# Noninteger Cycles Cosine Wave

Ken Kaiser

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$A := 2$

$T := 3$

$a := 1$

$$f_{avg1} := \frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \cos(a \cdot t)\, dt$$

$f_{avg1} = 1.33$

$$f_{avg2} := \frac{2 \cdot A}{a \cdot T} \cdot \sin\left(\frac{a \cdot T}{2}\right)$$

$f_{avg2} = 1.33$

$$f_{rms1} := \sqrt{\frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} (A \cdot \cos(a \cdot t))^2\, dt}$$

$f_{rms1} = 1.447$

$$f_{rms2} := A \cdot \sqrt{\frac{1}{2} + \frac{\cos\left(\frac{a \cdot T}{2}\right) \cdot \sin\left(\frac{a \cdot T}{2}\right)}{a \cdot T}}$$

$f_{rms2} = 1.447$

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$$\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\cos(\alpha\cdot t)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt\right]\cdot\frac{2}{T}$$

$$2\cdot\left(\sin\left(\frac{1}{2}\cdot\alpha\cdot T-\pi\cdot n\right)\cdot\alpha\cdot T+2\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T-\pi\cdot n\right)\cdot\pi\cdot n+\sin\left(\frac{1}{2}\cdot\alpha\cdot T+\pi\cdot n\right)\cdot\alpha\cdot T-2\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T+\pi\cdot n\right)\cdot\pi\cdot n\right)\cdot\frac{A}{\left(\alpha^2\cdot T^2-4\cdot\pi^2\cdot n^2\right)}$$

$$=\left[2\sin\left(\tfrac{1}{2}\alpha T-\pi n\right)\alpha T+2\sin\left(\tfrac{1}{2}\alpha T+\pi n\right)\alpha T\right]\frac{A}{\alpha^2T^2-4\pi^2n^2}$$

$$=4A\alpha T\,\frac{(-1)^n\sin\left(\frac{\alpha T}{2}\right)}{\alpha^2T^2-4\pi^2n^2}=4A\alpha T\,\frac{(-1)(-1)^n\sin\left(\frac{\alpha T}{2}\right)}{(-1)\left(4\pi^2n^2-\alpha^2T^2\right)}$$

$$=\boxed{4A\alpha T\,\frac{(-1)^{n-1}\sin\left(\frac{\alpha T}{2}\right)}{4\pi^2n^2-\alpha^2T^2}}$$

$\sin(\tfrac{1}{2}\alpha T-\pi n)=\sin(\tfrac{1}{2}\alpha$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha$

$=\sin(\tfrac{1}{2}\alpha T)\cos(-\pi n)+\cos(\tfrac{\alpha T}{2})$

$=\sin\left(\frac{\alpha T}{2}\right)\cos(-\pi n)$

$\cos(-\pi n)=(-1)^n$

$=(-1)^n\sin\left(\frac{\alpha T}{2}\right)$

$$\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\cos(\alpha\cdot t)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt\right]\cdot\frac{2}{T}$$

0

$$\frac{2\cdot A}{\alpha\cdot T}\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)+\sum_{n=1}^{3}4\cdot A\cdot\alpha\cdot T\cdot\left[\frac{(-1)^{n-1}\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot\pi^2\cdot n^2-\alpha^2\cdot T^2}\right]\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$

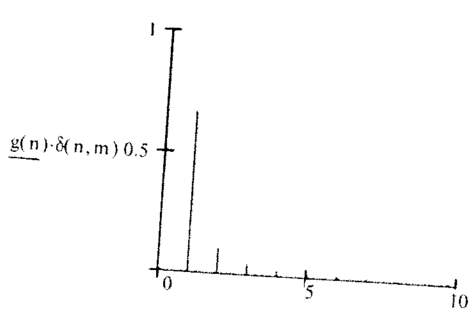
$$2\cdot\frac{A}{(\alpha\cdot T)}\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)+4\cdot A\cdot\alpha\cdot T\cdot\frac{\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(4\cdot\pi^2-\alpha^2\cdot T^2\right)}\cdot\cos\left(2\cdot\frac{\pi}{T}\cdot t\right)-4\cdot A\cdot\alpha\cdot T\cdot\frac{\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(16\cdot\pi^2-\alpha^2\cdot T^2\right)}\cdot\cos\left(4\cdot\frac{\pi}{T}\cdot t\right)+4\cdot A\cdot\alpha\cdot T\cdot\frac{\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(36\cdot\pi^2-\alpha^2\cdot T^2\right)}\cdot\cos\left(6\cdot\frac{\pi}{T}\cdot t\right)$$

$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$A := 1$

$$g(n) := \mathrm{if}\left[n = 0,\ \frac{2\cdot A}{4.2}\cdot\sin\left(\frac{4.2}{2}\right),\ \left|4\cdot A\cdot 4.2\cdot\left[\frac{(-1)^{n-1}\cdot\sin\left(\frac{4.2}{2}\right)}{4\cdot\pi^2\cdot n^2-17.64}\right]\right|\right]$$



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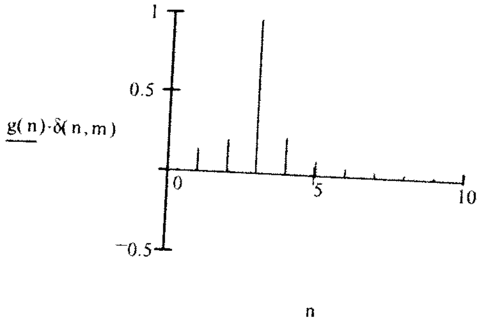
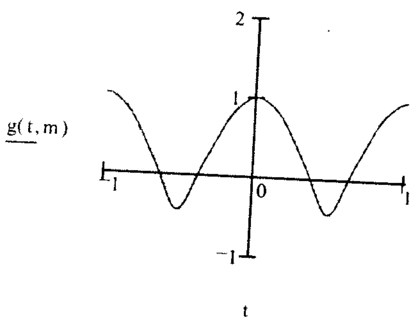
$$g(n) := \text{if}\left[n = 0, \frac{2\cdot A}{20.2}\cdot\sin\left(\frac{20.2}{2}\right), \left|4\cdot A\cdot 20.2\cdot\left[\frac{(-1)^{n-1}\cdot\sin\left(\frac{20.2}{2}\right)}{4\cdot\pi^2\cdot n^2 - 408.04}\right]\right|\right]$$



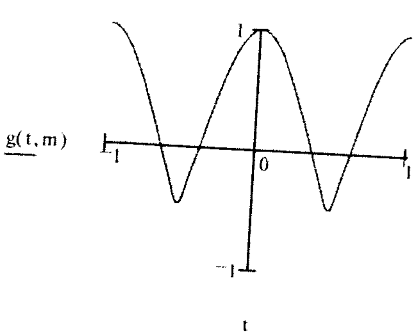

$$T := 1$$

$$g(t,m) := \frac{2\cdot A}{4.2}\cdot\sin\left(\frac{4.2}{2}\right) + \sum_{n=1}^{5} 4\cdot A\cdot 4.2\cdot\frac{(-1)^{n-1}\cdot\sin\left(\frac{4.2}{2}\right)}{4\cdot\pi^2\cdot n^2 - 17.64}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$

$$t := -T, -T + \frac{T}{1000} .. T$$




$$g(t,m) := \frac{2\cdot A}{4.2}\cdot\sin\left(\frac{4.2}{2}\right) + \sum_{n=1}^{10} 4\cdot A\cdot 4.2\cdot\frac{(-1)^{n-1}\cdot\sin\left(\frac{4.2}{2}\right)}{4\cdot\pi^2\cdot n^2 - 17.64}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$



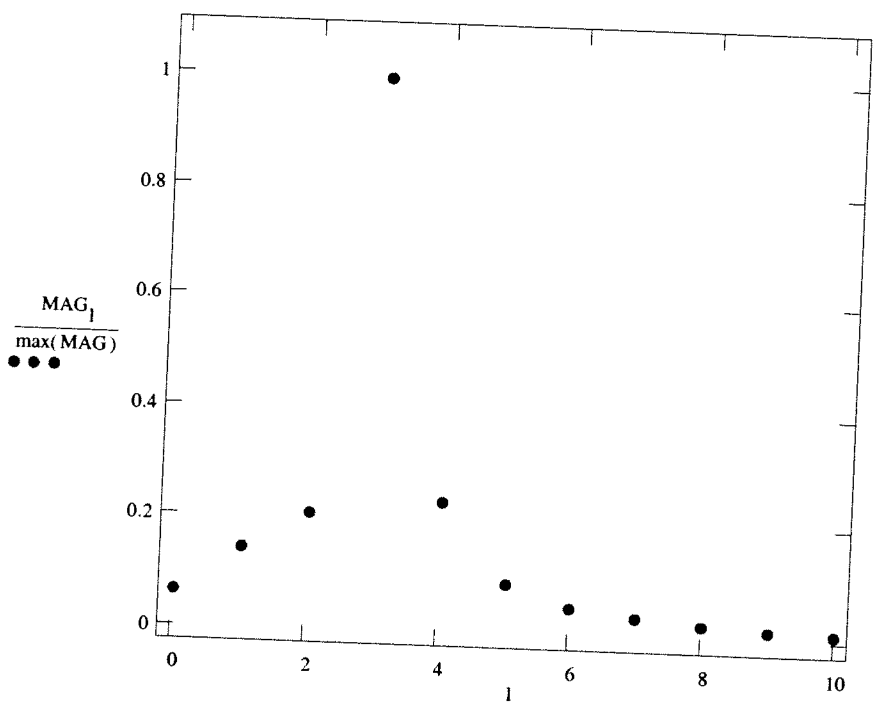
$k := 1 .. 10$ $\quad T := 0.5$ $\quad A := 2$ $\quad B := -2$ $\quad C := -1$

$l := 0 .. 10$ $\quad \alpha := 40.4$

$$M_k := \frac{4 \cdot A \cdot \alpha \cdot T}{4 \cdot \pi^2 \cdot k^2 - \alpha^2 \cdot T^2}$$

$$M_0 := \frac{2 \cdot A}{\alpha \cdot T}$$

$$MAG_l := \left| M_l \right|$$



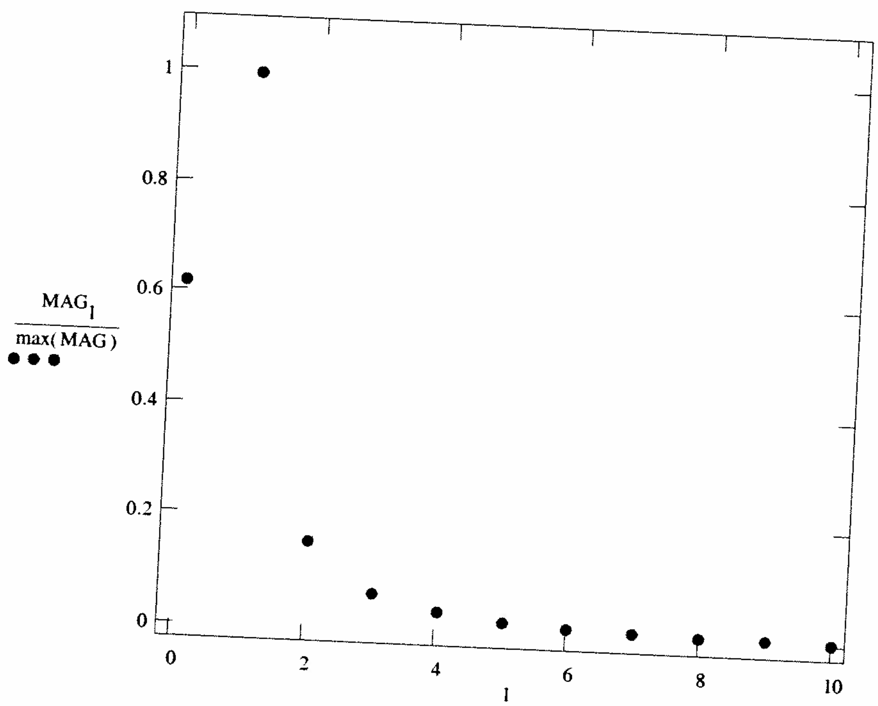
$k := 1..10$ $\quad T := 2$ $\quad A := 2$ $\quad B := -2$ $\quad C := -1$

$l := 0..10$ $\quad \alpha := 2.1$

$$M_k := \frac{4 \cdot A \cdot \alpha \cdot T}{4 \cdot \pi^2 \cdot k^2 - \alpha^2 \cdot T^2}$$

$$M_0 := \frac{2 \cdot A}{\alpha \cdot T}$$

$$MAG_l := \left| M_l \right|$$



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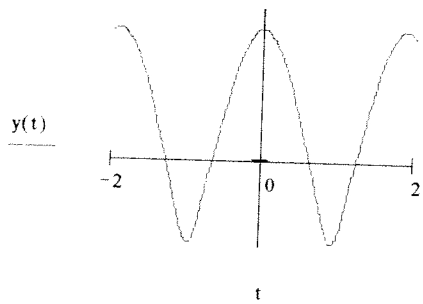
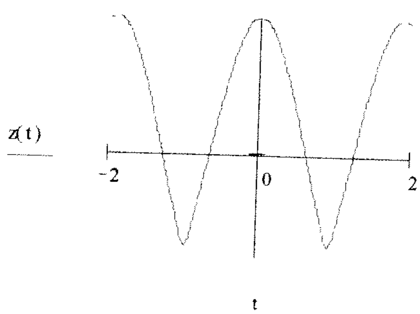
[redacted]

$t := -10, -9.99 .. 10 \quad A := 3 \qquad T := 1.9 \qquad \alpha T := 4.7$

$$y(t) := \frac{2\cdot 3}{4.7}\cdot \sin\left(\frac{4.7}{2}\right) + 4\cdot 3\cdot 4.7\cdot \sum_{n=1}^{5} \frac{(-1)^{n+1}\cdot \sin\left(\frac{4.7}{2}\right)}{4\cdot n^2\cdot \pi^2 - 4.7^2}\cdot \cos\left(\frac{2\cdot \pi\cdot n}{1.9}\cdot t\right)$$




$$z(t) := \frac{2\cdot 3}{4.7}\cdot \sin\left(\frac{4.7}{2}\right) + 4\cdot 3\cdot 4.7\cdot \sum_{n=1}^{10} \frac{(-1)^{n+1}\cdot \sin\left(\frac{4.7}{2}\right)}{4\cdot n^2\cdot \pi^2 - 4.7^2}\cdot \cos\left(\frac{2\cdot \pi\cdot n}{1.9}\cdot t\right)$$



## Solving for $a_n$

$$\frac{2}{T}\cdot\int_0^{T} A\cdot\delta\cdot(t \quad a)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\ \cos(2\cdot\pi\cdot n)\cdot T + 2\cdot a\cdot\sin(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)\cdot T\cdot A\cdot\frac{\delta}{\pi^2\cdot n^2} \quad \frac{1}{4}\cdot T^2\cdot A\cdot\frac{\delta}{\pi^2\cdot n^2}\right]$$

Simplify

## Solving for $b_n$

$$\frac{2}{T}\cdot\int_0^{T} A\cdot\delta\cdot(t \quad a)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\ \sin(2\cdot\pi\cdot n)\cdot T + 2\cdot a\cdot\cos(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\cdot T\cdot A\cdot\frac{\delta}{\pi^2\cdot n^2} \quad \frac{1}{2}\cdot T\cdot\frac{a}{\pi\cdot n}\cdot A\cdot\delta\right]$$

## Solving for $a_0$

$$\frac{2}{T}\cdot\int_0^{T} A\cdot\delta\cdot(t \quad a)\cdot\cos\left(\frac{2\cdot\pi\cdot 0}{T}\cdot t\right)dt$$

$$A\cdot\delta\cdot(\ T + 2\cdot a)$$

## Fourier Series

$$\frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\ \cos(2\cdot\pi\cdot n)\cdot T + 2\cdot a\cdot\sin(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)\cdot T\cdot A\cdot\frac{\delta}{\pi^2\cdot n^2} \quad \frac{1}{4}\cdot T^2\cdot A\cdot\frac{\delta}{\pi^2\cdot n^2}\right]\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) + \frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\ \sin(2\cdot\pi\cdot n)\cdot T + 2\cdot a\cdot\cos(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\cos\right.$$

$$1\cdot A\cdot\delta\cdot\ \cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\cos(2\cdot\pi\cdot n)\cdot T \quad 2\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot\sin(2\cdot\pi\cdot n)\cdot\pi\cdot n + 2\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)\cdot T \quad \cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot T\ \ \sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\sin(2\cdot\pi\cdot n)\cdot T\ \ 2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot c$$

$$A\cdot\delta\cdot\left(\frac{T}{2} \cdot 2\cdot a\right) \quad \sum_{n=1}^{4} \frac{1}{2}\cdot A\cdot\delta\cdot \cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\cos(2\cdot\pi\cdot n)\cdot T \quad 2\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot\sin(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)\cdot T \quad \cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot T \quad \sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\sin(2\cdot \ldots \quad \pi^2\cdot n^2 \quad \pi^2\cdot n^2$$

$$\frac{1}{2}\cdot A\cdot\delta\cdot(T \cdot 2\cdot a) \quad A\cdot\delta\cdot\frac{\sin\left(2\cdot\frac{\pi}{T}\cdot t\right)}{\pi}\cdot T \quad \frac{1}{2}\cdot A\cdot\delta\cdot\frac{\sin\left(4\cdot\frac{\pi}{T}\cdot t\right)}{\pi}\cdot T \quad \frac{1}{3}\cdot A\cdot\delta\cdot\frac{\sin\left(6\cdot\frac{\pi}{T}\cdot t\right)}{\pi}\cdot T \quad \frac{1}{4}\cdot A\cdot\delta\cdot\frac{\sin\left(8\cdot\frac{\pi}{T}\cdot t\right)}{\pi}\cdot T$$

**Graphs**

$T \quad 1$

$A \quad 10$

$m \quad 1, 0 .. 11$

$n \quad 0, 1 .. 10$

$g(n) \quad \text{if}\left(n<1, \frac{A}{T}, \frac{2\cdot A}{T}\right)$

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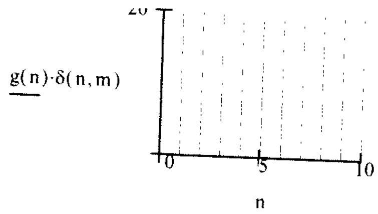


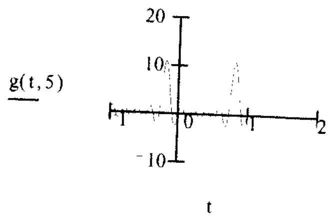

$a := .8$

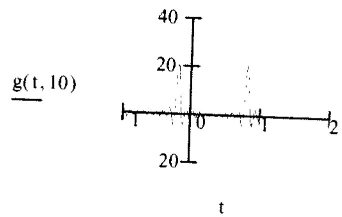
$T := 1$

$A := 1$

$t := -T, -T + \frac{T}{1000} .. T$

$$g(t,m) := \frac{A}{T} + \frac{2 \cdot A}{T} \cdot \sum_{n=1}^{m} \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot (t - a)\right)$$



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$$\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\cdot T\cdot A\cdot\frac{\delta}{\pi^2\cdot n^2} \quad \frac{1}{2}\cdot T\cdot\frac{a}{\pi\cdot n}\cdot A\cdot\delta\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$

$$\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot\cos(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\cdot T \quad 2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot\pi\cdot n$$

$$2\cdot\pi\cdot\frac{n}{T}\cdot t\cdot\sin(2\cdot\pi\cdot n)\cdot T \quad 2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot\cos(2\cdot\pi\cdot n)\cdot\pi\cdot n \quad 2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\cdot T \quad 2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\cdot a\cdot\pi\cdot n$$

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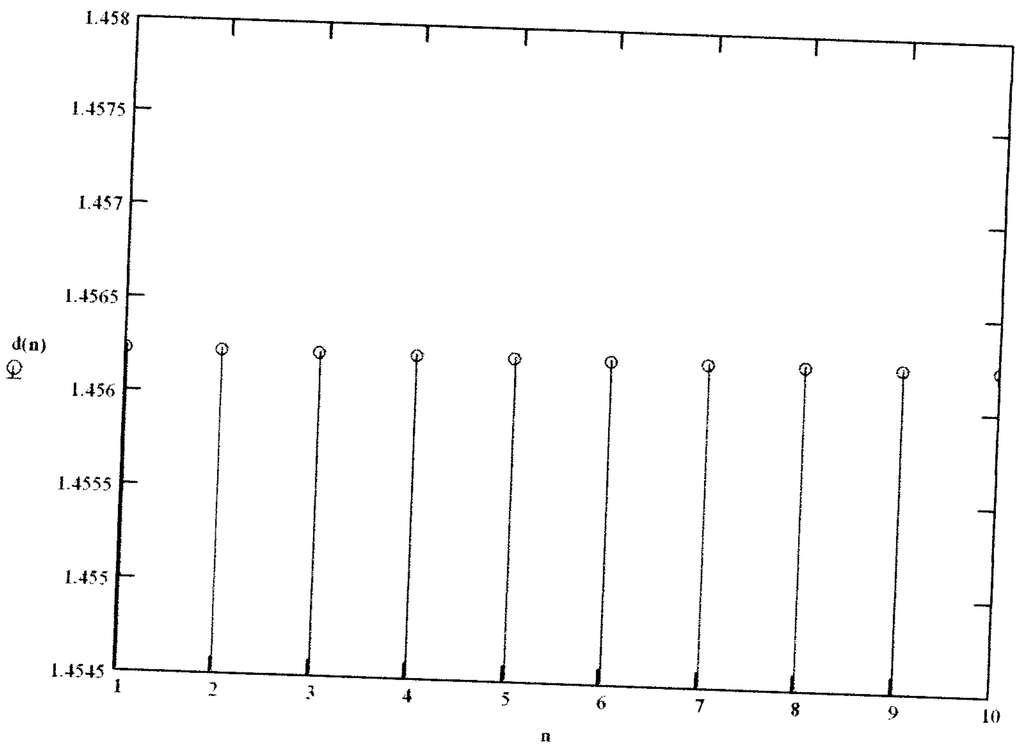
$a := .9$

$t := 1$

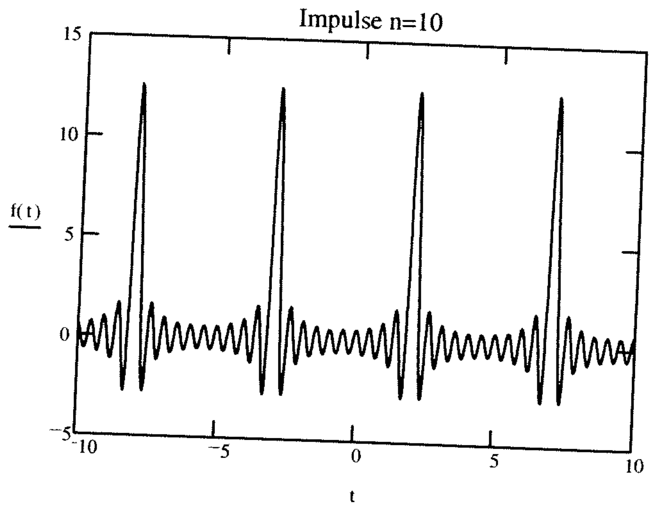
$n := 1, 2 .. 10$

$$d(n) := \frac{2 \cdot a}{t} \cdot \cos\left[\frac{2 \cdot \pi}{t} \cdot (t - a)\right]$$



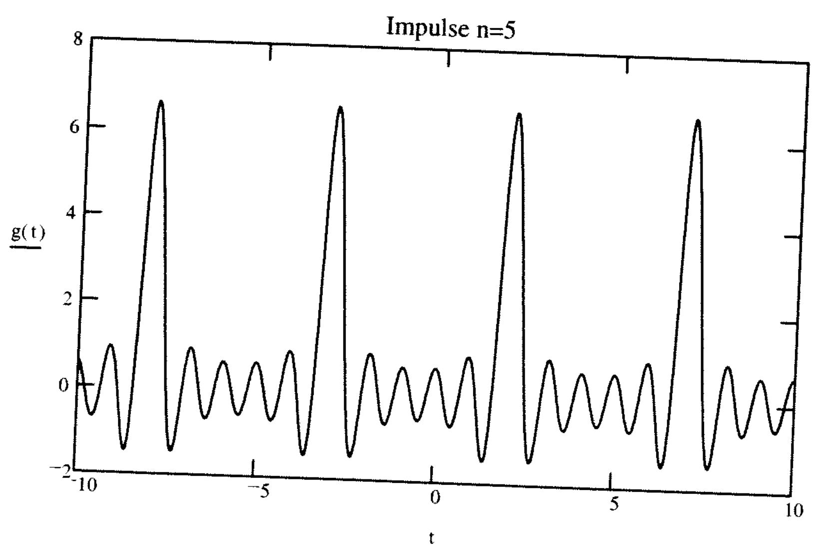
$A := 3 \quad \tau := 2 \quad T := 5 \quad n := 1..10 \quad t := -10, -9.99..10 \quad a := 2$

$$f(t) := \frac{A}{T} + \frac{2 \cdot A}{T} \cdot \sum_{n} \cos\left[\frac{2 \cdot \pi \cdot n}{T} \cdot (t - a)\right]$$




$A := 3 \quad \tau := 2 \quad T := 5 \quad n := 1..5 \quad t := -10, -9.99..10 \quad a := 2$

$$g(t) := \frac{A}{T} + \frac{2 \cdot A}{T} \cdot \sum_{n} \cos\left[\frac{2 \cdot \pi \cdot n}{T} \cdot (t - a)\right]$$



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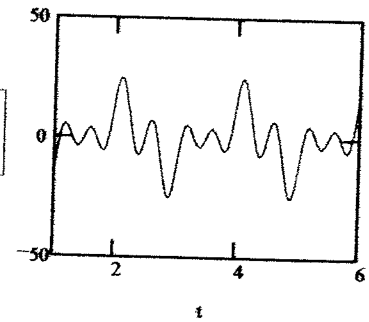
a = .2 case plotted in handout

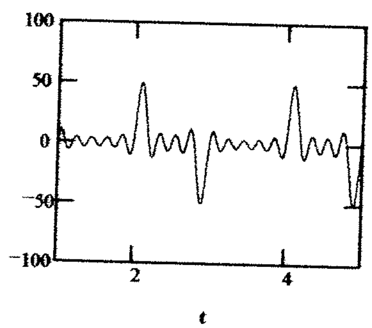
### *Alternating Impulse Train*

$A := 5 \quad T := 2 \quad t := 1, 1.0001 .. 8 \quad a := 0.1 \quad b := 0.9$

$$\frac{2 \cdot A}{T} \cdot \left[ \sum_{n=1}^{5} \cos\left[ \frac{2 \cdot \pi \cdot n}{T} \cdot (t - a) \right] - \cos\left[ \frac{2 \cdot \pi \cdot n}{T} \cdot (t - b) \right] \right]$$




$$\frac{2 \cdot A}{T} \cdot \left[ \sum_{n=1}^{10} \cos\left[ \frac{2 \cdot \pi \cdot n}{T} \cdot (t - a) \right] - \cos\left[ \frac{2 \cdot \pi \cdot n}{T} \cdot (t - b) \right] \right]$$




## ***Possible Error!***

**These graphs do not appear to be exact when compared to the previous page. Upon a rather close examination and comparison of the graphs there appears to be one more small oscillation between each peak oscillation. Look very closely at the graph when n=5.**

$$\frac{2}{T}\cdot\int_0^T \left[A\cdot\delta\cdot(t-a) - A\cdot\delta\cdot(t-b)\right]\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt$$

$$\frac{2}{T}\cdot\int_0^T$$

$$A\cdot\delta\cdot\sin(2\cdot\pi\cdot n)\cdot\frac{(a-b)}{\pi\cdot n}$$

$$\frac{2}{T}\cdot\int_0^T \left[A\cdot\delta\cdot(t-a) - A\cdot\delta\cdot(t-b)\right]\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt$$

$$A\cdot\delta\cdot\frac{a\cdot\cos(2\cdot\pi\cdot n) - a - b\cdot\cos(2\cdot\pi\cdot n) + b}{\pi\cdot n}$$

$$\sqrt{\left(A\cdot\delta\cdot\sin(2\cdot\pi\cdot n)\cdot\frac{(a-b)}{\pi\cdot n}\right)^2 + \left(A\cdot\delta\cdot\frac{a\cdot\cos(2\cdot\pi\cdot n) - a - b\cdot\cos(2\cdot\pi\cdot n) + b}{\pi\cdot n}\right)^2}$$

$$\frac{\sqrt{2\cdot A^2\cdot\delta^2\cdot a^2 - 4\cdot A^2\cdot\delta^2\cdot a\cdot b + 2\cdot A^2\cdot\delta^2\cdot b^2 - 2\cdot A^2\cdot\delta^2\cdot a^2\cdot\cos(2\cdot\pi\cdot n) + 4\cdot A^2\cdot\delta^2\cdot a\cdot\cos(2\cdot\pi\cdot n)\cdot b - 2\cdot A^2\cdot\delta^2\cdot b^2\cdot\cos(2\cdot\pi\cdot n)}}{\pi\cdot n}$$

$$i\cdot\sqrt{2}\cdot A\cdot\delta\cdot(a-b)\cdot\frac{\sqrt{-1+\cos(2\cdot\pi\cdot n)}}{\pi\cdot n}$$

= 0? all n?

$$i\cdot\sqrt{2}\cdot A\cdot\delta\cdot\frac{\sqrt{-2+2\cdot\cos(\pi\cdot n)^2}}{\pi\cdot n}\cdot a - i\cdot\sqrt{2}\cdot A\cdot\delta\cdot\frac{\sqrt{-2+2\cdot\cos(\pi\cdot n)^2}}{\pi\cdot n}\cdot b$$

doesn't look like my result

$$\frac{2\cdot A}{T}\cdot\sum_{n=-m}^{-1}\left(e^{\frac{-1\cdot2\cdot\pi\cdot n}{T}\cdot a} - e^{\frac{-1\cdot2\cdot\pi\cdot n}{T}\cdot b}\right)\cdot e^{\frac{1\cdot2\cdot\pi\cdot n}{T}\cdot t} + \frac{2\cdot A}{T}\cdot\sum_{n=m}^{1}\left(e^{\frac{-1\cdot2\cdot\pi\cdot n}{T}\cdot a} - e^{\frac{-1\cdot2\cdot\pi\cdot n}{T}\cdot b}\right)\cdot e^{\frac{1\cdot2\cdot\pi\cdot n}{T}\cdot t}$$

didn't we discuss the analytical approach involving $e^j$ fcn?

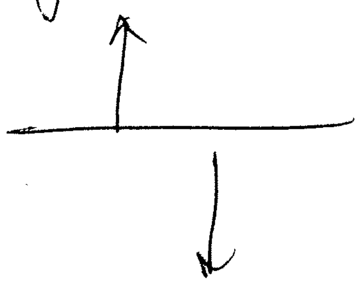
See back

$$F_N = \frac{1}{T}\int_0^T A\delta(t-a)e^{-j\frac{2\pi N}{T}t} + \frac{1}{T}\int_0^T -A\delta(t-b)e^{-j2\pi N}$$

$$= \frac{A}{T}\int_0^T \delta(t-a)e^{-j\frac{2\pi N}{T}a} + \frac{-A}{T}\int_0^T \delta(t-b)e^{-j\frac{2\pi N}{T}b}$$

$$= \frac{A}{T}e^{-j\frac{2\pi N}{T}a} - \frac{A}{T}e^{-j\frac{2\pi N}{T}b} \checkmark$$

avg = 0



equal and opposite cancel

N=0 in previous result

$a := .2 \quad T := 1$

$A := 10 \quad b := .8$

$n := 10$

$A := 1$

$t := -T, -T + \frac{T}{1000} .. T$

$\tau := \frac{T}{2}$

$t := -T, -T + \frac{T}{1000} .. T$

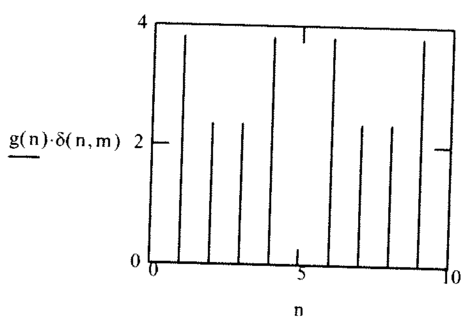
$\tau := \frac{T}{2}$

$$x(t,m) := \frac{A}{T} \cdot \left[ \sum_{n=-m}^{1} \left( e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot a} - e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot b} \right) \cdot e^{\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot t} \right] + \left[ \frac{A}{T} \cdot \sum_{n=-m}^{1} \left( e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot a} - e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot b} \right) \cdot e^{\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot t} \right]$$

$m := 1, 0 .. 11 \quad a := .2$

$n := 0, 1 .. 10 \quad b := .8$

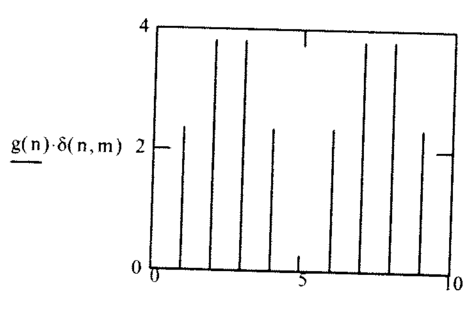
$$g(n) := \text{if}\left[ n<1, \frac{2 \cdot A}{T}, \left[ \frac{A}{T} \cdot \left( e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot a} - e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot b} \right) \cdot e^{\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot t} \right] \cdot 2 \right]$$




$m := 1, 0 .. 11 \quad a := .1$

$n := 0, 1 .. 10 \quad b := .9$

$$g(n) := \text{if}\left[ n<1, \frac{2 \cdot A}{T}, \left[ \frac{A}{T} \cdot \left( e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot a} - e^{-\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot b} \right) \cdot e^{\frac{1 \cdot 2 \cdot \pi \cdot n}{T} \cdot t} \right] \cdot 2 \right]$$

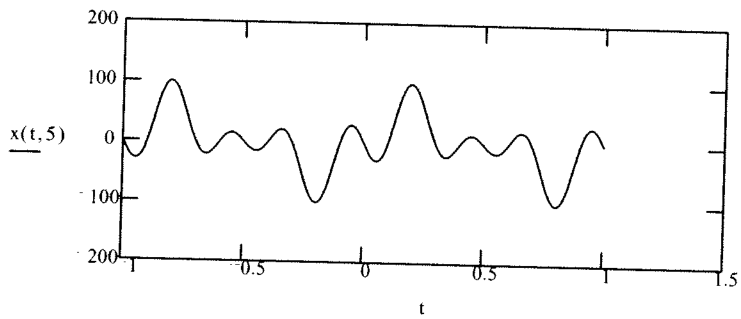


$T := 1 \quad a := .2 \quad b := .8$

$A := 10$

$\tau := \frac{T}{2} \quad t := -T, -T + \frac{T}{1000} .. T$

$$x(t,m) := \frac{2 \cdot A}{T} \cdot \sum_{n=1}^{5} \left( \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot (t - a)\right) - \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot (t - b)\right) \right)$$



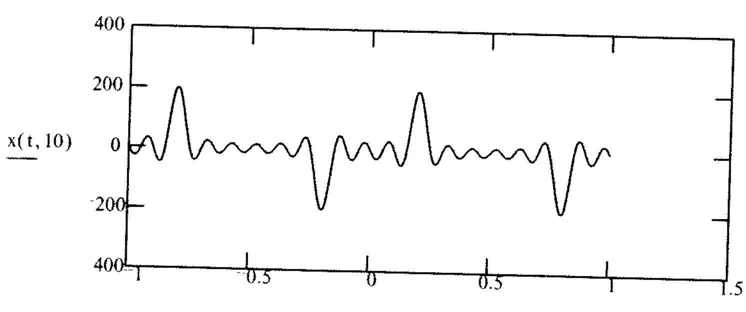

@ 5 samples

$T := 1 \quad a := .2 \quad b := .8$

$A := 10$

$\tau := \frac{T}{2} \quad t := -T, -T + \frac{T}{1000} .. T$

$$x(t,m) := \frac{2 \cdot A}{T} \cdot \sum_{n=1}^{10} \left( \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot (t - a)\right) - \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot (t - b)\right) \right)$$




@ 5 samples

## Hyperbolic Sine Wave Kaiser

[redacted]

$A := 2$

$T := 3$

$a := 1$

$$f_{avg1} := \frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \sinh(a \cdot t)\, dt$$

$$f_{avg2} := 0$$

$f_{avg1} = 0$

$f_{avg2} = 0$

$$f_{rms1} := \sqrt{\frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} (A \cdot \sinh(a \cdot t))^2\, dt}$$

$f_{rms1} = 2.163$

$$f_{rms2} := A \cdot \sqrt{\frac{\sinh(a \cdot T)}{2 \cdot a \cdot T} - \frac{1}{2}}$$

$f_{rms2} = 2.163$

**Determination of An**:

$$\frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\sinh(\alpha\cdot t)\cdot\cos\left(\frac{2\cdot n\cdot\pi}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{-1}{4}\cdot A\cdot T\cdot\frac{(-2\cdot\cos(n\cdot\pi)\cdot\alpha\cdot T-4\cdot n\cdot\pi\cdot\exp(\alpha\cdot T)\cdot\sin(n\cdot\pi)+4\cdot\sin(n\cdot\pi)\cdot\pi\cdot n-2\cdot\alpha\cdot T\cdot\exp(\alpha\cdot T)\cdot\cos(n\cdot\pi))}{\left[\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)\right]}+\frac{1}{4}\cdot A\cdot T\cdot\frac{(-2\cdot\cos(n\cdot\pi)\cdot\alpha\cdot T+4\cdot n\cdot\pi\cdot\exp(-\alpha\cdot T)\cdot\sin(n\cdot\pi)-4\cdot\sin(n\cdot\pi)\cdot\pi\cdot n}{\left[\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)\right]}\right.$$

$-2\alpha T e^{-\alpha T}\cdot\cos(n\pi)\,]$

0

**So the value of An is 0**.

**Determination Of Bn:**

$$\frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\sinh(\alpha\cdot t)\cdot\sin\left(\frac{2\cdot n\cdot\pi}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{-1}{4}\cdot A\cdot T\cdot\frac{(-2\cdot\sin(n\cdot\pi)\cdot\alpha\cdot T+4\cdot n\cdot\pi\cdot\exp(\alpha\cdot T)\cdot\cos(n\cdot\pi)-4\cdot\cos(n\cdot\pi)\cdot\pi\cdot n-2\cdot\alpha\cdot T\cdot\exp(\alpha\cdot T)\cdot\sin(n\cdot\pi))}{\left[\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)\right]}-\frac{1}{4}\cdot A\cdot T\cdot\frac{(-2\cdot\sin(n\cdot\pi)\cdot\alpha\cdot T-4\cdot n\cdot\pi\cdot\exp(-\alpha\cdot T)\cdot\cos(n\cdot\pi)+4\cdot\cos(n\cdot\pi)\cdot\pi\cdot n-}{\left[\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)\right]}\right.$$

$-2\alpha T e^{-\alpha T}\cdot\sin(n\pi))\,]$

$$2\cdot A\cdot\frac{\left(\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot\sin(n\cdot\pi)\cdot\alpha\cdot T-2\cdot\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot\cos(n\cdot\pi)\cdot\pi\cdot n+2\cdot\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot\cos(n\cdot\pi)\cdot\pi\cdot n+\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot\sin(n\cdot\pi)\cdot\alpha\cdot T\right)}{\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)}$$

As we know that the value of Sin(nπ)=0 and cos(nπ)=(-1)^n therefore on substituting these values in above equation, we get:

$$2\cdot A\cdot\frac{\left[\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot 0\cdot\alpha\cdot T-2\cdot\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot(-1)^n\cdot\pi\cdot n+2\cdot\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot(-1)^n\cdot\pi\cdot n+\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot 0\cdot\alpha\cdot T\right]}{\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)}$$

$$4\cdot A\cdot\pi\cdot n\cdot\frac{\left[(-1)^{(1+n)}\cdot\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)+\exp\left(\frac{-1}{2}\cdot\alpha\cdot T\right)\cdot(-1)^n\right]}{\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)}$$

$$=\frac{4A\pi n}{(T^2\alpha^2+4 n^2\pi^2)}(-1)^{1+n}\left[e^{\frac{\alpha T}{2}}-e^{-\frac{\alpha T}{2}}\right]$$

$$=\frac{4A\pi n}{(T^2\alpha^2+4 n^2\pi^2)}(-1)^{1+n}\cdot 2\sinh\left(\frac{\alpha T}{2}\right)$$

Taking $(-1)^{1+n}$ common.
We know from Euler's Identity
$\sinh\theta=\frac{e^{\theta}-e^{-\theta}}{2}$

$\therefore$ $e^{\frac{\alpha T}{2}}-e^{-\frac{\alpha T}{2}}=2\sinh\frac{\alpha T}{2}$

On simplifying the above expression we get,

$$4\cdot A\cdot\pi\cdot n\cdot\frac{(-1)^{n+1}\cdot 2\cdot\sinh\left(\frac{\alpha\cdot T}{2}\right)}{\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)}$$

$$8\cdot A\cdot\pi\cdot n\cdot(-1)^{(1+n)}\cdot\frac{\sinh\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)}$$

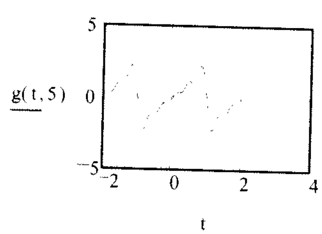
(This is the expression for Bn)

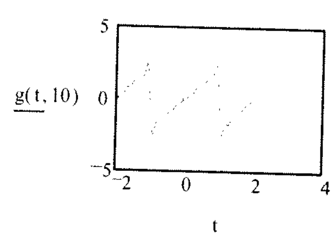
Therefore the series for this wave function is:

$A := 2 \quad \alpha := 1 \quad T := 2$

$t := -T, \left(-T+\frac{T}{1000}\right) .. T$

$$g(t,m) := \sum_{n=1}^{m}\left[8\cdot A\cdot\pi\cdot n\cdot(-1)^{(1+n)}\cdot\frac{\sinh\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(T^2\cdot\alpha^2+4\cdot n^2\cdot\pi^2\right)}\right]\cdot\sin\left(\frac{2\cdot n\cdot\pi}{T}\cdot t\right)$$



The Frequency Spectrum is:

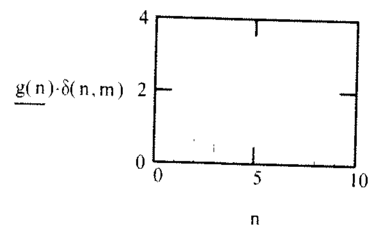
$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$$g(n) := \left| 8\cdot A\cdot\pi\cdot n\cdot(-1)^{(1+n)}\cdot\frac{\sinh\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(T^2\cdot\alpha^2 + 4\cdot n^2\cdot\pi^2\right)} \right|$$

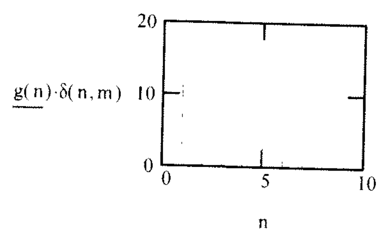
A) $\alpha := 2$

$$g(n) := \left| 8\cdot A\cdot\pi\cdot n\cdot(-1)^{(1+n)}\cdot\frac{\sinh\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(T^2\cdot\alpha^2 + 4\cdot n^2\cdot\pi^2\right)} \right|$$




B) $\alpha := 4$

$$g(n) := \left| 8\cdot A\cdot\pi\cdot n\cdot(-1)^{(1+n)}\cdot\frac{\sinh\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(T^2\cdot\alpha^2 + 4\cdot n^2\cdot\pi^2\right)} \right|$$



Hyperbolic Sine Wave pDs

n = 1

$$8A\pi \cdot \frac{1 \sinh\left(\frac{\alpha T}{2}\right)}{4 \cdot 1 \cdot \pi^2 + \alpha^2 T^2} \sin\left(\frac{2\pi t}{T}\right)$$

$$\frac{8A\pi \sinh\left(\frac{\alpha T}{2}\right)}{4\pi^2 + \alpha^2 T^2} \sin\left(\frac{2\pi t}{T}\right)$$

n = 2

$$8A\pi \cdot \frac{-1 \cdot 2 \cdot \sinh\left(\frac{\alpha T}{2}\right)}{4 \cdot 4 \cdot \pi^2 + \alpha^2 T^2} \sin\left(\frac{4\pi t}{T}\right)$$

$$-\frac{16 A\pi \sinh\left(\frac{\alpha T}{2}\right)}{16\pi^2 + \alpha^2 T^2} \sin\left(\frac{4\pi t}{T}\right)$$

n = 3

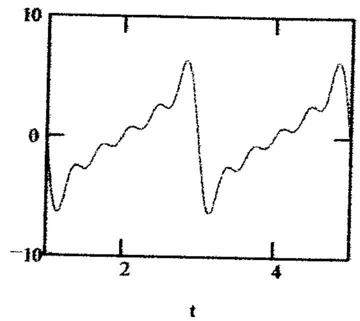
$$8A\pi \cdot \frac{1 \cdot 3 \cdot \sinh\left(\frac{\alpha T}{2}\right)}{4 \cdot 9 \cdot \pi^2 + \alpha^2 T^2} \sin\left(\frac{2\pi 3 t}{T}\right)$$

$$\frac{24 A\pi \sinh\left(\frac{\alpha T}{2}\right)}{36\pi^2 + \alpha^2 T^2} \sin\left(\frac{6\pi t}{T}\right)$$

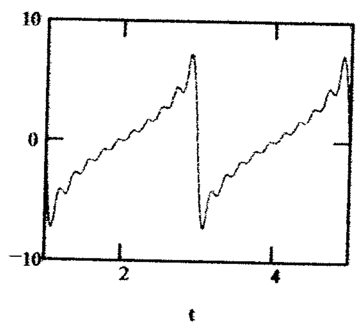
# *HyperBolic Sine Wave*

$A := 2 \quad T := 2 \quad \alpha := 2 \quad t := 1, 1.0001 .. 5$

$$8 \cdot A \cdot \pi \cdot \sum_{n=1}^{5} \frac{(-1)^{n+1} \cdot n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



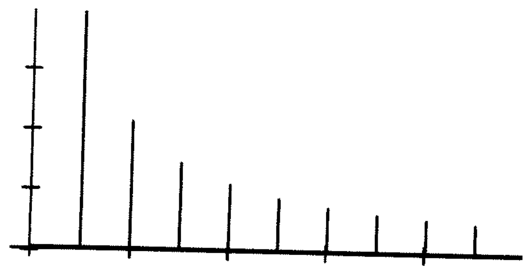

$$8 \cdot A \cdot \pi \cdot \sum_{n=1}^{10} \frac{(-1)^{n+1} \cdot n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



$m := -1, 0 .. 11$ $\alpha := 2$ T :

$n := 0, 1 .. 10$ $A := 1$

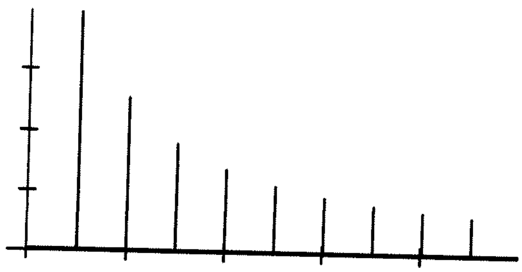
$$g(n) := \text{if}\left(n < 1, 0, 8 \cdot A \cdot \pi \cdot \left| \frac{n \cdot \sinh\left(\alpha \cdot \frac{T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \right| \right)$$



$m := -1, 0 .. 11$  $\alpha := 4$  $T := 1$

$n := 0, 1 .. 10$  $A := 1$

$$g(n) := \text{if}\left(n < 1, 0, 8 \cdot A \cdot \pi \cdot \left| \frac{n \cdot \sinh\left(\alpha \cdot \frac{T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \right| \right)$$

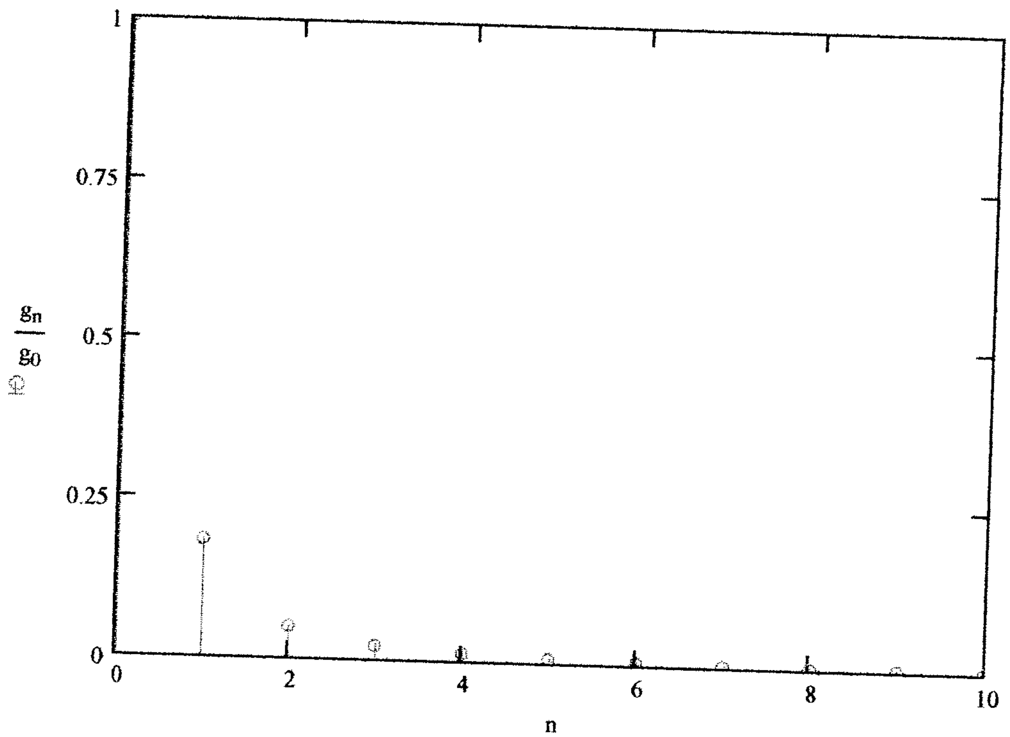


# Hyperbolic Cosine Wave (p.426)

$A := 1 \qquad T := 1 \quad \alpha := 2$

$$g_0 := \frac{2A}{\alpha \cdot T} \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right) \qquad n := 1, 2 .. 10 \qquad g_n := \left| 4 \cdot A \cdot \alpha \cdot T \cdot \frac{(-1)^n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{\left(4 \cdot n^2 \cdot \pi^2\right) + \alpha^2 T^2} \right|$$

$n := 0, 1 .. 10$



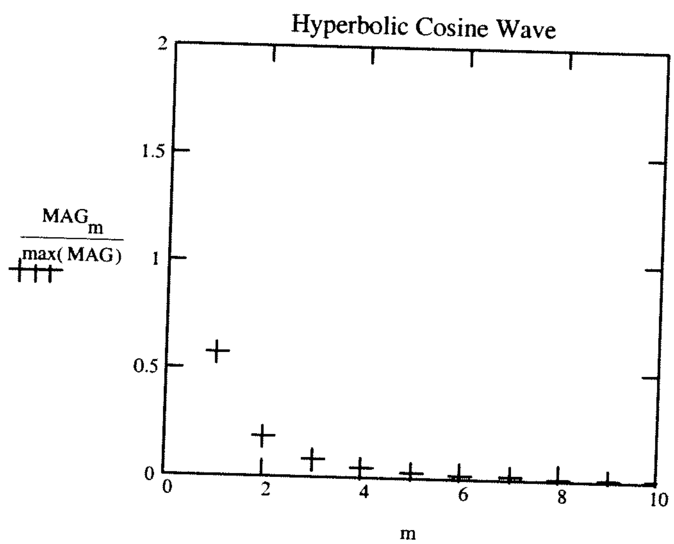
$n := 1..10 \quad A := 1 \quad k := \frac{1}{2} \quad \alpha := 2 \quad T := 2 \quad M_0 := 2 \cdot \frac{A}{\alpha \cdot T} \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right) \quad t := 2$

$m := 0..10$

$$M_n := \frac{4 \cdot \alpha \cdot A \cdot T \cdot (-1)^n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot t}{T}\right)$$

?

$MAG_m := |M_m|$




both cases?

$$fave7 := \left(2\cdot\frac{A}{\alpha\cdot T}\right)\cdot\sinh\left(\alpha\cdot\frac{T}{2}\right)$$

$$fave7 = 2.839$$

$$frms7 := A\cdot\sqrt{\left(\frac{1}{2}\right)+\left(\frac{\sinh(\alpha\cdot T)}{2\cdot\alpha\cdot T}\right)}$$

$$frms7 = 2.946$$

$$fave8 := \int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\cosh(\alpha\cdot t)\,dt\cdot\frac{1}{T}$$

$$fave8 = 2.839$$

$$frms8 := \sqrt{\int_{-\frac{T}{2}}^{\frac{T}{2}} (A\cdot\cosh(\alpha\cdot t))^2\,dt\cdot\frac{1}{T}}$$

$$frms8 = 2.946$$

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$$\frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}}\left(A\cdot\cosh(\alpha\cdot t)\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T}\right)\right)dt$$

$$A\cdot\frac{(-\alpha\cdot T+2i\cdot\exp(\alpha\cdot T)\cdot\exp(-2\cdot i\cdot\pi\cdot n)\cdot\pi\cdot n-2i\cdot\pi\cdot n+\alpha\cdot T\cdot\exp(\alpha\cdot T)-2i\cdot\pi\cdot n\cdot\exp(\alpha\cdot T)-\exp(-2\cdot i\cdot\pi\cdot n)\cdot\alpha\cdot T+2i\cdot\exp(-2\cdot i\cdot\pi}{\left[\exp\left(\frac{1}{2}\cdot\alpha\cdot T\right)\cdot\left[\exp(-i\cdot\pi\cdot n)\cdot\left(\alpha^2\cdot T^2+4\cdot\pi^2\cdot n^2\right)\right]\right]}$$

$$A\cdot\frac{(-1\cdot T+2i\cdot\exp(1\cdot T)\cdot\exp(-2\cdot i\cdot\pi\cdot 1)\cdot\pi\cdot 1-2i\cdot\pi\cdot 1+1\cdot T\cdot\exp(1\cdot T)-2i\cdot\pi\cdot 1\cdot\exp(1\cdot T)-\exp(-2\cdot i\cdot\pi\cdot 1)\cdot 1\cdot T+2i\cdot\exp(-2\cdot i}{\left[\exp\left(\frac{1}{2}\cdot 1\cdot T\right)\cdot\left[\exp(-i\cdot\pi\cdot 1)\cdot\left(1^2\cdot T^2+4\cdot\pi^2\cdot 1^2\right)\right]\right]}$$

If n=1, α=1....

$$-2\cdot A\cdot T\cdot(-1+\exp(T))\cdot\frac{\exp\left(\frac{-1}{2}\cdot T\right)}{\left(T^2+4\cdot\pi^2\right)}$$

bn = 0
even symmetry

$$-2\cdot A\cdot T\cdot\frac{-\exp\left(-\frac{T}{2}\right)+\exp\left(\frac{T}{2}\right)}{T^2+4\cdot\pi^2}$$

The above function must equal the following coefficient

$$4\cdot A\cdot\alpha\cdot T\cdot\frac{(-1)^n\cdot\sinh\left(\alpha\cdot\frac{T}{2}\right)}{4\cdot n^2\cdot\pi^2+\alpha^2\cdot T^2}$$ ← desired result

If n=1, α=1....

$$4 \cdot A \cdot 1 \cdot T \cdot \frac{(-1)^1 \cdot \sinh\left(1 \cdot \frac{T}{2}\right)}{4 \cdot 1^2 \cdot \pi^2 + 1^2 \cdot T^2}$$

$$4 \cdot T \cdot A \cdot \frac{\sinh\left(\frac{1}{2} \cdot T\right)}{\left(T^2 + 4 \cdot \pi^2\right)}$$

sinh(T/2) = [e^(T/2) + e^(-T/2)]/2

$$2 \cdot T \cdot A \cdot \frac{\exp\left(\frac{T}{2}\right) + \exp\left(\left(-\frac{T}{2}\right)\right)}{\left(T^2 + 4 \cdot \pi^2\right)}$$

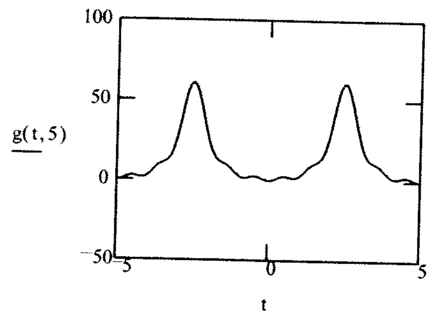
The function defined above equals the coefficient given in the problem set, hence the equation is verified.

$A := 1 \qquad T := 5$

$\tau := 1 \qquad \alpha := 2$

$t := -T, -T + \frac{T}{1000} .. T$

$$g(t,m) := 2 \cdot \frac{A}{\alpha \cdot T} \cdot \sinh\left(\alpha \cdot \frac{T}{2}\right) + 4 \cdot A \cdot \alpha \cdot T \cdot \sum_{n=1}^{m} \frac{(-1)^n \cdot \sinh\left(\alpha \cdot \frac{T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \cdot \cos\left(2 \cdot n \cdot \pi \cdot \frac{t}{T}\right)$$

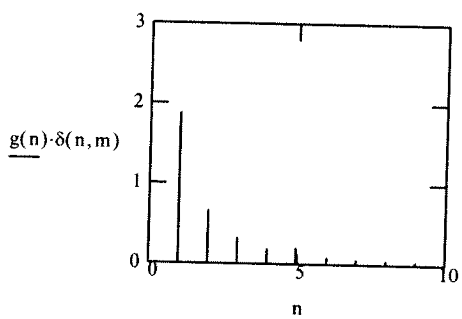



$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$\alpha := 1$

$$g(n) := \text{if}\left[n<1, \frac{2\cdot A}{\alpha\cdot T}\cdot\sinh\left(\alpha\cdot\frac{T}{2}\right), \left|4\cdot A\cdot\alpha\cdot T\cdot\frac{(-1)^n\cdot\sinh\left(\alpha\cdot\frac{T}{2}\right)}{4\cdot n^2\cdot\pi^2+\alpha^2\cdot T^2}\right|\right]$$



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n=1 $4A\alpha T \dfrac{(-1)^1 \sinh\left(\frac{\alpha T}{2}\right)}{4(1)^2\pi^2+\alpha^2T^2} \cos \dfrac{2\pi(1)t}{T}$

$$= -4A\alpha T \frac{\sinh\left(\frac{\alpha T}{2}\right)}{4\pi^2+\alpha^2T^2} \cos\left(\frac{2\pi t}{T}\right)$$

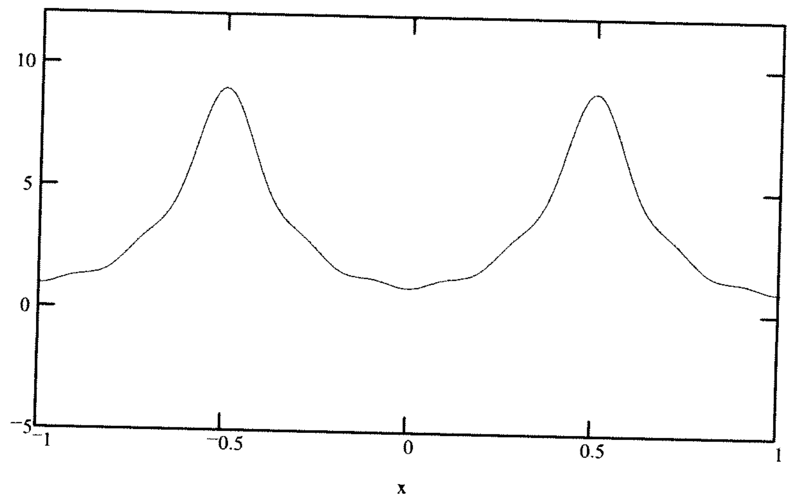
n=2 + 16 4

n=3 − 36 6

Ken Kaiser

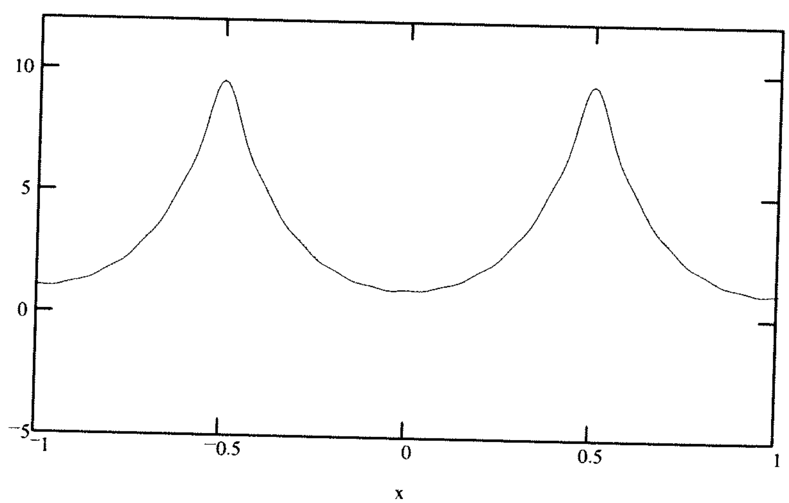
$n := 1, 2 .. 5$
$\alpha := 2$
$\tau := 3$

$$\frac{2}{\alpha\cdot\tau}\cdot\sinh\left(\frac{\alpha\cdot\tau}{2}\right)+4\cdot\alpha\cdot\tau\cdot\sum_{n}\frac{(-1)^{n}\cdot\sinh\left(\frac{\alpha\cdot\tau}{2}\right)}{4\cdot n^{2}\cdot\pi^{2}+\alpha^{2}\cdot\tau^{2}}\cdot\cos(2\cdot\pi\cdot n\cdot x)$$




$n := 1, 2 .. 10$
$\alpha := 2$
$\tau := 3$

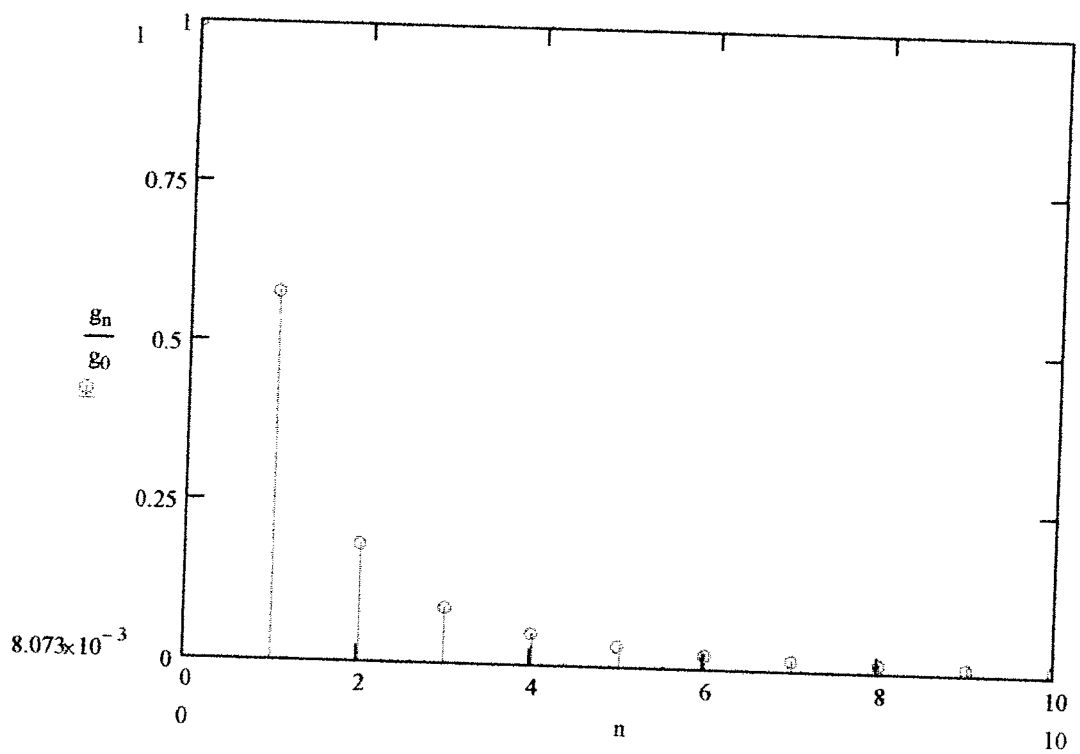
$$\frac{2}{\alpha\cdot\tau}\cdot\sinh\left(\frac{\alpha\cdot\tau}{2}\right)+4\cdot\alpha\cdot\tau\cdot\sum_{n}\frac{(-1)^{n}\cdot\sinh\left(\frac{\alpha\cdot\tau}{2}\right)}{4\cdot n^{2}\cdot\pi^{2}+\alpha^{2}\cdot\tau^{2}}\cdot\cos(2\cdot\pi\cdot n\cdot x)$$



$A := 1 \qquad T := 1 \quad \alpha := 4$

$g_0 := \frac{2A}{\alpha \cdot T} \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right) \qquad n := 1, 2 .. 10 \qquad g_n := \left| 4 \cdot A \cdot \alpha \cdot T \cdot \frac{(-1)^n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{\left(4 \cdot n^2 \cdot \pi^2\right) + \alpha^2 T^2} \right|$

$n := 0, 1 .. 10$



Natural Log of a Sine Wave, p427

An determination

$$\frac{2}{T}\cdot\int_{0}^{2\cdot\pi} -A\cdot\ln\left(2\cdot\sin\left(\frac{t}{2}\right)\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

Both integrals result in the error:

No closed form found for integral.

Bn determination

$$\frac{2}{T}\cdot\int_{0}^{2\cdot\pi} -A\cdot\ln\left(2\cdot\sin\left(\frac{t}{2}\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

As stated in class, because the integrals cannot be evaluated, the expression on p 427 will be assumed correct, and the first three Fourier series terms will be confirmed as will the magnitude and series plots.

Given expression is $A\cdot\sum_{n=1}^{\infty}\frac{1}{n}\cdot\cos(n\cdot t)$

n = 1 => $A\cdot\frac{1}{1}\cdot\cos(t) \quad = \quad A\cdot\cos(t)$

n = 2 => $A\cdot\frac{1}{2}\cdot\cos(2\ t) \quad = \quad \frac{A}{2}\cdot\cos(2\cdot t)$

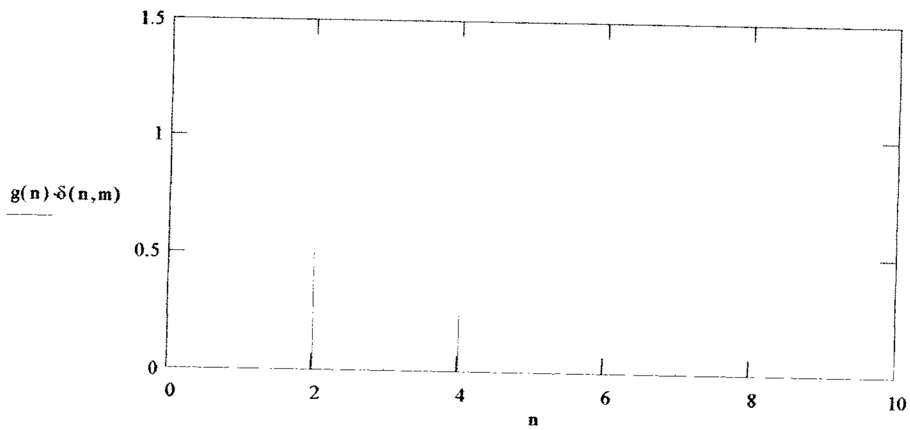
n = 3 => $A\cdot\frac{1}{3}\cdot\cos(3\cdot t) \quad = \quad \frac{A}{3}\cdot\cos(3\cdot t)$

These are the first three Fourier series terms that appear on p 427

## Magnitude Plots

$m := -1, 0 .. 11 \quad n := 0, 1 .. 10 \quad t := 2\cdot\pi \quad A := 1$

$$g(n) := \text{if}\left[n < 1, 0, \left| A\cdot\left(\frac{1}{n}\cdot\cos(n\cdot t)\right)\right|\right]$$




$$fo = \frac{1}{T} = \frac{1}{2\cdot\pi}$$

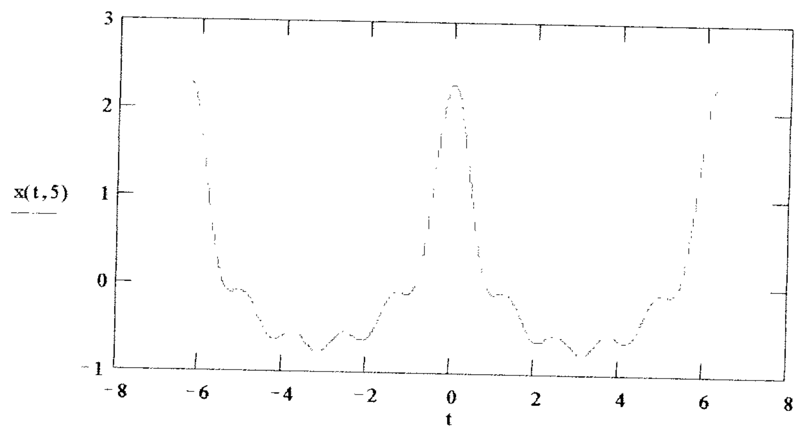
This is the same magnitude plot that appears on page 427.

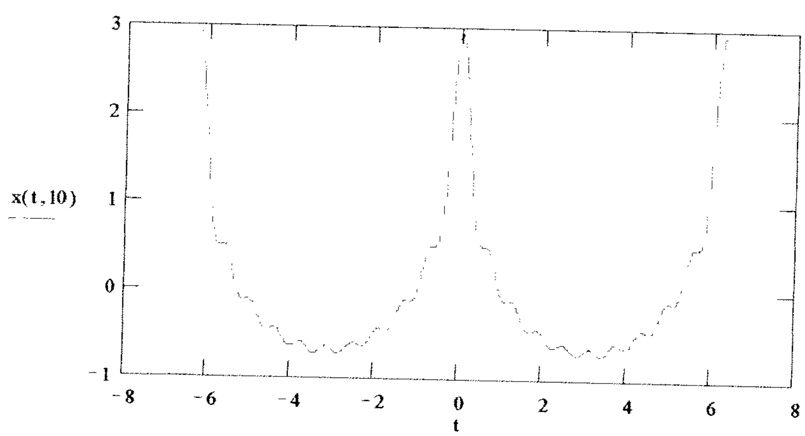
Fourier series plots

$T := 2\cdot\pi \qquad \tau := 1 \qquad m := -1, 0 .. 11$

$t := -T, -T + \frac{T}{1000} .. T \qquad n := 0, 1 .. 10$

$$x(t,m) := A\cdot \sum_{n=1}^{m} \frac{1}{n}\cdot \cos(n\cdot t)$$




These magnitude plots verify those that appear on p 427.

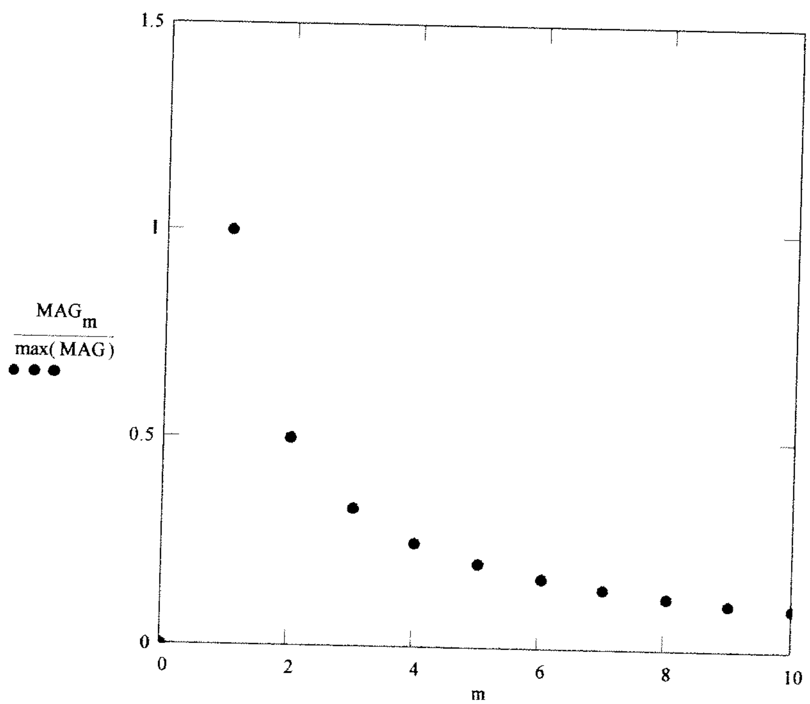
EMC [redacted] Ch 13 pg 427

$n := 1, 2 .. 10$ $A := 1$

$m := 0, 1 .. 10$

$M_n := \frac{A}{n}$

$M_0 := 0$ $MAG_m := |M_m|$



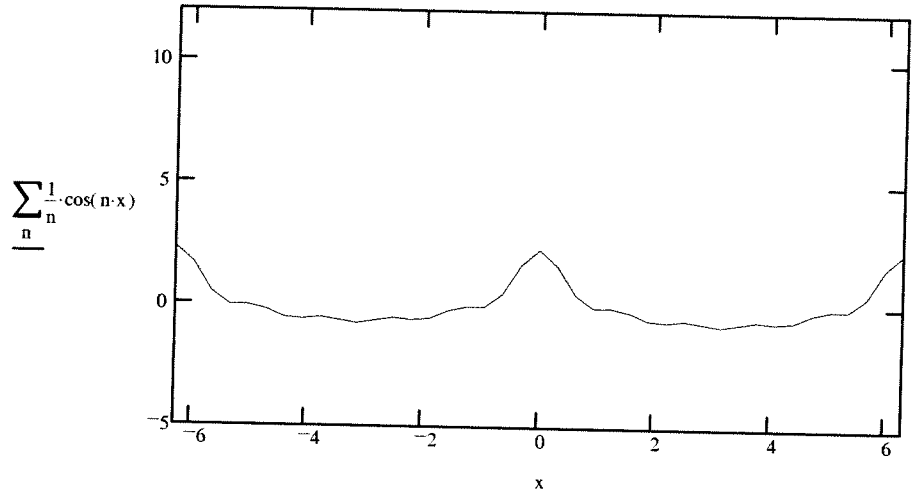
427

$n=1 \Rightarrow A \frac{1}{1} \cos t = A\cos(t)$

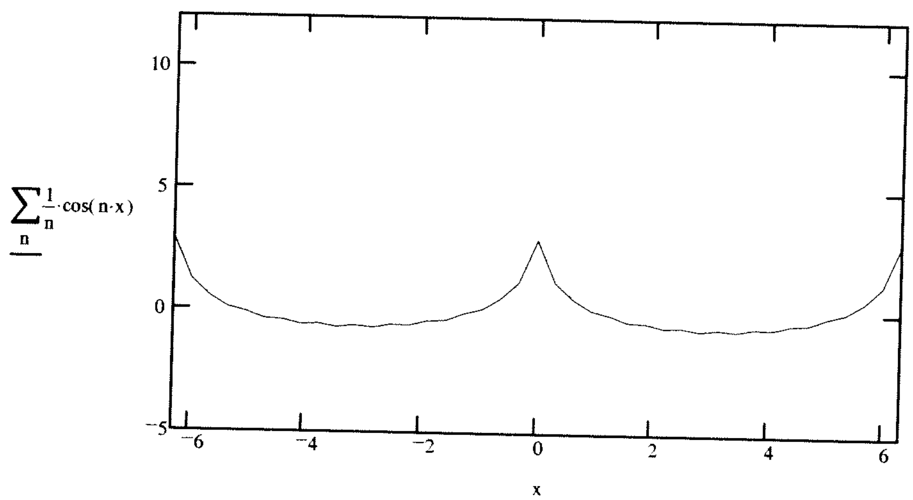
$n=2 \quad A \frac{1}{2} \cos(2t) = \frac{A}{2}\cos(2t)$

$n=3 \quad A \frac{1}{3} \cos(3t) = \frac{A}{3}\cos(3t)$

$n := 1, 2 .. 5$
$x := -2\cdot\pi, -1.9\cdot\pi .. 2\cdot\pi$




$n := 1, 2 .. 10$
$x := -2\cdot\pi, -1.9\cdot\pi .. 2\cdot\pi$



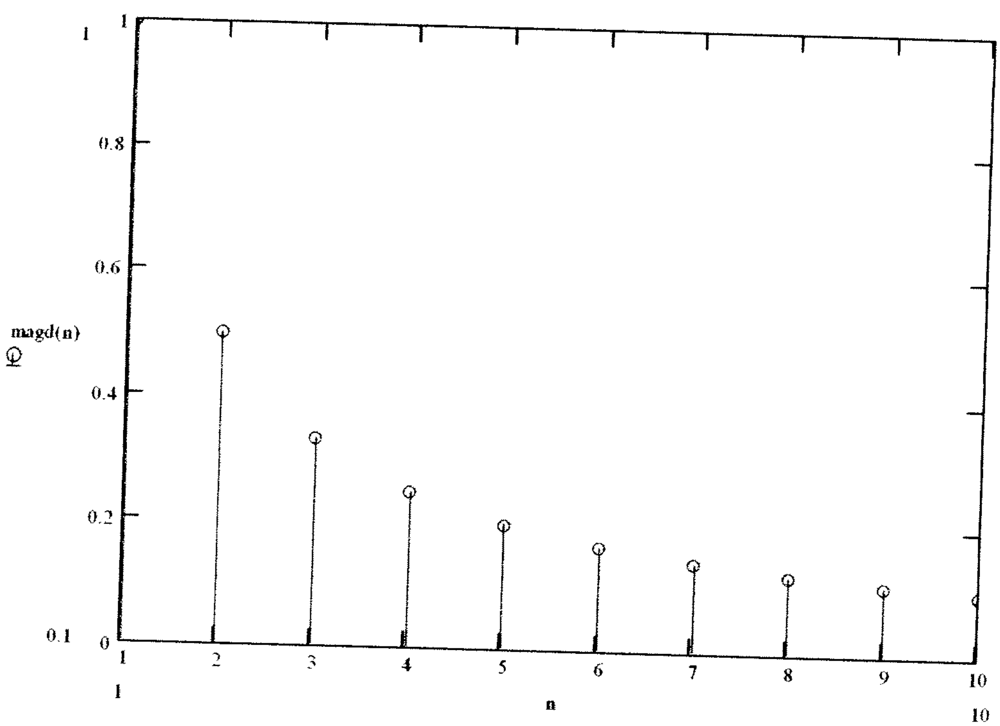
chapter 13 spectra of natural log of a cosine wave#1 page 428

$a := 1$

$n := 1, 2 .. 10$

$$d(n) := \frac{a \cdot (-1)^{n+1}}{n}$$

$$magd(n) := |d(n)|$$



Function

$$A\cdot\sum_{n=1}^{5}\frac{(1)^{n+1}}{n}\cdot\cos(n\cdot t)$$

No Bn coeffecient due to symmetry of plot

An coeffecient

$$\frac{2}{T}\cdot\int_{\pi}^{\pi} A\cdot\ln\left(2\cdot\cos\frac{t}{2}\right) dt$$

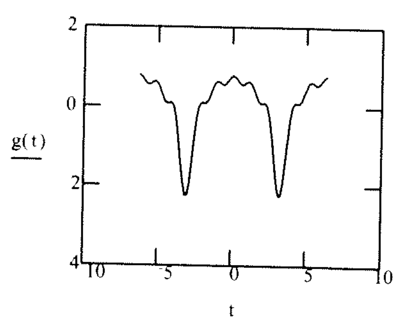
Error when evaluating "no closed form found for integral"

Here "n" is equal to 5

Ken Kaiser

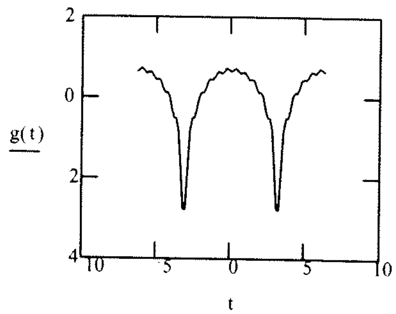
$A\ 1 \qquad t\ \ 2\cdot\pi,\ 6.13..2\cdot\pi$

$$g(t)\quad A\cdot\sum_{n=1}^{5}\frac{(1)^{n+1}}{n}\cdot\cos(n\cdot t)$$




Here "n" is equal to 10

$$g(t) \quad A\cdot\sum_{n=1}^{10} \frac{(\ 1)^{n+1}}{n}\cdot\cos(n\cdot t)$$




## Spectra

$A \quad 1 \qquad m \quad 1, 0 .. 11 \qquad n \quad 1, 2 .. 10$

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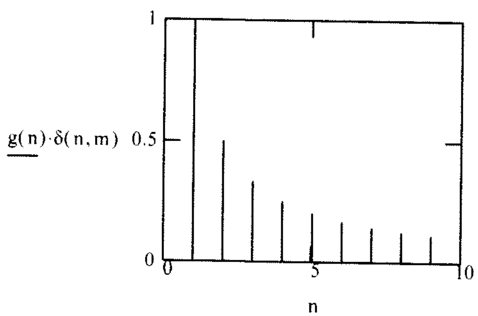
$$g(n) \quad \frac{(-1)^{n+1}}{n}\cdot\cos(n\cdot t)\cdot A$$



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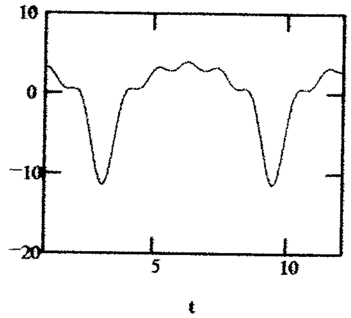
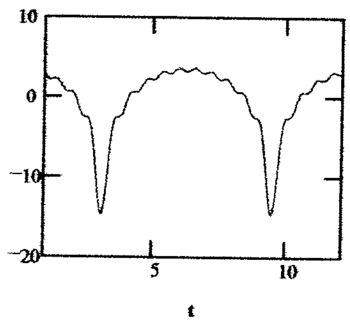
# Natural Log of a Cosine Wave #1

$A := 5 \quad t := 1, 1.0001 .. 15$

$A \cdot \sum_{n=1}^{5} \frac{(-1)^{n+1}}{n} \cdot \cos(n \cdot t)$




$A \cdot \sum_{n=1}^{10} \frac{(-1)^{n+1}}{n} \cdot \cos(n \cdot t)$




$N = \omega_0 = \frac{2\pi}{T}$

$1 \quad \frac{2\pi}{T} = \frac{2\pi}{2\pi}$

$T =$

$\omega_0 = N$

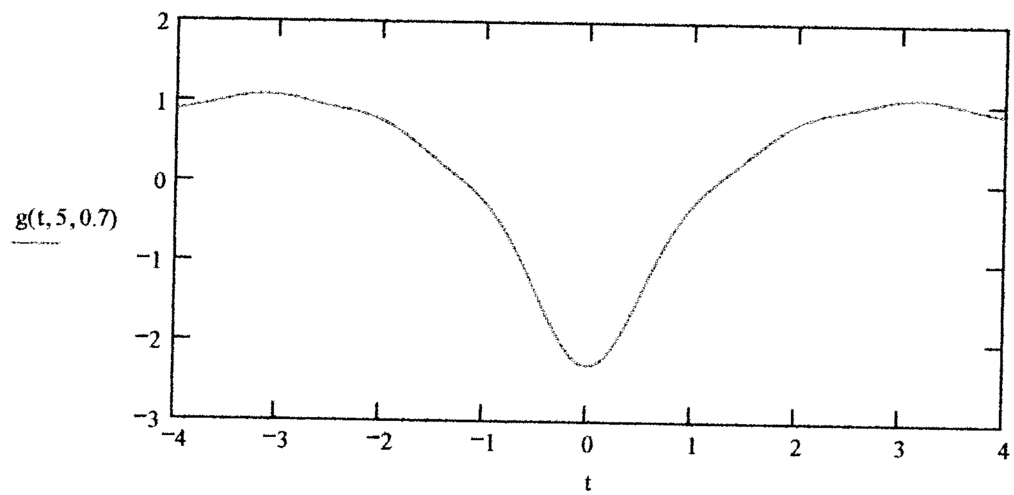
$\omega_0 = 1 = \frac{2\pi}{T}$

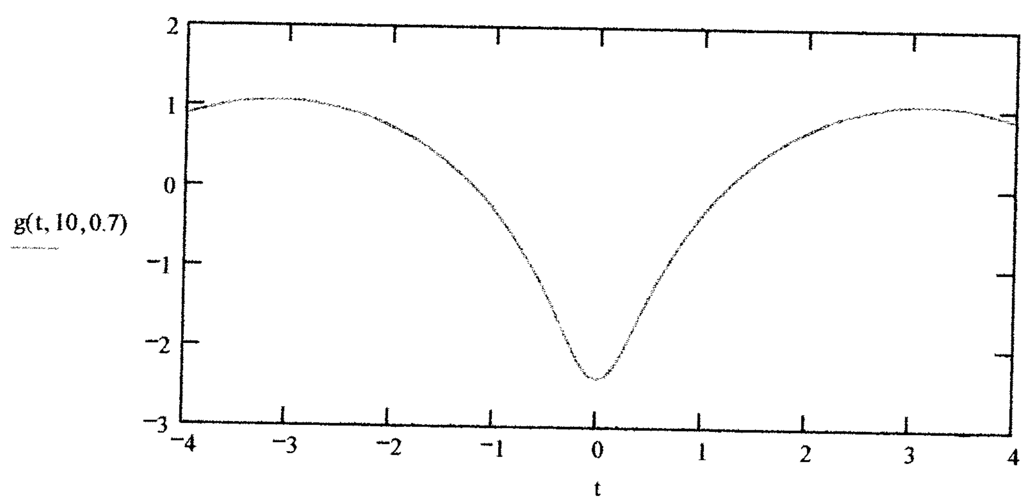
$T = 2\pi$

# Natural Log of a Cosine Wave #2 (p.429)

$A := 1 \qquad T := 1 \qquad t := -4T, -4T + \frac{8T}{100} .. 4T$

$$g(t, M, \alpha) := -2A \cdot \sum_{n=1}^{M} \frac{\alpha^n}{n} \cos(n \cdot t)$$



$$-2\cdot A\cdot \sum_{n=1}^{3} \frac{\alpha^n}{n}\cdot\cos(n\cdot t)$$

$$-2\cdot\alpha\cdot A\cdot\cos(t)-\alpha^2\cdot A\cdot\cos(2\cdot t)-\frac{2\cdot\alpha^3\cdot A}{3}\cdot\cos(3\cdot t)$$

$$\frac{-2\cdot A\cdot \sum_{n=1}^{3} \frac{\alpha^n}{n}\cdot\cos(n\cdot t)}{-2\cdot\alpha\cdot A\cdot\cos(t)-\alpha^2\cdot A\cdot\cos(2\cdot t)-\frac{2\cdot\alpha^3\cdot A}{3}\cdot\cos(3\cdot t)}$$

1 ratio of ↑

✓

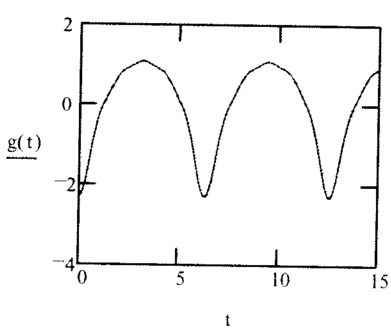
$\alpha := 0.7$

$A := 1$

$m := 0, 1 .. 10$

$t := 0, .1 .. 15$

$$g(t) := -2\cdot A\cdot \sum_{n=1}^{5} \frac{\alpha^n}{n}\cdot\cos(n\cdot t)$$




n=5

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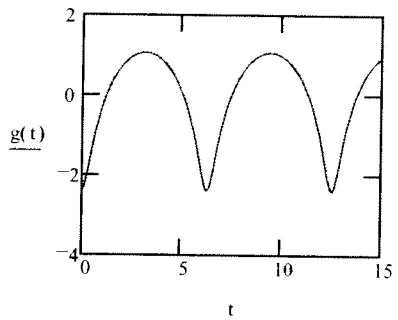
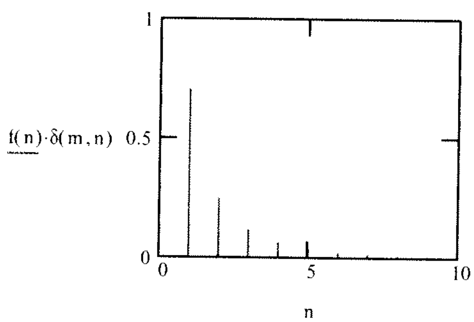
$$g(t) := -2\cdot A\cdot \sum_{n=1}^{10} \frac{\alpha^n}{n}\cdot \cos(n\cdot t)$$




n=10

$n := 1, 2 .. 10$

$$f(n) := \frac{\alpha^n}{n}$$




α = .27 case.

[redacted]

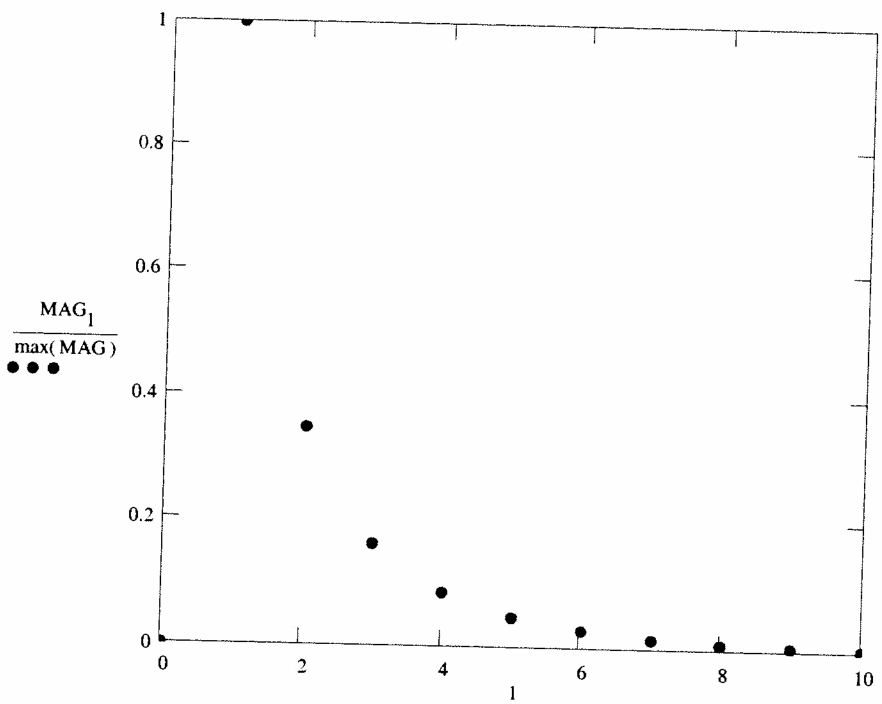
$k := 1 .. 10$ ~~T := 0.5 A := 2 B := -2 C := 1~~

$l := 0 .. 10$ $\alpha := 0.7$

$$M_k := \frac{-2 \cdot \alpha^k \cdot A}{k}$$

$$M_0 := 0$$

$$MAG_l := |M_l|$$



[redacted]

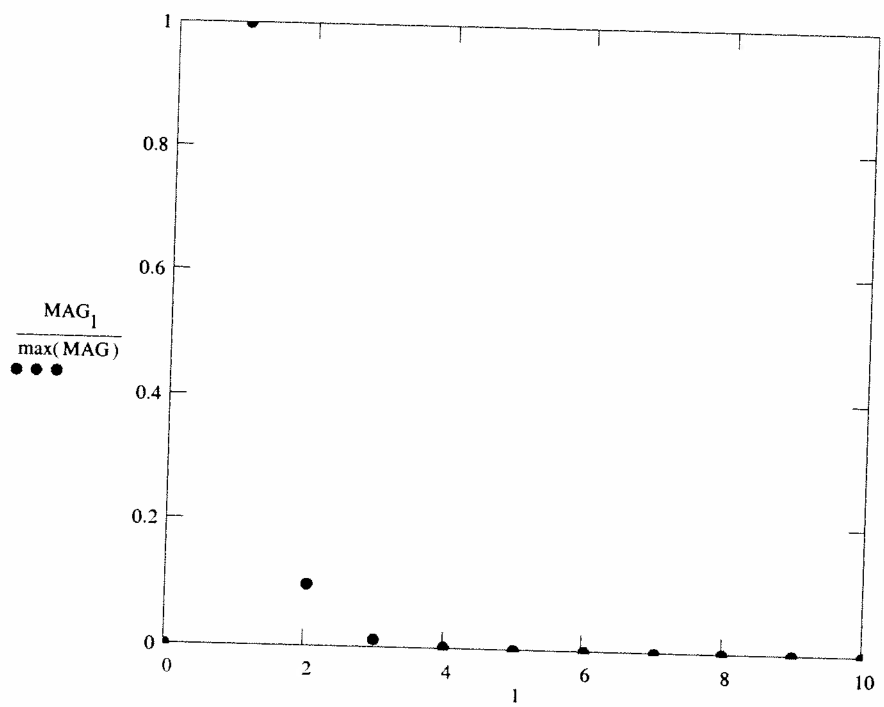
$k := 1 .. 10$ $T := 0.5$ $A := 2$ $B := -2$ $C := -1$

$l := 0 .. 10$ $\alpha := 0.2$

$$M_k := \frac{-2 \cdot \alpha^k \cdot A}{k}$$

$$M_0 := 0$$

$$MAG_l := \left| M_l \right|$$



$k := 1 .. 10$

$T := 2$ $A := 2$

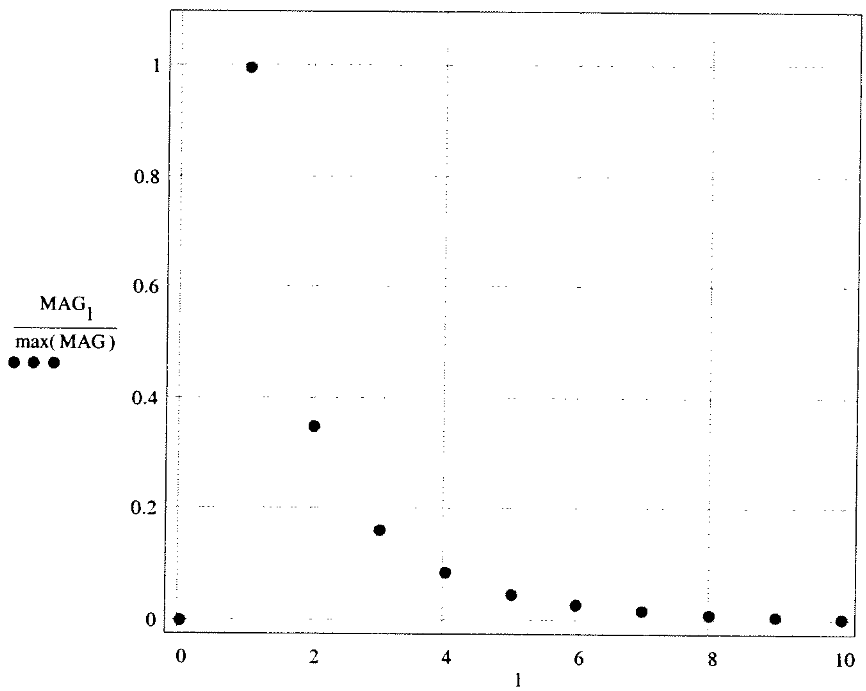
$l := 0 .. 10$

$\alpha := 0.7$

$M_k := -2 \cdot A \cdot \frac{\alpha^k}{k}$

$M_0 := 0$

$MAG_l := \left| M_l \right|$



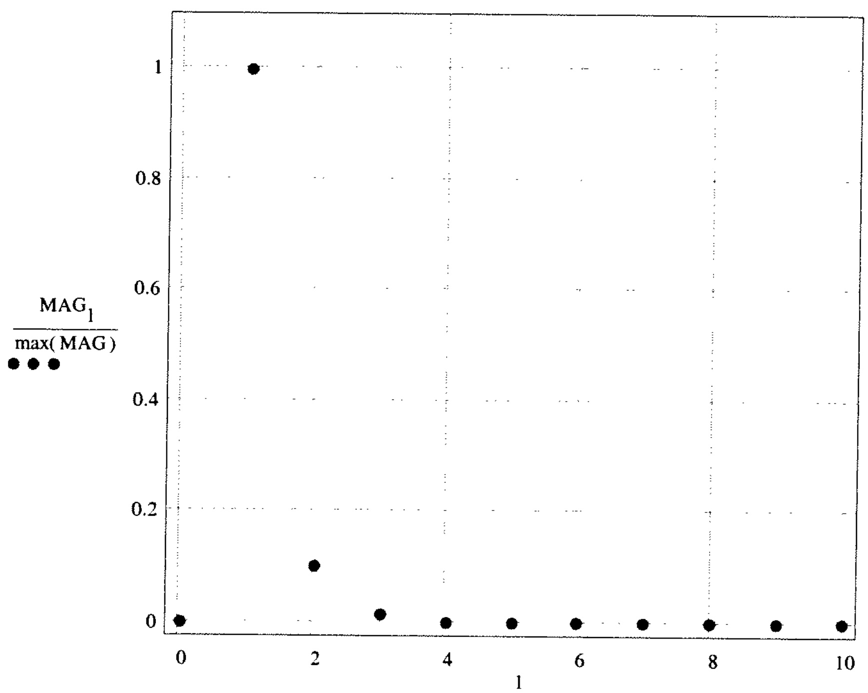
$k := 1 .. 10$ $\quad T := 2 \quad A := 2 \quad B := -2 \quad C := -1$

$l := 0 .. 10$ $\quad \alpha := 0.2$

$M_k := -2 \cdot A \cdot \frac{\alpha^k}{k}$

$M_0 := 0$

$MAG_l := \left| M_l \right|$

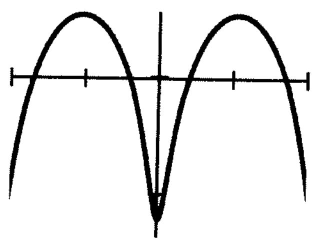


$T := 2\cdot\pi$ $\qquad A := 1 \qquad \alpha := 0.7$

$t := -\frac{4}{2}\cdot T, -\frac{4}{2}\cdot T + \frac{T}{1000} .. \frac{3}{2}\cdot T$

$f(t) := A\cdot\ln\left(1 - 2\cdot\alpha\cdot\cos(t) + \alpha^2\right)$

$g(t) := \mathrm{if}\left(-\frac{T}{2} \le t \le \frac{T}{2}, f(t), \mathrm{if}\left(-\frac{3.0}{2}\cdot T < t < -\frac{T}{2}, f(t+T), \mathrm{if}\left(\frac{T}{2} < t < \frac{3}{2}\cdot T, f(t-T), 0\right)\right)\right)$



$$\frac{\left[\int_{-\pi}^{\pi} \left(A\cdot\ln\left(1 - 2\cdot\alpha\cdot\cos(t) + \alpha^{2}\right)\right)\cdot\cos\left(2\cdot\pi\cdot\frac{1}{T}\cdot t\right)\,dt\right]\cdot 2}{\pi\cdot 2}$$

$$\frac{16}{T^2}\cdot t\cdot\left(\frac{T}{2}-t\right)$$

$$-t\cdot\left(\frac{T}{2}-t\right)$$

$$\frac{16}{T^2}\cdot\left(\frac{1}{2}\cdot T-t\right)-\frac{16}{T^2}\cdot t=0$$

$$\frac{1}{4}\cdot T$$

$$\frac{16}{T^2}\cdot\frac{T}{4}\cdot\left(\frac{T}{2}-\frac{T}{4}\right)$$

1

$$A \cdot t \cdot \left(\frac{T}{2} - t\right) \cdot \left(\frac{T}{2} + t\right)$$

$$A \cdot \left(\frac{1}{2} \cdot T - t\right) \cdot \left(\frac{1}{2} \cdot T + t\right) - A \cdot t \cdot \left(\frac{1}{2} \cdot T + t\right) + A \cdot t \cdot \left(\frac{1}{2} \cdot T - t\right) = 0$$

$$\begin{pmatrix} \frac{-1}{6} \cdot \sqrt{3} \cdot T \\ \frac{1}{} \sqrt{3} \cdot T \end{pmatrix}$$

$$\frac{1}{36} \cdot A \cdot \sqrt{3} \cdot T^3 \quad \frac{T}{2} - \frac{1}{6} \cdot \sqrt{3} \cdot T\Bigg) \cdot \left(\frac{T}{2} + \frac{1}{6} \cdot \sqrt{3} \cdot T\right)$$

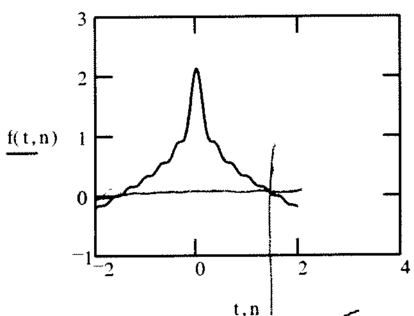
[redacted]
Natural Log of a Tangent Wave #1

$A := 1$

$n := 10$

$t := -2, -1.99 .. 2$

$$f(t,n) := A \cdot \sum_{m=1}^{n} \frac{1}{(2 \cdot m) - 1} \cdot \cos(((2 \cdot m) - 1) \cdot t)$$



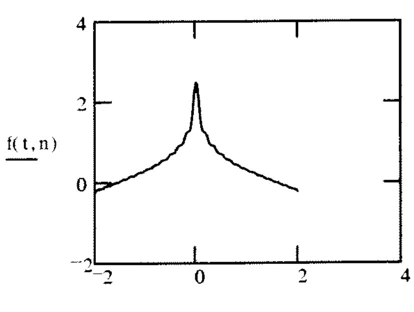

$\frac{\pi}{2} \approx 1.5$

$A := 1$

$n := 20$

$t := -2, -1.99 .. 2$

$$f(t,n) := A \cdot \sum_{m=1}^{n} \frac{1}{(2 \cdot m) - 1} \cdot \cos(((2 \cdot m) - 1) \cdot t)$$

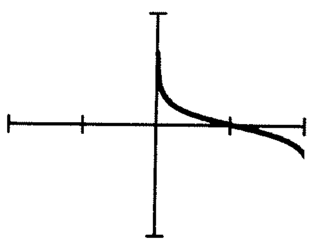


$T := \pi$ $\qquad A := 1 \qquad \alpha := 2$

$t := -\frac{4}{2}\cdot T, -\frac{4}{2}\cdot T + \frac{T}{1000} .. \frac{3}{2}\cdot T$

$f(t) := -\frac{A}{2}\cdot \ln\left(\tan\left(\frac{t}{2}\right)\right)$

$g(t) := \text{if}\left(0 \le t \le T, f(t), \text{if}\left(-\frac{2.0}{2}\cdot T < t < 0, f(t + T + \pi), \text{if}\left(T < t < \frac{4}{2}\cdot T, f(t - T), 0\right)\right)\right)$



the extra pie inserted because of the quadrant the tangent must be in for negative t--notice that the series consists entirely of cosines which must be an even function

$$\sqrt{\frac{\int_0^{\pi} \left(-\frac{A}{2}\cdot \ln\left(\tan\left(\frac{t}{2}\right)\right)\right)^2 dt}{\pi}} \qquad \frac{\int_0^{\pi} \left(-\frac{A}{2}\cdot \ln\left(\tan\left(\frac{t}{2}\right)\right)\right) dt}{\pi}$$

$$\frac{\left[\int_0^{\pi}\left(\frac{-A}{2}\cdot\ln\left(\tan\left(\frac{t}{2}\right)\right)\right)\cdot\cos\left(2\cdot\pi\cdot\frac{2.5}{\pi}\cdot t\right)dt\right]\cdot 2}{\pi}$$

$.2\cdot A$

$\frac{16}{T^2}\cdot t\cdot\left(\frac{T}{2}-t\right)$

$-t\cdot\left(\frac{T}{2}-t\right)$

$\frac{16}{T^2}\cdot\left(\frac{1}{2}\cdot T-t\right)-\frac{16}{T^2}\cdot t = 0$

$\frac{1}{4}\cdot T$

$\frac{16}{T^2}\cdot\frac{T}{4}\cdot\left(\frac{T}{2}-\frac{T}{4}\right)$

1

$$A \cdot t \cdot \left(\frac{T}{2} - t\right) \cdot \left(\frac{T}{2} + t\right)$$

$$A \cdot \left(\frac{1}{2} \cdot T - t\right) \cdot \left(\frac{1}{2} \cdot T + t\right) - A \cdot t \cdot \left(\frac{1}{2} \cdot T + t\right) + A \cdot t \cdot \left(\frac{1}{2} \cdot T - t\right) = 0$$

$$\begin{pmatrix} \frac{-1}{6} \cdot \sqrt{3} \cdot T \\ \frac{1}{6} \cdot \sqrt{3} \cdot T \end{pmatrix}$$

$$\frac{1}{36} \cdot A \cdot \sqrt{3} \cdot T^3 \quad \frac{T}{2} - \frac{1}{6} \cdot \sqrt{3} \cdot T\right) \cdot \left(\frac{T}{2} + \frac{1}{6} \cdot \sqrt{3} \cdot T\right)$$

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$m := -1, 0 .. 11$

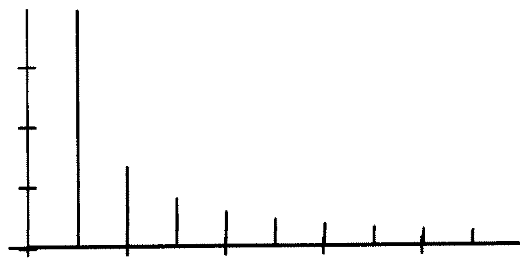
$n := 0, 1 .. 10$

$A := 1$

$\alpha := 2$

$T := 1$

$$g(n) := \text{if}\left(n < 1, 0, \frac{A}{2 \cdot n - 1}\right)$$



$$\frac{2}{T}\cdot\int_{0}^{\pi}\frac{A}{2}\cdot\ln\left(\tan\left(\frac{t}{2}\right)\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$ ←

$$\frac{2}{T}\cdot\int_{0}^{\pi}\frac{A}{2}\cdot\ln\left(\tan\left(\frac{t}{2}\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$ ←

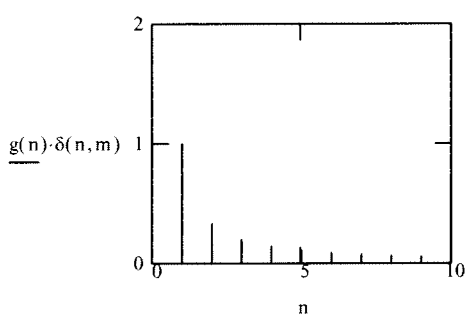
} NO CLOSED FORM FOUND FOR INTEGRAL!

$A := 2 \quad t := 0$

$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$g(n) := \mathrm{if}\left(n<1, A, \frac{1}{2\cdot n-1}\cdot\cos((2\cdot n-1)\cdot t)\right)$

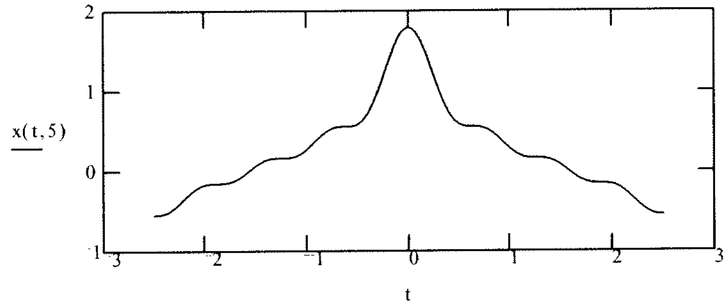



$T := 2.5$

$A := 1 \quad t := -T, -T+\frac{T}{1000} .. T$

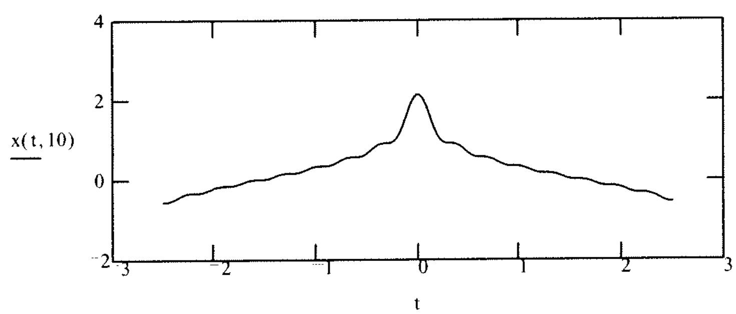
$x(t,m) := A\cdot\cos(t) - \frac{A}{3}\cdot\cos(3\cdot t) + \frac{A}{5}\cdot\cos(5\cdot t)$

$$x(t,m) = A\cdot\sum_{n=1}^{5}\frac{1}{2\cdot n-1}\cdot\cos((2\cdot n-1)\cdot t)$$




@ 5 samples

$$x(t,m) = A\cdot\sum_{n=1}^{10}\frac{1}{2\cdot n-1}\cdot\cos((2\cdot n-1)\cdot t)$$




@ 5 samples

**Natural Log of a Tangent Wave #2: pg. 431**

**Just by observing the waveform, it is obvious that this functions is odd, therefore a(n) coefficients are equal to 0.**

**b(n) coefficient:**

$$\left[\frac{2}{\pi}\cdot\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{-A}{2}\cdot\ln\left(\tan\left(\frac{\pi}{4}-\frac{t}{2}\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

**Unable to evaluate:**
**No closed form found for integral**

Ken Kaiser

**Instructor's Equation:**

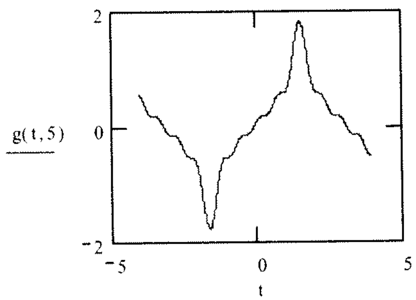
$$A\cdot\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2\cdot n-1}\cdot\sin((2\cdot n-1)\cdot t)$$

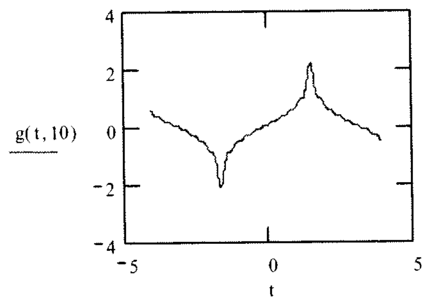
$T := 1$

$A := 1$

$t := -(4\cdot T), -(4\cdot T)+\frac{T}{100} .. 4\cdot T$

$$g(t,m) := A \cdot \sum_{n=1}^{m} \frac{(-1)^{n-1}}{2 \cdot n - 1} \cdot \sin((2 \cdot n - 1) \cdot t)$$



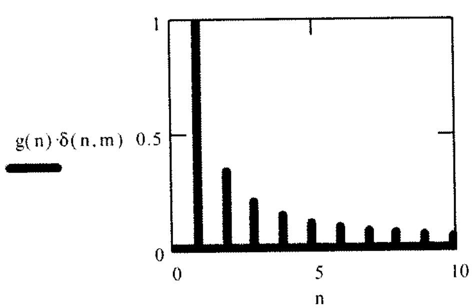

Ken Kaiser

$m := -1, 0 .. 11$

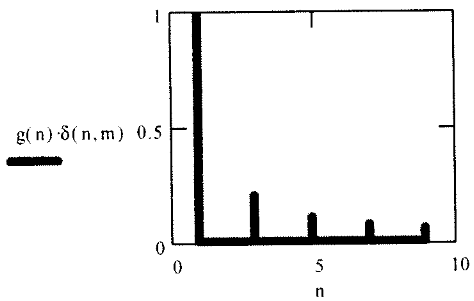
$n := 0, 1 .. 10$

$A := 1$

$$g(n) := \text{if}\left[n < 1, 0, \left|\frac{(-1)^{n-1}}{2 \cdot n - 1}\right|\right]$$



$n := 1, 3 .. 9$

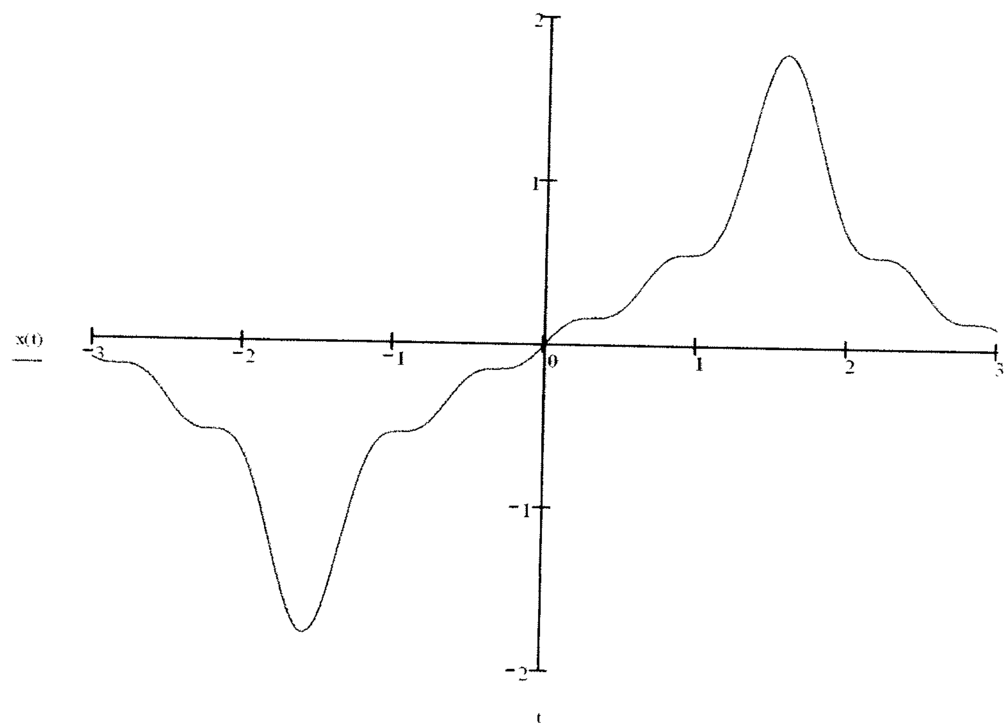



Ken Kaiser

Ken Kaiser

$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 5$

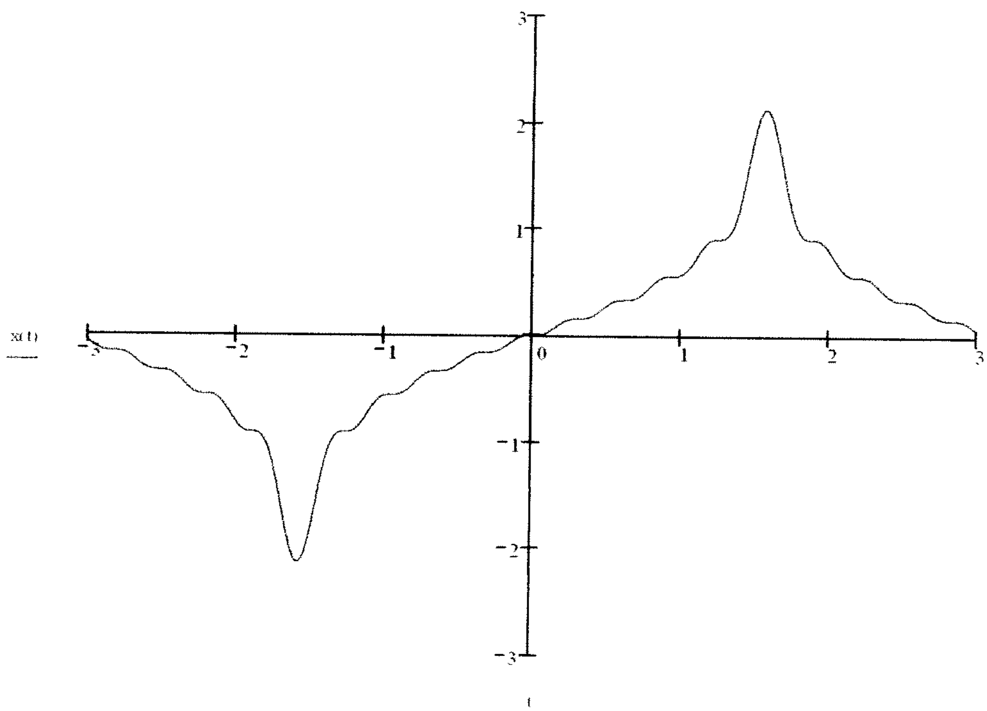
$$x(t) := A \cdot \sum_{n=1}^{N} \frac{(-1)^{n-1}}{2 \cdot n - 1} \cdot \sin[(2n-1)t]$$



Ken Kaiser

$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 10$

$$x(t) := A \cdot \sum_{n=1}^{N} \frac{(-1)^{n-1}}{2 \cdot n - 1} \cdot \sin[(2n-1)t]$$



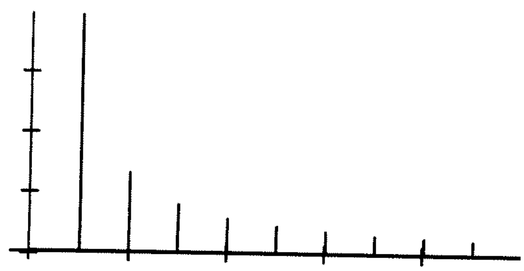
Ken Kaiser

$m := -1, 0 .. 11$

$\alpha := 2$ $\quad$ T : =(

$n := 0, 1 .. 10$ $\quad$ $A := 1$

$$g(n) := \text{if}\left(n < 1, 0, \frac{A}{2 \cdot n - 1}\right)$$



# p432: Arctangent of Trigonometric Function Wave #1

## No closed form for integrals

$a_n$

$n = 1$

$$\frac{2}{T}\cdot\left[\int_{-\pi}^{\pi} A\cdot \operatorname{atan}\left[\frac{\alpha\cdot\sin(t)}{1-(\alpha\cdot\cos(t))}\right]\cdot\cos\left(\frac{2\cdot\pi\cdot 1}{T}\cdot t\right)dt\right]$$

$b_n$

$$\frac{2}{T}\cdot\left[\int_{-\pi}^{\pi} A\cdot \operatorname{atan}\left[\frac{\alpha\cdot\sin(t)}{1-(\alpha\cdot\cos(t))}\right]\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

$$A\cdot\sum_{n=1}^{\infty}\frac{\alpha^{n}}{n}\cdot\sin(n\cdot t)$$

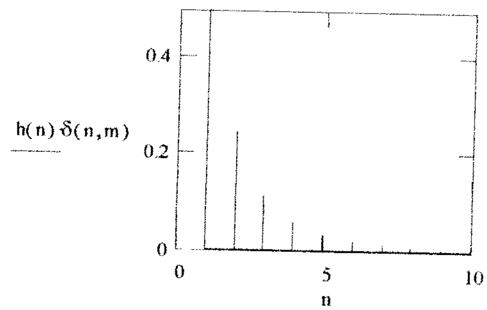
$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$\alpha := 0.7$

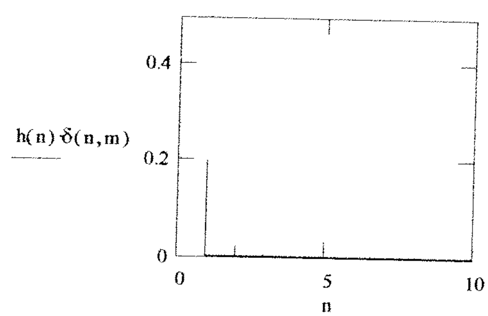
$h(n) := \text{if}\left(n < 0, 0, \left|\frac{\alpha^n}{n}\right|\right)$

no dc offset




$\alpha := 0.2$

$h(n) := \text{if}\left(n < 0, 0, \left|\frac{\alpha^n}{n}\right|\right)$



$\alpha := 0.7$

$n := 1 .. 10$

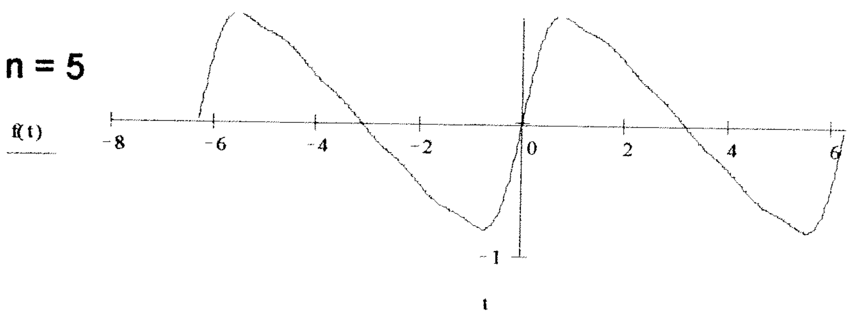
$t := -2\cdot\pi, -6.26 .. 2\cdot\pi$

$A := 1$

$T := 1$

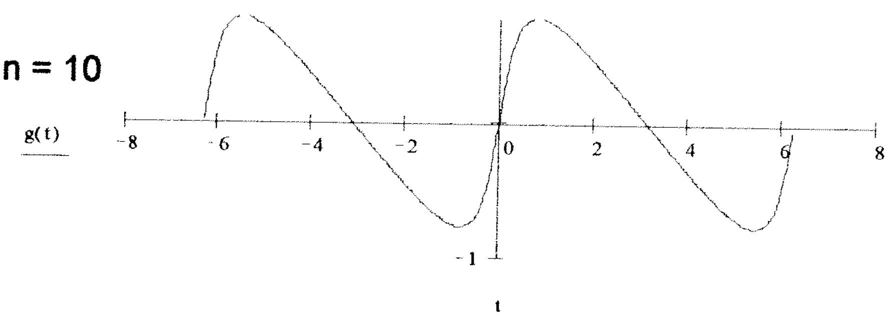
$$f(t) := A\cdot\sum_{n=1}^{5}\frac{\alpha^n}{n}\cdot\sin(n\cdot t)$$

**n = 5**




$$g(t) := A\cdot\sum_{n=1}^{10}\frac{\alpha^n}{n}\cdot\sin(n\cdot t)$$

**n = 10**



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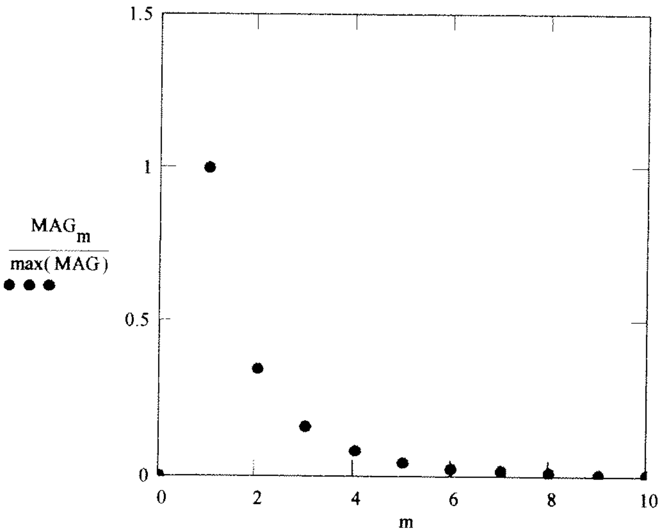
$\alpha := .7 \quad A := 1$

$n := 1, 2 .. 10$

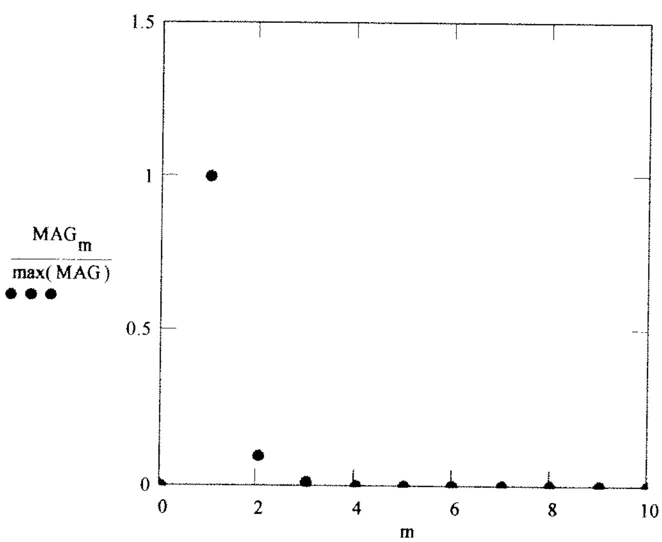
$m := 0, 1 .. 10 \qquad M_n := \frac{\alpha^n}{n} \cdot A \qquad M_0 := 0 \qquad MAG_m := |M_m|$




$\alpha := .2 \quad A := 1$

$n := 1, 2 .. 10$

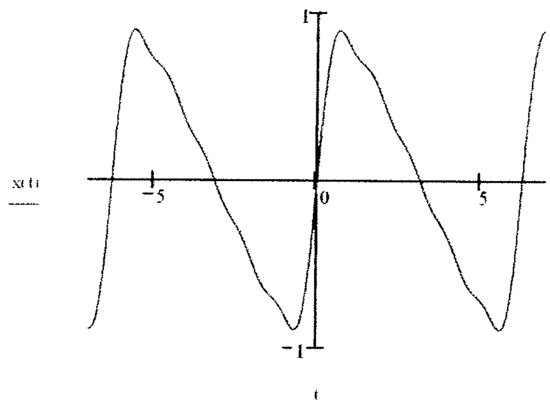
$m := 0, 1 .. 10 \qquad M_n := \frac{\alpha^n}{n} \cdot A \qquad M_0 := 0 \qquad MAG_m := |M_m|$



Ken Kaiser

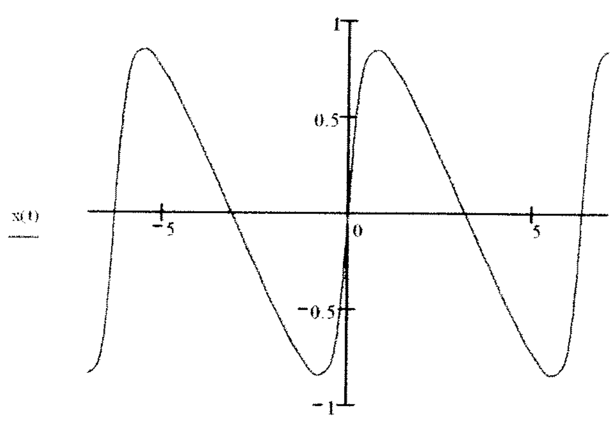
$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 5 \qquad \alpha := .75$

$$x(t) := A \cdot \sum_{n=1}^{N} \frac{\alpha^n}{n} \cdot \sin(n t)$$




$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 10 \qquad \alpha := .75$

$$x(t) := A \cdot \sum_{n=1}^{N} \frac{\alpha^n}{n} \cdot \sin(n t)$$

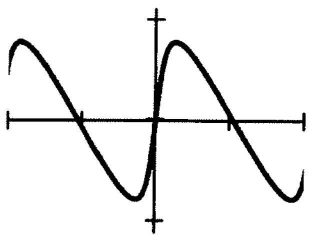


$$T := 2\cdot\pi \qquad A := 1 \qquad \alpha := 0.7$$

$$t := -\frac{4}{2}\cdot T, -\frac{4}{2}\cdot T + \frac{T}{1000} .. \frac{3}{2}\cdot T$$

$$f(t) := A\cdot \text{atan}\left(\frac{\alpha\cdot\sin(t)}{1 - \alpha\cdot\cos(t)}\right)$$

$$g(t) := \text{if}\left(-\frac{T}{2} \le t \le \frac{T}{2}, f(t), \text{if}\left(-\frac{3.0}{2}\cdot T < t < -\frac{T}{2}, f(t+T), \text{if}\left(\frac{T}{2} < t < \frac{3}{2}\cdot T, f(t-T), 0\right)\right)\right)$$



$$\frac{\int_{-\pi}^{\pi} A\cdot \text{atan}\left(\frac{\alpha\cdot\sin(t)}{1 - \alpha\cdot\cos(t)}\right) dt}{2\cdot\pi}$$

$$\sqrt{\frac{\int_{-\pi}^{\pi} \left(A\cdot \text{atan}\left(\frac{\alpha\cdot\sin(t)}{1 - \alpha\cdot\cos(t)}\right)\right)^2 dt}{2\cdot\pi}}$$

$$A\cdot \text{atan}\left(\frac{\alpha\cdot\sin(t)}{1 - \alpha\cdot\cos(t)}\right)$$

$$A\cdot\frac{\left[\alpha\cdot\frac{\cos(t)}{(1 - \alpha\cdot\cos(t))} - \alpha^2\cdot\frac{\sin(t)^2}{(1 - \alpha\cdot\cos(t))^2}\right]}{\left[\quad ^2 \quad \sin(t)^2 \quad\right]} = 0$$

$$\left\lfloor 1 + \alpha^{2}\cdot\frac{\sin(t)}{(1-\alpha\cdot\cos(t))^{2}} \right\rfloor$$

$$\operatorname{acos}(\alpha)$$

$$A\cdot\operatorname{atan}\left(\frac{\alpha\cdot\sin(\operatorname{acos}(\alpha))}{1-\alpha\cdot\cos(\operatorname{acos}(\alpha))}\right)$$

$$-i\cdot A\cdot\operatorname{atanh}\left(\frac{\alpha}{\sqrt{-1+\alpha^{2}}}\right)$$

$$-i\cdot A\cdot\operatorname{atanh}\left(\frac{\alpha}{\sqrt{-1+\alpha^{2}}}\right)$$

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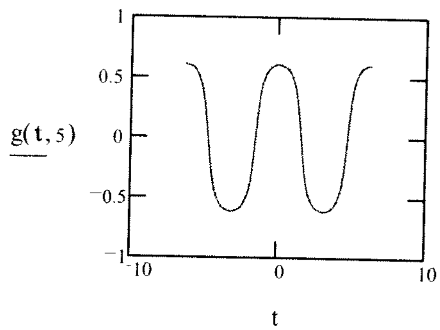
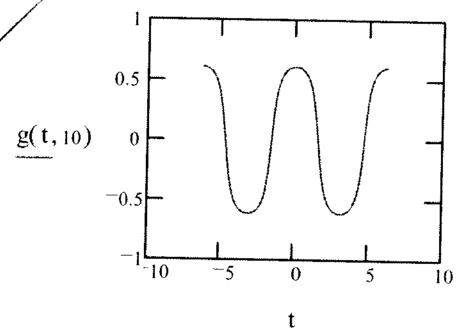
Ken Kaiser

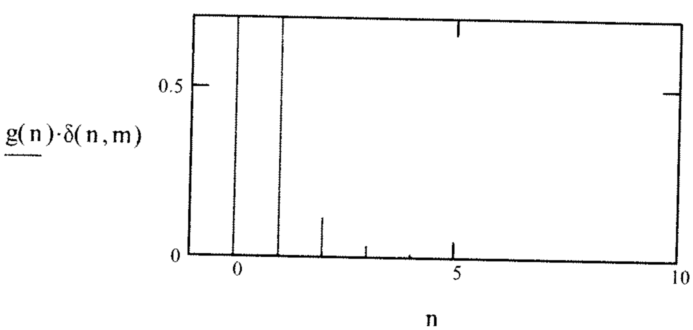
# Arctangent of Trig. Function Wave #3

$\alpha := 0.7$ $\quad A := 1$ $\quad t := -2\cdot\pi, -2\cdot\pi + \frac{2\cdot\pi}{10000} .. 2\cdot\pi$ $\quad g(t,m) := A\cdot\left[\sum_{n=1}^{m}\left[\frac{(-1)^{n-1}\cdot\alpha^{(2\cdot n-1)}}{2\cdot n-1}\cdot\cos((2\cdot n-1)\cdot t)\right]\right]$

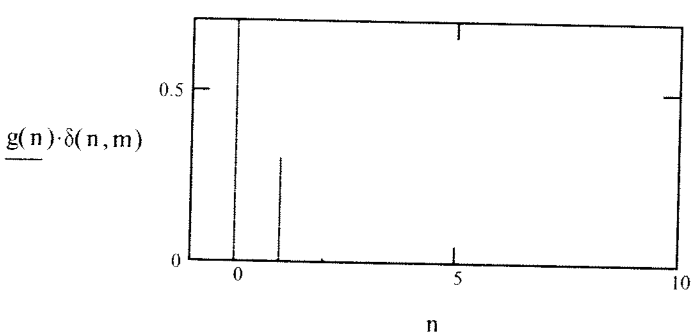



$n := 0,1..10$ $\quad m := -1,0..11$ $\quad A := 1$ $\quad g(n) := \left|\frac{(-1)^{n-1}\cdot\alpha^{(2\cdot n-1)}}{2\cdot n-1}\cdot A\right|$



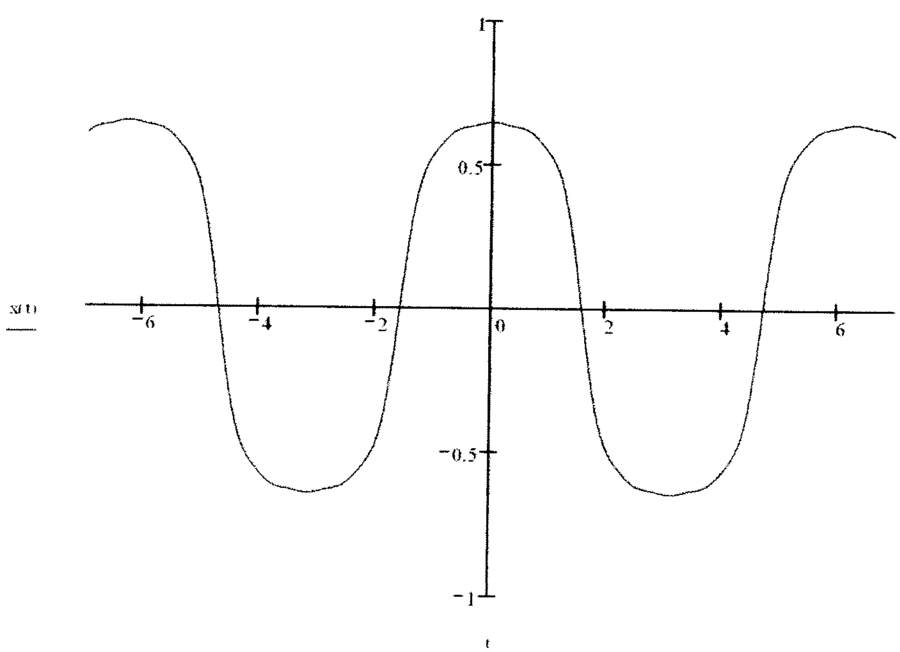

$\alpha := 0.3$ $\quad g(n) := \left|\frac{(-1)^{n-1}\cdot\alpha^{(2\cdot n-1)}}{2\cdot n-1}\cdot A\right|$



Ken Kaiser

$A := 1$ $\quad f := \frac{1}{2 \cdot \pi}$ $\quad N := 5$ $\quad \alpha := .75$

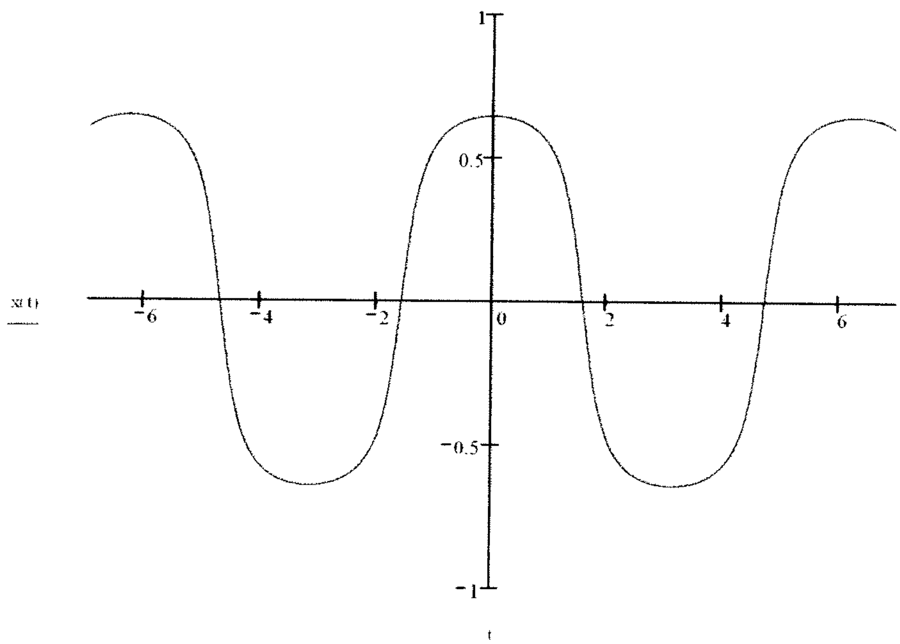
$$x(t) := A \cdot \sum_{n=1}^{N} \frac{(-1)^{n-1} \alpha^{2n-1}}{2n-1} \cdot \cos[(2n-1) \cdot t]$$



Ken Kaiser

$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 10 \qquad \alpha := .75$

$$x(t) := A \cdot \sum_{n=1}^{N} \frac{(-1)^{n-1} \alpha^{2n-1}}{2n-1} \cdot \cos[(2n-1) \cdot t]$$

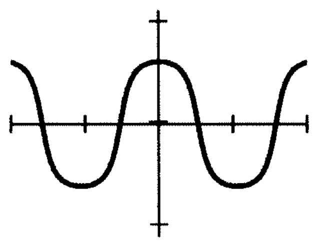


$T := 2\cdot\pi$ $\qquad A := 1 \qquad \alpha := 0.7$

$$t := -\frac{4}{2}\cdot T, -\frac{4}{2}\cdot T + \frac{T}{1000} .. \frac{3}{2}\cdot T$$

$$f(t) := \frac{A}{2}\cdot \operatorname{atan}\left(\frac{2\cdot\alpha\cdot\cos(t)}{1-\alpha^2}\right)$$

$$g(t) := \operatorname{if}\left(-\frac{T}{2} \le t \le \frac{T}{2}, f(t), \operatorname{if}\left(-\frac{3.0}{2}\cdot T < t < -\frac{T}{2}, f(t+T), \operatorname{if}\left(\frac{T}{2} < t < \frac{3}{2}\cdot T, f(t-T), 0\right)\right)\right)$$



$$\frac{\displaystyle\int_{-\pi}^{\pi} \frac{A}{2}\cdot \operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^2}\right) dt}{2\cdot\pi}$$

$$\frac{1}{2}\cdot\frac{\left[\left[\frac{-1}{4}\cdot \operatorname{dilog}\left(\frac{-1}{\alpha}\right) - \frac{1}{4}\cdot\ln\left(\frac{1}{\alpha}\right)\cdot\ln(-\alpha-1) + \frac{1}{4}\cdot\operatorname{dilog}(-\alpha+1) + \frac{1}{4}\cdot\operatorname{dilog}(-\alpha) + \frac{1}{4}\cdot i\cdot\pi\cdot\ln(-\alpha+1) - \frac{1}{4}\cdot\operatorname{dilog}\left[\frac{(-1+\alpha)}{\alpha}\right] - \frac{1}{4}\cdot i\cdot\pi\cdot\ln(-1+\alpha) - \frac{1}{4}\cdot i\cdot\pi\cdot\ln\right.\right.}{}$$

$$\frac{1}{4}\cdot i\cdot A\cdot\left(\ln(-\alpha+1) - \ln(-1+\alpha) + \ln(\alpha)\right)$$

$$\sqrt{\frac{\int_{-\pi}^{\pi}\left(A\cdot \operatorname{atan}\left(\frac{\alpha\cdot \sin(t)}{1-\alpha\cdot \cos(t)}\right)\right)^{2}dt}{2\cdot\pi}}$$

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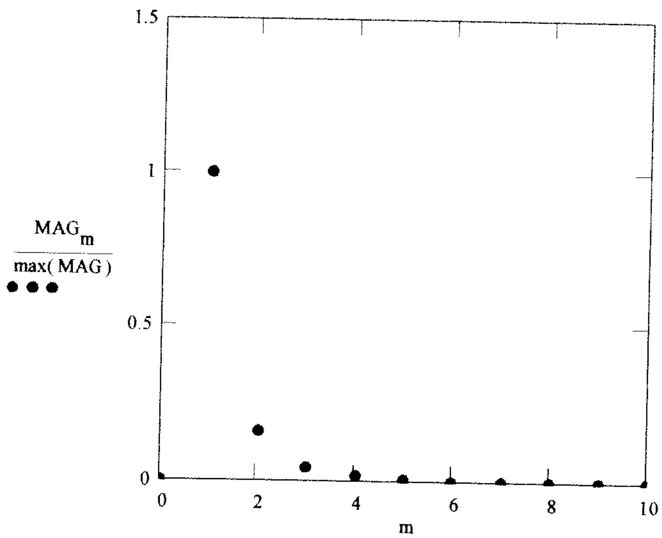
$\alpha := .7 \quad A := 1$

$n := 1, 2 .. 10$

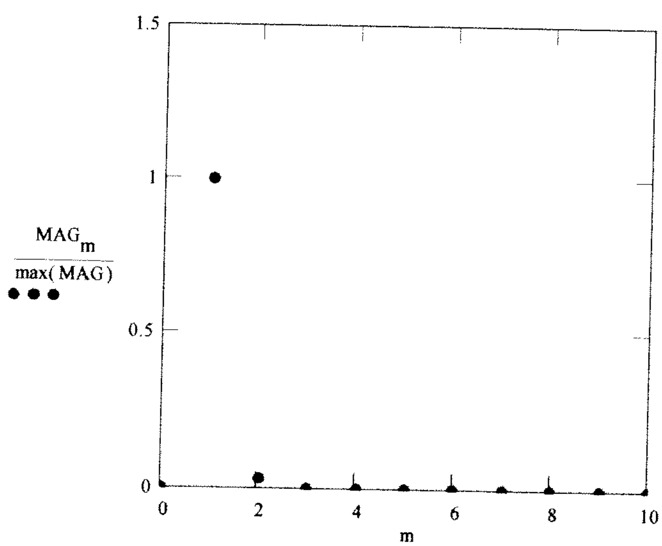
$m := 0, 1 .. 10 \qquad M_n := \frac{\alpha^{(2 \cdot n - 1)}}{2 \cdot n - 1} \cdot A \qquad M_0 := 0 \qquad MAG_m := |M_m|$




$\alpha := .3 \quad A := 1$

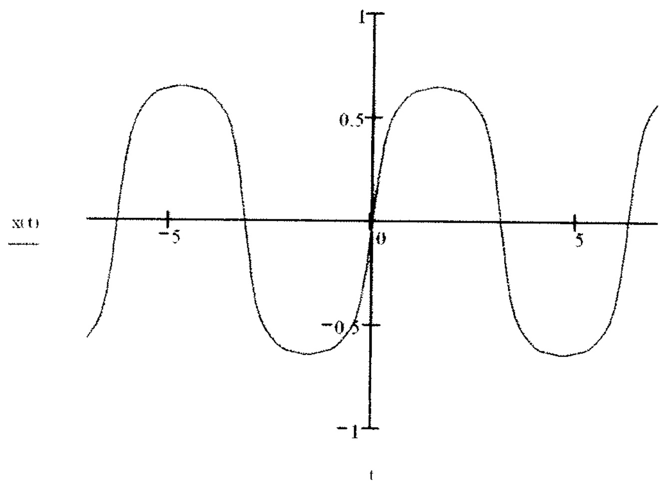
$n := 1, 2 .. 10$

$m := 0, 1 .. 10 \qquad M_n := \frac{\alpha^{(2 \cdot n - 1)}}{2 \cdot n - 1} \cdot A \qquad M_0 := 0 \qquad MAG_m := |M_m|$



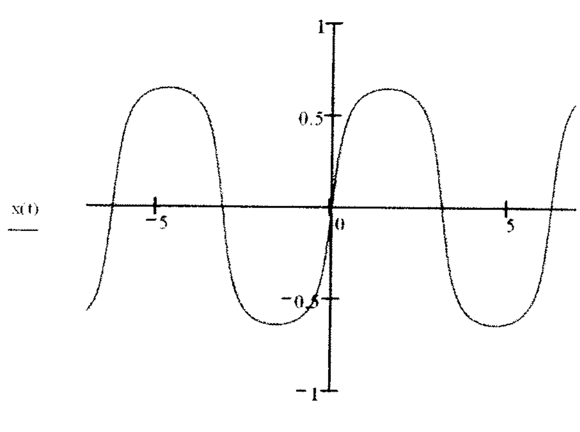
$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 5 \qquad \alpha := .75$

$$x(t) := A \cdot \sum_{n=1}^{N} \frac{\alpha^{2n-1}}{2n-1} \cdot \sin[(2n-1) \cdot t]$$




$A := 1 \qquad f := \frac{1}{2 \cdot \pi} \qquad N := 10 \qquad \alpha := .75$

$$x(t) := A \cdot \sum_{n=1}^{N} \frac{\alpha^{2n-1}}{2n-1} \cdot \sin[(2n-1) \cdot t]$$



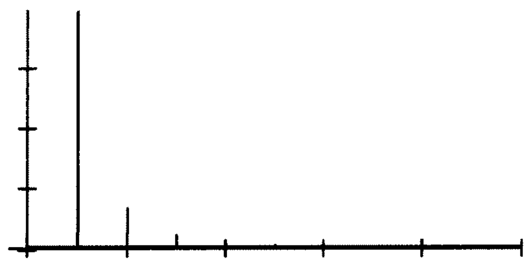
$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$A := 1$

$\alpha := 0.7$    $T := 1$

$$g(n) := \mathrm{if}\left(n < 1, 0, \frac{A \cdot \alpha^{2 \cdot n - 1}}{2 \cdot n - 1}\right)$$

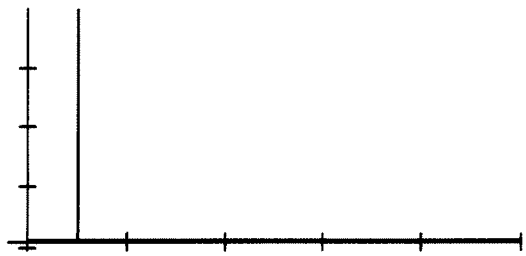


$m := -1, 0 .. 11$

$n := 0, 1 .. 10$ $\quad A := 1$

$\alpha := 0.3 \quad T := 1$

$$g(n) := \text{if}\left(n < 1, 0, \frac{A \cdot \alpha^{2 \cdot n - 1}}{2 \cdot n - 1}\right)$$

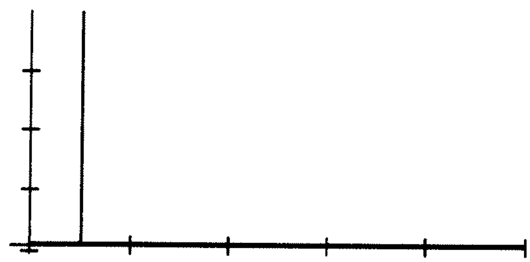


$\alpha := 0.3 \qquad T := 1$

$m := -1, 0 .. 11$

$n := 0, 1 .. 10 \qquad A := 1$

$$g(n) := \mathrm{if}\left(n < 1, 0, \frac{A \cdot \alpha^{2 \cdot n - 1}}{2 \cdot n - 1}\right)$$

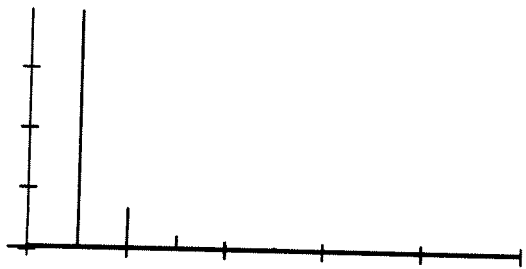


Ken Kaiser

$m := -1, 0 .. 11$ $\alpha := 0.7$ $T := 1$

$n := 0, 1 .. 10$ $A := 1$

$$g(n) := \text{if}\left(n < 1, 0, \frac{A \cdot \alpha^{2 \cdot n - 1}}{2 \cdot n - 1}\right)$$



Proof of P345

An terms:

$$\frac{2}{T}\cdot\int_{-\pi}^{0}\frac{A}{2}\cdot\operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^{2}}\right)\cdot\cos\left(\frac{\pi\cdot 2\cdot n\cdot t}{T}\right)dt+\frac{1}{\pi}\cdot\int_{0}^{\pi}\frac{A}{2}\cdot\operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^{2}}\right)\cdot\cos\left(\frac{\pi\cdot 2\cdot n}{T}\cdot t\right)dt$$

**MathCAD can not symbolically solve this equation**

Bn terms:

$$\frac{2}{T}\cdot\int_{-\pi}^{0}\frac{A}{2}\cdot\operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^{2}}\right)\cdot\sin\left(\frac{\pi\cdot 2\cdot n\cdot t}{T}\right)dt+\frac{1}{\pi}\cdot\int_{0}^{\pi}\frac{A}{2}\cdot\operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^{2}}\right)\cdot\sin\left(\frac{\pi\cdot 2\cdot n}{T}\cdot t\right)dt$$

**MathCAD can not symbolically solve this equation**

**Therefore, we must verify the first three terms of the standard form and their plots as shown below;**

$$A\cdot\sum_{n=1}^{3}\frac{\alpha^{2\cdot n-1}}{2\cdot n-1}\cdot\sin[(2\cdot n-1)\cdot t]$$

$$A\cdot\alpha\cdot\sin(t)+\frac{1}{3}\cdot A\cdot\alpha^{3}\cdot\sin(3\cdot t)+\frac{1}{5}\cdot A\cdot\alpha^{5}\cdot\sin(5\cdot t)$$

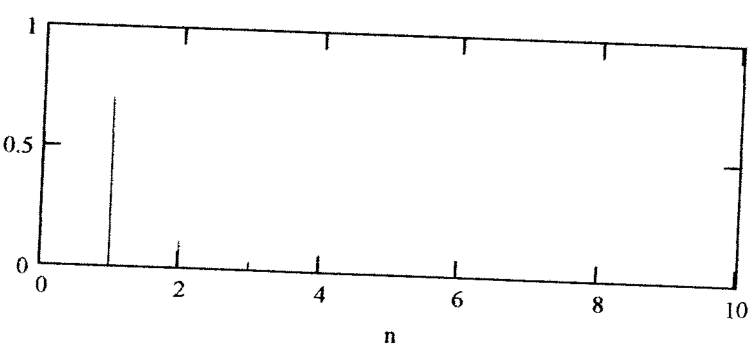
$\alpha := 0.7$

$A := 1$

$m := -1, 0 .. 11$ **$\alpha$ = 0.7 in this example**

$n := 1, 2 .. 10$

$A\cdot\frac{\alpha^{2\cdot n-1}}{((2\cdot n-1))}\cdot\delta(n,m)$



$\alpha := 0.3$

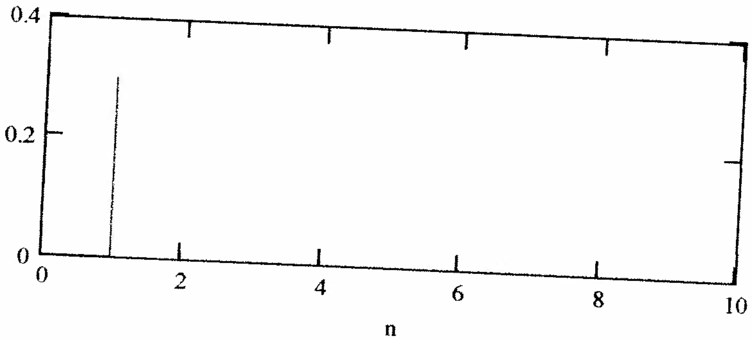
$A := 1$

$m := -1, 0 .. 11$

$n := 1, 2 .. 10$

$\alpha = 0.3$ in this example

$A \cdot \frac{\alpha^{2 \cdot n - 1}}{((2 \cdot n - 1))} \cdot \delta(n, m)$



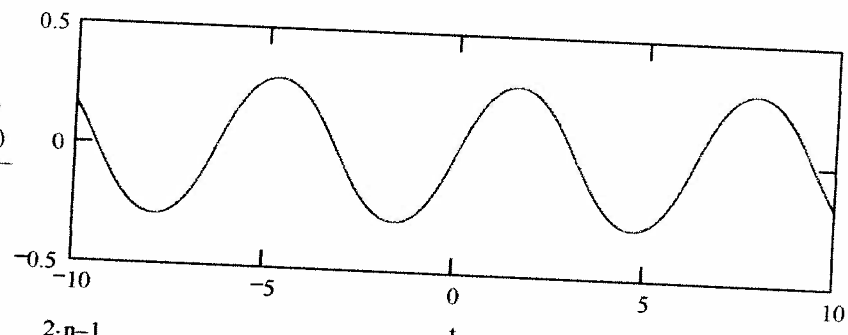
$n := 1 .. 10$

$t := -10, -9.99 .. 10$

$$g(t) := A \cdot \sum_{n=1}^{5} \frac{\alpha^{2 \cdot n - 1}}{2 \cdot n - 1} \cdot \sin[(2 \cdot n - 1) \cdot t]$$

n = 5

g(t)



$$f(t) := A \cdot \sum_{n=1}^{10} \frac{\alpha^{2 \cdot n - 1}}{2 \cdot n - 1} \cdot \sin[(2 \cdot n - 1) \cdot t]$$

f(t)

n = 10