$$V_s \cdot L \cdot w \cdot \frac{\left(2 \cdot L \cdot w - i \cdot R_1\right)}{\left(4 \cdot R_s \cdot w^2 \cdot L^2 - 5 \cdot i \cdot R_s \cdot w \cdot L \cdot R_2 + w^2 \cdot L^2 \cdot R_2 - i \cdot R_s \cdot w \cdot L \cdot R_1 - R_s \cdot R_2 \cdot R_1 + w^2 \cdot L^2 \cdot R_1 - 2 \cdot i \cdot L \cdot w \cdot R_2 \cdot \right.}$$

$$+\sqrt{-1}\cdot\left[w\cdot L\cdot\left(I_2 - I_1\right)\right] + \sqrt{-1}\cdot w\cdot L\cdot I_2 + \sqrt{-1}\cdot\left[w\cdot L\cdot\left(I_2 - I_1\right)\right] + \sqrt{-1}\cdot\left[w\cdot L\cdot\left(I_2 - I_3\right)\right]\Big] = 0$$

$$_1\Big)$$

$$_1 - L^2\cdot w^2\cdot R_1 + 2\cdot i\cdot w\cdot L\cdot R_2\cdot R_1\Big)$$

$$\cdot\left(2\cdot i\cdot w\cdot L + 3\cdot R_2\right)$$

$$^2\cdot R_1 + 2\cdot i\cdot w\cdot L\cdot R_2\cdot R_1\Big)$$

$$\cdot R_1 - L^2\cdot w^2\cdot R_1 + 2\cdot i\cdot w\cdot L\cdot R_2\cdot R_1\Big)\Bigg]$$

$$\cdot\left(2\cdot L\cdot w - 3\cdot i\cdot R_2\right)$$

$$i\cdot L\cdot w\cdot R_2\cdot R_1\Big)$$

$$2\cdot i\cdot L\cdot w\cdot R_2\cdot R_1\Big)$$

$$\Bigg[V_s\cdot\overline{\quad}\;\Big(4\cdot R$$

$$\Big(-5\cdot i\cdot L\cdot w\cdot$$

$$\Big(-4\cdot R_s\cdot w^2$$

$$\underline{\cdot w \cdot R_2 \cdot R_1\Big)}$$

$$-\cdot\left(2\cdot L\cdot w - 3\cdot i\cdot R_2\right)$$

$$)$$

$$-\cdot\left(2\cdot L\cdot w - 3\cdot i\cdot R_2\right) - i\cdot V_s\cdot L\cdot w\cdot\frac{\left(-3\cdot R_2 + R_1\right)}{\left(4\cdot R_s\cdot w^2\cdot L^2 - 5\cdot i\cdot R_s\cdot w\cdot L\cdot R_2 + w^2\cdot L^2\cdot R_2 - i\cdot R_s\cdot w\cdot L\cdot R_1 - R_s\cdot R_2\cdot R_1 + w^2\cdot\right.}$$

$$)$$

$$\overline{{}_1)}$$

$$\frac{V_s}{\frac{(-5\cdot i\cdot L\cdot w\cdot R_2 - i\cdot L\cdot w\cdot R_1 + 4\cdot w^2\cdot L^2 - R_2\cdot R_1)}{{}_s\cdot w^2\cdot L^2 - 5\cdot i\cdot R_s\cdot w\cdot L\cdot R_2 + w^2\cdot L^2\cdot R_2 - i\cdot R_s\cdot w\cdot L\cdot R_1 - R_s\cdot R_2\cdot R_1 + w^2\cdot L^2\cdot R_1 - 2\cdot i\cdot L\cdot w\cdot R_2\cdot R_1)}]}$$

$$\frac{4}{\cdot R_2 - i\cdot L\cdot w\cdot R_1 + 4\cdot w^2\cdot L^2 - R_2\cdot R_1)}\cdot R_s\cdot w^2\cdot L^2 - 5\cdot\frac{i}{(-5\cdot i\cdot L\cdot w\cdot R_2 - i\cdot L\cdot w\cdot R_1 + 4\cdot w^2\cdot L^2 - R_2\cdot R_1)}\cdot R_s\cdot w\cdot L\cdot$$

$${}^{2}\cdot L^2 + 5\cdot i\cdot R_s\cdot w\cdot L\cdot R_2 - w^2\cdot L^2\cdot R_2 + i\cdot R_s\cdot w\cdot L\cdot R_1 + R_s\cdot R_2\cdot R_1 - w^2\cdot L^2\cdot R_1 + 2\cdot i\cdot L\cdot w\cdot R_2\cdot R_1)$$

$$\left(5 \cdot i \cdot L \cdot w \cdot R_2 + i \cdot L \cdot w \cdot R_1 - 4 \cdot w^2 \cdot L^2 + R_2 \cdot R_1\right)$$

$$L^{2}\cdot R_{1} - 2\cdot i\cdot L\cdot w\cdot R_{2}\cdot R_{1}\Big)$$

$$R_2 + \frac{1}{\left(-5 \cdot i \cdot L \cdot w \cdot R_2 - i \cdot L \cdot w \cdot R_1 + 4 \cdot w^2 \cdot L^2 - R_2 \cdot R_1\right)} \cdot w^2 \cdot L^2 \cdot R_2 - \frac{i}{\left(-5 \cdot i \cdot L \cdot w \cdot R_2 - i \cdot L \cdot w \cdot R_1 + 4 \cdot w^2 \cdot L^2 - R\right.}$$

$$\frac{}{{}_2\cdot R_1)}\cdot R_S\cdot w\cdot L\cdot R_1-\frac{1}{\left(-5\cdot i\cdot L\cdot w\cdot R_2-i\cdot L\cdot w\cdot R_1+4\cdot w^2\cdot L^2-R_2\cdot R_1\right)}\cdot R_S\cdot R_2\cdot R_1+\frac{1}{(-5\cdot i\cdot L\cdot w\cdot R_2-i\cdot L\cdot w\cdot R_1\cdot}$$

$$\frac{\quad}{+ 4\cdot w^{2}\cdot L^{2} - R_{2}\cdot R_{1})}\cdot w^{2}\cdot L^{2}\cdot R_{1} - 2\cdot\frac{i}{\left(-5\cdot i\cdot L\cdot w\cdot R_{2} - i\cdot L\cdot w\cdot R_{1} + 4\cdot w^{2}\cdot L^{2} - R_{2}\cdot R_{1}\right)}\cdot L\cdot w\cdot R_{2}\cdot R_{1}$$

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27-10

Given

$$2 \cdot I_1 \cdot R_{eq} - I_2 \cdot R_{eq} = V_s$$

$$Z_c \cdot I_2 + \frac{V_s}{2} + R_{eq} \cdot (I_2 - I_1) = 0$$

$$\text{Find}(I_1, I_2) \rightarrow \begin{pmatrix} \frac{1}{2} \cdot \frac{V_s}{R_{eq}} \\ 0 \end{pmatrix}$$

Given

$$2 \cdot I_x \cdot R_{eq} - I_y \cdot R_{eq} = V_s$$

$$Z_c \cdot I_y + R_{eq} \cdot (I_y - I_x) = 0$$

$$\text{Find}(I_x, I_y) \rightarrow \begin{bmatrix} V_s \cdot \frac{(Z_c + R_{eq})}{[(2 \cdot Z_c + R_{eq}) \cdot R_{eq}]} \\ \frac{V_s}{(2 \cdot Z_c + R_{eq})} \end{bmatrix}$$

$$\frac{V_s}{(2 \cdot Z_c + R_{eq})} - V_s \cdot \frac{(Z_c + R_{eq})}{[(2 \cdot Z_c + R_{eq}) \cdot R_{eq}]}$$

$$-V_s \cdot \frac{Z_c}{[(2 \cdot Z_c + R_{eq}) \cdot R_{eq}]}$$

Given

$$-V_s + I_x \cdot (R_s + R_{eq}) + (I_x - I_y) \cdot R_{eq} = 0$$

$$Z_c \cdot I_y + (I_y - I_x) \cdot R_{eq} = 0$$

$$\mathrm{Find}(I_x, I_y) \rightarrow \begin{bmatrix} \dfrac{V_s}{\left(R_s \cdot Z_c + R_s \cdot R_{eq} + 2 \cdot R_{eq} \cdot Z_c + R_{eq}^{\,2}\right)} \cdot (Z_c + R_{eq}) \\ V_s \cdot \dfrac{R_{eq}}{\left(R_s \cdot Z_c + R_s \cdot R_{eq} + 2 \cdot R_{eq} \cdot Z_c + R_{eq}^{\,2}\right)} \end{bmatrix}$$

$$V_s \cdot \frac{R_{eq}}{\left(R_s \cdot Z_c + 2 \cdot Z_c \cdot R_{eq} + R_s \cdot R_{eq} + R_{eq}^{\,2}\right)} - V_s \cdot \frac{(Z_c + R_{eq})}{\left(R_s \cdot Z_c + 2 \cdot Z_c \cdot R_{eq} + R_s \cdot R_{eq} + R_{eq}^{\,2}\right)}$$

differential mode

$$\frac{V_s \cdot \dfrac{(Z_c + R_{eq})}{\left(R_s \cdot Z_c + 2 \cdot Z_c \cdot R_{eq} + R_s \cdot R_{eq} + R_{eq}^{\,2}\right)} - \left[V_s \cdot \dfrac{R_{eq}}{\left(R_s \cdot Z_c + 2 \cdot Z_c \cdot R_{eq} + R_s \cdot R_{eq} + R_{eq}^{\,2}\right)} - V_s \cdot \dfrac{}{\left(R_s \cdot Z_c + \right.}\right.}{2}$$

$$\frac{1}{2} \cdot V_s \cdot \frac{(2 \cdot Z_c + R_{eq})}{\left(R_s \cdot Z_c + 2 \cdot Z_c \cdot R_{eq} + R_s \cdot R_{eq} + R_{eq}^{\,2}\right)}$$

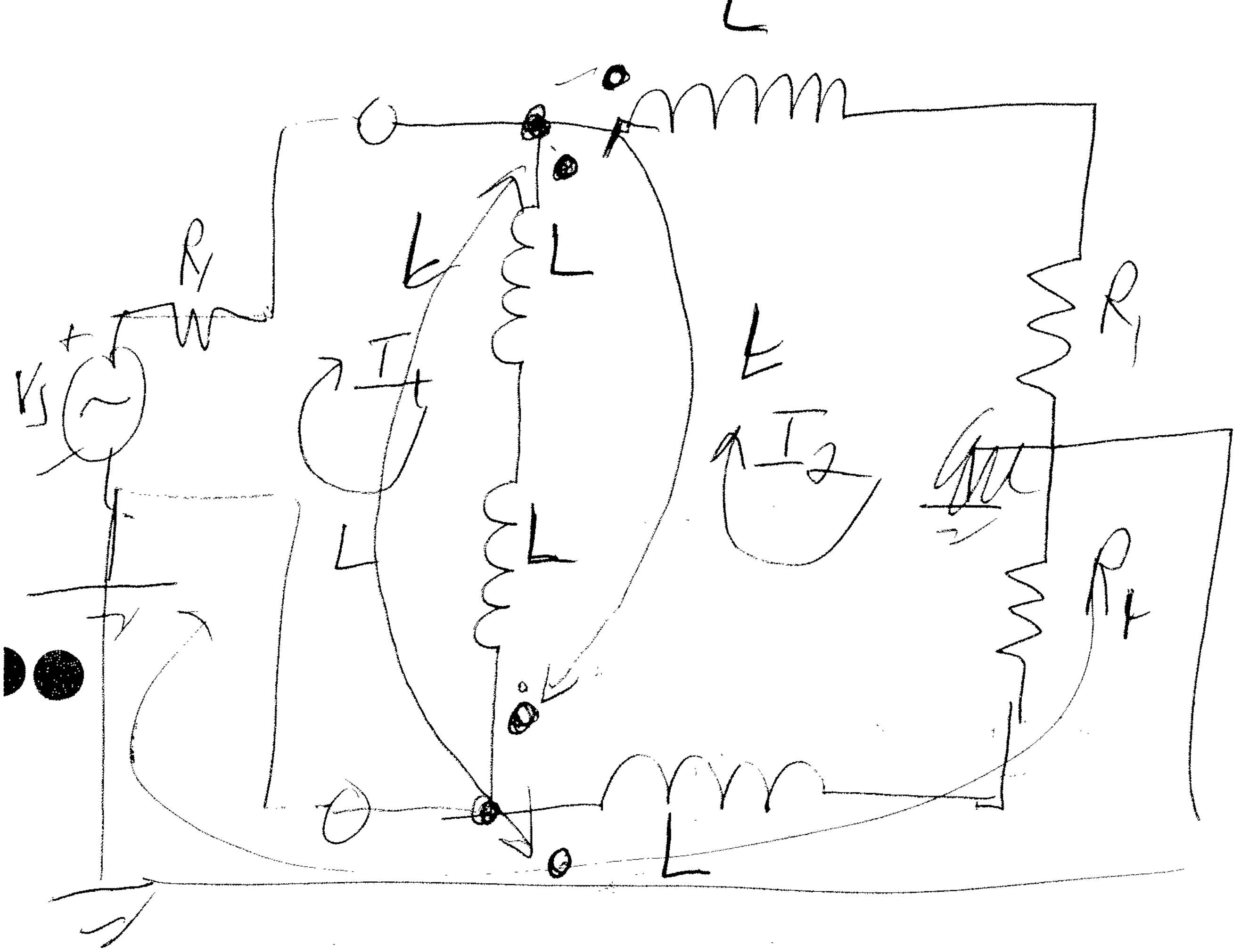
$$\left.\frac{\left(Z_c + R_{eq}\right)}{2 \cdot Z_c \cdot R_{eq} + R_s \cdot R_{eq} + R_{eq}^{\;2}\right)}\right]$$

$$\begin{array}{r} I_1 = I_c + I_d \\ + \quad I_2 = I_c - I_d \\ \hline I_1 + I_2 = 2I_c \end{array}$$

$$I_c = \frac{I_1 + I_2}{2}$$

$$\begin{array}{r} I_1 = I_c + I_d \\ -I_2 = -I_c + I_d \\ \hline I_1 - I_2 = 2I_d \end{array}$$

$$I_c = \frac{I_1 - I_2}{2}$$



$$(1)\quad -V_s + R_s I_1 + j\omega L(I_1 - I_2) - j\omega L I_2 + j\omega L(I_2 + I_2)$$

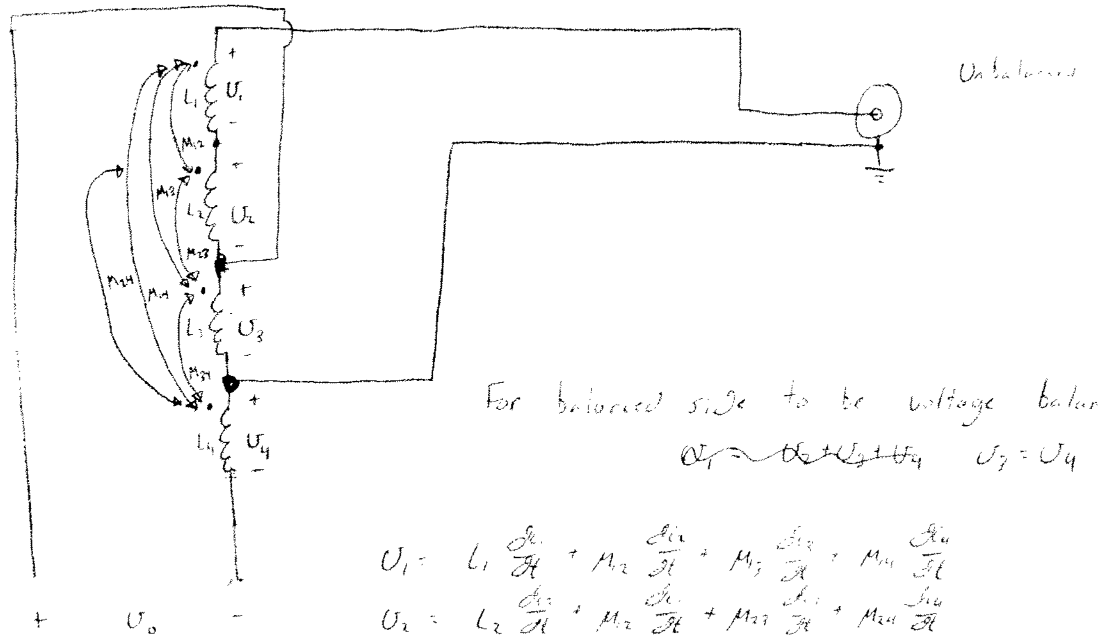
$$-j\omega L I_2 = 0$$

$$(2)\quad j\omega L I_2 + j\omega L(I_2 - I_1) + I_2 R_1 + I_2 R_1$$

[redacted]

Section 26

(#1) Repeat the analy. contained in Problem 26.1 for another voltage balun.



For balanced side to be voltage balanced:

~~$v_1 = v_2 + v_3 + v_4$~~ $v_3 = v_4$

$$v_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} + M_{14} \frac{di_4}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} + M_{23} \frac{di_3}{dt} + M_{24} \frac{di_4}{dt}$$

$$v_3 = L_3 \frac{di_3}{dt} + M_{34} \frac{di_4}{dt} + M_{23} \frac{di_2}{dt} + M_{13} \frac{di_1}{dt}$$

$$v_4 = L_4 \frac{di_4}{dt} + M_{34} \frac{di_3}{dt} + M_{24} \frac{di_2}{dt} + M_{14} \frac{di_1}{dt}$$

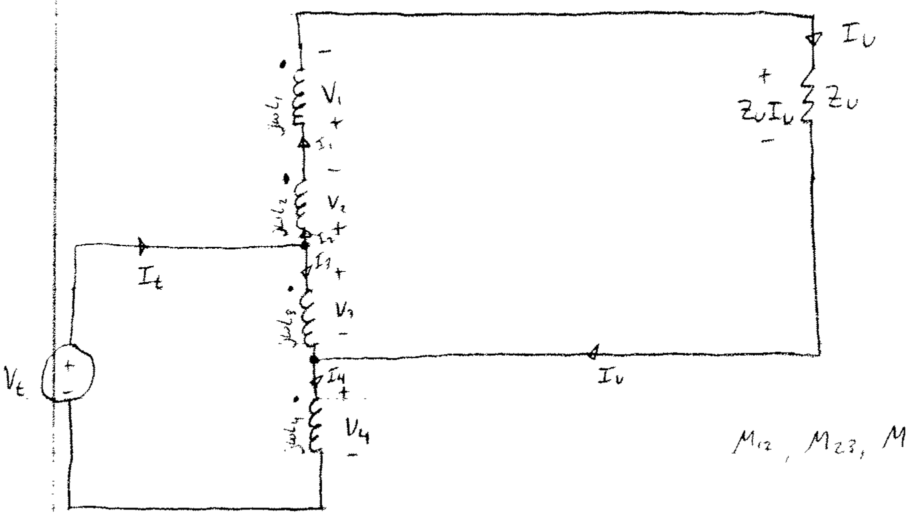
If coupling perfect & $L_1 = L_2 = L_3 = L_4 = L \Rightarrow M = L$

$$v_3 = L\left(\frac{di_3}{dt} + \frac{di_4}{dt} + \frac{di_2}{dt} + \frac{di_1}{dt}\right)$$

$$v_4 = L\left(\frac{di_4}{dt} + \frac{di_3}{dt} + \frac{di_2}{dt} + \frac{di_1}{dt}\right)$$

∴ balanced side is voltage balanced

Apply test voltage to balanced side



$I_1 = I_2 = I_u$

$I_4 = I_t$

$I_t = I_2 + I_3 = I_u + I_3$

$I_t = I_3 + I_u$

$M_{12}, M_{23}, M_{34}, M_{13}, M_{14}, M_{24}$

Voltage Loops:

① $-V_t + j\omega L_3 I_3 + j\omega M_{34} I_4 - j\omega M_{23} I_2 - j\omega M_{13} I_1 + j\omega L_4 I_4 + j\omega M_{34} I_3 - j\omega M_{24} I_2 - j\omega M_{14} I_1 = 0$

② $Z_u I_u - j\omega L_3 I_3 - j\omega M_{34} I_4 + j\omega M_{23} I_2 + j\omega M_{13} I_1 + j\omega L_2 I_2 + j\omega M_{12} I_1 - j\omega M_{23} I_3 - j\omega M_{24} I_4 +$
$j\omega L_1 I_1 + j\omega M_{12} I_2 - j\omega M_{13} I_3 - j\omega M_{14} I_4 = 0$

Subst. $M = L$

① $-V_t + j\omega L I_3 + j\omega L I_4 - j\omega L I_2 - j\omega L I_1 + j\omega L I_4 + j\omega L I_3 - j\omega L I_2 - j\omega L I_1 = 0$

② $Z_u I_u - j\omega L I_3 - j\omega L I_4 + j\omega L I_2 + j\omega L I_1 + j\omega L I_2 + j\omega L I_1 - j\omega L I_3 - j\omega L I_4 +$
$j\omega L I_1 + j\omega L I_2 - j\omega L I_3 - j\omega L I_4 = 0$

Simplify:

① $V_t = j\omega 2L\left(-\overset{I_u}{\cancel{I_1}} - \overset{I_u}{\cancel{I_2}} + \overset{I_t\,I_u}{\cancel{I_3}} + \overset{I_t}{\cancel{I_4}}\right) = j\omega 2L(3I_u + 2I_t)$

$V_t = -j\omega 6L I_u + j\omega 4L I_t$

② $Z_u I_u + j\omega 3L I_1 + j\omega 3L I_2 - j\omega 3L I_3 - j\omega 3L I_4 = 0$

$Z_u I_u + j\omega 3L I_u + j\omega 3L I_u - j\omega 3L I_t + j\omega 3L I_u - j\omega 3L I_t = 0$

$Z_u I_u + j\omega 9L I_u - j\omega 6L I_t = 0 \qquad I_u(Z_u + j\omega 9L) = j\omega 6L I_t$

$$I_u = \frac{j\omega 6L I_t}{Z_u + j\omega 9L}$$

$$V_t = +j\omega 4L I_t - j\omega 6L\left(\frac{j\omega 6L I_t}{Z_u + j\omega 9L}\right)$$

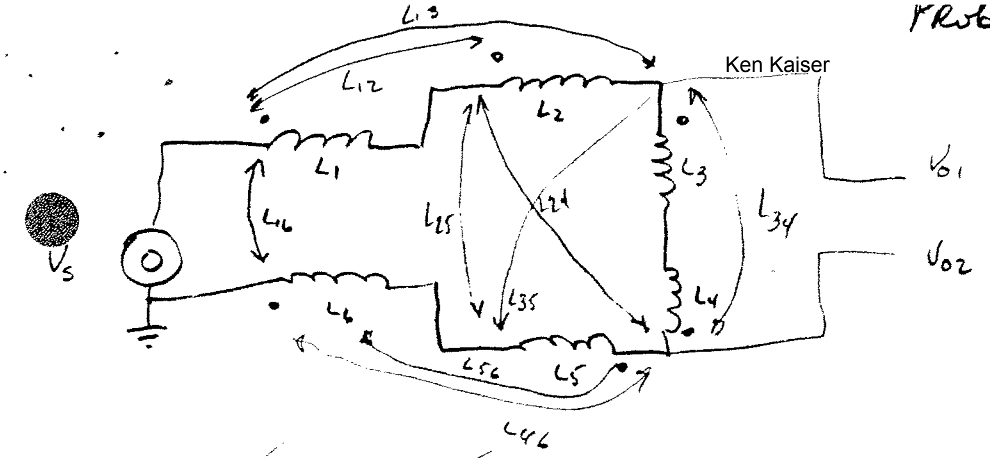
$$V_t = \frac{+j\omega 4L I_t(Z_u) + \cancel{j\omega 4L I_t(j\omega 9L)} - \cancel{j\omega 6L(j\omega 6L I_t)}}{Z_u + j\omega 9L} = \frac{+j\omega 4L I_t(Z_u)}{Z_u + j\omega 9L}$$

if $|j\omega 9L| \gg Z_u$ $\qquad Z_b = \dfrac{V_t}{I_t} \approx \dfrac{j\omega 4L Z_u}{j\omega 9L} = \dfrac{4Z_u}{9}$

$Z_b \approx \frac{4}{9} Z_u$

$Z_b = .4Z_u$

PRoblem 162



Ken Kaiser

BALUN

$$\frac{V_s}{I} = L_1 + L_2 + L_{12} + L_{21} - L_{16} + L_3 + L_{13} + L_{23} + L_{31} - L_{25} - L_{35} + L_{32} - L_{34} - L_{24} + L_4 - L_{42} - L_{43} + L_5 +$$
$$L_{45} - L_{53} - L_{52} + L_{54} + L_{46} + L_{56} + L_6 + L_{65} + L_{64} - L_{61} = 8L$$

$$V_{02} = I\left(L_6 + L_{56} + L_{46} + L_{65} + L_{64} + L_5 - L_{61} - L_{25} - L_{35} + L_{45} - L_{34} - L_{24}\right)$$
$$V_{02} = I(2L) = (2L)\frac{V_s}{8L} = \frac{V_s}{4}$$

V_{01} voltage across L_1 and L_2 ✓

$$V_{01} = I\big(L_6 + L_{56} + L_{46} + L_{65} + L_5 + L_{64} - L_{61} - L_{25} - L_{35} + L_{45} - L_{34} - L_{24} - L_{53} + L_{23} + L_{13} + L_3 + L_4 - L$$

[illegible]$_1$ $= I(4L) = \frac{V_s}{8L}(4L) = \frac{V_s}{2}$

$$V_{01} - V_{02} = \frac{V_s}{4}$$

$\frac{2}{6}$

$$Z_b = \frac{V_t}{I_1 + I_2}$$

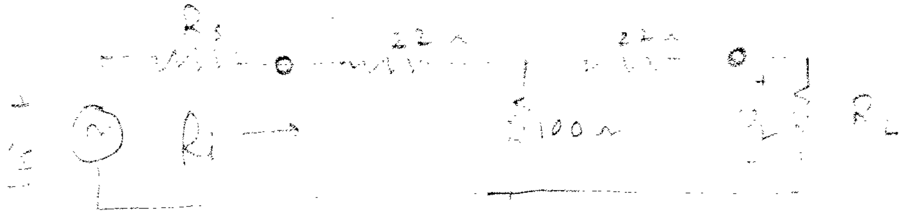
$$V_t = (J\omega L_3 + J\omega L_4 - J\omega L_{34})I_1 - (J\omega L_{35} + J\omega L_{24} - J\omega L_{23} - J\omega L_{45} - J\omega L_{46} - J\omega L_{13})I_2$$
$$= J\omega L\, I_1 + 2J\omega L\, I_2$$

$$V_t = (J\omega L_2 + J\omega L_{12} + J\omega L_{13} + J\omega L_1 - J\omega L_{16} + Z_u - J\omega L_{25} + J\omega L_6 - J\omega L_{61} + J\omega L_{56} + J\omega L_5 + J\omega L_{65} - J\omega L_{52})I$$
$$- (J\omega L_{53} + J\omega L_{42} - J\omega L_{45} - J\omega L_{46} - J\omega L_{32} - J\omega L_{31})I_1$$
$$= 4J\omega L\, I_2 + Z_u I_2 + 2J\omega L\, I_1$$

[illegible]

Ken Kaiser [redacted]

9. A matching pad provides standard coupling to [illegible] [illegible] even a source [illegible] ranges from a [illegible] variety of loads.



[illegible] of the T matching pad is placed in all balanced system, it will unbalance the system as it is not symmetrical with respect to 0 V [illegible]

The input resistance

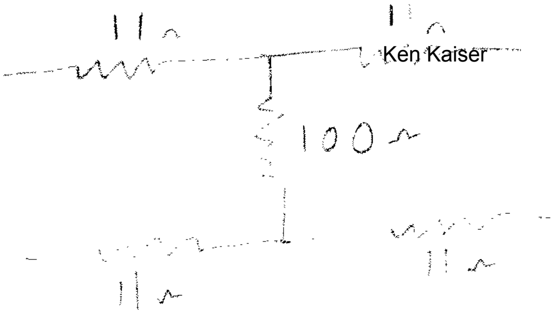
$$R_i = \{(R_L + 22) \| 100\} + 22$$

$$v_L = \frac{v_s}{(R_s + R_i)} \cdot \frac{100 R_L}{(122 + R_L)}$$

$$\therefore \frac{v_L}{v_s} = \frac{100 R_L}{[\{(R_L + 22) \| 100\} + 22 + R_s] [illegible]}$$

A balanced version of the pad is as follows [illegible] symmetrical and its input impedance and attenuation is identical to the unbalanced version.

Ken Kaiser



A balanced load that center is ground [illegible] as following.