Ken Kaiser

Critically Damped Exponential Wave

$n := 1 .. 10$ $\qquad T := 2 \quad A := 2 \quad t := 1 \quad \alpha := 7 \cdot T$

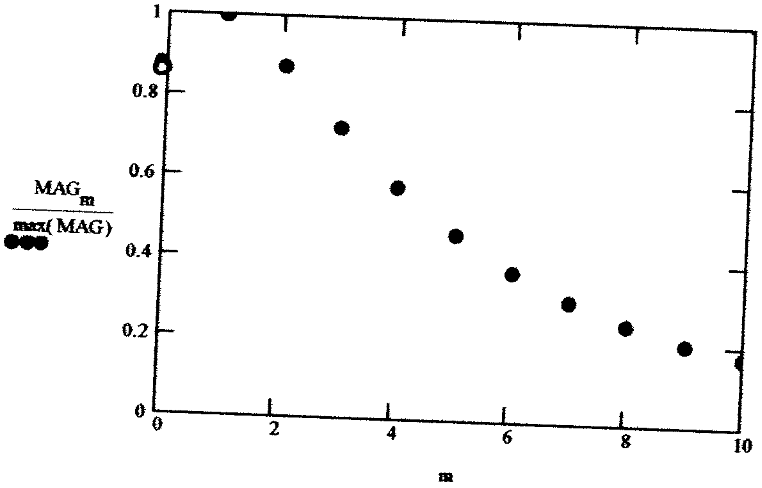
$m := \cancel{1} .. 10$

$\bigcirc\!\!\!\!0$

$$M_n := \frac{2 \cdot A \cdot e^1}{(7 \cdot T) \cdot T} \cdot \frac{1}{1 + \left[\frac{2 \cdot \pi \cdot n}{(7 \cdot T) \cdot T} \right]^2}$$

$$M_0 := \frac{A \cdot e^1}{(7 \cdot T) \cdot T}$$

$$MAG_m := \left| M_m \right|$$



Ken Kaiser

Critically Damped Exponential Wave

$n := 1 .. 10$ $T := 2$ $A := 2$ $t := 1$ $\alpha := 14 \cdot T$

$m := 1 .. 10$

$m = 0 \ldots 10$

$T >> \frac{1}{\alpha}$

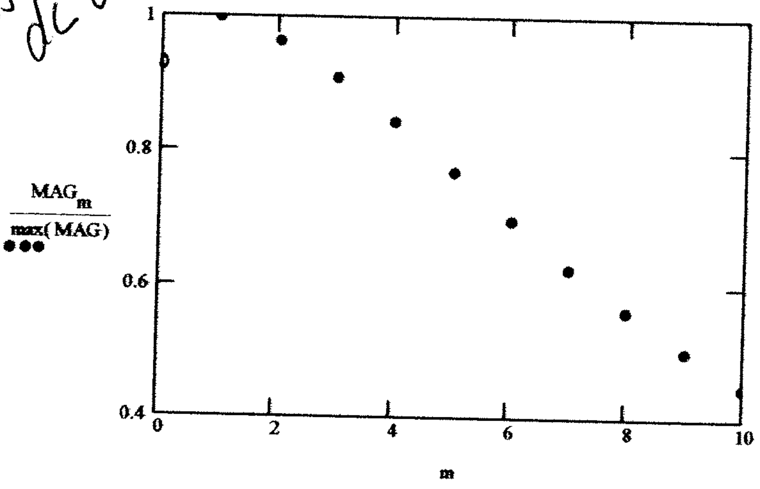
$2 >> \frac{1}{14(2)}$ ✓

$$M_n := \frac{2 \cdot A \cdot e^1}{(14 \cdot T) \cdot T} \cdot \frac{1}{1 + \left[\frac{2 \cdot \pi \cdot n}{(14 \cdot T) \cdot T}\right]^2}$$ ✓

$$M_0 := \frac{A \cdot e^1}{(14 \cdot T) \cdot T}$$ ✓ ✓

$MAG_m := |M_m|$

Missin dc valu



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I used these values to varify DC

Assume These Values:

$A := 8 \quad t := 1 \quad T := 1 \quad \alpha := 7 \cdot T$

DC

$$F := \frac{A \cdot e^{1}}{7 \cdot T \cdot T}$$

$|F| = 3.107$ ✓

f_0

$$F := \frac{2 \cdot A \cdot e^{1}}{7 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{2 \cdot \pi}{7 \cdot T \cdot T}\right)^{2}}$$

$|F| = 3.441$ ✓

$2f_0$

$$F := \frac{2 \cdot A \cdot e^{1}}{7 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{4 \cdot \pi}{7 \cdot T \cdot T}\right)^{2}}$$

$|F| = 1.471$ ✓

$3f_0$

$$F := \frac{2 \cdot A \cdot e^{1}}{7 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{6 \cdot \pi}{7 \cdot T \cdot T}\right)^{2}}$$

$|F| = 0.753$ ✓

$4f_0$

$$F := \frac{2 \cdot A \cdot e^{1}}{7 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{8 \cdot \pi}{7 \cdot T \cdot T}\right)^{2}}$$

$|F| = 0.447$ ✓

$5f_0$ $$F := \frac{2\cdot A\cdot e^{1}}{7\cdot T\cdot T}\cdot\frac{1}{1+\left(\frac{10\cdot\pi}{7\cdot T\cdot T}\right)^{2}}$$

$|F| = 0.294$ ✓

$6f_0$ $$F := \frac{2\cdot A\cdot e^{1}}{7\cdot T\cdot T}\cdot\frac{1}{1+\left(\frac{12\cdot\pi}{7\cdot T\cdot T}\right)^{2}}$$

$|F| = 0.207$ ✓

$7f_0$ $$F := \frac{2\cdot A\cdot e^{1}}{7\cdot T\cdot T}\cdot\frac{1}{1+\left(\frac{14\cdot\pi}{7\cdot T\cdot T}\right)^{2}}$$

$|F| = 0.153$ ✓

$8f_0$ $$F := \frac{2\cdot A\cdot e^{1}}{7\cdot T\cdot T}\cdot\frac{1}{1+\left(\frac{16\cdot\pi}{7\cdot T\cdot T}\right)^{2}}$$

$|F| = 0.118$ ✓

$9f_0$ $$F := \frac{2\cdot A\cdot e^{1}}{7\cdot T\cdot T}\cdot\frac{1}{1+\left(\frac{18\cdot\pi}{7\cdot T\cdot T}\right)^{2}}$$

$|F| = 0.094$ ✓

$10f_0$ $$F := \frac{2\cdot A\cdot e^{1}}{7\cdot T\cdot T}\cdot\frac{1}{1+\left(\frac{20\cdot\pi}{7\cdot T\cdot T}\right)^{2}}$$

$|F| = 0.076$ ✓

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$A := 8 \qquad t := 1 \qquad T := 1 \qquad \alpha := 14 \cdot T$

$$F := \frac{A \cdot e^{1}}{14 \cdot T \cdot T}$$

$$|F| = 1.553$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{2 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 2.586$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{4 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 1.72$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{6 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 1.104$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{8 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.736$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{10 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.515$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{12 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.377$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{14 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.286$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{16 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.224$$

$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{18 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.179$$

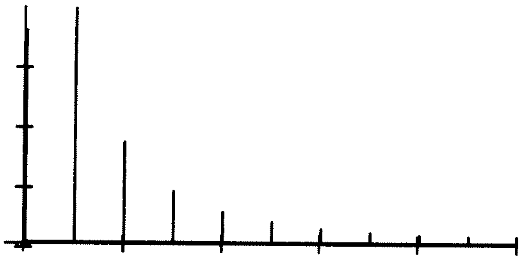
$$F := \frac{2 \cdot A \cdot e^{1}}{14 \cdot T \cdot T} \cdot \frac{1}{1 + \left(\frac{20 \cdot \pi}{14 \cdot T \cdot T}\right)^{2}}$$

$$|F| = 0.147$$

$m := -1, 0 .. 11$ $\quad T := 1$ $\quad \tau := \frac{T}{2}$ $\quad A := 1$ $\quad \alpha := 7 \cdot T$

$n := 0, 1 .. 10$

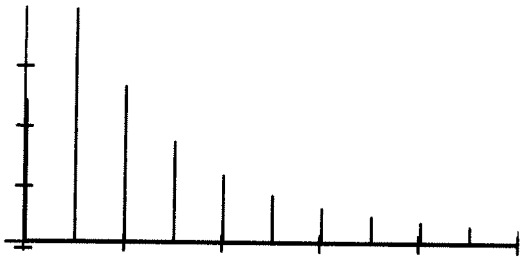
$$g(n) := \text{if}\left[n < 1, A \cdot \frac{e}{\alpha \cdot T}, \left| 2 \cdot \left[A \cdot \frac{e}{\alpha \cdot T} \cdot \frac{1}{1 + \left(2 \cdot \pi \cdot \frac{n}{\alpha \cdot T}\right)^2} \right] \right| \right]$$



$m := -1, 0 .. 11$ $\quad T := 1 \quad \tau := \frac{T}{2} \quad A := 1 \quad \alpha := 14 \cdot T$

$n := 0, 1 .. 10$

$$g(n) := \mathrm{if}\left[n < 1, A\cdot\frac{e}{\alpha\cdot T}, \left|2\cdot\left[A\cdot\frac{e}{\alpha\cdot T}\cdot\frac{1}{1 + \left(2\cdot\pi\cdot\frac{n}{\alpha\cdot T}\right)^{2}}\right]\right|\right]$$



$T \to 0$

$$\frac{1}{T}\cdot\left[\exp(\alpha\cdot T)\cdot(\alpha\cdot T+1)\cdot A\cdot\frac{\exp(1)}{\alpha}+A\cdot\frac{\exp(1)}{\alpha}\right]$$

$$A\cdot\frac{\exp(\alpha\cdot T+1)\cdot\alpha\cdot T+\exp(\alpha\cdot T-1)\quad\exp(1)}{(\alpha\cdot T)}$$

KAISER!

$$\frac{1}{T}\cdot\int_{0}^{T}\left(A\cdot e\cdot t\cdot\alpha\cdot e^{-\alpha\cdot t}\right)\cdot e^{\sqrt{-1}\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t}\,dt$$

$$\left|\frac{T\cdot A\cdot\alpha\cdot\exp(1-\alpha\cdot T)\cdot\exp(2\cdot i\cdot\pi\cdot n)-T^{2}\cdot A\cdot\alpha^{2}\cdot\exp(1-\alpha\cdot T)\cdot\exp(-2\cdot i\cdot\pi\cdot n)-2i\cdot T\cdot A\cdot\alpha\cdot\exp(1-\alpha\cdot T)\cdot\exp(2\cdot i\cdot\pi\cdot n)\cdot\pi\cdot n+T\cdot A\cdot\alpha\cdot\exp(1)}{\left(\alpha^{2}\cdot T^{2}+4i\cdot\pi\cdot n\cdot\alpha\cdot T\quad 4\cdot\pi^{2}\cdot n^{2}\right)}\right|$$

$$T\cdot A\cdot\alpha\cdot\sqrt{\frac{-2\cdot\exp(2-\alpha\cdot T)\cdot\overset{1}{\cancel{\cos(2\cdot\pi\cdot n)}}-2\cdot T\cdot\alpha\cdot\exp(2-\alpha\cdot T)\cdot\overset{1}{\cancel{\cos(2\cdot\pi\cdot n)}}+4\cdot\exp(2-2\cdot\alpha\cdot T)\cdot\pi^{2}\cdot n^{2}-4\cdot\exp(2-\alpha\cdot T)\cdot\sin(2\cdot\pi\cdot n)\cdot\pi\cdot n+\exp(2)+\exp(2-2\cdot\alpha\cdot T)+2\cdot T\cdot\alpha\cdot\exp(2-}{\left(\alpha^{2}\cdot T^{2}+4\cdot\pi^{2}\cdot n^{2}\right)}}$$

$\exp(\cdot\alpha T)+T^2\alpha^2 e^{2-2\alpha T}$

$T >> \frac{1}{\alpha}$

$T\alpha >> 1 \quad e^{-T\alpha}$ small

$e^{2\alpha T} \simeq e^2 e^{-\alpha t}$ small

$C_N = 2|F_N|$ ✓

$$\approx TA\alpha\sqrt{\frac{e^2}{\alpha^2T^2+4\pi^2N^2}} = \frac{TA\alpha e}{\alpha^2T^2+4\pi^2N^2}$$

$$= \frac{eA\alpha T}{\alpha^2T^2\left[1+\frac{4\pi^2N^2}{\alpha^2T^2}\right]} = \frac{eA}{\alpha T\left[1+\frac{4\pi^2N^2}{\alpha^2T^2}\right]} = \frac{eA}{\alpha T\left[1+\left(\frac{2\pi N}{\alpha T}\right)^2\right]}$$ ✓

Critically Damped Exponential Wave (pg. 414)

Solving for a_n

$$\frac{2}{T}\cdot\int_{0}^{T} A\cdot e^{1}\cdot t\cdot\alpha\cdot e^{\alpha\cdot t}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\ T^{2}\cdot\exp\ \alpha\cdot T\cdot\ 4\cdot T\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot\pi^{2}\cdot n^{2}\quad 8\cdot\pi^{3}\cdot n^{3}\cdot\sin\ 2\cdot\pi\cdot n\quad T^{3}\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha^{3}\quad 4\cdot\pi\cdot n\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot T\quad 4\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\pi^{2}\cdot n^{2}\quad T^{2}\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\quad 2\cdot\pi\cdot n\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\alpha$$

Solving for b_n

$$\frac{2}{T}\cdot\int_{0}^{T} A\cdot e^{1}\cdot t\cdot\alpha\cdot e^{\alpha\cdot t}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\ T^{2}\cdot\exp\ \alpha\cdot T\cdot\ 8\cdot\pi^{3}\cdot n^{3}\cdot\cos\ 2\cdot\pi\cdot n\ + T^{2}\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\ +\ 2\cdot\pi\cdot n\cdot\cos\ 2\cdot\pi\cdot n\ \cdot T^{2}\cdot\alpha^{2}\quad 4\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\pi^{2}\cdot n^{2} + 4\cdot T\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot\pi^{2}\cdot n^{2} - 4\cdot\pi\cdot n\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot T\quad T^{3}\cdot\sin\ 2\cdot\pi\cdot n$$

Solving for c_n

$$\frac{2}{T}\cdot\ T^{2}\cdot\exp\ \alpha\cdot T\cdot\ 4\cdot T\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot\pi^{2}\cdot n^{2}\quad 8\cdot\pi^{3}\cdot n^{3}\cdot\sin\ 2\cdot\pi\cdot n\quad T^{3}\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha^{3}\quad 4\cdot\pi\cdot n\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot T\quad 4\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\pi^{2}\cdot n^{2}\quad T^{2}\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\quad 2\cdot\pi\cdot n\cdot\sin\ 2\cdot\pi\cdot n$$

$$\frac{4}{T^{2}}\cdot\ T^{2}\cdot\exp\ \alpha\cdot T\cdot\ 4\cdot T\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot\pi^{2}\cdot n^{2}\quad 8\cdot\pi^{3}\cdot n^{3}\cdot\sin\ 2\cdot\pi\cdot n\quad T^{3}\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha^{3}\quad 4\cdot\pi\cdot n\cdot\sin\ 2\cdot\pi\cdot n\ \cdot\alpha\cdot T\quad 4\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\pi^{2}\cdot n^{2}\quad T^{2}\cdot\cos\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\quad 2\cdot\pi\cdot n\cdot\sin\ 2\cdot\pi\cdot n$$

Graphs

$A \quad 1$

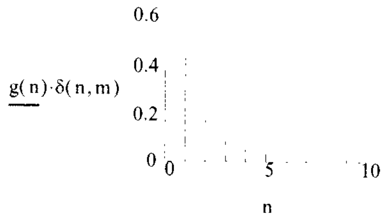
$m \quad 1, 0 .. 11$

$n \quad 0, 1 .. 10$

$\alpha \quad 7$

$T \quad 1$

$$g(n) \quad \text{if}\left(n<1, \frac{A\cdot e^{1}}{\alpha\cdot T}, \frac{2\cdot A\cdot e^{1}}{\alpha\cdot T}\cdot\frac{1}{1+\left(\frac{2\cdot\pi\cdot n}{\alpha\cdot T}\right)^{2}}\right)$$



$A \quad 1$

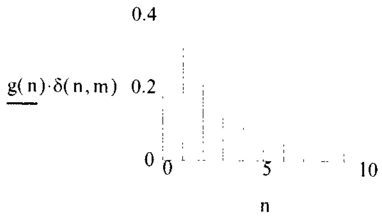
$m \quad -1, 0 .. 11$

$n \quad 0, 1 .. 10$

$\alpha \quad 14$

$T \quad 1$

$$g(n) \quad \text{if}\left(n<1, \frac{A\cdot e^{1}}{\alpha\cdot T}, \frac{2\cdot A\cdot e^{1}}{\alpha\cdot T}\cdot\frac{1}{1+\left(\frac{2\cdot\pi\cdot n}{\alpha\cdot T}\right)^{2}}\right)$$



$$\cdot n\cdot \sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\cdot T^{2}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ 4\cdot\pi^{2}\cdot n^{2}}^{2}\ T^{2}\cdot\ \alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}$$

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$$\cdot T^{3}\cdot \sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{3}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}\ 4\cdot T^{3}\cdot\pi\cdot n\cdot\alpha^{2}\cdot A\cdot \frac{\exp(1)}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}$$

$$2\cdot\pi\cdot n\cdot \sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\cdot T^{2}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}\ T^{2}\cdot\ \alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}\ ^{2}\ \frac{2}{T}\cdot\ T^{2}\cdot \exp\ \ \alpha\cdot T\ \cdot\ 8\cdot\pi^{3}\cdot n^{3}\cdot \cos\ 2\cdot\pi\cdot n\ \ T^{2}\cdot \sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\ \ 2\cdot\pi\cdot n\cdot \cos\ 2$$

$$\cdot\pi\cdot n\cdot \sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\cdot T^{2}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}\ T^{2}\cdot\ \alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}\ \cdot A\cdot \exp(1)\cdot \frac{\alpha}{\alpha^{2}\cdot T^{2}\ \ 4\cdot\pi^{2}\cdot n^{2}}^{2}\ ^{2}\ \frac{4}{T^{2}}\cdot\ T^{2}\cdot \exp\ \ \alpha\cdot T\ \cdot\ 8\cdot\pi^{3}\cdot n^{3}\cdot \cos\ 2\cdot\pi\cdot n\ \ T^{2}\cdot \sin\ 2\cdot\pi\cdot n\ \cdot\alpha^{2}\ \ 2\cdot\pi\cdot n\cdot \cos\ 2\cdot\pi\cdot n$$

$$\cdot\pi\cdot n\;\;\cdot T^2\cdot\alpha^2\quad 4\cdot\sin\;2\cdot\pi\cdot n\;\;\cdot\pi^2\cdot n^2\;\; 4\cdot T\cdot\sin\;2\cdot\pi\cdot n\;\;\cdot\alpha\cdot\pi^2\cdot n^2\;\; 4\cdot\pi\cdot n\cdot\cos\;2\cdot\pi\cdot n\;\;\cdot\alpha\cdot T\;\; T^3\cdot\sin\;2\cdot\pi\cdot n\;\;\cdot\alpha^3\;\;\cdot A\cdot\exp(1)\cdot\frac{\alpha}{\alpha^2\cdot T^2\;\;4\cdot\pi^2\cdot n^2}{}^2\;\; 4\cdot T^3\cdot\pi\cdot n\cdot\alpha^2\cdot A\cdot\frac{\exp(1)}{\alpha^2\cdot T^2\;\;4\cdot\pi^2\cdot n^2{}^2}{}^2$$

$$\cdot T^2\cdot\alpha^2\quad 4\cdot\sin\;2\cdot\pi\cdot n\;\;\cdot\pi^2\cdot n^2\;\; 4\cdot T\cdot\sin\;2\cdot\pi\cdot n\;\;\cdot\alpha\cdot\pi^2\cdot n^2\;\; 4\cdot\pi\cdot n\cdot\cos\;2\cdot\pi\cdot n\;\;\cdot\alpha\cdot T\;\; T^3\cdot\sin\;2\cdot\pi\cdot n\;\;\cdot\alpha^3\;\;\cdot A\cdot\exp(1)\cdot\frac{\alpha}{\alpha^2\cdot T^2\;\;4\cdot\pi^2\cdot n^2}{}^2\;\; 4\cdot T^3\cdot\pi\cdot n\cdot\alpha^2\cdot A\cdot\frac{\exp(1)}{\alpha^2\cdot T^2\;\;4\cdot\pi^2\cdot n^2{}^2}{}^2$$

Sawtooth Modulated Wave # 2

F avg = 0

```
> (1/3)*int(2*((2*t/3)-1)*cos(2*Pi*t/3),  t=0..3);
                                   0
> evalf(.44*2);
                                  .88
> evalf(sqrt((1/3)*int((2*((2*t/3)-1)*cos(2*Pi*t/3))^2,  t=0..3)));
                              .8763491602
```

Both F avg and F rms had simalar from both methods

Critically Damped Exponential Wave

```
> evalf((2*exp(1))/(21*3));
                             .08629466122
> evalf((1/3)*(int((41*exp(1)*exp(-21*t)*t),  t=0..3)));
                             .08424002642
> evalf(exp(1)*2/(2*sqrt(3)));
                              1.569400746
> evalf(sqrt((1/3)*(int((2*exp(1)*exp(-t)*t)^2,  t=0..3))));
                              1.519996176
```

Both F avg and F rms had simalar from both methods, I had to reset a to 21 as stated on the sheet

Double-sided Exponential Wave

```
> evalf(4*(1-exp(-1/2)));
                              1.573877361
> evalf((1/3)*int(2*exp(-abs(t)/3),  t=(-3/2)..(3/2)));
                              1.573877361
> evalf(2*sqrt(1-exp(-1)));
                              1.590120195
> evalf(sqrt((1/3)*int((2*exp(-abs(t)/3))^2,  t=(-3/2)..(3/2))));
                              1.590120195
```

Both F avg and F rms had simalar from both methods

Even symmetrical Trapexiodal Wave with DC Offset

```
> evalf(2*((3/4)+.5)/3);
                             .8333333333
> (1/3)*(int((2),  t=0..(3/8))+int((4*((3/8)+0.5-t)),
  t=(3/8)..((3/8)+0.5))+int((4*(t-2.5+(3/8))),
  t=2.125..2.625)+int((2),  t=2.625..3));
                             .8333333334
> evalf(2*sqrt((9/4+1)/9));
```

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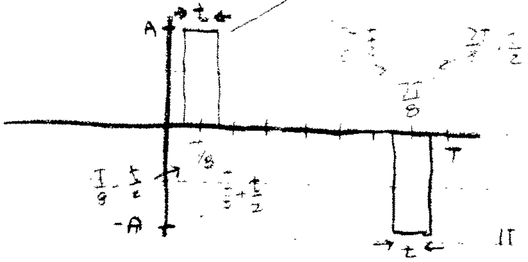
TOPIC 16

1)

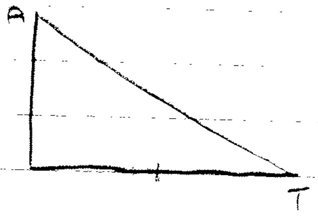


IT is A From $T/8 - \frac{t}{2}$ To $T/8 + \frac{t}{2}$

IT is $-A$ From $\frac{2T}{8} - \frac{t}{2}$ to $\frac{2T}{8} + \frac{t}{2}$

It is 0 everywhere else over T

2)



$\Delta A = 0 - A$ $\quad m = \frac{-A}{T}$ $\quad$ using $y = mx + b$

$\Delta T = T - 0$

$$y = \frac{-A}{T} t + A$$

$$= \frac{A}{T}(-t + T) = \frac{A}{T}(T - t)$$

3) <u>critical points</u>

1) $t = 0$: $Ae^{1} 0 \alpha e^{-\alpha 0} = 0$

2) $t = T$: $Ae^{1} T \alpha e^{-\alpha T} = Ae^{1} T \alpha 0 = 0$

$f'(t) = Ae^{1} \alpha e^{-\alpha t} + (-\alpha A e^{1} t \alpha e^{-\alpha t})$

$f'(1/\alpha) = Ae^{1} \alpha e^{-\alpha/\alpha} + (-\alpha A e^{1} \frac{1}{\alpha} \alpha e^{-\alpha/\alpha})$

$= A\alpha - A\alpha$

$= 0$

This shows $t = 0$, $t = T$, $t = 1/\alpha$ are all critical points

over

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$$\frac{1}{T}\cdot\int_{0}^{T} A\cdot e^{1}\cdot t\cdot\alpha\cdot e^{-\alpha\cdot t}\,dt$$

$$T \gg \frac{1}{\alpha}$$

$$A\cdot\frac{\exp(-\alpha\cdot T+1)\cdot\alpha\cdot T+\exp(-\alpha\cdot T+1)-\exp(1)}{\alpha\cdot T} \qquad -A\cdot\frac{(0\cdot 0-e^{1})}{(\alpha\cdot T)}$$

$$=\boxed{\frac{Ae^{1}}{\alpha T}}$$

$$\sqrt{\frac{1}{T}\cdot\int_{0}^{T}\left(A\cdot e^{1}\cdot t\cdot\alpha\cdot e^{-\alpha\cdot t}\right)^{2}dt}$$

$$\frac{1}{2}\cdot i\cdot\frac{\left(2\cdot T\cdot\exp(-2\cdot\alpha\cdot T+2)\cdot A\cdot\alpha+\exp(-2\cdot\alpha\cdot T+2)\cdot A+2\cdot T^{2}\cdot\exp(-2\cdot\alpha\cdot T+2)\cdot A\cdot\alpha^{2}-\exp(2)\cdot A\right)}{\sqrt{T}\cdot\sqrt{2\cdot\exp(-2\cdot\alpha\cdot T+2)\cdot\alpha\cdot T+\exp(-2\cdot\alpha\cdot T+2)+2\cdot\exp(-2\cdot\alpha\cdot T+2)\cdot\alpha^{2}\cdot T^{2}-\exp(2)}\cdot\sqrt{\alpha}}$$

$$=\frac{1}{2}\frac{(0+0+0+e^{2}\cdot A)}{\sqrt{T}\cdot\sqrt{0+0+0-e^{2}}\cdot\sqrt{\alpha}}$$

$$=\frac{1}{2}\frac{-e^{2}A}{\sqrt{T}\cdot -e^{\frac{2}{2}}\cdot\sqrt{\alpha}} = \boxed{\frac{1}{2}\frac{Ae}{\sqrt{T\alpha}}}$$

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[redacted]
1pm ECE 210

Double Sided Exponential Wave

$$\sqrt{\left(\frac{1}{T}\right)\cdot\left[\int_{-\frac{T}{2}}^{0}\left(A\cdot e^{\alpha\cdot\frac{t}{T}}\right)^2 dt+\int_{0}^{\frac{T}{2}}\left(A\cdot e^{-\alpha\cdot\frac{t}{T}}\right)^2 dt\right]}$$

✓

$$\frac{1}{\sqrt{T}}\cdot\sqrt{\frac{1}{\alpha}\cdot T\cdot A^2-\frac{\exp\left(-\frac{1}{2}\cdot\alpha\right)^2}{\alpha}\cdot T\cdot A^2}\quad = \text{Frms} = A\sqrt{\frac{1-\left(e^{-\alpha/2}\right)^2}{\alpha}} = A\sqrt{\frac{1-e^{-\alpha}}{\alpha}}$$

$$\left(\frac{1}{T}\right)\cdot\left[\int_{-\frac{T}{2}}^{0}A\cdot e^{\alpha\cdot\frac{t}{T}}\,dt+\int_{0}^{\frac{T}{2}}A\cdot e^{-\alpha\cdot\frac{t}{T}}\,dt\right]$$

$$\frac{1}{T}\cdot\left(\frac{2}{\alpha}\cdot T\cdot A-2\cdot\frac{\exp\left(-\frac{1}{2}\cdot\alpha\right)}{\alpha}\cdot T\cdot A\right)\quad = \text{Favg}$$

$$-2\cdot A\cdot\frac{\left(-1+\exp\left(-\frac{1}{2}\cdot\alpha\right)\right)}{\alpha}\quad = \text{Favg} = 2A\left(\frac{1-e^{(-\alpha/2)}}{\alpha}\right)$$

✓

Ken Kaiser

$$f_{avg} = \frac{1}{T} \cdot \int_{0}^{\frac{T}{2}} A \cdot e^{\frac{\alpha}{T} \cdot t}\, dt - \frac{1}{T} \cdot \int_{\frac{T}{2}}^{0} A \cdot e^{-\frac{\alpha}{T} \cdot t}\, dt$$

$$f_{avg} = \frac{2}{T} \cdot \frac{1}{\alpha} \cdot T \cdot \exp\left(\frac{1}{2} \cdot \alpha\right) \cdot A - \frac{1}{\alpha} \cdot T \cdot A$$

$$f_{avg} = 2 \cdot A \cdot \frac{\exp\left(\frac{1}{2} \cdot \alpha\right) - 1}{\alpha}$$

$$\boxed{f_{avg} = 2 \cdot A \cdot \frac{1 - \exp\left(\frac{1}{2} \cdot \alpha\right)}{\alpha}}$$

$$f_{rms} = \sqrt{\frac{1}{T} \cdot \left[\int_{0}^{\frac{T}{2}} \left(A \cdot e^{\frac{\alpha}{T} \cdot t}\right)^{2} dt - \int_{\frac{T}{2}}^{0} \left(A \cdot e^{\frac{\alpha}{T} \cdot t}\right)^{2} dt\right]}$$

$$f_{rms} = \sqrt{\frac{1}{T} \cdot \frac{1}{\alpha} \cdot T \cdot \exp\left(\frac{1}{2} \cdot \alpha\right)^{2} \cdot A^{2} - \frac{1}{\alpha} \cdot T \cdot A^{2}}$$

$$i \cdot \frac{\frac{\exp(\alpha) \cdot A - A}{\alpha \cdot \left(\exp(\alpha) - 1\right)}}{A \cdot \frac{1 - e^{-\alpha}}{\alpha}}$$

My answer as simplified by Mathcad

divided by

your answer on the handout

$$i \cdot \frac{\exp(-\alpha) \cdot A \quad A}{\left(\exp(\alpha) - 1\right) \cdot A \cdot \left(1 - \exp(\alpha)\right)}$$

Simplify this answer once more ....

1 Because the division of my answer by your answer equals one our answers must be the same therefore

$$\boxed{f_{rms} = A \cdot \sqrt{\frac{1 \quad e^{\alpha}}{\alpha}}}$$

Sawtooth Modulated Wave # 2

F avg = 0

```
> (1/3)*int(2*((2*t/3)-1)*cos(2*Pi*t/3),  t=0..3);
                                  0
> evalf(.44*2);
                                 .88
> evalf(sqrt((1/3)*int((2*((2*t/3)-1)*cos(2*Pi*t/3))^2,  t=0..3)));
                             .8763491602
```

Both F avg and F rms had simalar from both methods

Critically Damped Exponential Wave

```
> evalf((2*exp(1))/(21*3));
                             .08629466122
> evalf((1/3)*(int((41*exp(1)*exp(-21*t)*t),  t=0..3)));
                             .08424002642
> evalf(exp(1)*2/(2*sqrt(3)));
                             1.569400746
> evalf(sqrt((1/3)*(int((2*exp(1)*exp(-t)*t)^2,  t=0..3))));
                             1.519996176
```

Both F avg and F rms had simalar from both methods, I had to reset a to 21 as stated on the sheet

Double-sided Exponential Wave

```
> evalf(4*(1-exp(-1/2)));
                             1.573877361
> evalf((1/3)*int(2*exp(-abs(t)/3),  t=(-3/2)..(3/2)));
                             1.573877361
> evalf(2*sqrt(1-exp(-1)));
                             1.590120195
> evalf(sqrt((1/3)*int((2*exp(-abs(t)/3))^2,  t=(-3/2)..(3/2))));
                             1.590120195
```

Both F avg and F rms had simalar from both methods

Even symmetrical Trapexiodal Wave with DC Offset

```
> evalf(2*((3/4)+.5)/3);
                             .8333333333
> (1/3)*(int((2),  t=0..(3/8))+int((4*((3/8)+0.5-t)),
  t=(3/8)..((3/8)+0.5))+int((4*(t-2.5+(3/8))),
  t=2.125..2.625)+int((2),  t=2.625..3));
                             .8333333334
> evalf(2*sqrt((9/4+1)/9));
```

Ken Kaiser

Double-Side Exponential Wave

[redacted]

$A := 2$

$B := 5$

$C := 5$

$T := 3$

$a := 1$

$k := 0.4$

$t_o := 0.5$

$$f_{avg} := \frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{a}{T} \cdot |t|} \, dt \qquad f_{avg} = 1.574$$

$$f_{avg2} := 2 \cdot A \cdot \left(\frac{1 - e^{-\frac{a}{2}}}{a} \right) \qquad f_{avg2} = 1.574$$

$$f_{rms} := \sqrt{\frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A \cdot e^{-\frac{a}{T} \cdot |t|} \right)^2 dt} \qquad f_{rms} = 1.59$$

$$f_{rms2} := A \cdot \sqrt{\frac{1 - e^{-a}}{a}} \qquad f_{rms2} = 1.59$$

Double-Sided Exponential: pg. 415

a(n) coefficient:

$$\frac{4}{T}\cdot\int_{0}^{\frac{T}{2}} A\cdot e^{\frac{-\alpha}{T}\cdot t}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$4\cdot A\cdot\frac{\left(-\exp\left(\frac{-1}{2}\cdot\alpha\right)\cdot\alpha\cdot\cos(\pi\cdot n)+2\cdot\exp\left(\frac{-1}{2}\cdot\alpha\right)\cdot\pi\cdot n\cdot\sin(\pi\cdot n)+\alpha\right)}{\left(\alpha^{2}+4\cdot\pi^{2}\cdot n^{2}\right)}$$

$$4\cdot A\cdot\left[\frac{-e^{\frac{-\alpha}{2}}\cdot\alpha\cdot(-1)^{n}+\alpha}{\alpha^{2}+4\cdot\pi^{2}\cdot n^{2}}\right] \qquad = \qquad 4\cdot A\cdot\alpha\cdot\frac{\frac{e^{\frac{-\alpha}{2}}}{(-1)}\cdot(-1)^{n}+1}{\alpha^{2}+4\cdot\pi^{2}\cdot n^{2}}$$

$$= \qquad 4\cdot A\cdot\alpha\cdot\left[\frac{1+(-1)^{n-1}\cdot e^{\frac{-\alpha}{2}}}{\alpha^{2}+4\cdot\pi^{2}\cdot n^{2}}\right]$$

PAGE 415

a(0) coefficient:

reason of Symmetry ✓

$$\frac{4}{T}\cdot\int_{0}^{\frac{T}{2}} A\cdot e^{\frac{-\alpha}{T}\cdot t}\cdot\cos\left(\frac{2\cdot\pi\cdot 0}{T}\cdot t\right)dt$$

$$-4\cdot A\cdot\frac{\left(\exp\left(\frac{-1}{2}\cdot\alpha\right)-1\right)}{\alpha} \quad = \quad 4\cdot A\cdot\left(\frac{1-e^{\frac{-\alpha}{2}}}{\alpha}\right)$$

b(n) coeffcient is equal to zero per page 409 table in chapter 14:

Fourier Series:

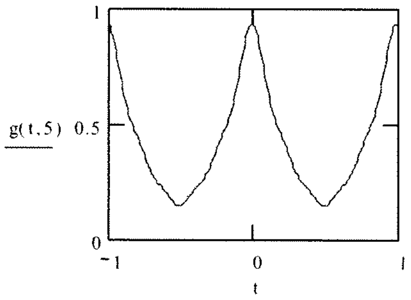
$T := 1$

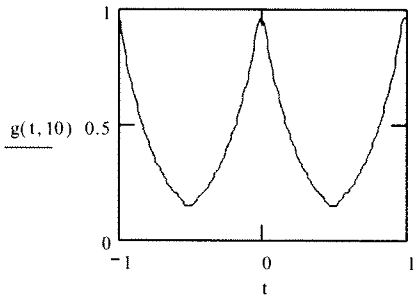
$A := 1$

$t := -T, -T+\frac{T}{100} .. T$

$\alpha := 4$

$$g(t,m) := 4\cdot A\cdot\frac{1-e^{\frac{-\alpha}{2}}}{2\cdot\alpha} + \sum_{n=1}^{m} 4\cdot A\cdot\alpha\cdot\left[\frac{1+(-1)^{n-1}\cdot e^{\frac{-\alpha}{2}}}{\alpha^2+4\cdot\pi^2\cdot n^2}\right]\cdot\cos\left(\frac{2\cdot n\cdot\pi}{T}\cdot t\right)$$



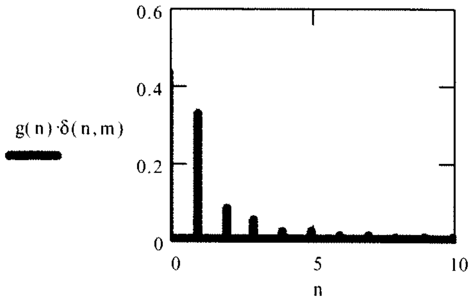


$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$A := 1$

$$g(n) := \mathrm{if}\left[n<1, \left|4\cdot A\cdot\frac{1-e^{\frac{-\alpha}{2}}}{2\cdot\alpha}\right|, 4\cdot A\cdot\alpha\cdot\left[\frac{1+(-1)^{n-1}\cdot e^{\frac{-\alpha}{2}}}{\alpha^{2}+4\cdot\pi^{2}\cdot n^{2}}\right]\right]$$



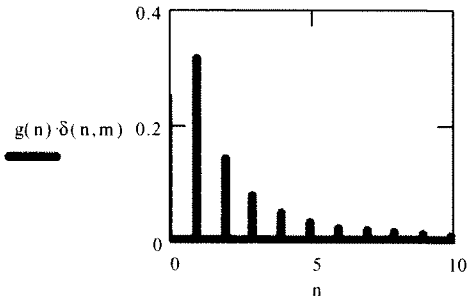
$\alpha := 8$

$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$A := 1$

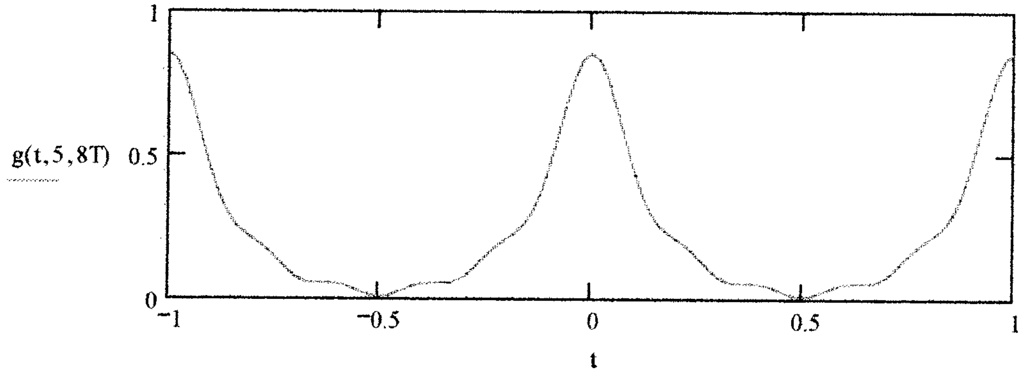
$$g(n) := \mathrm{if}\left[n < 1, \left| 4 \cdot A \cdot \frac{1 - e^{\frac{-\alpha}{2}}}{2 \cdot \alpha} \right|, 4 \cdot A \cdot \alpha \cdot \left[\frac{1 + (-1)^{n-1} \cdot e^{\frac{-\alpha}{2}}}{\alpha^2 + 4 \cdot \pi^2 \cdot n^2} \right]\right]$$

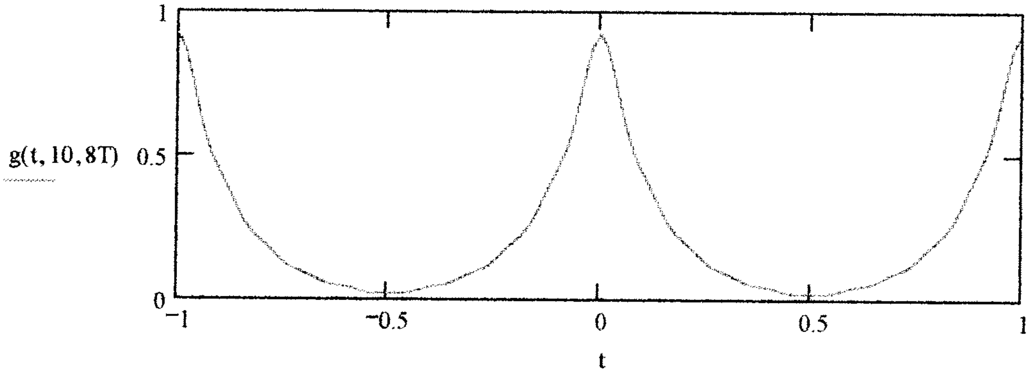


Double-Sided Exponential Wave (p.414)

$A := 1$ $T := 1$ $t := -T, -T + \frac{2T}{100} .. T$

$$g(t,M,\alpha) := 2A\cdot\left(\frac{1 - e^{\frac{-\alpha}{2}}}{\alpha}\right) + 4\alpha\cdot\sum_{n=1}^{M} \frac{\left[1 + (-1)^{(n-1)} e^{\frac{-\alpha}{2}}\right]}{\left(\alpha^2 + 4\pi^2 n^2\right)} \cos\left(\frac{2\pi n}{T} t\right)$$

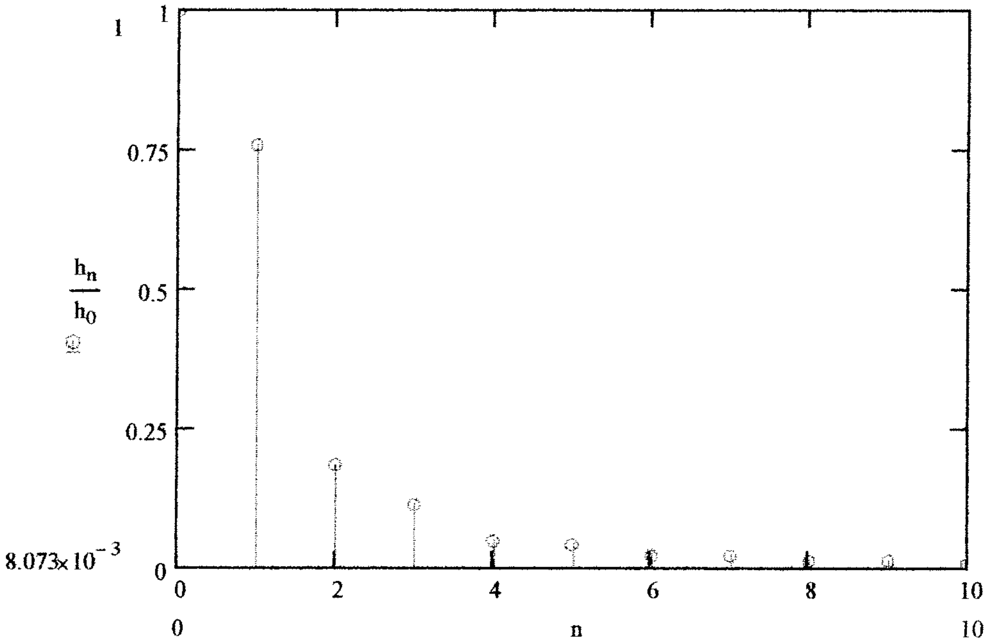




Ken Kaiser

$\alpha := 4T$ $\qquad h_0 := 2\cdot\frac{A\cdot\left(1 - e^{\frac{-\alpha}{2}}\right)}{\alpha}$ $\qquad n := 1, 2 .. 10$ $\qquad h_n := 4\cdot\alpha\cdot A\cdot\left[\frac{1 + (-1)^{n-1}\cdot e^{\frac{-\alpha}{2}}}{\alpha^2 + 4\cdot\pi^2\cdot n^2}\right]$

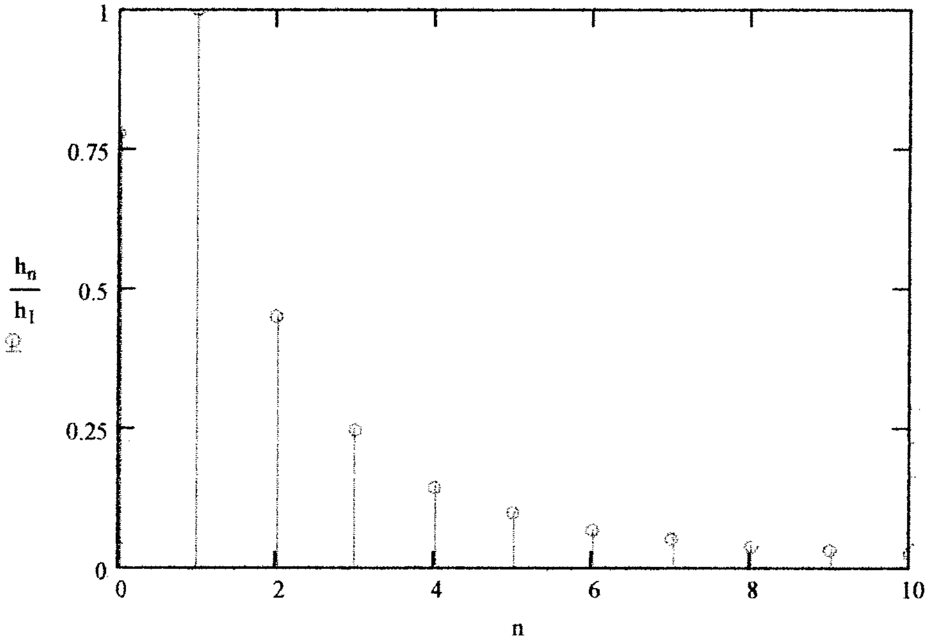
$n := 0, 1 .. 10$



$\alpha := 8T$

$$h_0 := 2\cdot\frac{A\cdot\left(1 - e^{\frac{-\alpha}{2}}\right)}{\alpha} \qquad n := 1,2..10 \qquad h_n := 4\cdot\alpha\cdot A\cdot\left[\frac{1 + (-1)^{n-1}\cdot e^{\frac{-\alpha}{2}}}{\alpha^2 + 4\cdot\pi^2\cdot n^2}\right]$$

$n := 0,1..10$



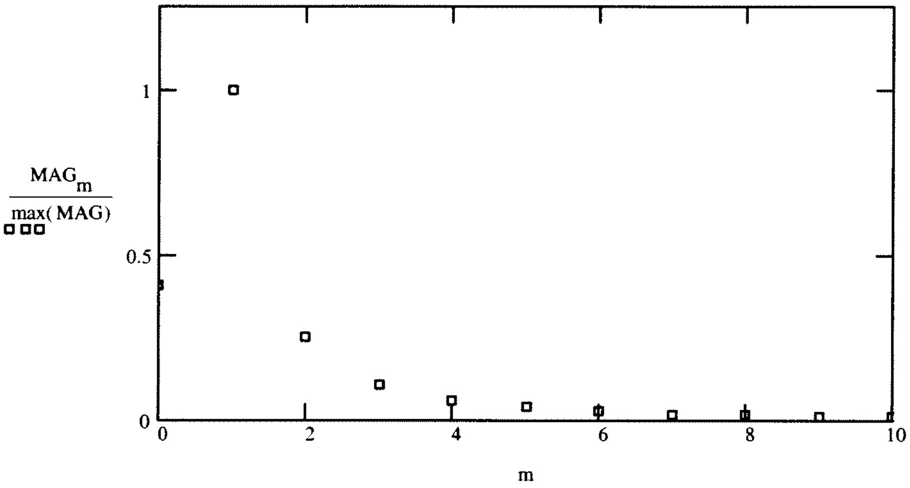
$n := 1 .. 10 \qquad t := 4 \quad T := 2$

$m := 0 .. 10 \quad A := 1 \quad \alpha := 4 \cdot T$

$$M_n := 4 \cdot \alpha \cdot A \cdot \left[\frac{1 + (-1)^{n-1} \cdot e^{-\frac{\alpha}{2}}}{\alpha^2 + 4 \cdot \pi^2 \cdot n^2} \right] \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) \qquad M_0 := 2 \cdot A \cdot \left(\frac{1 - e^{-\frac{\alpha}{2}}}{\alpha} \right)$$

$$MAG_m := \left| M_m \right|$$

Double - Sided Exponential Wave



[redacted]

Ken Kaiser

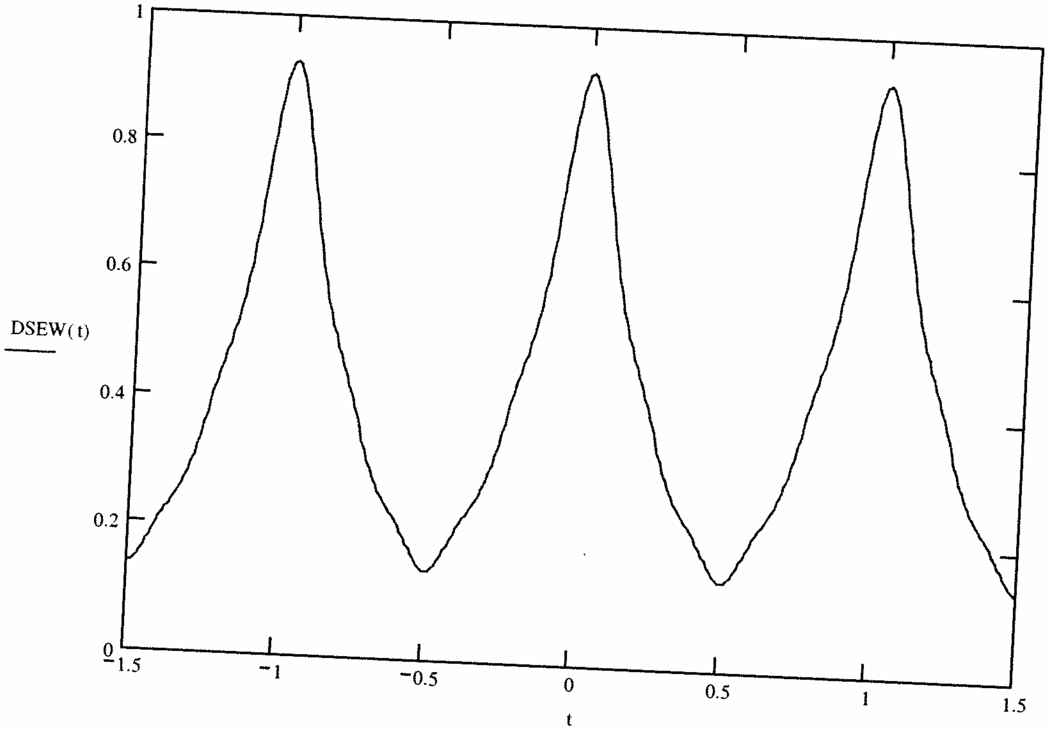
$A := 1$

$T := 1$

$\alpha := 4 \cdot T$

$t := -1.5, -1.49 .. 1.5$

$$DSEW(t) := 2 \cdot A \cdot \left(\frac{1 - e^{\frac{-\alpha}{2}}}{\alpha} \right) + 4 \cdot \alpha \cdot A \cdot \sum_{n=1}^{5} \frac{\left[1 + (-1)^{(n-1)} \cdot e^{\frac{-\alpha}{2}} \right]}{\left(\alpha^2 + 4 \cdot \pi^2 \cdot n^2 \right)} \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot t}{T} \right)$$



Ken Kaiser

plot of parabolic wave spectra p416

$a := 1$

$t := 1$

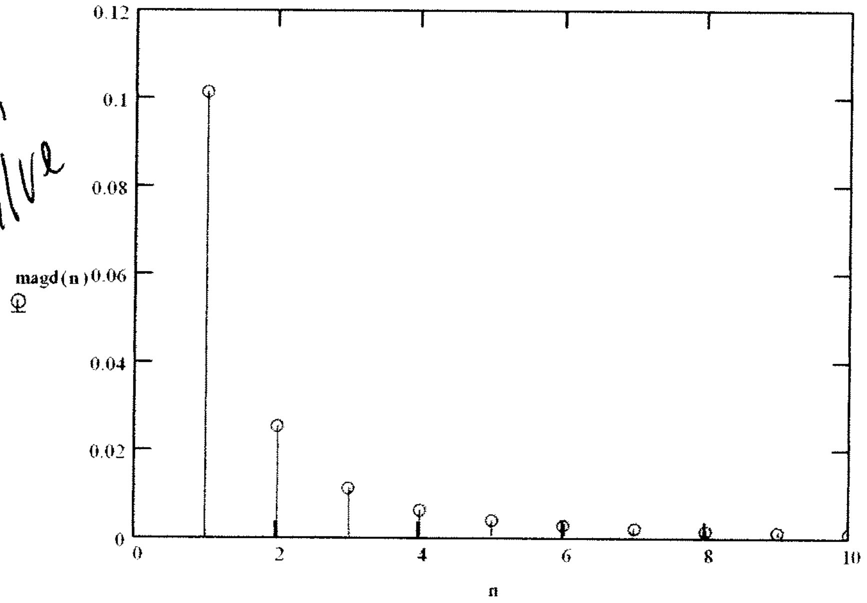
$n := 0, 1 .. 10$

$$d(n) := \frac{(-1)^n}{n^2}$$

$$a(n) := \left[\frac{\left(a \cdot t^2\right)}{\pi^2} \cdot d(n)\right]$$

$$magd(n) := |a(n)|$$

Missing dc value



Ken Kaiser

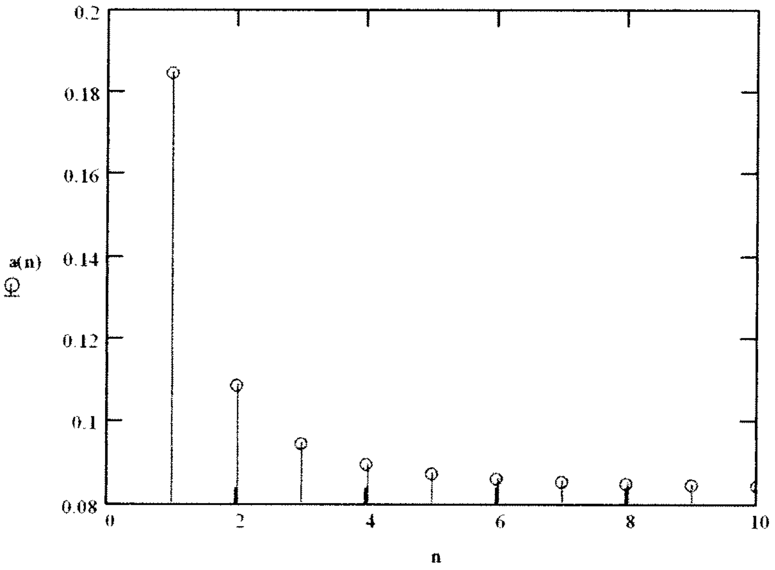
$a := 1$

$t := 1$

$n := 0, 1 .. 10$

$$d(n) := \frac{(-1)^n}{n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{t}\right)$$

$$a(n) := \frac{\left(a \cdot t^2\right)}{12} + \left[\frac{\left(a \cdot t^2\right)}{\pi^2} \cdot \left|d(n)\right|\right]$$



Ken Kaiser

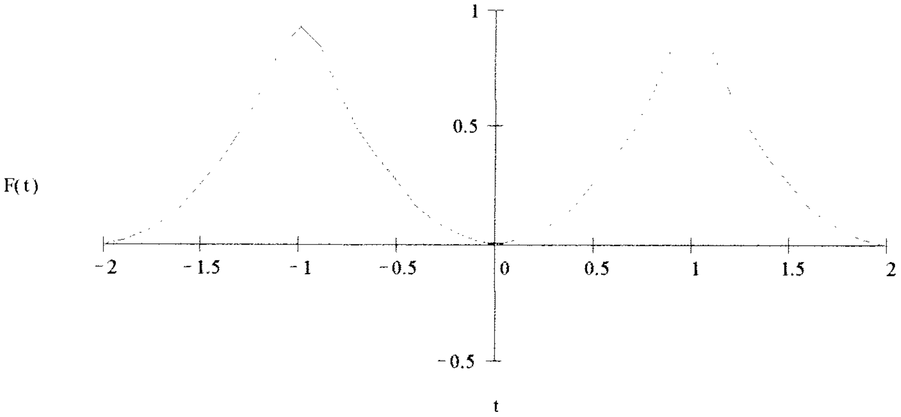
[redacted]

#3

$A := 1$ $\qquad$ $m := 5$

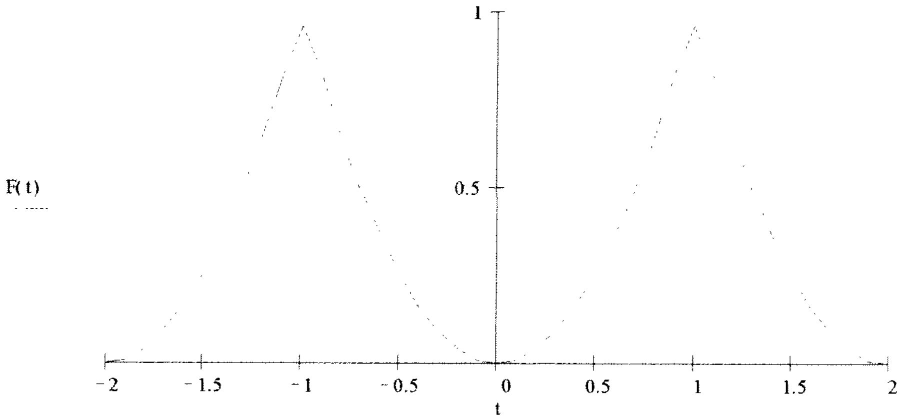
$T := 2$ $\qquad$ $t := -2, -1.9 .. 2$

$$F(t) := \frac{A \cdot T^2}{12} + \frac{A \cdot T^2}{\pi^2} \cdot \sum_{n=1}^{m} \frac{(-1)^n}{n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



$m := 10$

$$F(t) := \frac{A \cdot T^2}{12} + \frac{A \cdot T^2}{\pi^2} \cdot \sum_{n=1}^{m} \frac{(-1)^n}{n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



✓ p416 : PARABOLIC WAVE

a_n

$$\frac{2}{T}\cdot\int_{\frac{T}{2}}^{\frac{T}{2}} A\cdot t^{2}\cdot\cos\frac{2\cdot\pi\cdot n}{T}\cdot t\ dt$$

$$\frac{1}{2}\cdot T^{2}\cdot\ 2\cdot\sin\ \pi\cdot n\ \ \pi^{2}\cdot n^{2}\cdot\sin\ \pi\cdot n\ -\ 2\cdot\pi\cdot n\cdot\cos\ \pi\cdot n\ \cdot\frac{A}{\pi^{3}\cdot n^{3}} \qquad = \frac{AT^{2}}{\pi^{2}}\cdot\frac{(-1)^{n}}{n^{2}}$$

b_n

$$\frac{2}{T}\cdot\int_{\frac{T}{2}}^{\frac{T}{2}} A\cdot t^{2}\cdot\sin\frac{2\cdot\pi\cdot n}{T}\cdot t\ dt$$

$$0$$

✓ ✓ ✓

$$\frac{A\cdot T^{2}}{12}\ \sum_{n=1}^{3}\frac{1}{\cancel{2}}\cdot T^{2}\cdot\ \overset{0}{\cancel{2\cdot\sin\ \pi\cdot n}}\ -\ \overset{0}{\cancel{\pi^{2}\cdot n^{2}\cdot\sin\ \pi\cdot n}}\ \ \overset{(-1)}{2\cdot\cancel{\pi}\cdot\cancel{n}\cdot\cancel{\cos\ \pi\cdot n}}\ \cdot\frac{A}{\pi^{\cancel{3}2}\cdot n^{\cancel{3}2}}\cdot\cos\ 2\cdot\pi\cdot n\cdot\frac{t}{T}$$

✓ $$\frac{1}{12}\cdot A\cdot T^{2}\ \ \frac{T^{2}}{\pi^{2}}\cdot A\cdot\cos\ 2\cdot\frac{\pi}{T}\cdot t\ \ \frac{1}{4}\cdot\frac{T^{2}}{\pi^{2}}\cdot A\cdot\cos\ 4\cdot\frac{\pi}{T}\cdot t\ \ \frac{1}{9}\cdot\frac{T^{2}}{\pi^{2}}\cdot A\cdot\cos\ 6\cdot\frac{\pi}{T}\cdot t$$

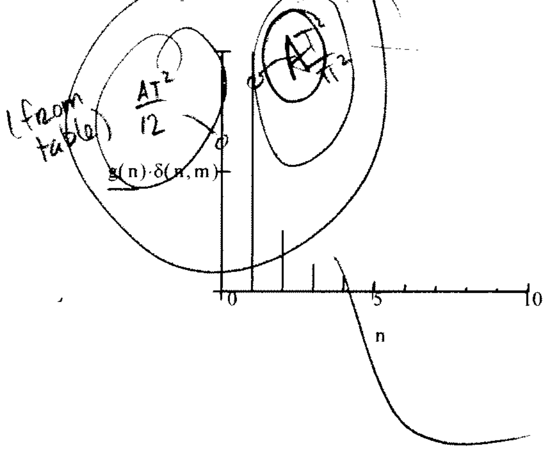
$$\left(\frac{AT^{2}}{12}\right)+\frac{AT^{2}}{\pi^{2}}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n}}{n^{2}}\cdot\cos\left(2\pi n\frac{t}{T}\right)\right)$$ ✓

Ken Kaiser

m · 1,0..11

n · 0,1..10

$g(n)\quad \text{if}\left(n<1,\frac{1}{12},\frac{1}{2}\cdot 2\cdot\sin\,\pi\cdot n\quad \pi^2\cdot n^2\cdot\sin\,\pi\cdot n\quad - 2\cdot\pi\cdot n\cdot\cos\,\pi\cdot n\cdot\frac{1}{\pi^3\cdot n^3}\right)$



$\rightarrow \quad \frac{AT^2}{\pi^2}\cdot\frac{(-1)^n}{n^2} \qquad |n=1| \rightarrow \left|-\frac{AT^2}{\pi^2}\right|$ ✓ ?

Did I forget A's? in my spectral lines

n · 1..10

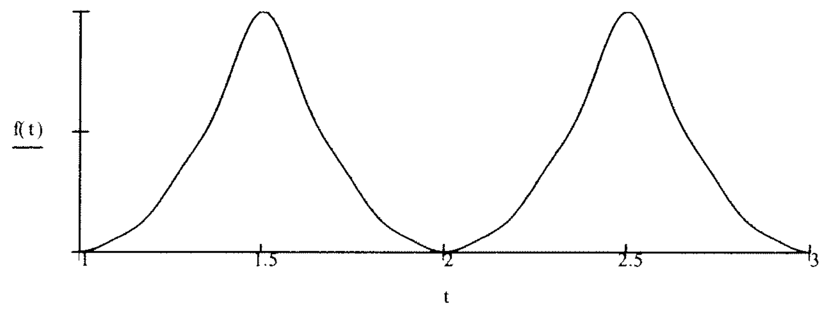
t · 1,1.01..3

A · 1

T · 1

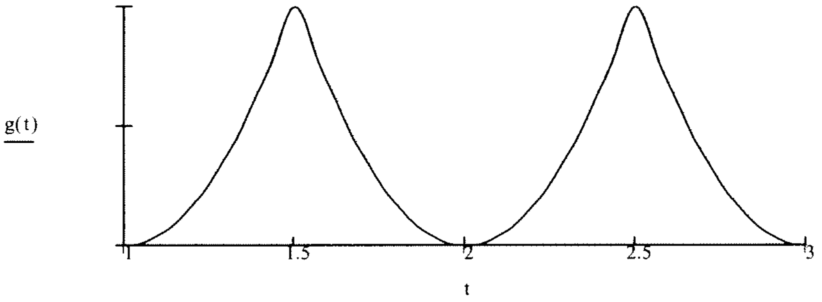
n = 5

$f(t)\ :=\ \frac{A\cdot T^2}{12}\ -\ \sum_{n=1}^{5}\frac{1}{2}\cdot T^2\cdot\ 2\cdot\sin\,\pi\cdot n\ -\pi^2\cdot n^2\cdot\sin\,\pi\cdot n\ -2\cdot\pi\cdot n\cdot\cos\,\pi\cdot n\ \cdot\frac{A}{\pi^3\cdot n^3}\cdot\cos\,2\cdot\pi\cdot n\cdot\frac{t}{T}$



n = 10

$$g(t)\ \frac{A\cdot T^2}{12}\ \sum_{n=1}^{10} \frac{1}{2}\cdot T^2\cdot\ 2\cdot\sin\ \pi\cdot n\ -\ \pi^2\cdot n^2\cdot\sin\ \pi\cdot n\ -\ 2\cdot\pi\cdot n\cdot\cos\ \pi\cdot n\ \cdot\ \frac{A}{\pi^3\cdot n^3}\ \cdot\cos\ 2\cdot\pi\cdot n\cdot\frac{t}{T}$$

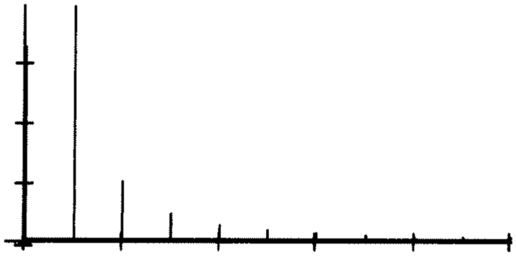


Ken Kaiser

$m := -1, 0 .. 11$ $\quad T := 2$

$n := 0, 1 .. 10$ $\quad A := 1$

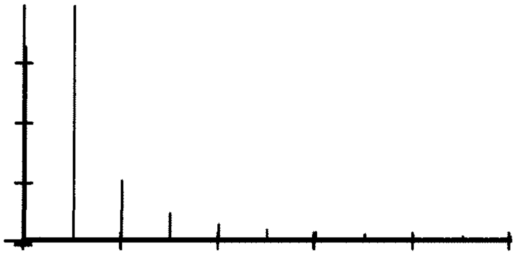
$$g(n) := \text{if}\left(n < 1, \frac{T^2}{12}, \frac{T^2 \cdot A}{\pi^2} \cdot \frac{1}{n^2}\right)$$



$m := -1, 0 .. 11$ $\quad T := 2$

$n := 0, 1 .. 10$ $\quad A := 1$

$$g(n) := \mathrm{if}\left(n < 1, \frac{T^2}{12}, \frac{T^2 \cdot A}{\pi^2} \cdot \frac{1}{n^2}\right)$$



$$\frac{\displaystyle\int_{\frac{-T}{2}}^{\frac{T}{2}} A \cdot t^2 \, dt}{T} \qquad \sqrt{\frac{\displaystyle\int_{\frac{-T}{2}}^{\frac{T}{2}} \left(A \cdot t^2\right)^2 dt}{T}}$$

$$\frac{1}{20} \cdot \sqrt{5} \cdot \left(T^4 \cdot A^2\right)^{\left(\frac{1}{2}\right)}$$

$$\frac{1}{12} \cdot T^2 \cdot A$$

Half Parabolic Wave

Proof of P417

An terms:

$$\frac{2}{T}\cdot\int_{0}^{T} A\cdot t^2\cdot\cos\frac{2\cdot\pi\cdot n}{T}\cdot t \; dt$$

= 0 @ N=0

$$\frac{1}{2}\cdot T^2\cdot \quad \sin 2\cdot\pi\cdot n \quad 2\cdot\pi^2\cdot n^2\cdot\sin 2\cdot\pi\cdot n \quad \boxed{2\cdot\pi\cdot n\cdot\cos 2\cdot\pi\cdot n} \quad \cdot\frac{A}{\pi^3\cdot n^3}$$

$$\frac{1}{2}\cdot T^2\cdot \quad 2\cdot\pi\cdot n\cdot 1 \quad \cdot\frac{A}{\pi^3\cdot n^3}$$

evaluate N=1 separately

$$\frac{T^2}{\pi^2\cdot n^2}\cdot A \qquad = An$$

$$\frac{0}{2\cdot n}$$ ~~A0 = 0 due to L'Hospital's rule~~

I obtained

$$a_0 = \frac{2t^2 A}{3} \checkmark$$

$$\frac{a_0}{2} = \frac{T^2 A}{3}$$

Bn terms:

$$\frac{2}{T}\cdot\int_{0}^{T} A\cdot t^2\cdot\sin\frac{2\cdot\pi\cdot n}{T}\cdot t \; dt$$

$$\frac{2}{T}\cdot\frac{1}{4}\cdot T^3\cdot \quad \cos 2\cdot\pi\cdot n \quad - 2\cdot\pi^2\cdot n^2\cdot\cos 2\cdot\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\sin 2\cdot\pi\cdot n \quad \cdot\frac{A}{\pi^3\cdot n^3} \quad \frac{1}{4}\cdot T^3\cdot\frac{A}{\pi^3\cdot n^3}$$

$$\frac{1}{2}\cdot T^2\cdot A\cdot\frac{\cos 2\cdot\pi\cdot n \quad 2\cdot\pi^2\cdot n^2\cdot\cos 2\cdot\pi\cdot n \quad 2\cdot\pi\cdot n\cdot\sin 2\cdot\pi\cdot n \quad 1}{\pi^3\cdot n^3}$$

$$\frac{1}{2}\cdot T^2\cdot A\cdot\frac{1 \quad 2\cdot\pi^2\cdot n^2\cdot 1 \quad 2\cdot\pi\cdot n\cdot 0 \quad 1}{\pi^3\cdot n^3}$$

$$T^2\cdot\frac{A}{\pi\cdot n} \qquad = Bn\ term$$

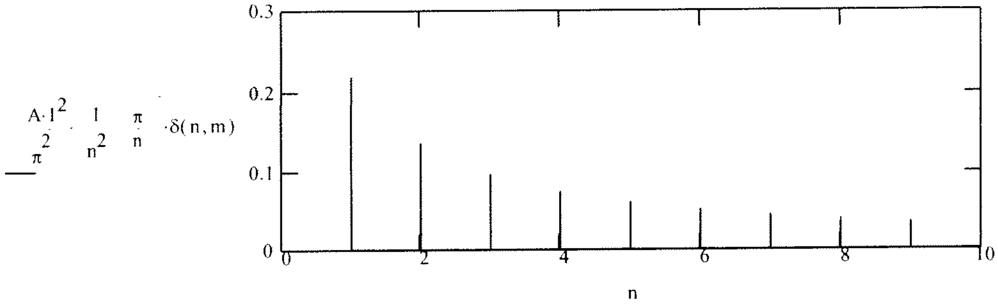
Plug An & Bn back into the approximation to get:

$$\frac{A\cdot T^2}{\pi^2}\cdot\sum_{n=1}^{3}\frac{1}{n^2}\cdot\cos\ \frac{2\cdot n\cdot\pi}{1}\cdot t\quad \frac{\pi}{n}\cdot\sin\ \frac{2\cdot\pi\cdot n}{1}\cdot t$$

$A\quad 1$

$m\quad 1,0..11$

$n\quad 1,2..10$



$$\frac{A\cdot T^2}{\pi^2}\cdot\sum_{n=1}^{3}\frac{1}{n^2}\cdot\cos\ \frac{2\cdot n\cdot\pi}{T}\cdot t\quad \frac{\pi}{n}\cdot\sin\ \frac{2\cdot\pi\cdot n}{T}\cdot t$$

$$\frac{1}{36}\cdot A\cdot T^2\cdot\ \frac{36\cdot\cos\ 2\cdot\frac{\pi}{T}\cdot t\quad 36\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad 9\cdot\cos\ 4\cdot\frac{\pi}{T}\cdot t\quad 18\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad 4\cdot\cos\ 6\cdot\frac{\pi}{T}\cdot t\quad 12\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t\ \cdot\pi}{\pi^2}$$

$$A\cdot T^2\cdot\ \frac{36\cdot\cos\ 2\cdot\frac{\pi}{T}\cdot t\quad 36\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad 9\cdot\cos\ 4\cdot\frac{\pi}{T}\cdot t\quad 18\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad 4\cdot\cos\ 6\cdot\frac{\pi}{T}\cdot t\quad 12\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t\ \cdot\pi}{36\cdot\pi^{\ 2}}$$

$$\frac{A\cdot T^2}{\pi^2}\cdot\ \cos\ 2\cdot\frac{\pi}{T}\cdot t\quad \sin\ 2\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad \frac{1}{4}\cdot\cos\ 4\cdot\frac{\pi}{T}\cdot t\quad \frac{1}{2}\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad \frac{1}{9}\cdot\cos\ 6\cdot\frac{\pi}{T}\cdot t\quad \frac{1}{3}\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t\ \cdot\pi$$

$$\frac{A\cdot T^2}{\pi^2}\cdot\cos\ 2\cdot\frac{\pi}{T}\cdot t\quad \frac{A\cdot T^2}{\pi^2}\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad \frac{A\cdot T^2}{4\cdot\pi^{\ 2}}\cdot\cos\ 4\cdot\frac{\pi}{T}\cdot t\quad \frac{A\cdot T^2}{2\cdot\pi^{\ 2}}\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t\ \cdot\pi\quad \frac{A\cdot T^2}{9\cdot\pi^{\ 2}}\cdot\cos\ 6\cdot\frac{\pi}{T}\cdot t\quad \frac{A\cdot T^2}{3\cdot\pi^{\ 2}}\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t\ \cdot\pi$$

These are the first 3 terms of the sequence +....

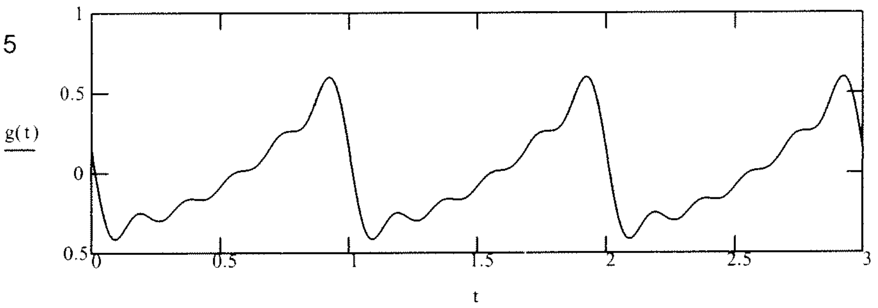
Ken Kaiser

$n \quad 1..10$

$t \quad 0,0.01..3$

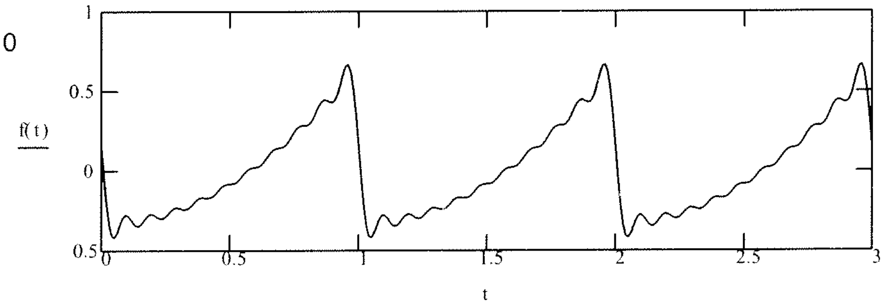
$$g(t) \quad \sum_{n=1}^{5} \quad 1^{2}\cdot \quad \frac{A}{\pi\cdot n} \quad \cdot\sin \quad \frac{2\cdot\pi\cdot n}{1}\cdot t \quad \cdot \quad \frac{1^{2}}{\pi^{2}\cdot n^{2}} \quad \cdot A \quad \cdot\cos \quad \frac{2\cdot\pi\cdot n}{1}\cdot t$$

n = 5



$$f(t) \quad \sum_{n=1}^{10} \quad 1^{2}\cdot \quad \frac{A}{\pi\cdot n} \quad \cdot\sin \quad \frac{2\cdot\pi\cdot n}{1}\cdot t \quad \cdot \quad \frac{1^{2}}{\pi^{2}\cdot n^{2}} \quad \cdot A \quad \cdot\cos \quad \frac{2\cdot\pi\cdot n}{1}\cdot t$$

n = 10



$k := 1 .. 10$

$T := 15$ $A := 9$

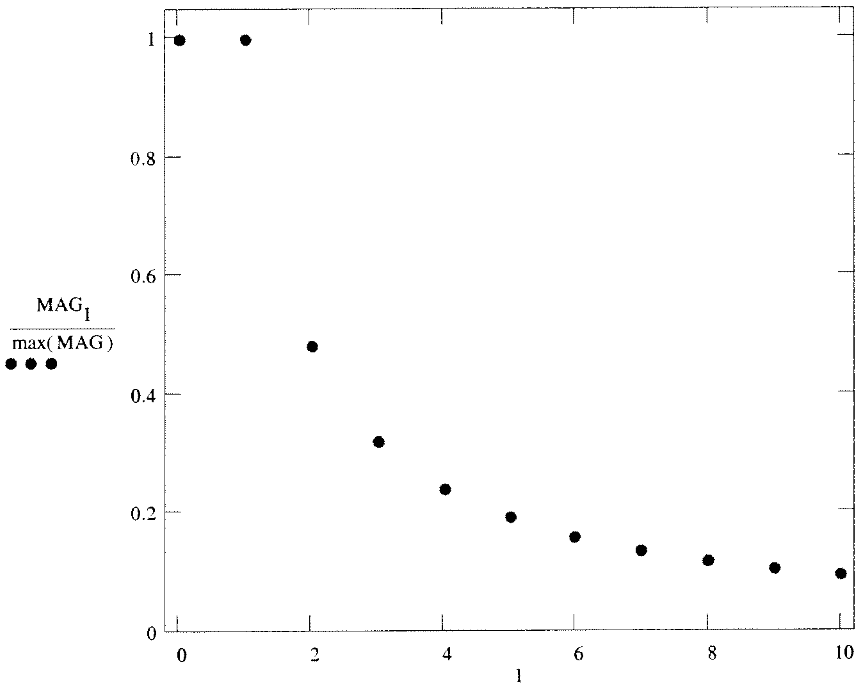
$l := 0 .. 10$

$\alpha := 10$

$$M_k := \sqrt{\left[\frac{A \cdot T^2}{(\pi)^2 \cdot k^2}\right]^2 + \left(\frac{A \cdot T^2}{k \cdot \pi}\right)^2}$$

$$M_0 := \frac{A \cdot T^2}{3}$$

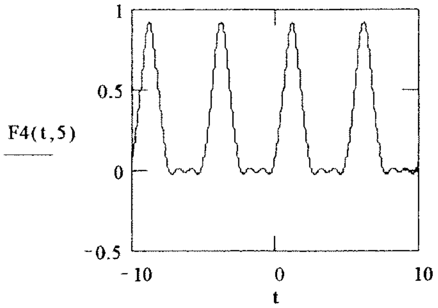
$$MAG_l := \left| M_l \right|$$

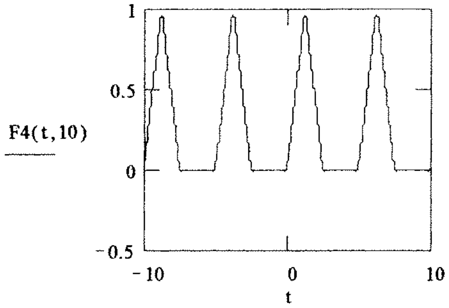


$$j := \sqrt{-1}$$

page 387 - Half-Rectified Triangular Wave #2

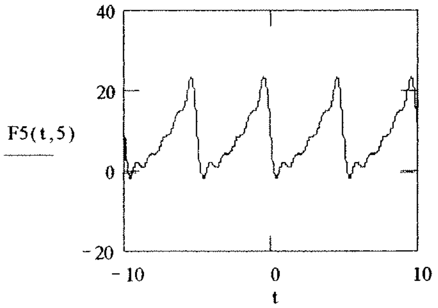
$$F4(t,m) := \frac{A}{4} + \left[\frac{A}{\pi^2}\cdot\left[\sum_{n=-m}^{-1} \frac{\left(2\cdot e^{-j\cdot\frac{\pi\cdot n}{2}} - 1\right) + (-1)^{n-1}}{n^2}\cdot e^{j\cdot\frac{2\cdot\pi\cdot n}{T}\cdot t}\right]\right] + \frac{A}{\pi^2}\cdot\left[\sum_{n=1}^{m} \frac{\left(2\cdot e^{-j\cdot\frac{\pi\cdot n}{2}} - 1\right) + (-1)^{n-1}}{n^2}\right]$$

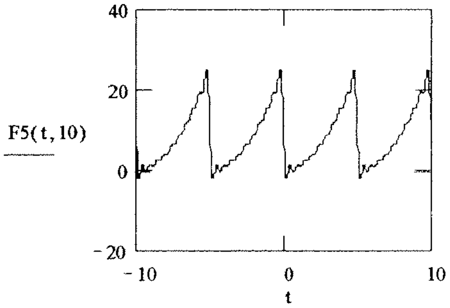




page 417 - Half Parabolic Wave

$$F5(t,m) := \frac{A\cdot T^2}{3} + \frac{A\cdot T^2}{\pi^2}\cdot\left[\sum_{n=1}^{m} \frac{1}{n^2}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) - \frac{\pi}{n}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]$$





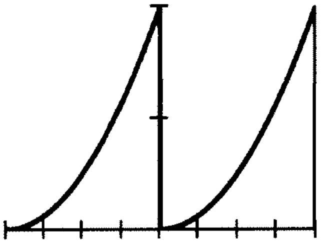
$T := 2$

$A := 1$

$$t := -\frac{4}{2}\cdot T, -\frac{4}{2}\cdot T + \frac{T}{1000} .. \frac{3}{2}\cdot T$$

$$f(t) := t^2$$

$$g(t) := \text{if}\left(0 \le t \le T, f(t), \text{if}\left(-\frac{4}{2}\cdot T < t < -0, f(t+T), \text{if}\left(T < t < \frac{4}{2}\cdot T, f(t-T), 0\right)\right)\right)$$



0

$$\frac{\int_0^T t^2\,dt}{T}$$

$$\frac{1}{3}\cdot T^2$$

$$\sqrt{\frac{\int_0^T t^4\,dt}{T}}$$

$$\frac{1}{5}\cdot\sqrt{5}\cdot T^2$$

$$\frac{\int_0^T t^2\cdot\cos\left(2\cdot\pi\cdot\frac{0}{T}\cdot t\right)dt\cdot 2}{T}$$

$$\frac{2}{3}\cdot T^2$$

$$\frac{\int_0^T t^2\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)dt\cdot 2}{T}$$

$$\frac{1}{2}\cdot T^2\cdot\frac{\left(-\sin(2\cdot\pi\cdot n)+2\cdot\pi^2\cdot n^2\cdot\sin(2\cdot\pi\cdot n)+2\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\right)}{\left(\pi^3\cdot n^3\right)}$$

$$\frac{\displaystyle\int_0^T t^2\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)dt\cdot 2}{T}$$

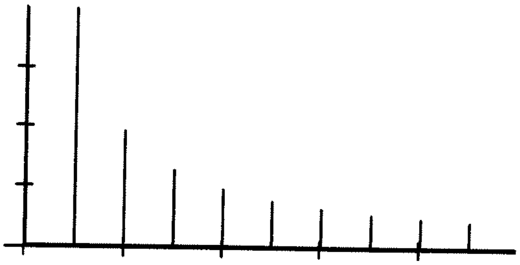
$$\frac{-1}{2}\cdot T^2\cdot\frac{\left(-\cos(2\cdot\pi\cdot n)+2\cdot\pi^2\cdot n^2\cdot\cos(2\cdot\pi\cdot n)-2\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)+1\right)}{\left(\pi^3\cdot n^3\right)}$$

Ken Kaiser

$m := -1, 0 .. 11$ $T := 2$

$n := 0, 1 .. 10$ $A := 1$

$$g(n) := \text{if}\left[n < 1, \frac{T^2}{3}, \sqrt{\left(T^2 \cdot \frac{A}{\pi^2 \cdot n^2}\right)^2 + \left(T^2 \cdot \frac{A}{\pi \cdot n}\right)^2}\right]$$



Ken Kaiser

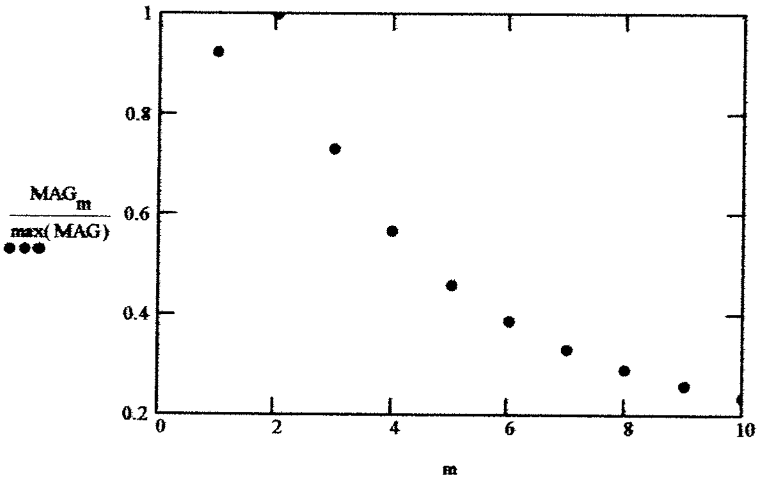
Cubic Wave

$n := 1 .. 10$ $\qquad T := 2 \quad A := 2 \quad t := 1$

$m := 1 .. 10$

$$M_n := \frac{A \cdot T^3}{4 \cdot \pi} \cdot \frac{(-1)^{(n+1)}}{n} \cdot \left(1 - \frac{6}{\pi^2 \cdot n^2}\right)$$

$$MAG_m := \left|M_m\right|$$



✓

Ken Kaiser

Assume these values:

$A := 8$ $T := 2$

f_0 $$F := \frac{A\cdot T^3}{4\cdot\pi}\cdot\left(1-\frac{6}{\pi^2}\right)$$

$|F| = 1.997$ ✓

$2f_0$ $$F := \frac{A\cdot T^3}{8\cdot\pi}\cdot\left[1-\frac{3}{(\pi)^2\cdot 2}\right]$$

$|F| = 2.159$ *MISTAKE ON GRAPH

$3f_0$ $$F := \frac{A\cdot T^3}{12\cdot\pi}\cdot\left[1-\frac{2}{(\pi)^2\cdot 3}\right]$$

$|F| = 1.583$ ✓

$4f_0$ $$F := \frac{A\cdot T^3}{16\cdot\pi}\cdot\left[1-\frac{3}{(\pi)^2\cdot 8}\right]$$

$|F| = 1.225$ ✓

$5f_0$ $$F := \frac{A\cdot T^3}{20\cdot\pi}\cdot\left[1-\frac{6}{(\pi)^2\cdot 25}\right]$$

$|F| = 0.994$ ✓

$6f_0$ $$F := \frac{A\cdot T^3}{24\cdot\pi}\cdot\left[1-\frac{6}{(\pi)^2\cdot 36}\right]$$

$|F| = 0.834$ ✓

$7f_0$ $$F := \frac{A\cdot T^3}{28\cdot\pi}\cdot\left[1-\frac{6}{(\pi)^2\cdot 49}\right]$$

$|F| = 0.719$ ✓

$8f_0$ $$F := \frac{A \cdot T^3}{32 \cdot \pi} \cdot \left[1 - \frac{6}{(\pi)^2 \cdot 64}\right]$$

$$|F| = 0.631$$ ✓

$7f_0$ $$F := \frac{A \cdot T^3}{36 \cdot \pi} \cdot \left[1 - \frac{6}{(\pi)^2 \cdot 81}\right]$$

$$|F| = 0.562$$ ✓

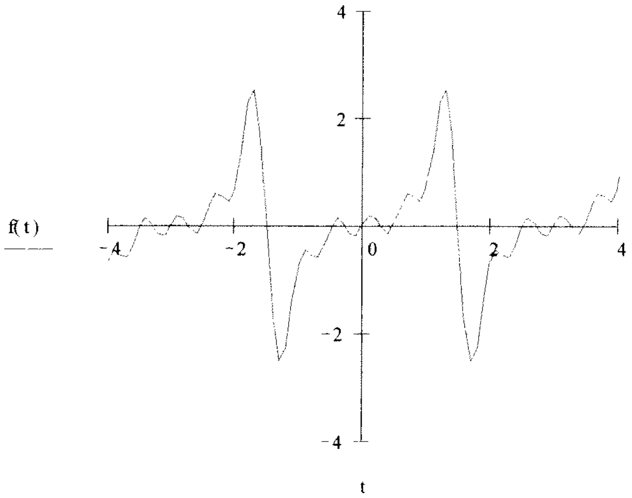
$10f_0$ $$F := \frac{A \cdot T^3}{40 \cdot \pi} \cdot \left[1 - \frac{6}{(\pi)^2 \cdot 100}\right]$$

$$|F| = 0.506$$ ✓

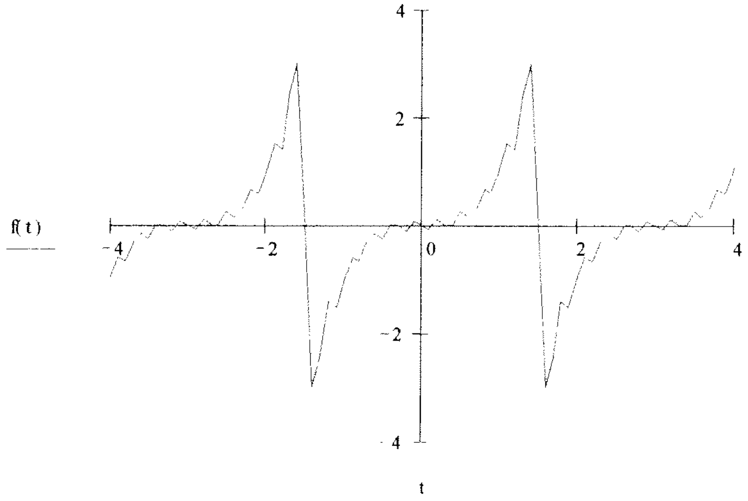
EMO[redacted] Ch 13 pg 418

$A := 1 \qquad T := 3 \qquad t := -10, -9.9 .. 10$

$$f(t) := \frac{A \cdot T^3}{4 \cdot \pi} \cdot \sum_{n=1}^{5} \frac{(-1)^{n+1}}{n} \cdot \left(1 - \frac{6}{\pi^2 \cdot n^2}\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



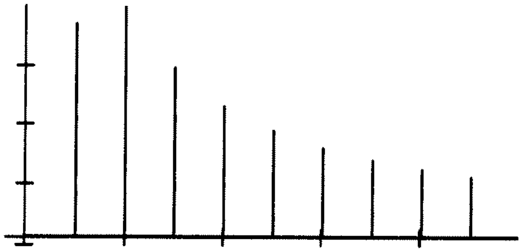
$$f(t) := \frac{A \cdot T^3}{4 \cdot \pi} \cdot \sum_{n=1}^{10} \frac{(-1)^{n+1}}{n} \cdot \left(1 - \frac{6}{\pi^2 \cdot n^2}\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



$m := -1, 0 .. 11$ $T := 2$

$n := 0, 1 .. 10$ $A := 1$

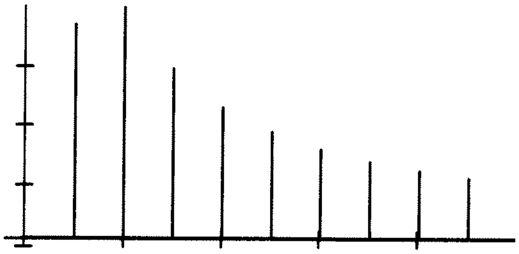
$$g(n) := \mathrm{if}\left(n < 1, 0, \frac{T^3}{\pi \cdot 4} \cdot \frac{1}{n} \cdot \left|1 - \frac{6}{n^2 \cdot \pi^2}\right|\right)$$



$m := -1, 0 .. 11$ $T := 2$

$n := 0, 1 .. 10$ $A := 1$

$$g(n) := \text{if}\left(n < 1, 0, \frac{T^3}{\pi \cdot 4} \cdot \frac{1}{n} \cdot \left|1 - \frac{6}{n^2 \cdot \pi^2}\right|\right)$$



Ken Kaiser

Cubic wave

$A := 2$

$B := 4$

$C := 5$

$T := 3$

$a := 1$

$k := 0.4$

$$f_{avg} := \frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot t^3 \, dt \qquad f_{avg} = 0 \qquad f_{avg2} := 0$$ ✓

$$f_{rms} := \sqrt{\frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A \cdot t^3\right)^2 dt} \qquad f_{rms} = 2.551 \qquad f_{rms2} := \frac{A \cdot T^3}{8 \cdot \sqrt{7}}$$ = 2,55 ✓

Ken Kaiser

Problem #1 - Square Wave with Time Shift

$$f_{avg} = \frac{1}{T}\cdot\left(\int_{0}^{t_0} -A\,dt + \int_{t_0}^{t_0+\frac{T}{2}} A\,dt + \int_{t_0+\frac{T}{2}}^{T} A\,dt\right) = 0$$

$$f_{rms} = \sqrt{\frac{1}{T}\cdot\left(\int_{0}^{t_0} (-A)^2\,dt + \int_{t_0}^{t_0+\frac{T}{2}} A^2\,dt + \int_{t_0+\frac{T}{2}}^{T} (-A)^2\,dt\right)} = A$$

Problem #2 - Cubic Wave

$$f_{avg} = \frac{1}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot t^3\,dt = 0$$

$$f_{rms} = \sqrt{\frac{1}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A\cdot t^3\right)^2\,dt} = \frac{1}{56}\cdot\sqrt{7}\cdot T^3\cdot A \approx .047\,T^3 A$$

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$$\frac{2}{T}\cdot\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot t^3\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

$(-1)^N$ $\quad 0 \quad$ $(-1)^N$ $\quad 0$

$$\frac{1}{4}\cdot T^3\cdot\left(6\cdot\pi\cdot n\cdot\cos(\cancel{\pi\cdot n}) - 6\cdot\sin(\cancel{\pi\cdot n})\ \boxed{-\pi^3}\cdot n^3\cdot\cos(\cancel{\pi\cdot n}) + 3\cdot\pi^2\cdot n^2\cdot\sin(\cancel{\pi\cdot n})\right)\cdot\frac{A}{(\pi^4\cdot n^4)}$$

$$\frac{1}{4}\cdot T^3\cdot\left(6\cdot\pi\cdot 1\cdot\cos(\pi\cdot 1) - 6\cdot\sin(\pi\cdot 1) - \pi^3\cdot 1^3\cdot\cos(\pi\cdot 1) + 3\cdot\pi^2\cdot 1^2\cdot\sin(\pi\cdot 1)\right)\cdot\frac{A}{(\pi^4\cdot 1^4)} \qquad \frac{1}{4}\cdot\frac{T^3}{\pi^3}\cdot(-6+\pi^2)\cdot A$$

$$\frac{1}{4}\cdot T^3\cdot\left(6\cdot\pi\cdot 2\cdot\cos(\pi\cdot 2) - 6\cdot\sin(\pi\cdot 2) - \pi^3\cdot 2^3\cdot\cos(\pi\cdot 2) + 3\cdot\pi^2\cdot 2^2\cdot\sin(\pi\cdot 2)\right)\cdot\frac{A}{(\pi^4\cdot 2^4)} \qquad \frac{-1}{16}\cdot\frac{T^3}{\pi^3}\cdot(-3+2\cdot\pi^2)\cdot A$$

$$\frac{1}{4}\cdot T^3\cdot\left(6\cdot\pi\cdot 3\cdot\cos(\pi\cdot 3) - 6\cdot\sin(\pi\cdot 3) - \pi^3\cdot 3^3\cdot\cos(\pi\cdot 3) + 3\cdot\pi^2\cdot 3^2\cdot\sin(\pi\cdot 3)\right)\cdot\frac{A}{(\pi^4\cdot 3^4)} \qquad \frac{1}{36}\cdot\frac{T^3}{\pi^3}\cdot(-2+3\cdot\pi^2)\cdot A$$

$$\frac{1}{4}\cdot T^3\cdot\left(6\cdot\pi\cdot 4\cdot\cos(\pi\cdot 4) - 6\cdot\sin(\pi\cdot 4) - \pi^3\cdot 4^3\cdot\cos(\pi\cdot 4) + 3\cdot\pi^2\cdot 4^2\cdot\sin(\pi\cdot 4)\right)\cdot\frac{A}{(\pi^4\cdot 4^4)} \qquad \frac{-1}{128}\cdot\frac{T^3}{\pi^3}\cdot(-3+8\cdot\pi^2)\cdot A$$

$$\frac{A}{\pi^4 N^4}\frac{1}{4}T^3\left(6\pi N(-1)^N \mp \pi^3 N^3(-1)^N\right)$$

$$\frac{AT^3}{4\pi}\frac{1}{N}\left[\frac{6\pi N(-1)^N}{N^3\pi^3} \mp \frac{\pi^3 N^3(-1)^N}{N^3\pi^3}\right]$$

$$\frac{AT^3}{4\pi}\frac{(-1)^N}{N}\left[\frac{6}{\pi^2N^2} \mp 1\right]$$

$$\frac{AT^3}{4\pi}\frac{(-1)^{N+1}}{N}\left[1-\frac{6}{\pi^2N^2}\right]$$

$$\cancel{-\frac{1}{16}\frac{T^3}{\pi^3}3\pi^2}\left(\cancel{\frac{1}{\pi^2}}\right.$$

$$-\frac{1}{16}\frac{T^3}{\pi^3}2\pi^2\left(1-\frac{3}{2\pi^2}\right)$$

$$-\frac{AT^3}{8\pi}\left(1-\frac{3}{2\pi^2}\right)$$

$$\frac{T^3}{36\pi^{\cancel{3}}}3\pi^{\cancel{2}}\left(1-\frac{2}{3\pi^2}\right)$$

Ken Kaiser

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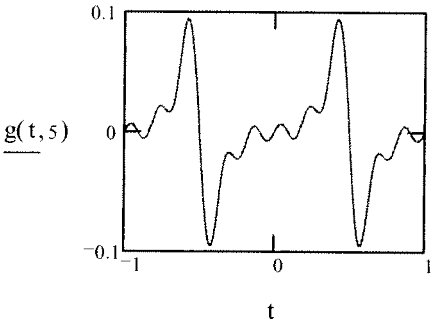
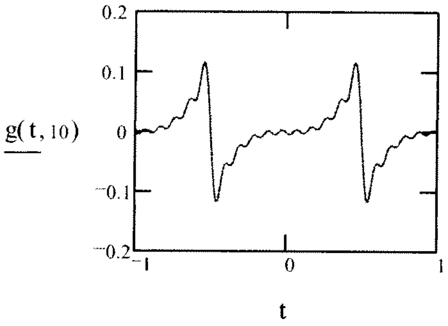
$T := 1 \qquad A := 1 \qquad t := -T, -T + \frac{T}{10000} .. T$

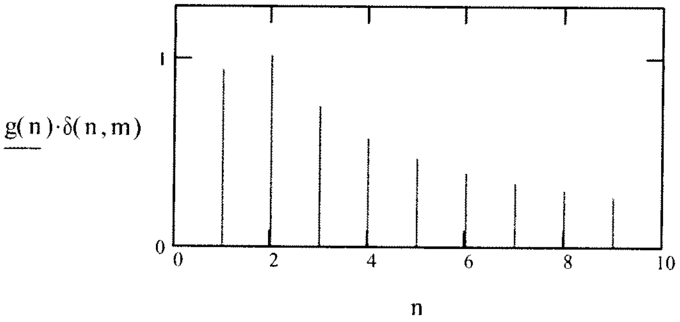
$$g(t,m) := \frac{A\cdot T^3}{4\cdot\pi}\cdot\sum_{n=1}^{m}\frac{(-1)^{n+1}}{n}\cdot\left(1-\frac{6}{\pi^2\cdot n^2}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$





$n := 1, 2 .. 10 \qquad m := -1, 0 .. 11 \qquad A := 30$

$$g(n) := \left|\frac{A\cdot T^3}{4\cdot\pi}\cdot\left[\frac{(-1)^{n+1}}{n}\cdot\left(1-\frac{6}{\pi^2\cdot n^2}\right)\right]\right|$$



Quadratic Wave #1
p.419

Ken Kaiser

Solving for a_n and b_n.

$$\frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}}\left(At^2+B\cdot t+C\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\frac{\left(2\cdot At^2\cdot\sin(\pi\cdot n)\cdot\pi\cdot n+B\cdot\pi\cdot n\cdot\sin(\pi\cdot n)\cdot T+B\cdot\cos(\pi\cdot n)\cdot T+2\cdot C\cdot\sin(\pi\cdot n)\cdot\pi\cdot n\right)}{\left(\pi^2\cdot n^2\right)}+\frac{1}{4}\cdot T\cdot\frac{\left(2\cdot At^2\cdot\sin(\pi\cdot n)\cdot\pi\cdot n-B\cdot\pi\cdot n\cdot\sin(\pi\cdot n)\cdot\right.}{\left(\pi^2\cdot\right.}\right.$$

$$2\cdot\frac{\sin(\pi\cdot n)}{(\pi\cdot n)}\cdot\left(At^2+C\right)$$

My check Method

$$\frac{1}{2}\left[\frac{2A\pi N(-1)^N T^2}{\pi^3 N^3}\right]$$

$$\frac{A(-1)T^2}{\pi^2 N^2}$$

$$\frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}}\left(A\cdot t^2+B\cdot t+C\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{-1}{8}\cdot T\cdot\frac{\left(-2\cdot A\cdot\cos(\pi\cdot n)\cdot T^2+4\cdot C\cdot\cos(\pi\cdot n)\cdot\pi^2\cdot n^2+A\cdot\pi^2\cdot n^2\cdot\cos(\pi\cdot n)\cdot T^2-2\cdot A\cdot\pi\cdot n\cdot\sin(\pi\cdot n)\cdot T^2+2\cdot B\cdot\pi^2\cdot n^2\cdot\cos(\pi\cdot n)\cdot T-2\cdot B\cdot\pi\cdot n\cdot\right.}{\left(\pi^3\cdot n^3\right)}\right.$$

✓ ✓ ✓

$$-T\cdot\frac{B}{\left(\pi^2\cdot n^2\right)}\cdot\left(\pi\cdot n\cdot\cos(\pi\cdot n)-\cancel{\sin(\pi\cdot n)}\right)\quad 0$$

$$=-\frac{B}{\pi^2 n^2}\left(\pi n(-1)^n\right)=\frac{-BT}{\pi N}(-1)^N$$

?

expansion

m := -1, 0 .. 11

n := 1, 2 .. 10

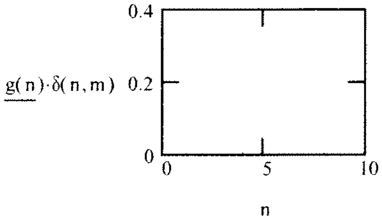
A := 2

$\cancel{T := 1}$

$$g(n) := \cancel{\left|\frac{(-1)^n\cdot A\cdot T^2}{\pi^2\cdot n^2}\right|}$$

?

$$\sqrt{a_N^2+b_N^2}$$



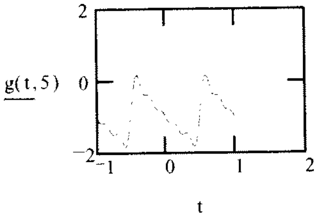
Ken Kaiser

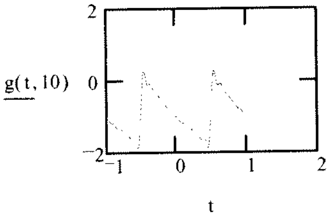
$T := 1 \qquad A := 1$

$t := -T, -T + \frac{T}{1000} .. T \qquad C := -1$

$B := -2$

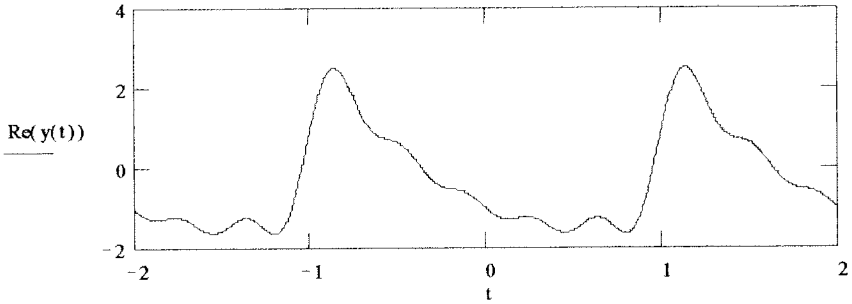
$$g(t,m) := \frac{A \cdot T^2}{12} + C + \left[\sum_{n=1}^{m} \frac{(-1)^n \cdot A \cdot T^2}{\pi^2 \cdot n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) - \frac{(-1)^n \cdot B \cdot T}{\pi \cdot n} \cdot \sin\left[\left(\frac{2 \cdot \pi \cdot n}{T}\right) \cdot t\right]\right]$$



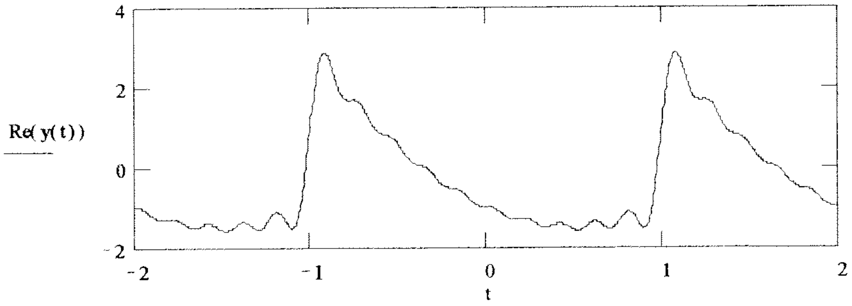


$A := 2 \qquad B := -2 \qquad C := -1 \qquad T := 2$

$$y(t) := \frac{A\cdot T^2}{12} + C + \left[\sum_{n=1}^{5}\left[\frac{(-1)^n\cdot A\cdot T^2}{\pi^2\cdot n^2}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) - \left[\frac{(-1)^n\cdot B\cdot T}{\pi\cdot n}\right]\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]\right]$$



$$y(t) := \frac{A\cdot T^2}{12} + C + \left[\sum_{n=1}^{10}\left[\frac{(-1)^n\cdot A\cdot T^2}{\pi^2\cdot n^2}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) - \left[\frac{(-1)^n\cdot B\cdot T}{\pi\cdot n}\right]\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]\right]$$



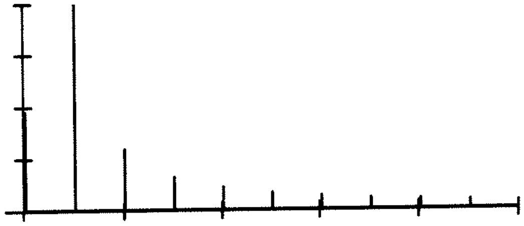
Ken Kaiser

$m := -1, 0 .. 11$ $T := 2$

$A := 8$ $B := -2$ $C := -1$

$n := 0, 1 .. 10$

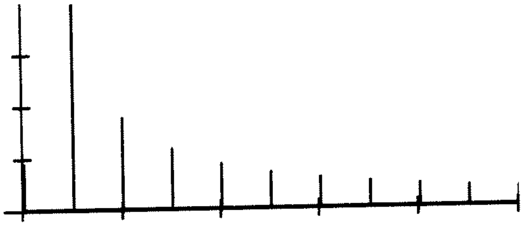
$$g(n) := \mathrm{if}\left[n < 1, \left|A \cdot \frac{T^2}{12} + C\right|, \sqrt{\left[\frac{(-1)^n \cdot A \cdot T^2}{\pi^2 \cdot n^2}\right]^2 + \left[\frac{(-1)^n \cdot B \cdot T}{-\pi \cdot n}\right]^2}\right]$$



$m := -1, 0 .. 11$ $T := 2$ $A := 2$ $B := -2$ $C := -1$

$n := 0, 1 .. 10$

$$g(n) := \mathrm{if}\left[n < 1, \left|A \cdot \frac{T^2}{12} + C\right|, \sqrt{\left[\frac{(-1)^n \cdot A \cdot T^2}{\pi^2 \cdot n^2}\right]^2 + \left[\frac{(-1)^n \cdot B \cdot T}{-\pi \cdot n}\right]^2}\right]$$



3.

$$f_{rms} = \sqrt{\left[\left(\frac{1}{T}\right)\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}}\left[A\cdot\left(t^2\right)+B\cdot t+C\right]^2\,dt\right]}$$

$$\frac{1}{\left(60\cdot\sqrt{T}\right)}\cdot\sqrt{600\cdot C\cdot A\cdot T^3+3600\cdot C^2\cdot T+45\cdot A^2\cdot T^5+300\cdot B^2\cdot T^3}$$

$$\frac{1}{60}\cdot\sqrt{3}\cdot\sqrt{5}\cdot\sqrt{40\cdot C\cdot A\cdot T^2+240\cdot C^2+3\cdot A^2\cdot T^4+20\cdot B^2\cdot T^2}$$

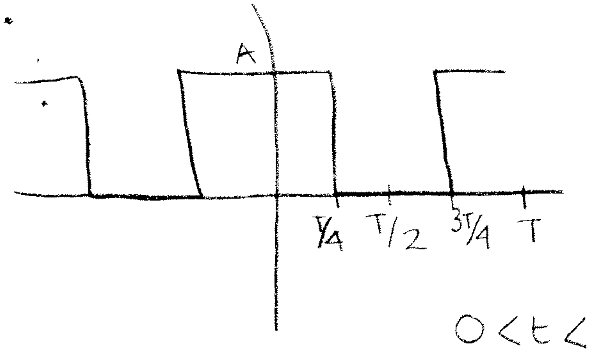
$$f_{avg} = \left(\frac{1}{T}\right)\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}}A\cdot\left(t^2\right)+B\cdot t+C\,dt$$

$$\frac{1}{T}\cdot\left(C\cdot T+\frac{1}{12}\cdot A\cdot T^3\right)$$

$$C+\frac{1}{12}\cdot A\cdot T^2$$

Ken Kaiser

①

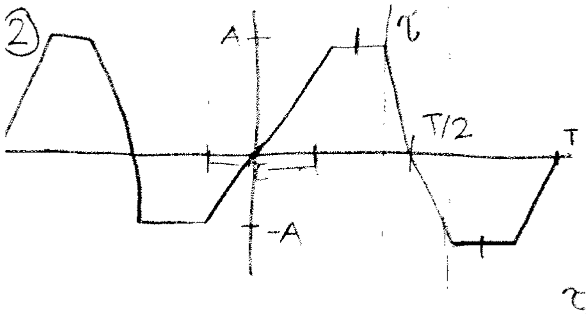


$0 < t < T/4 = A$ const. straight line @ A

$T/4 < t < 3T/4 = 0$ const. straight line @ 0

$3T/4 < t < T = A$ const. straight line @ A

②



$0 < t < \tau/2 =$ slope $= \frac{\text{rise}}{\text{run}} = \frac{2A}{\tau}$

$\tau/2 < t < T/2 - \tau/2 = A$ const. @ top of wave

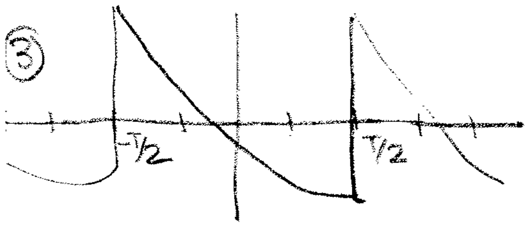
$T/2 - \tau/2 < t < T/2 + \tau/2 = \underset{\text{slope}\rightarrow}{\frac{-2A}{\tau}}\left(t - \overset{\leftarrow\text{intercept}}{T/2}\right)$

$= \frac{2A}{\tau}\left(T/2 - t\right)$

$T/2 + \tau/2 < t < T - \tau/2 = -A$ const @ bottom of wave

$T - \tau/2 < t < T = \underset{\nearrow\text{slope}}{\frac{2A}{\tau}}\left(t - \overset{\leftarrow\text{intercept}}{T}\right)$

③



$-\frac{T}{2} < t < T/2 = At^2 + bt + C$

since it is a parabolic wave

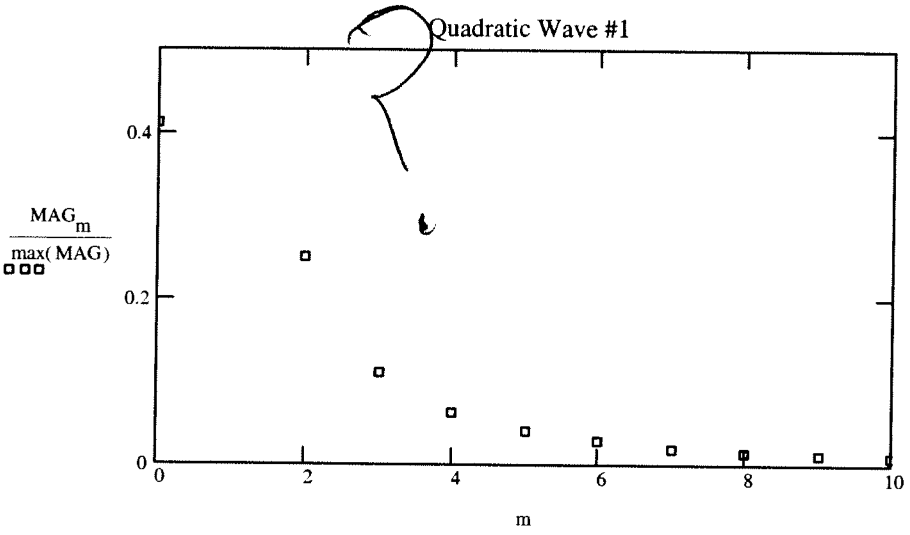
Ken Kaiser

$n := 1..10 \quad T := 2 \quad C := -1$

$m := 0..10 \quad A := 2 \quad t := 4 \quad B := -2$

$$M_n := \frac{(-1)^n \cdot A \cdot T^2}{\pi^2 \cdot n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) - \frac{(-1)^n \cdot B \cdot T}{\pi \cdot n} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) \qquad M_0 := \frac{A \cdot T^2}{12} + C$$

$$MAG_m := \left|M_m\right|$$



$$fave5 := \frac{A \cdot T^2}{12} + C$$

$$fave5 = 6.5$$

$$frms5 := \frac{\sqrt{\left(40 \cdot A \cdot C \cdot T^2\right) + \left(240 \cdot C^2\right) + \left(3 \cdot A^2 \cdot T^4\right) + \left(20 \cdot B^2 \cdot T^2\right)}}{4 \cdot \sqrt{15}}$$

$$frms5 = 7.487$$

$$fave6 := \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A \cdot t^2 + B \cdot t + C\right) dt \cdot \frac{1}{T}$$

$$fave6 = 6.5$$

$$frms6 := \sqrt{\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A \cdot t^2 + B \cdot t + C\right)^2 dt \cdot \frac{1}{T}}$$

$$frms6 = 7.487$$

Quadratic Wave 2

$A := 2$
$B := 4$
$C := 5$
$T := 3$
$\tau := 0.5$
$k := 0.4$

a)

$$Favg1 := \frac{A \cdot T^2}{3} + \frac{B \cdot T}{2} + C$$

$Favg1 = 17$

b)

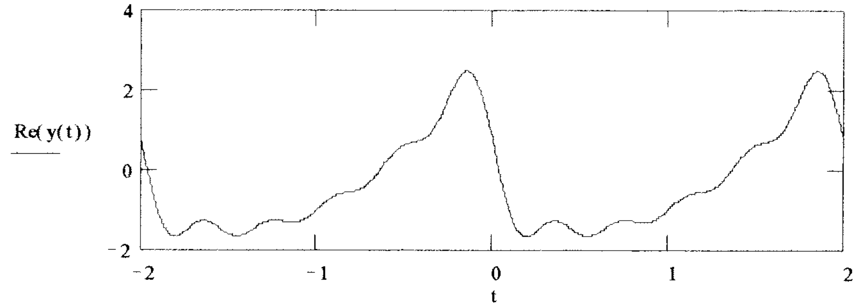
$$Favg1 := \frac{\int_0^T A \cdot t^2 + B \cdot t + C \, dt}{T}$$

$Favg1 = 17$

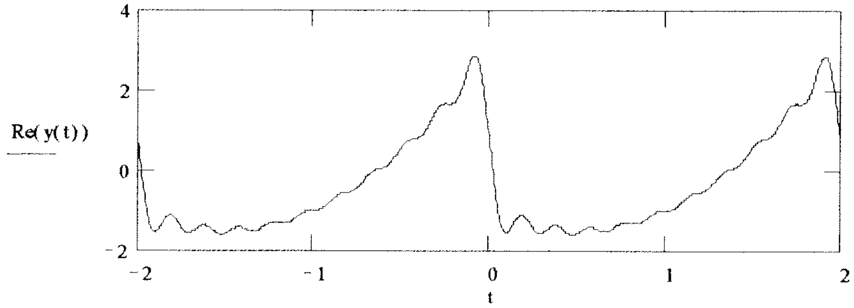
c)
The results from parts *a* and *b* are equal.

$A := 2 \qquad B := -2 \qquad C := -1 \qquad T := 2$

$$y(t) := \frac{A\cdot T^2}{3} + \frac{B\cdot T}{2} + C + \left[\sum_{n=1}^{5}\left[\frac{A\cdot T^2}{\pi^2\cdot n^2}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) - \left(\frac{A\cdot T^2 + B\cdot T}{\pi\cdot n}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]\right]$$



$$y(t) := \frac{A\cdot T^2}{3} + \frac{B\cdot T}{2} + C + \left[\sum_{n=1}^{10}\left[\frac{A\cdot T^2}{\pi^2\cdot n^2}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) - \left(\frac{A\cdot T^2 + B\cdot T}{\pi\cdot n}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]\right]$$



Page 420, Quadratic Wave #2

$$a_n \qquad \frac{2}{T}\cdot\int_0^T (A\cdot t^2+B\cdot t+C)\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T}\right)dt$$

$$\frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\frac{\left(2\cdot A\cdot\pi^2\cdot n^2\cdot\sin(2\cdot\pi\cdot n)\cdot T^2+2\cdot C\cdot\sin(2\cdot\pi\cdot n)\cdot\pi^2\cdot n^2+2\cdot A\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\cdot T^2+2\cdot B\cdot\pi^2\cdot n^2\cdot\sin(2\cdot\pi\cdot n)\cdot T-A\cdot\sin(2\cdot\pi\cdot n)\cdot T^2+B\cdot\pi\cdot n\cdot\cos(2\cdot\pi\cdot n)\cdot T\right)}{\left(\pi^3\cdot n^3\right)}-\frac{1}{4}\cdot\frac{B}{\left(\pi^2\cdot n^2\right)}\cdot T^2\right]$$

$$\frac{2}{T}\cdot\left[\frac{1}{4}\cdot T\cdot\frac{(2\cdot A\cdot\pi\cdot n\cdot T^2+B\cdot\pi\cdot n\cdot T)}{\left(\pi^3\cdot n^3\right)}-\frac{1}{4}\cdot\frac{B}{\left(\pi^2\cdot n^2\right)}\cdot T^2\right]$$

$$\frac{T^2}{\left(\pi^2\cdot n^2\right)}\cdot A$$

Ken Kaiser

$$b_n \qquad \frac{2}{T}\cdot\left[\int_0^T (A\cdot t^2+B\cdot t+C)\cdot\sin\left(2\cdot\pi\cdot n\cdot\frac{t}{T}\right)dt\right]$$

$$\frac{2}{T}\cdot\left[\frac{-T}{4}\cdot\frac{\left(2\cdot A\cdot\pi^2\cdot n^2\cdot\cos(2\cdot\pi\cdot n)\cdot T^2+2\cdot C\cdot\cos(2\cdot\pi\cdot n)\cdot\pi^2\cdot n^2-2\cdot A\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)\cdot T^2+2\cdot B\cdot\pi^2\cdot n^2\cdot\cos(2\cdot\pi\cdot n)\cdot T-A\cdot\cos(2\cdot\pi\cdot n)\cdot T^2-B\cdot\pi\cdot n\cdot\sin(2\cdot\pi\cdot n)\cdot T\right)}{\left(\pi^3\cdot n^3\right)}+\frac{T}{4}\cdot\frac{\left(-A\cdot T^2+2\cdot C\cdot\pi^2\cdot n^2\right)}{\left(\pi^3\cdot n^3\right)}\right]$$

$$\frac{2}{T}\cdot\left[\frac{-T}{4}\cdot\frac{\left[\left(2\cdot A\cdot\pi^2\cdot n^2\cdot T^2+2\cdot C\cdot\pi^2\cdot n^2+2\cdot B\cdot\pi^2\cdot n^2\cdot T\right)-A\cdot T^2\right]}{\left(\pi^3\cdot n^3\right)}+\frac{T}{4}\cdot\frac{\left(-A\cdot T^2+2\cdot C\cdot\pi^2\cdot n^2\right)}{\left(\pi^3\cdot n^3\right)}\right]$$

$$\frac{-T}{(\pi\cdot n)}\cdot(A\cdot T+B)$$

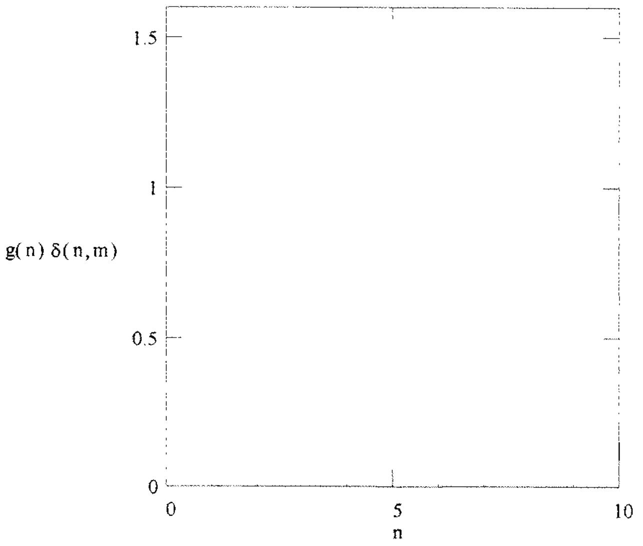
$$\left(\frac{A\cdot T^2}{3}+\frac{B\cdot T}{2}+C\right)+\left[\sum_{n=1}^{3}\left[\frac{A\cdot T^2}{\left(\pi^2\cdot n^2\right)}\cdot\cos\left(\frac{2\cdot\pi\cdot n\cdot t}{T}\right)-\frac{\left(A\cdot T^2+B\cdot T\right)}{(\pi\cdot n)}\cdot\sin\left(\frac{2\cdot\pi\cdot n\cdot t}{T}\right)\right]\right]$$

$$\frac{A\cdot T^2}{3}+\frac{B\cdot T}{2}+C+A\cdot\frac{T^2}{\pi^2}\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)-\frac{\left(A\cdot T^2+B\cdot T\right)}{\pi}\cdot\sin\left(2\cdot\pi\cdot\frac{t}{T}\right)+\frac{A\cdot T^2}{4\cdot\pi^2}\cdot\cos\left(4\cdot\pi\cdot\frac{t}{T}\right)-\frac{1}{2}\cdot\frac{\left(A\cdot T^2+B\cdot T\right)}{\pi}\cdot\sin\left(4\cdot\pi\cdot\frac{t}{T}\right)+\frac{A\cdot T^2}{9\cdot\pi^2}\cdot\cos\left(6\cdot\pi\cdot\frac{t}{T}\right)-\frac{1}{3}\cdot\frac{\left(A\cdot T^2+B\cdot T\right)}{\pi}\cdot\sin\left(6\cdot\pi\cdot\frac{t}{T}\right)$$

$n = 0,1..10 \quad m = -1,0..11 \quad B = -2 \quad A = 2 \quad C = -1 \quad t = 1,1.01..5 \quad T = 2$

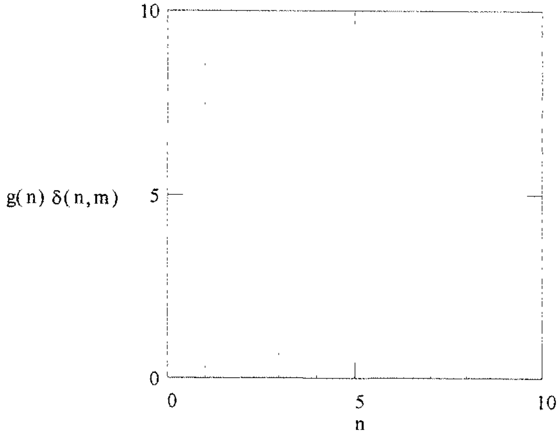
$$g(n) = \text{if}\left[n<1,\frac{A\cdot T^2}{3}+\frac{B\cdot T}{2}+C,\sqrt{\left(\frac{A\cdot T^2}{\pi^2\cdot n^2}\right)^2+\left(\frac{A\cdot T^2+B\cdot T}{\pi\cdot n}\right)^2}\right]$$

Ken Kaiser



$n = 0,1..10 \quad m = -1,0..11 \quad A = 8 \quad B = -2 \quad C = -1 \quad t = 1,1.01..5 \quad T = 2$

$$g(n) = \text{if}\left[n<1,\frac{A\cdot T^2}{3}+\frac{B\cdot T}{2}+C,\sqrt{\left(\frac{A\cdot T^2}{\pi^2\cdot n^2}\right)^2+\left(\frac{A\cdot T^2+B\cdot T}{\pi\cdot n}\right)^2}\right]$$

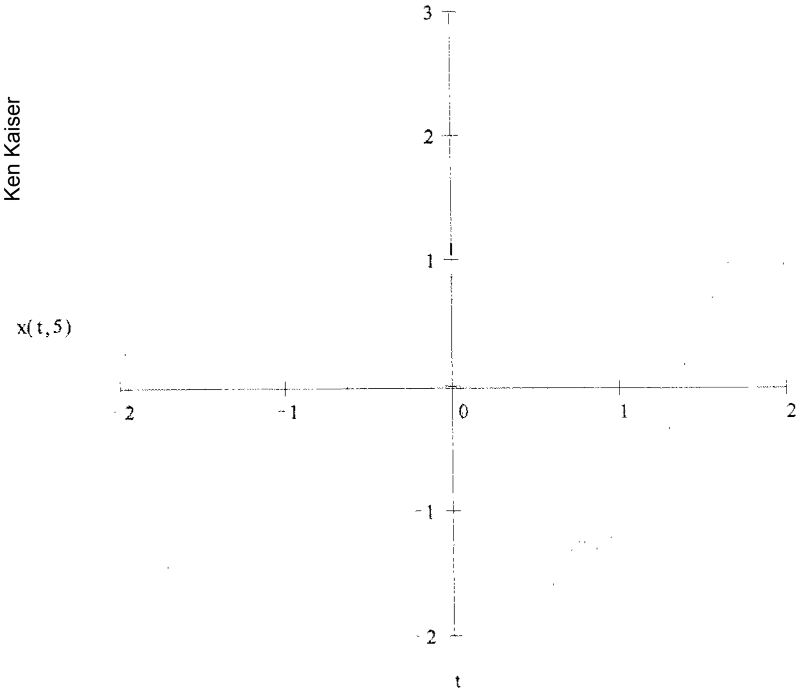


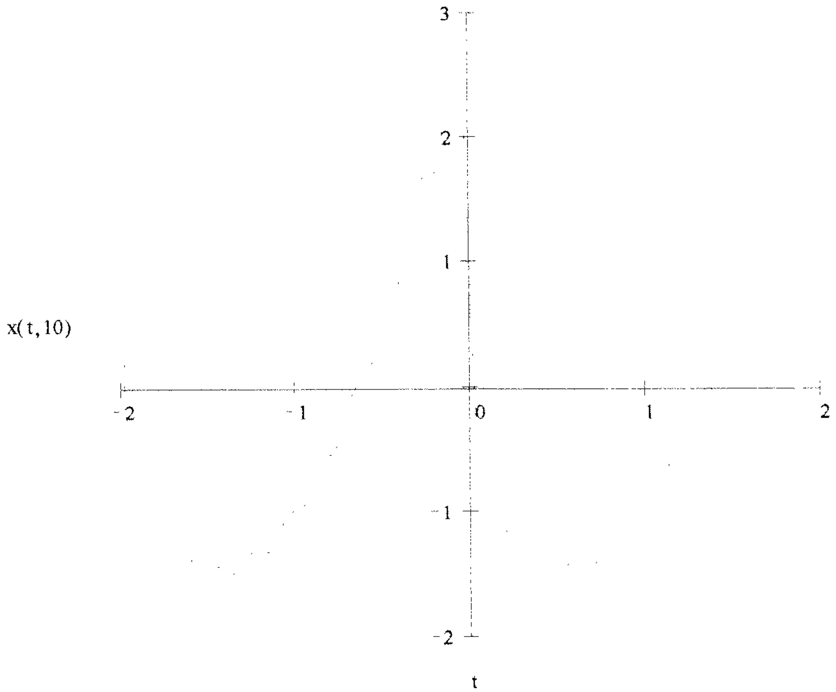
$m = -1, 0 .. 11 \quad n = 0, 1 .. 10 \quad T = 2 \quad A = 2 \quad B = -2 \quad C = -1$

$$x(t,m) = \left(\frac{A\cdot T^2}{3} + \frac{B\cdot T}{2} + C\right) + \left[\sum_{n=1}^{m}\left[\frac{A\cdot T^2}{(\pi^2\cdot n^2)}\cdot\cos\left(\frac{2\cdot\pi\cdot n\cdot t}{T}\right) - \frac{(A\cdot T^2 + B\cdot T)}{(\pi\cdot n)}\cdot\sin\left(\frac{2\cdot\pi\cdot n\cdot t}{T}\right)\right]\right]$$

$t = -T, -T + \frac{T}{1000} .. T$

Ken Kaiser





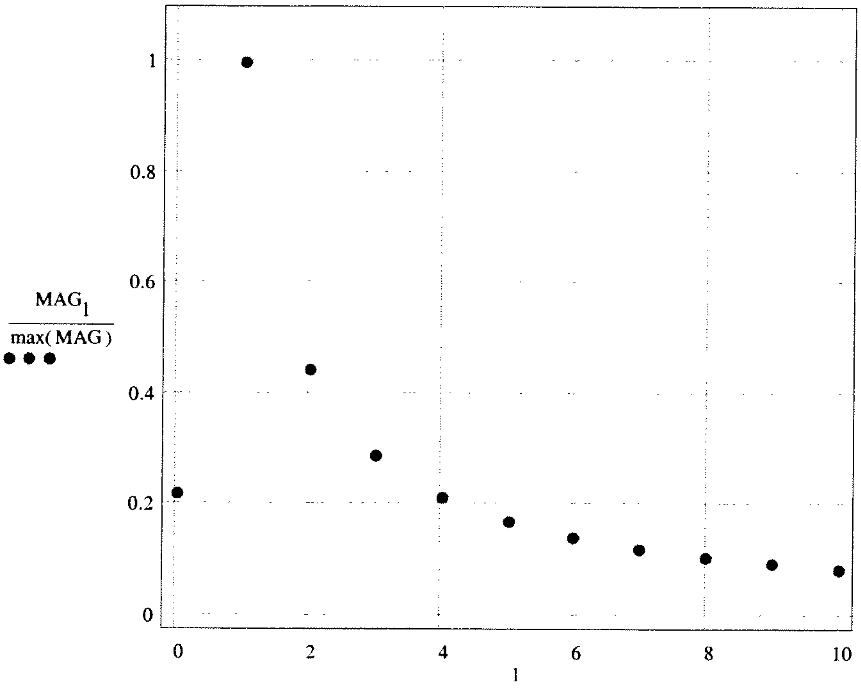
$k := 1..10$ $\qquad T := 2 \qquad A := 2 \qquad B := -2 \qquad C := -1$

$l := 0..10$ $\qquad \alpha := 0.2$

$$M_k := \sqrt{\left(\frac{A \cdot T^2}{\pi^2 \cdot k^2}\right)^2 + \left(\frac{A \cdot T^2 + B \cdot T}{\pi \cdot k}\right)^2}$$

$$M_0 := \frac{A \cdot T^2}{3} + \frac{B \cdot T}{2} + C$$

$$MAG_l := \left| M_l \right|$$



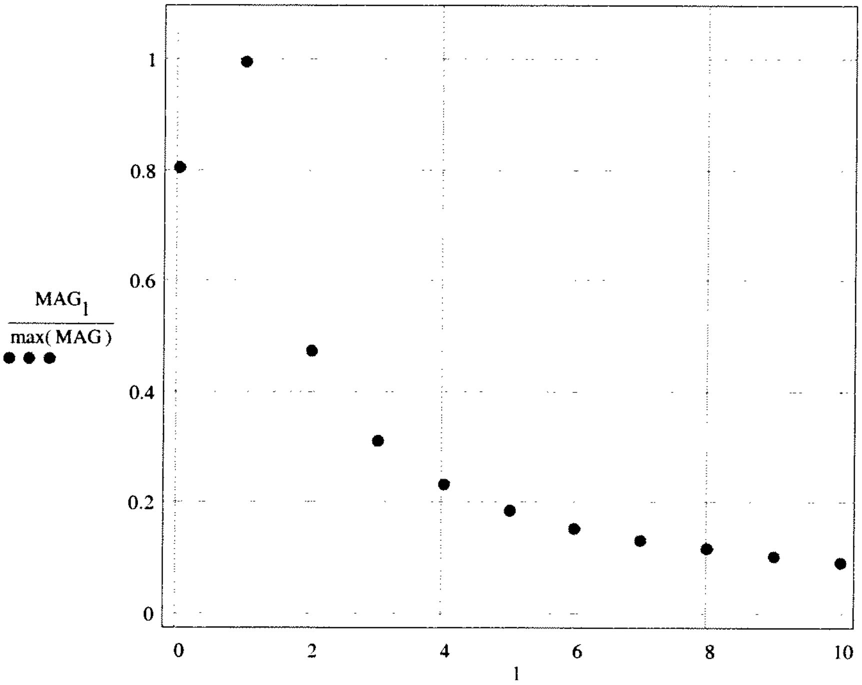
$k := 1..\ 10$ $\qquad T := 2 \qquad A := 8 \qquad B := -2 \qquad C := -1$

$l := 0..\ 10$ $\qquad \alpha := 0.2$

$$M_k := \sqrt{\left(\frac{A \cdot T^2}{\pi^2 \cdot k^2}\right)^2 + \left(\frac{A \cdot T^2 + B \cdot T}{\pi \cdot k}\right)^2}$$

$$M_0 := \frac{A \cdot T^2}{3} + \frac{B \cdot T}{2} + C$$

$$MAG_l := \left| M_l \right|$$



$A := 2$
$T := 3$
$a := 1$

Noninteger Cycles Sine Wave

Ken Kaiser
Tyler Clarke

$$f_{avg1} := \frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \sin(a \cdot t)\, dt \qquad f_{avg2} := 0$$

$$f_{avg1} = 0 \qquad f_{avg2} = 0$$

$$f_{rms1} := \sqrt{\frac{1}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} (A \cdot \sin(a \cdot t))^2\, dt}$$

$$f_{rms1} = 1.381$$

$$f_{rms2} := A \cdot \sqrt{\frac{1}{2} - \frac{\cos\left(\frac{a \cdot T}{2}\right) \cdot \sin\left(\frac{a \cdot T}{2}\right)}{a \cdot T}}$$

$$f_{rms2} = 1.381$$

Ken Kaiser

③ $f(t) = A\sin(\alpha t)$, $-\frac{T}{2} < t < \frac{T}{2}$ where $\frac{\alpha T}{2\pi} \neq$ integer

$f(0) = 0$

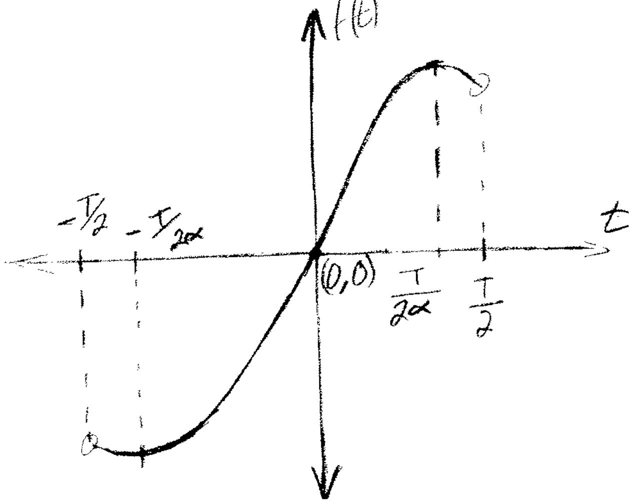
$f\left(\frac{T}{2}\right) =$ undefined

$f\left(-\frac{T}{2}\right) =$ undefined

$f'(t) = A\alpha\cos(\alpha t)$ max/min of function

if $\alpha t = \frac{T}{2}$ or $-\frac{T}{2}$ ...

$t = \frac{T}{2\alpha}$ or $-\frac{T}{2\alpha}$

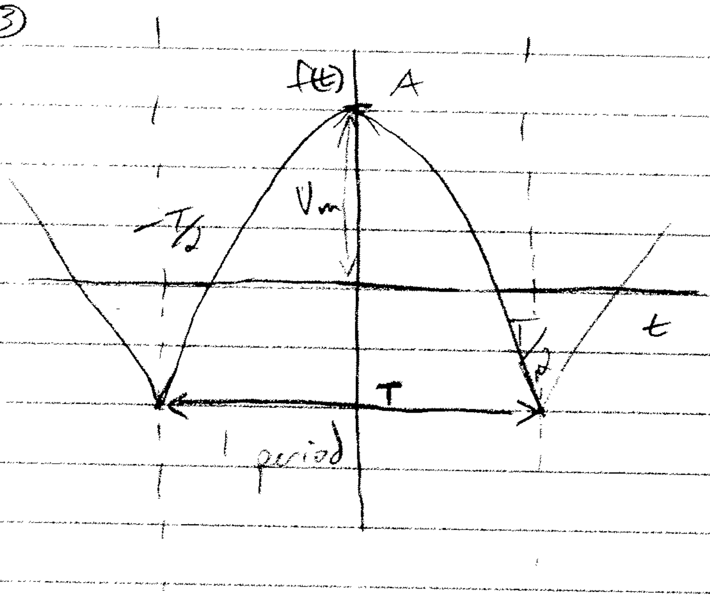


if $\alpha > 1$

if $\alpha < 1$

then the max & min would not be inside, closer to t=0, the unefined points at $\frac{T}{2}$ & $-\frac{T}{2} = t$

③



generic sinusoidal equation: $V_m \cos(\omega t + \phi)$

$\omega = 2\pi f$, $f = \frac{1}{T}$ $\phi = 0$ in this case because a cosine function has a peak at $t=0$ if $\phi=0$

$\omega = \frac{2\pi}{T} \equiv \alpha$

$\therefore$ for this function $f(t) = A\cos(\alpha t + 0)$

#3

f_{ave}

$$\frac{\int_{\frac{-T}{2}}^{\frac{T}{2}} A\cdot\cos(\alpha\cdot t)\,dt}{T} \to 2\cdot\frac{\sin\left(\frac{1}{2}\cdot T\cdot\alpha\right)}{\alpha}\cdot\frac{A}{T}$$

✓ is different from given value of:

$$\frac{AT}{2\alpha}\sin\left(\frac{\alpha T}{2}\right)$$

f_{rms}

$$\sqrt{\frac{\int_{\frac{-T}{2}}^{\frac{T}{2}} (A\cdot\cos(\alpha\cdot t))^2\,dt}{T}} \to \frac{1}{2}\cdot\sqrt{2}\cdot\sqrt{\left[A^2\cdot\frac{\left(2\cdot\cos\left(\frac{1}{2}\cdot T\cdot\alpha\right)\cdot\sin\left(\frac{1}{2}\cdot T\cdot\alpha\right)+T\cdot\alpha\right)}{(\alpha\cdot T)}\right]}$$

$$= A\sqrt{\frac{2}{4}\left(\frac{2\cos\left(\frac{T\alpha}{2}\right)\sin\left(\frac{T\alpha}{2}\right)}{\alpha T}+\frac{T\alpha}{\alpha T}\right)}$$

$$= A\sqrt{\frac{1}{2}\left(\frac{2\cos\left(\frac{T\alpha}{2}\right)\sin\left(\frac{T\alpha}{2}\right)}{\alpha T}+1\right)}$$

$$= A\sqrt{\frac{\cos\left(\frac{\alpha T}{2}\right)\sin\left(\frac{\alpha T}{2}\right)}{\alpha T}+\frac{1}{2}}$$

→ given value

Pg. 421 (1)

$$An \quad \frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\sin(\alpha\cdot t)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

0

$$Bn \quad \frac{2}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\sin(\alpha\cdot t)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

$$-2\cdot\left(-\sin\left(\frac{1}{2}\cdot\alpha\cdot T-\pi\cdot n\right)\cdot\alpha\cdot T-2\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T-\pi\cdot n\right)\cdot\pi\cdot n+\sin\left(\frac{1}{2}\cdot\alpha\cdot T+\pi\cdot n\right)\cdot\alpha\cdot T-2\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T+\pi\cdot n\right)\cdot\pi\cdot n\right)\cdot\frac{A}{((\alpha\cdot T+2\cdot\pi\cdot n)\cdot(\alpha\cdot T-2\cdot\pi}$$

$$-2\cdot\left(-\sin\left(\frac{1}{2}\cdot\alpha\cdot T-\pi\cdot n\right)\cdot\alpha\cdot T-2\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T-\pi\cdot n\right)\cdot\pi\cdot n+\sin\left(\frac{1}{2}\cdot\alpha\cdot T+\pi\cdot n\right)\cdot\alpha\cdot T-2\cdot\sin\left(\frac{1}{2}\cdot\alpha\cdot T+\pi\cdot n\right)\cdot\pi\cdot n\right)\cdot\frac{A}{\left(\alpha^2\cdot T^2-4\cdot\pi^2\cdot n^2\right)}$$

Hand Calc's to show -> $\dfrac{8\cdot A\cdot\pi\cdot(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2-\alpha^2\cdot T^2}$ ✓

$$-\sin\left(\tfrac{1}{2}\alpha T-\pi n\right)\alpha T+\sin\left(\tfrac{1}{2}\alpha T+\pi n\right)$$
$$-\sin\left(\tfrac{1}{2}\alpha T+\pi n\right)\alpha T+\sin\left(\tfrac{1}{2}\alpha T+\pi n\right)$$
$$=0$$

"- πn" shift equals "πn" shift; its just the difference between shifting left and right.

$$-2\sin\left(\tfrac{1}{2}\alpha T-\pi n\right)\cdot\pi n-2\sin\left(\tfrac{1}{2}\alpha T+\pi n\right)\cdot\pi$$

180° phase shift 180° phase

$$-4(-1)^n\sin\left(\frac{\alpha T}{2}\right)\cdot\pi n$$

When n = 1, (odd) the phase will shift 180° and the answer will switch signs. When n = 0, 2, 4, the shift will be 2π, and the sign will be unchanged.

$$\left(-4(-1)^n\sin\left(\frac{\alpha T}{2}\right)\cdot\pi n\right)\frac{-2A}{\alpha^2T^2-4\pi^2n^2}$$

$$=\frac{8A\pi(-1)^n\cdot n\cdot\sin\left(\frac{\alpha T}{2}\right)}{\alpha^2T^2-4n^2\pi^2}$$

$$=\frac{8A\pi(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha T}{2}\right)}{4n^2\pi^2-\alpha^2T^2}$$

$$\sum_{n=1}^{3}\frac{8\cdot A\cdot\pi\cdot(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2-\alpha^2\cdot T^2}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$

$$\boxed{8\cdot A\cdot\pi\cdot\frac{\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(4\cdot\pi^2-\alpha^2\cdot T^2\right)}\cdot\sin\left(2\cdot\frac{\pi}{T}\cdot t\right)-16\cdot A\cdot\pi\cdot\frac{\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(16\cdot\pi^2-\alpha^2\cdot T^2\right)}\cdot\sin\left(4\cdot\frac{\pi}{T}\cdot t\right)+24\cdot A\cdot\pi\cdot\frac{\sin\left(\frac{1}{2}\cdot\alpha\cdot T\right)}{\left(36\cdot\pi^2-\alpha^2\cdot T^2\right)}\cdot\sin\left(6\cdot\frac{\pi}{T}\cdot t\right)}$$

multiply top + btm by $(-1)^1$ to get in the form given on Pg. 389.

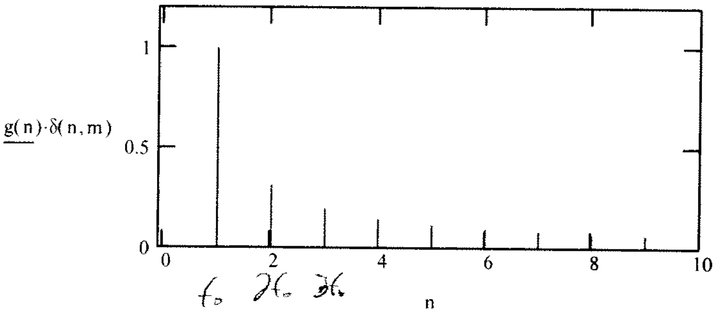
Ken Kaiser

Pg. 421(2)

$m := -1, 0 .. 11 \quad A := 1 \quad \alpha := 4.2 \quad T := 1$

$n := 0, 1 .. 10$

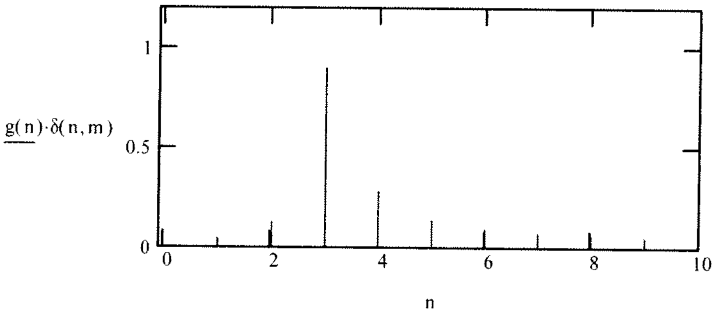
$$g(n) := \mathrm{if}\left[n<1, 0, \left|\frac{8\cdot A\cdot\pi\cdot(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2-\alpha^2\cdot T^2}\right|\right]$$



$\alpha\cdot T = 4.2$

$\alpha := 20.2$

$$g(n) := \mathrm{if}\left[n<1, 0, \left|\frac{8\cdot A\cdot\pi\cdot(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2-\alpha^2\cdot T^2}\right|\right]$$



$\alpha\cdot T = 20.2$

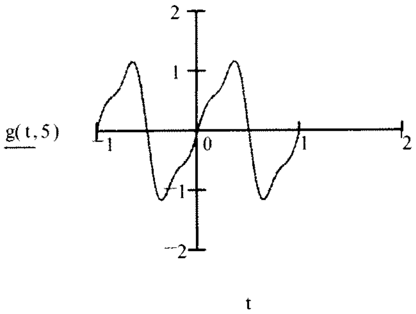
Pg 421

Ken Kaiser

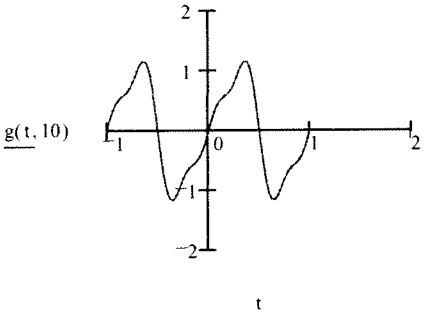
$$T := 1 \qquad t := -T, -T + \frac{T}{1000} .. T \qquad \alpha := 4.2$$

$$g(t,m) := \sum_{n=1}^{3} \frac{8 \cdot A \cdot \pi \cdot (-1)^{n+1} \cdot n \cdot \sin\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 - \alpha^2 \cdot T^2} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$

n=5



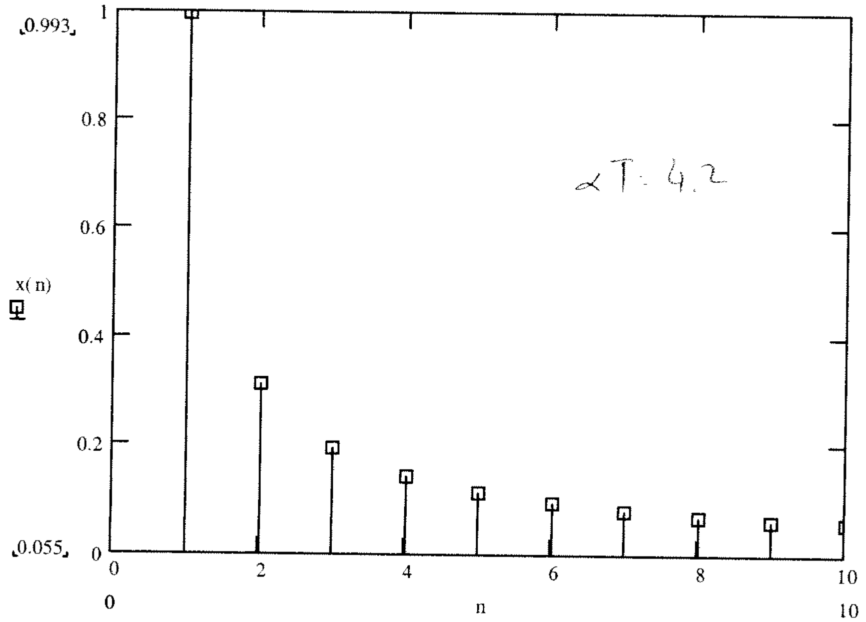
n=10



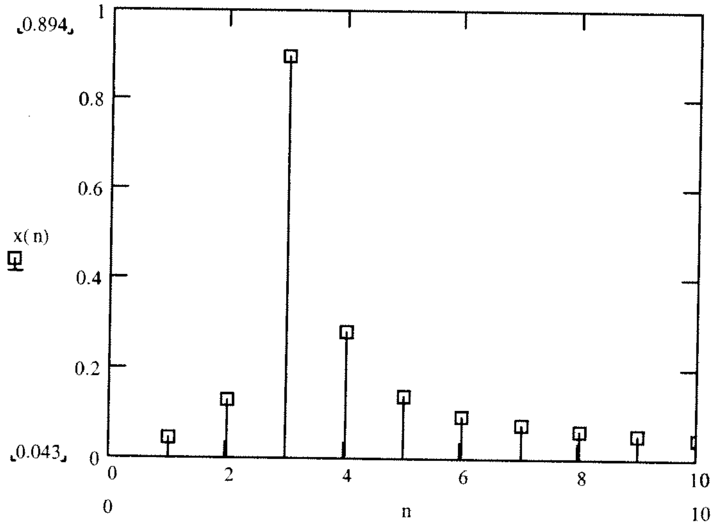
Values of α and T affect the appearance of the graphs.

Ken Kaiser

Sine Wave



αT: 4.2



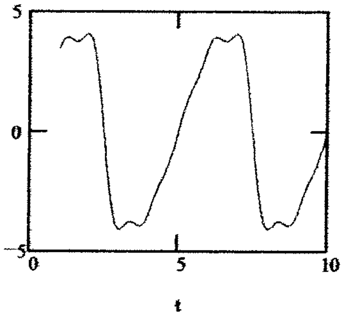
αT=20.2

Ken Kaiser

Noninteger Cycles Sine Wave

$A := 4 \qquad T := 5 \qquad \tau := 2 \qquad t := 1, 1.0001 .. 10 \qquad w := 1 \qquad \alpha := 1$

$$8\cdot A\cdot\pi\left[\sum_{n=1}^{5}\frac{(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^{2}\cdot\pi^{2}-\alpha^{2}\cdot T^{2}}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]$$



$$8\cdot A\cdot\pi\cdot\sum_{n=1}^{10}\frac{(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^{2}\cdot\pi^{2}-\alpha^{2}\cdot T^{2}}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$