p400: Quarter-Rectified Cosine Wave

F_n

$$\frac{1}{T}\cdot\int_0^{\frac{T}{4}} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}\,dt+\frac{1}{T}\cdot\int_{\frac{T}{4}}^{T}(0)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}\,dt$$

$$\frac{AT}{2\pi T}\;\frac{-e^{-j\frac{\pi n}{2}}-jN}{N^2-1}$$

$$\frac{A}{2\pi}\left[\frac{jnTe^{-j\frac{\pi n}{2}}}{1-n^2}\right]$$

$$\frac{1}{T}\cdot\left[\overset{\downarrow}{\frac{-1}{(2\cdot\pi)}}\cdot T\cdot\exp\left(\frac{-1}{2}\cdot i\cdot\pi\cdot n\right)\cdot\frac{A}{\left(n^2-1\right)}\overset{\downarrow}{-}\frac{1}{2}\cdot\frac{i}{\pi}\cdot T\cdot n\cdot\frac{A}{\left(n^2-1\right)}\right]$$

$$=\frac{jn\cancel{+}e^{j\frac{\pi n}{2}}}{1-n^2}$$

$n := -1$

$$\frac{1}{T}\cdot\int_0^{\frac{T}{4}} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot -1}{T}\cdot t\right)}\,dt+\frac{1}{T}\cdot\int_{\frac{T}{4}}^{T}(0)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot -1}{T}\cdot t\right)}\,dt$$

$$\frac{1}{\cancel{T}}\cdot\left(\frac{1}{8}\cdot\cancel{T}\cdot A+\frac{1}{4}\cdot i\cdot\cancel{T}\cdot\frac{A}{\pi}\right)\checkmark$$

$$=\frac{A}{8}\left(1+\frac{2j}{\pi}\right)\checkmark$$

$n := 1$

$$\frac{1}{T}\cdot\int_0^{\frac{T}{4}} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot 1}{T}\cdot t\right)}\,dt+\frac{1}{T}\cdot\int_{\frac{T}{4}}^{T}(0)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot 1}{T}\cdot t\right)}\,dt$$

$$\frac{1}{\cancel{T}}\cdot\left(\frac{1}{8}\cdot\cancel{T}\cdot A-\frac{1}{4}\cdot i\cdot\cancel{T}\cdot\frac{A}{\pi}\right)\checkmark$$

$$\frac{A}{8}\left(1-\frac{2j}{\pi}\right)\checkmark$$

$n := 0$

$$\frac{1}{T}\cdot\int_0^{\frac{T}{4}} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot 0}{T}\cdot t\right)}\,dt+\frac{1}{T}\cdot\int_{\frac{T}{4}}^{T}(0)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot 0}{T}\cdot t\right)}\,dt$$

$$\frac{1}{(2\cdot\pi)}\cdot A \checkmark \checkmark$$

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$$\frac{1}{(2\cdot\pi)}\cdot A+\frac{A}{8}\cdot\left(1+\sqrt{-1}\cdot\frac{2}{\pi}\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\pi}{T}\cdot t\right)}+\frac{A}{8}\cdot\left(1-\sqrt{-1}\cdot\frac{2}{\pi}\right)\cdot e^{\sqrt{-1}\cdot\left(\frac{2\pi}{T}\cdot t\right)}+\left[\sum_{n=2}^{\infty}\frac{n\cdot\sqrt{-1}+e^{-\sqrt{-1}\cdot\frac{\pi\cdot n}{2}}}{1-n^2}\cdot e^{\sqrt{-1}\cdot\left(\frac{2\pi\cdot n}{T}\cdot t\right)}+\sum_{n=-\infty}^{-2}\frac{n\cdot\sqrt{-1}+e}{1-}\right.$$

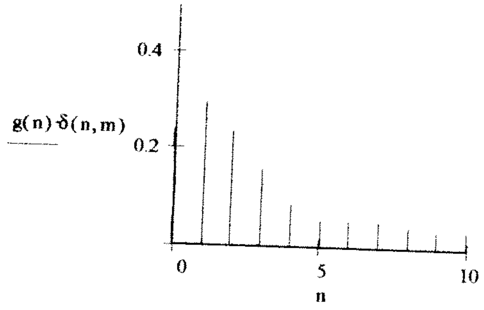
$T := 1$

$A := 1$

$m := -1, 0 .. 11$

$n := 0, 1 .. 10$

$$g(n) := \mathrm{if}\left[n\le 0,\frac{1}{2\cdot\pi},\mathrm{if}\left[n\le 1,2\cdot\left|\frac{1}{8}\cdot\left(1-\sqrt{-1}\cdot\frac{2}{\pi}\right)\right|,2\cdot\left|\frac{1}{T}\cdot\left[\frac{-1}{(2\cdot\pi)}\cdot T\cdot\exp\left(\frac{-1}{2}\cdot i\cdot\pi\cdot n\right)\cdot\frac{A}{\left(n^2-1\right)}-\frac{1}{2}\cdot\frac{i}{\pi}\cdot T\cdot n\cdot\frac{A}{\left(n^2-1\right)}\right]\right|\right]\right]$$



$n = 0, \ \frac{A}{2\pi}$

$$\frac{{}^{-\sqrt{-1}\cdot\frac{\pi\cdot n}{2}}}{n^{2}}\cdot e^{\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}\Bigg|$$

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$n := 1 .. 10$

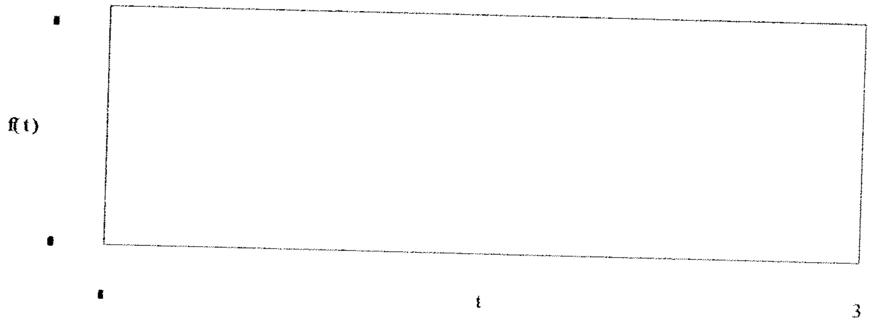
$t := 1, 1.01 .. 5$

$A := 1$

$T := 1$

$n = 5$

$$f(t) := \frac{1}{(2\cdot\pi)}\cdot A + \frac{A}{8}\cdot\left(1+\sqrt{-1}\cdot\frac{2}{\pi}\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\pi}{T}\cdot t\right)} + \frac{A}{8}\cdot\left(1-\sqrt{-1}\cdot\frac{2}{\pi}\right)\cdot e^{\sqrt{-1}\cdot\left(\frac{2\pi}{T}\cdot t\right)} + \left[\sum_{n=2}^{5}\frac{1}{T}\cdot\left[\frac{-1}{(2\cdot\pi)}\cdot T\cdot\exp\left(\frac{-1}{2}\cdot i\cdot\pi\cdot n\right)\cdot\frac{A}{\left(n^2-1\right)} - \frac{1}{2}\cdot\frac{i}{\pi}\cdot\right.\right.$$



complex result won't plot

Need $\sum_{N=5}^{-2}$ and $\sum_{N=2}^{5}$ ✓

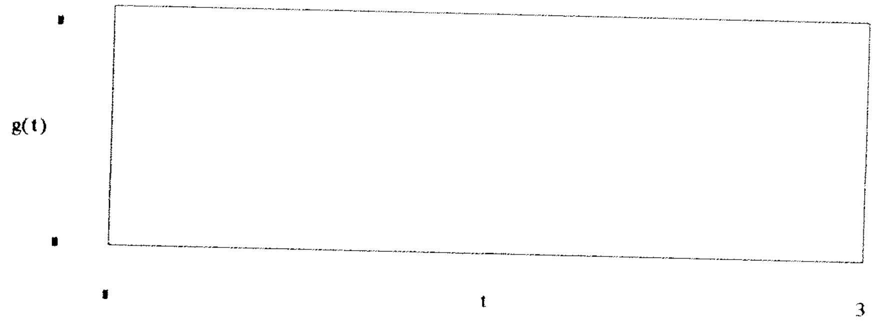
Summation as shown in table

I also checked against exact eqn - numerically

$$\Gamma\cdot n\cdot\frac{A}{\left(n^2-1\right)}\Bigg]\cdot e^{\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}+\sum_{n=-5}^{-2}\frac{1}{T}\cdot\left[\frac{-1}{(2\cdot\pi)}\cdot T\cdot\exp\left(\frac{-1}{2}\cdot i\cdot\pi\cdot n\right)\cdot\frac{A}{\left(n^2-1\right)}-\frac{1}{2}\cdot\frac{i}{\pi}\cdot T\cdot n\cdot\frac{A}{\left(n^2-1\right)}\right]\cdot e^{\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}\Bigg]$$

n = 10

$$g(t) = \frac{1}{(2\cdot\pi)}\cdot A + \frac{A}{8}\cdot\left(1+\sqrt{-1}\cdot\frac{2}{\pi}\right)\cdot e^{-\sqrt{-1}\cdot\left(\frac{2\cdot\pi}{T}\cdot t\right)} + \frac{A}{8}\cdot\left(1-\sqrt{-1}\cdot\frac{2}{\pi}\right)\cdot e^{\sqrt{-1}\cdot\left(\frac{2\cdot\pi}{T}\cdot t\right)} + \Bigg[\sum_{n=2}^{10}\frac{1}{T}\cdot\bigg[\frac{-1}{(2\cdot\pi)}\cdot T\cdot \exp\left(\frac{-1}{2}\cdot i\cdot\pi\cdot n\right)\cdot\frac{A}{\left(n^{2}-1\right)} - \frac{1}{2}\cdot\frac{i}{\pi}$$



$$\cdot T\cdot n\cdot\frac{A}{\left(n^{2}-1\right)}\Bigg]\cdot e^{\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}+\sum_{n=-10}^{-2}\frac{1}{T}\cdot\left[\left[\frac{-1}{(2\cdot\pi)}\cdot T\cdot\exp\left(\frac{1}{2}\cdot i\ \ \cdot\pi\cdot n\right)\cdot\frac{A}{\left(n^{2}-1\right)}-\frac{1}{2}\cdot\frac{i}{\pi}\cdot T\cdot n\cdot\frac{A}{\left(n^{2}-1\right)}\right]\cdot e^{\sqrt{-1}\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}\right]$$

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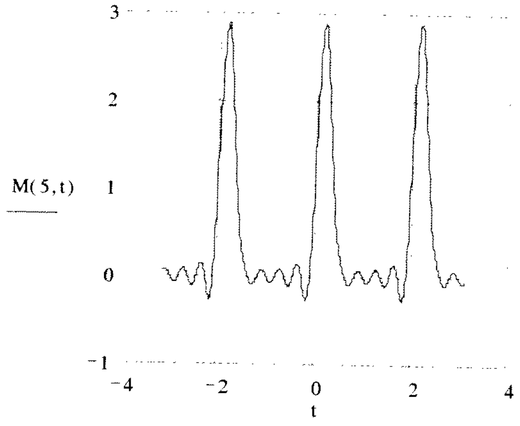
$j := \sqrt{-1}$ $\quad T := 2 \quad A := 2.98 \quad \tau := \frac{T}{3} \quad t := -\pi, -\pi + 0.01 .. \pi$

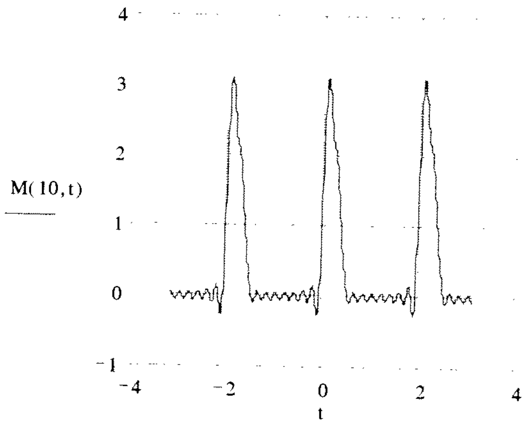
$$f(n,t) := \frac{A}{2\cdot\pi}\cdot\frac{j\cdot n + e^{-j\cdot\frac{\pi\cdot n}{2}}}{1-n^2}\cdot e^{j\cdot\frac{2\cdot\pi\cdot n}{T}\cdot t} + \frac{A}{2\cdot\pi}\cdot\frac{j\cdot -n + e^{-j\cdot\frac{\pi\cdot -n}{2}}}{1-n^2}\cdot e^{j\cdot\frac{2\cdot\pi\cdot -n}{T}\cdot t}$$

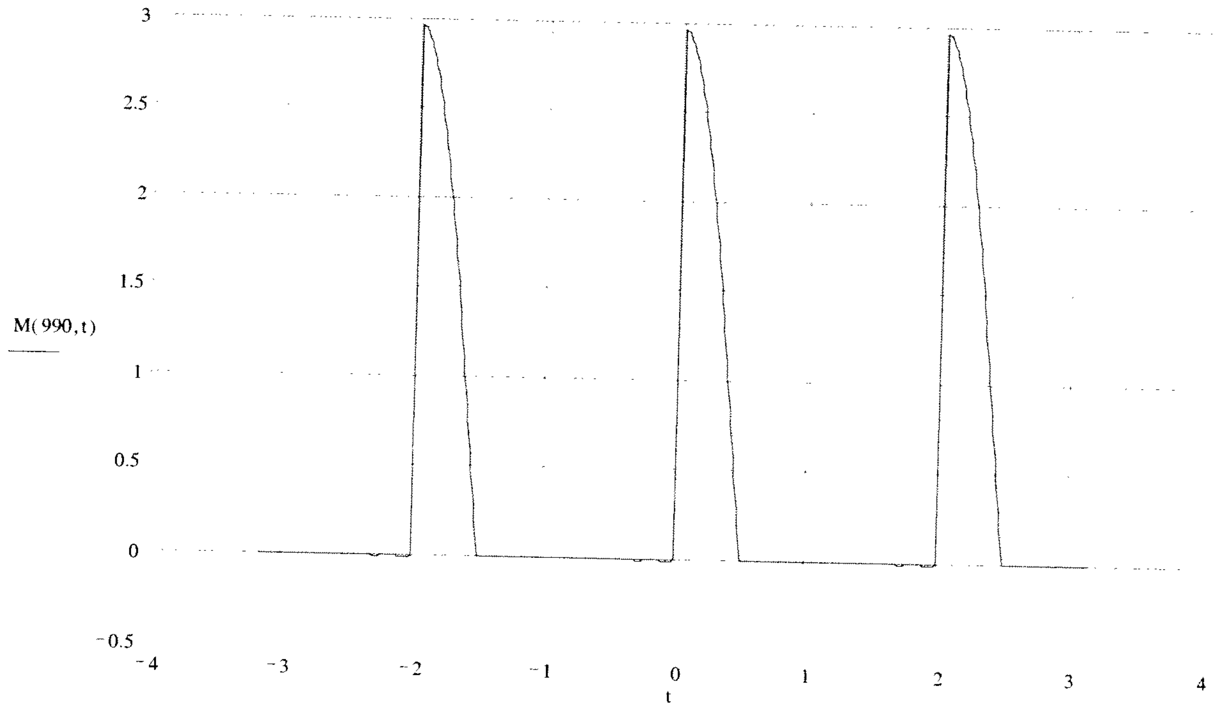
$$M_0 := \frac{A}{2\cdot\pi} \qquad M1(t) := \frac{A}{8}\cdot\left[\left(1 - j\cdot\frac{2}{\pi}\right)\cdot e^{j\cdot\frac{2\cdot\pi}{T}\cdot t}\right] + \frac{A}{8}\cdot\left[\left(1 + j\cdot\frac{2}{\pi}\right)\cdot e^{-j\cdot\frac{2\cdot\pi}{T}\cdot t}\right]$$

$$M(k,t) := \sum_{g=2}^{k} f(g,t) + M_0 + M1(t)$$

Interesting note: To eliminate the downward spike at the beginning of each quarter-cosine, it is necessary to use more than 980 terms. 990 works, 980 isn't enough.







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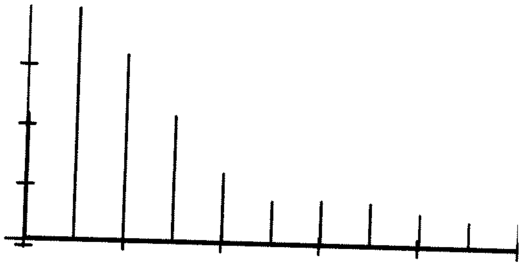
$m := -1, 0 .. 11$ $\qquad$ $j := \sqrt{-1}$

$n := 0, 1 .. 10$ $\qquad$ $A := 1$

$$g(n) := \mathrm{if}\left[n < 1, \frac{A}{2\cdot\pi}, \mathrm{if}\left[n < 2, \left|\frac{A}{8}\cdot\left(\frac{2}{\pi}\cdot j + 1\right)\right|\cdot 2, \frac{2\cdot A}{2\cdot\pi}\cdot\left|\frac{j\cdot n + e^{-j\cdot\pi\cdot\frac{n}{2}}}{1 - n^2}\right|\right]\right]$$



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HALF PERIOD COSINE WAVE

$$f_{avg} = \frac{1}{(T)} \cdot \int_{0}^{T} A \cdot \cos\left(\frac{\pi}{T} \cdot t\right) dt$$

$$\to 0$$

NOTICE: T/2 was changed to T. Function limits given in problem were incorrect.

$$f_{rms} = \left[\frac{1}{T} \cdot \int_{0}^{T} \left(A \cdot \cos\left(\frac{\pi}{T} \cdot t\right)\right)^{2} dt\right]^{\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \sqrt{2} \cdot A = \frac{\sqrt{2}}{2} A \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{A}{\sqrt{2}} = 0.71 A$$

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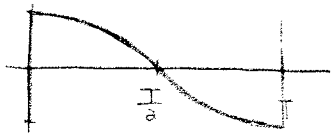
HALF - PEROID COSINE WAVE



if we plug 0 into the equation $A \cos\left(\frac{\pi}{T} t\right)$ the point goes to A

at $\frac{T}{2}$ the graph would display 0

and at T the graph will display -A, so the equation matches the graph

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ECE-210
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F_{avg} for Half-Peroid Cosine Wave

$$\frac{1}{T}\cdot\int_0^T A\cdot\cos\left(\frac{\pi}{T}\cdot t\right)dt$$

0

F_{rms} for Half-Period Cosine Wave

$$\sqrt{\left[\frac{1}{T}\cdot\left[\int_0^T \left(A\cdot\cos\left(\frac{\pi}{T}\cdot t\right)\right)^2 dt\right]\right]}$$

$\frac{1}{2}\cdot\sqrt{2}\cdot A$

$\approx 0.7071\cdot A$

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$A := 2$

$B := 4$

$C := 5$

$T := 3$

$a := 1$

$\tau := 0.5$

$k := 0.4$

$f_{avg} := 0$

$$f_{rms} := \frac{A}{\sqrt{2}}$$

$f_{avg} = 0$

$f_{rms} = 1.414$

$$f(t) := A \cdot \cos\left(\frac{\pi}{T} \cdot t\right)$$

$$avg := \frac{1}{T} \cdot \int_0^T f(t)\, dt$$

$avg = 0$

$$rms := \sqrt{\frac{1}{T} \cdot \int_0^T f(t)^2\, dt}$$

$rms = 1.414$

Proof of P401

An terms:

$$\frac{2}{T}\cdot\int_{0}^{T} A\cdot\cos\frac{\pi}{T}\cdot t\ \cdot\cos\frac{2\cdot\pi\cdot n}{T}\cdot t\ dt$$

$$\sin\ \pi\cdot(\ 1 - 2\cdot n)\ \cdot\ 2\cdot\sin\ \pi\cdot(\ 1\ \cdot\ 2\cdot n)\ \cdot n\quad \sin\ \pi\cdot(1\ \cdot\ 2\cdot n)\ -\ 2\cdot\sin\ \pi\cdot(1\ \cdot\ 2\cdot n)\ \cdot n\ \cdot\ \frac{A}{\pi\cdot((\ 1\ \cdot\ 2\cdot n)\cdot(1 - 2\cdot n))}$$

$$4\cdot\sin\ 2\cdot\pi\cdot n\ \cdot n\cdot\frac{A}{\pi\cdot\ \ 1 - 4\cdot n^2}$$

$$4\cdot 0\cdot n\cdot\frac{A}{\pi\cdot\ \ 1 - 4\cdot n^2}$$

0 This also implies that A0 = 0

Bn terms:

$$\frac{2}{T}\cdot\int_{0}^{T} A\cdot\cos\frac{\pi}{T}\cdot t\ \cdot\sin\frac{2\cdot\pi\cdot n}{T}\cdot t\ dt$$

$$\frac{2}{T}\cdot\frac{1}{2}\cdot T\cdot\ \cos\ \pi\cdot(1 - 2\cdot n)\ \cdot\ 2\cdot\cos\ \pi\cdot(1 - 2\cdot n)\ \cdot n\ \cdot\ \cos\ \pi\cdot(\ 1 - 2\cdot n)\ -\ 2\cdot\cos\ \pi\cdot(\ 1 - 2\cdot n)\ \cdot n\ \cdot\ \frac{A}{\pi\cdot((1 - 2\cdot n)\cdot(\ 1\ \cdot\ 2\cdot n))}$$

$$4\cdot n\cdot A\cdot\frac{\cos\ 2\cdot\pi\cdot n\ \ \cdot\ 1}{\pi\cdot\ \ 1 - 4\cdot n^2}$$

$$4\cdot n\cdot A\cdot\frac{(1\ \cdot\ 1)}{\pi\cdot\ \ 1\ \cdot\ 4\cdot n^2}$$

$$8\cdot A\cdot\frac{n}{\pi\cdot\ \ 1\ \cdot\ 4\cdot n^2}$$

Plug An & Bn back into the approximation to get:

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$n \quad 1..3$

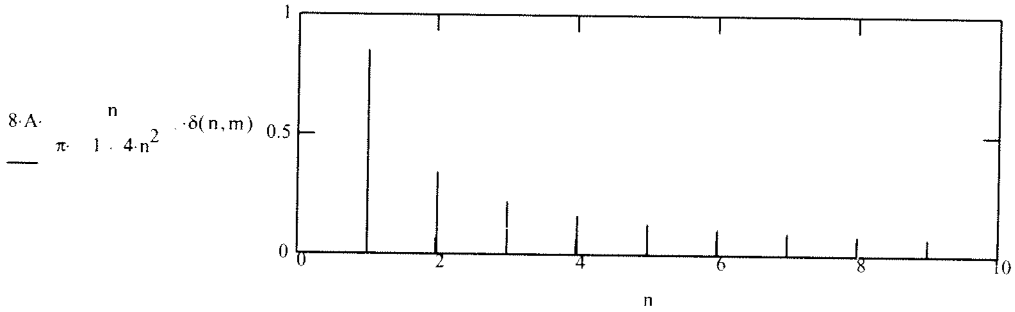
$$\sum_{n=1}^{3} 8\cdot A\cdot \frac{n}{\pi\cdot \quad 1 \cdot 4\cdot n^{2}} \cdot\sin \frac{2\cdot\pi\cdot n}{T}\cdot t$$

$$\frac{8}{105}\cdot A\cdot \frac{35\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t \ - \ 14\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t \ - \ 9\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t}{\pi}$$

$A \quad 1$

$m \quad 1,0..11$

$n \quad 0,1..10$



$$\sum_{n=1}^{3} 8\cdot A\cdot \frac{n}{\pi\cdot \quad 1 \cdot 4\cdot n^{2}} \cdot\sin \frac{2\cdot\pi\cdot n}{T}\cdot t$$

$$\frac{A\cdot 8}{\pi}\cdot \sum_{n=1}^{3} \frac{n}{1 \cdot 4\cdot n^{2}} \cdot\sin \frac{2\cdot\pi\cdot n}{T}\cdot t$$

$$\frac{8}{105}\cdot A\cdot \frac{35\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t \ - \ 14\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t \ - \ 9\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t}{\pi}$$

$$\frac{8\cdot A}{105\cdot\pi}\cdot \ 35\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t \ - \ 14\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t \ - \ 9\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t$$

$$\frac{8\cdot 35\cdot A}{105\cdot\pi}\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t \quad + \quad \frac{8\cdot 14\cdot A}{105\cdot\pi}\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t \quad + \quad \frac{8\cdot 9\cdot A}{105\cdot\pi}\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t \quad +\ldots$$

$$\frac{8}{3}\cdot\frac{A}{\pi}\cdot\sin\ 2\cdot\frac{\pi}{T}\cdot t \quad + \quad \frac{16}{15}\cdot\frac{A}{\pi}\cdot\sin\ 4\cdot\frac{\pi}{T}\cdot t \quad + \quad \frac{24}{35}\cdot\frac{A}{\pi}\cdot\sin\ 6\cdot\frac{\pi}{T}\cdot t \quad +\ldots$$

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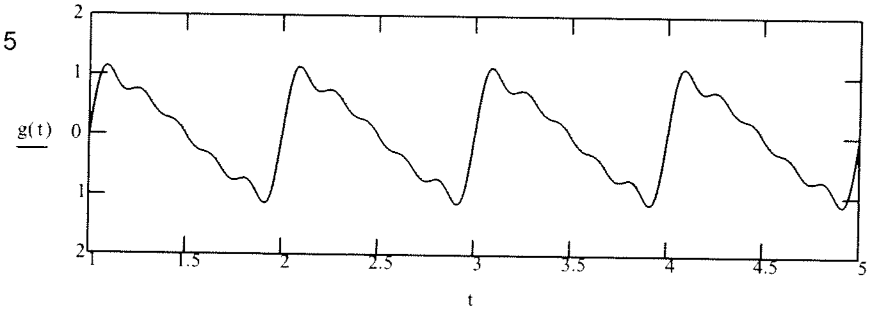
n 1 .. 10

t 1, 1.01 .. 5

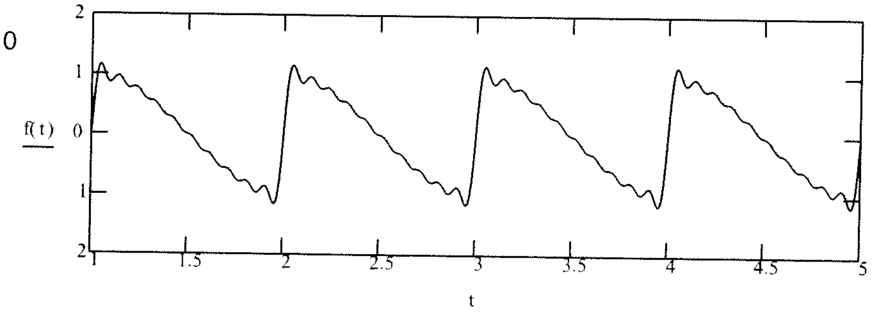
$$g(t)\quad \frac{A\cdot 8}{\pi}\cdot\sum_{n=1}^{5}\frac{n}{1-4\cdot n^{2}}\cdot\sin\frac{2\cdot\pi\cdot n}{1}\cdot t$$

n = 5



$$f(t)\quad \frac{A\cdot 8}{\pi}\cdot\sum_{n=1}^{10}\frac{n}{1\cdot 4\cdot n^{2}}\cdot\sin\frac{2\cdot\pi\cdot n}{1}\cdot t$$

n = 10



P. 401

n=1 $$\left(\frac{8A}{\pi}\right)\left(\frac{1}{4-1}\right)\sin\frac{2\pi}{T}t = \frac{8A}{3\pi}\sin\frac{2\pi}{T}t$$ Ved

n=2 $$\frac{8A}{\pi}\left(\frac{2}{16-1}\right)\sin\left(\frac{4\pi}{T}t\right) = \frac{16A}{15}\sin\frac{4\pi}{T}t$$ Ved

n=3 $$\frac{8A}{\pi}\left(\frac{3}{4\cdot 9-1}\right)\sin\left(\frac{6\pi}{T}t\right) = \frac{24A}{35\pi}\sin\left(\frac{6\pi}{T}t\right)$$ Ved

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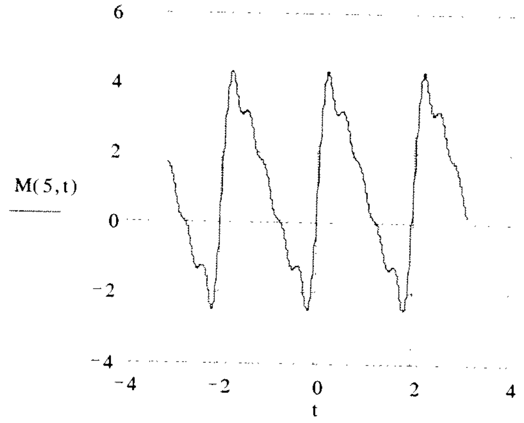
$T := 2$ $\qquad A := 2.98$ $\qquad \tau := \frac{T}{3}$ $\qquad t := -\pi, -\pi + 0.01 .. \pi$

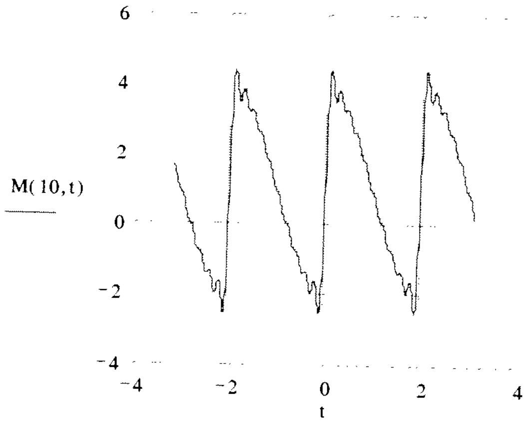
$$f(n,t) := \frac{8 \cdot A}{\pi} \cdot \frac{n}{4 \cdot n^2 - 1} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$

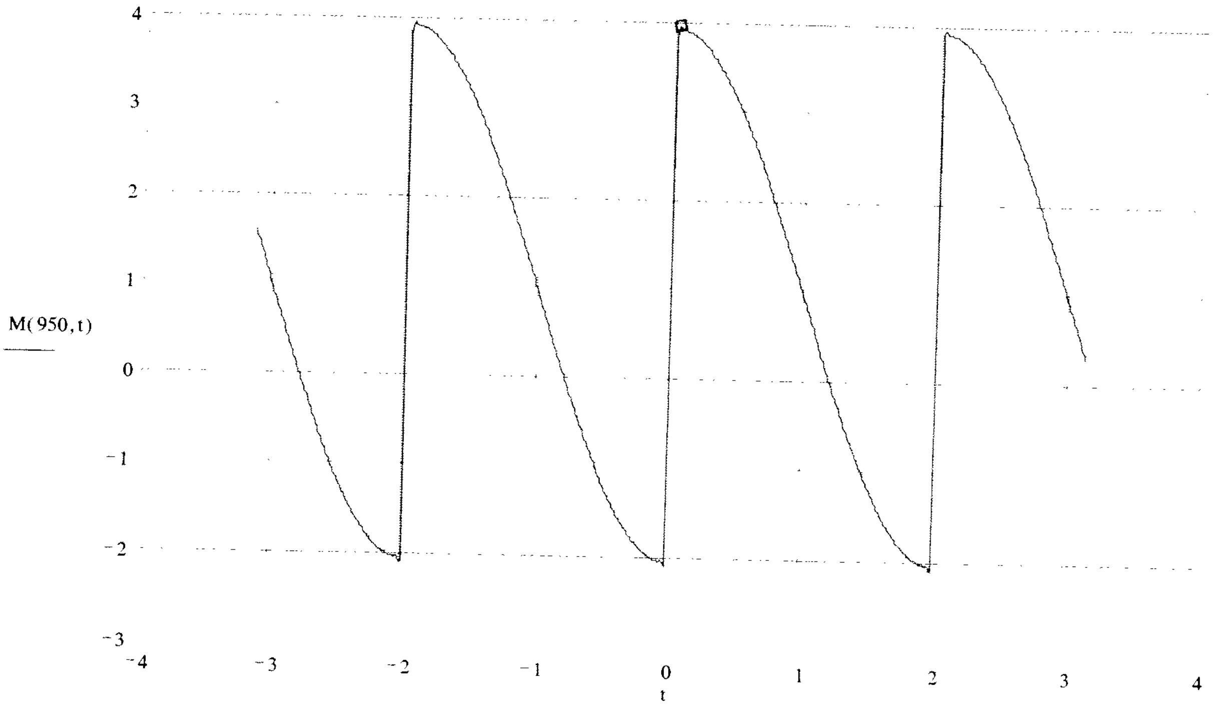
Note: Around 950 terms are necessary for it to look "good" on the large graph.

$$M_0 := \frac{A}{\pi}$$

$$M(k,t) := \sum_{g=1}^{k} f(g,t) + M_0$$



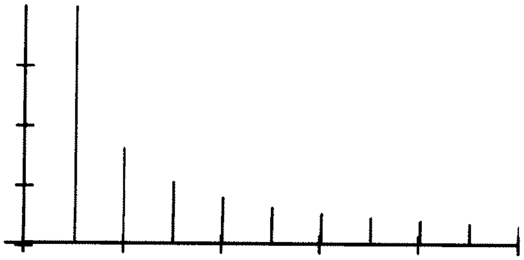




$m := -1, 0 .. 11$

$n := 0, 1 .. 10$ $A := 1$

$$g(n) := \mathrm{if}\left(n < 1, 0, 8\cdot\frac{A}{\pi}\cdot\frac{n}{4\cdot n^2 - 1}\right)$$



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Topic # 16

Fractional Rectified Cosine Wave

$A := 2 \qquad T := 3 \qquad k := .25$

a)

$$F_{avg} := \left(\frac{A}{\pi}\right)\cdot\left(\frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)$$

$$F_{rms} := \left(\frac{A}{\sqrt{2\cdot\pi}}\right)\cdot\left[\frac{\sqrt{4\cdot\pi\cdot k + 2\cdot\pi\cdot k\cdot\cos(4\cdot\pi\cdot k) - \left(\frac{3}{2}\right)\cdot\sin(4\cdot\pi\cdot k)}}{1-\cos(2\cdot\pi\cdot k)}\right]$$

$F_{avg} = 0.637$

$F_{rms} = 1$

b)

$$F_{avg2} := \left(\frac{1}{T}\right)\cdot\int_0^{k\cdot T}\frac{A\cdot\cos\left[\left(\frac{2\cdot\pi}{T}\right)\cdot t\right] - A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\,dt + \left(\frac{1}{T}\right)\cdot\int_{k\cdot T}^{T-k\cdot T} 0\,dt + \left(\frac{1}{T}\right)\cdot\int_{T-k\cdot T}^{T}\frac{A\cdot\cos\left[\left(\frac{2\cdot\pi}{T}\right)\cdot t\right] - A\cdot\cos(2\cdot\pi\cdot}{1-\cos(2\cdot\pi\cdot k)}$$

✓

$F_{avg2} = 0.637$

$$F_{rms2} := \sqrt{\left[\left(\frac{1}{T}\right)\cdot\int_0^{k\cdot T}\left[\frac{A\cdot\cos\left[\left(\frac{2\cdot\pi}{T}\right)\cdot t\right] - A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right]^2 dt + \left(\frac{1}{T}\right)\cdot\int_{k\cdot T}^{T-k\cdot T} 0^2\,dt + \left(\frac{1}{T}\right)\cdot\int_{T-k\cdot T}^{T}\left[\frac{A\cdot\cos\left[\left(\frac{2\cdot\pi}{T}\right)\cdot t\right] - A\cdot}{1-\cos(2\cdot\pi\cdot}\right.\right.}$$

✓

$F_{rms2} = 1$

c) Both parts a & b match 100%

* ~~Long~~ or wide Equations

whats missing on previous page

k)

— dt

$$\left.\frac{\cos(2\cdot\pi\cdot k)}{k)}\right]^{2} dt \Bigg]$$

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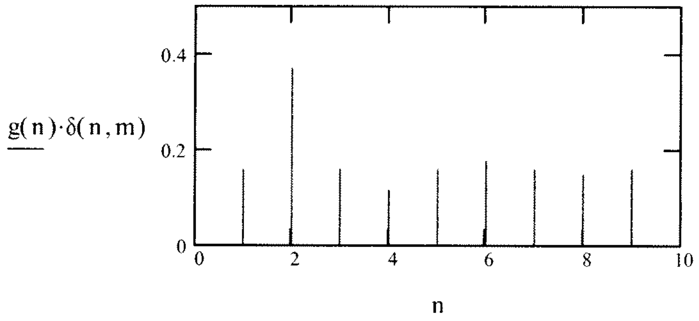
$n := 0,1..10 \quad m := -1,0..11 \quad A := 1 \quad k := 0.25$ ✓

$$a_0 := \frac{A}{\pi}\cdot\frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}$$

?

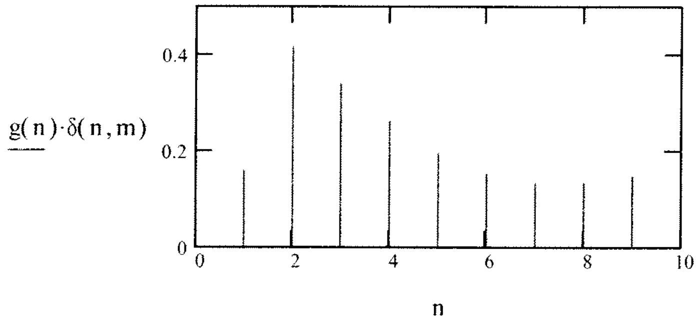
$$g(n) := \left|\left[\frac{2\cdot A}{\pi}\cdot\frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\cdot\frac{\sin(2\cdot\pi\cdot n\cdot k)\cdot\cos(2\cdot\pi\cdot k) - (n\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot n\cdot k))}{n\cdot(n^2-1)\cdot(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))}\right] + \frac{a_0}{2}\right|$$

If
Statement
needed



$k := \frac{1}{8}$

$$g(n) := \left|\left[\frac{2\cdot A}{\pi}\cdot\frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\cdot\frac{\sin(2\cdot\pi\cdot n\cdot k)\cdot\cos(2\cdot\pi\cdot k) - (n\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot n\cdot k))}{n\cdot(n^2-1)\cdot(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))}\right] \cancel{+ \frac{a_0}{2}}\right|$$



$$\frac{\displaystyle\int_{0}^{k\cdot T}\left(\frac{A\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)-A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)\cdot e^{\sqrt{-1}\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t}\,dt+\int_{T-k\cdot T}^{T}\left(\frac{A\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right)-A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)\cdot e^{-\sqrt{-1}\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t}\,dt}{T}$$

$$\left|\frac{\frac{1}{2}\cdot T\cdot\exp(-2\cdot i\cdot\pi\cdot n\cdot k)\cdot A\cdot\frac{(-i\cdot\cos(2\cdot\pi\cdot k)+n\cdot\sin(2\cdot\pi\cdot k))}{\left[\pi\cdot\left[(n^2-1)\cdot\left[(-1+\cos(2\cdot\pi\cdot k))\cdot n\right]\right]\right]}-\frac{1}{2}\cdot i\cdot T\cdot A\cdot\frac{(-n^2+\cos(2\cdot\pi\cdot k)\cdot n^2-\cos(2\cdot\pi\cdot k))}{\left[\pi\cdot\left[(n^2-1)\cdot\left[(-1+\cos(2\cdot\pi\cdot k))\cdot n\right]\right]\right]}+\frac{1}{2}\cdot i\cdot T\cdot\exp(-2\cdot i\cdot\pi\cdot n)\cdot A\cdot\frac{(-n^2+\cos(2\cdot\pi\cdot k)\cdot n^2-\cos(2}{\left[\pi\cdot\left[(n^2-1)\cdot\left[(-1+\cos(2\cdot\pi\cdot k)\right.\right.\right.}}{T}\right.$$

$A = 1$

$k = 0.25 \qquad T = 3$

$$G(n) = \left|\frac{\left[\frac{1}{2}\cdot T\cdot\exp(-2\cdot i\cdot\pi\cdot n\cdot k)\cdot A\cdot\frac{(-i\cdot\cos(2\cdot\pi\cdot k)+n\cdot\sin(2\cdot\pi\cdot k))}{\left[\pi\cdot\left[(n^2-1)\cdot\left[(-1+\cos(2\cdot\pi\cdot k))\cdot n\right]\right]\right]}-\frac{1}{2}\cdot i\cdot T\cdot A\cdot\frac{(-n^2+\cos(2\cdot\pi\cdot k)\cdot n^2-\cos(2\cdot\pi\cdot k))}{\left[\pi\cdot\left[(n^2-1)\cdot\left[(-1+\cos(2\cdot\pi\cdot k))\cdot n\right]\right]\right]}+\frac{1}{2}\cdot i\cdot T\cdot\exp(-2\cdot i\cdot\pi\cdot n)\cdot A\cdot\frac{(-n^2+\cos(2\cdot\pi\cdot k)\cdot n}{\left[\pi\cdot\left[(n^2-1)\cdot\left[(-1+\right.\right.\right.}\right.}{T}\right.$$

$G(2) = 0.212 \qquad G(3) = 0 \qquad G(4) = 0.042 \qquad G(5) = 0 \qquad G(6) = 0.018$

$$y1(n) = \left|\frac{2\cdot A}{\pi}\cdot\frac{\sin(2\cdot\pi\cdot k)-2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\cdot\frac{\sin(2\cdot\pi\cdot k\cdot n)\cdot\cos(2\cdot\pi\cdot k)-n\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot k\cdot n)}{(n^2-1)\cdot n\cdot(\sin(2\cdot\pi\cdot k)-2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))}\right|$$

$y1(1) = 0$

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$y1(2) = 0.212$

$y1(3) = 0$

$y1(4) = 0.042$

$y1(5) = 0$ $y1(6) = 0.018$

$$\frac{\displaystyle\int_{0}^{k\cdot T} \frac{A\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right) - A\cdot\cos(2\cdot\pi\cdot k)}{1 - \cos(2\cdot\pi\cdot k)}\,dt + \int_{T - k\cdot T}^{T} \frac{A\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T}\right) - A\cdot\cos(2\cdot\pi\cdot k)}{1 - \cos(2\cdot\pi\cdot k)}\,dt}{T}$$

$$\frac{\left[\frac{1}{2}\cdot A\cdot T\cdot\frac{(-\sin(2\cdot\pi\cdot k) + 2\cdot\cos(2\cdot\pi\cdot k)\cdot\pi\cdot k)}{[\pi\cdot(-1 + \cos(2\cdot\pi\cdot k))]} + A\cdot\cos(2\cdot\pi\cdot k)\cdot\frac{T}{(-1 + \cos(2\cdot\pi\cdot k))} + \frac{1}{2}\cdot A\cdot T\cdot\frac{[-\sin[2\cdot\pi\cdot(-1 + k)] - 2\cdot\cos(2\cdot\pi\cdot k)\cdot\pi + 2\cdot\cos(2\cdot\pi\cdot k)\cdot\pi\cdot k]}{[\pi\cdot(-1 + \cos(2\cdot\pi\cdot k))]}\right]}{T}$$

$$A\cdot\frac{(-\sin(2\cdot\pi\cdot k) + 2\cdot\cos(2\cdot\pi\cdot k)\cdot\pi\cdot k)}{[\pi\cdot(-1 + \cos(2\cdot\pi\cdot k))]}$$

402

Ken Kaiser

[redacted]

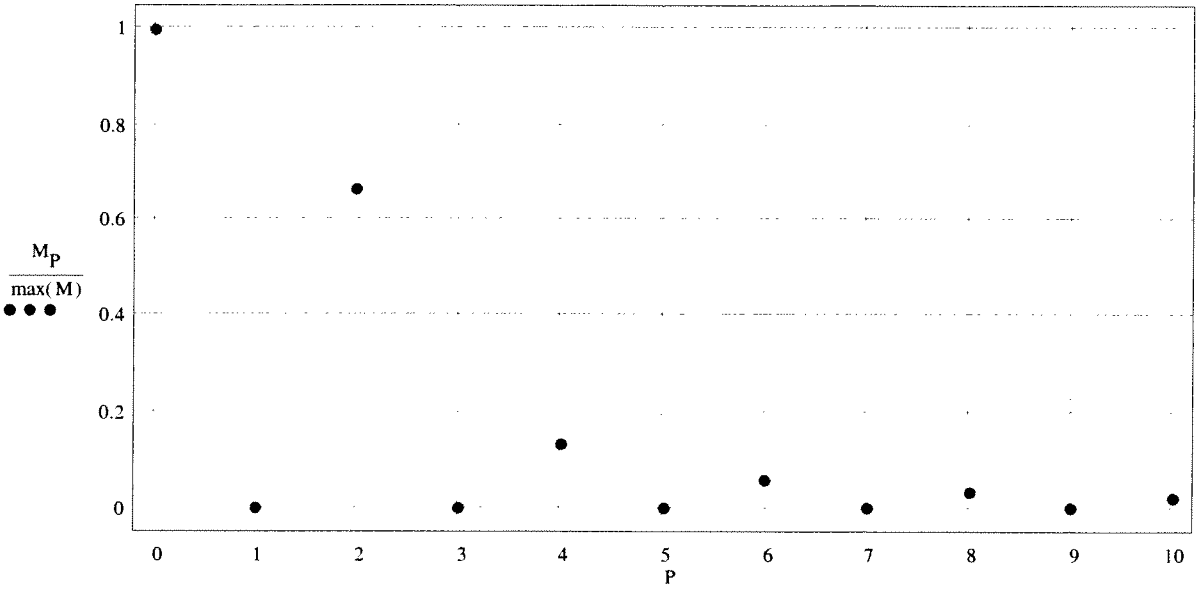
$T := 2 \qquad A := 2.98 \qquad \tau := \frac{T}{3} \qquad t := -\pi, -\pi + 0.01 .. \pi$

$N := 1..\ 10 \qquad k := \frac{1}{4} \qquad P := 0..\ 10$

$$f(n) := \frac{2 \cdot A}{\pi} \cdot \frac{(\sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k))}{1 - \cos(2 \cdot \pi \cdot k)} \cdot \left[\frac{(\sin(2 \cdot \pi \cdot n \cdot k) \cdot \cos(2 \cdot \pi \cdot k)) - n \cdot \sin(2 \cdot \pi \cdot k) \cdot \cos(2 \cdot \pi \cdot n \cdot k)}{n \cdot \left(n^2 - 1\right) \cdot (\sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k))} \right]$$

$$M_0 := \left| \frac{A}{\pi} \cdot \frac{(\sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k))}{1 - \cos(2 \cdot \pi \cdot k)} \right|$$

$$M_N := | f(N) |$$



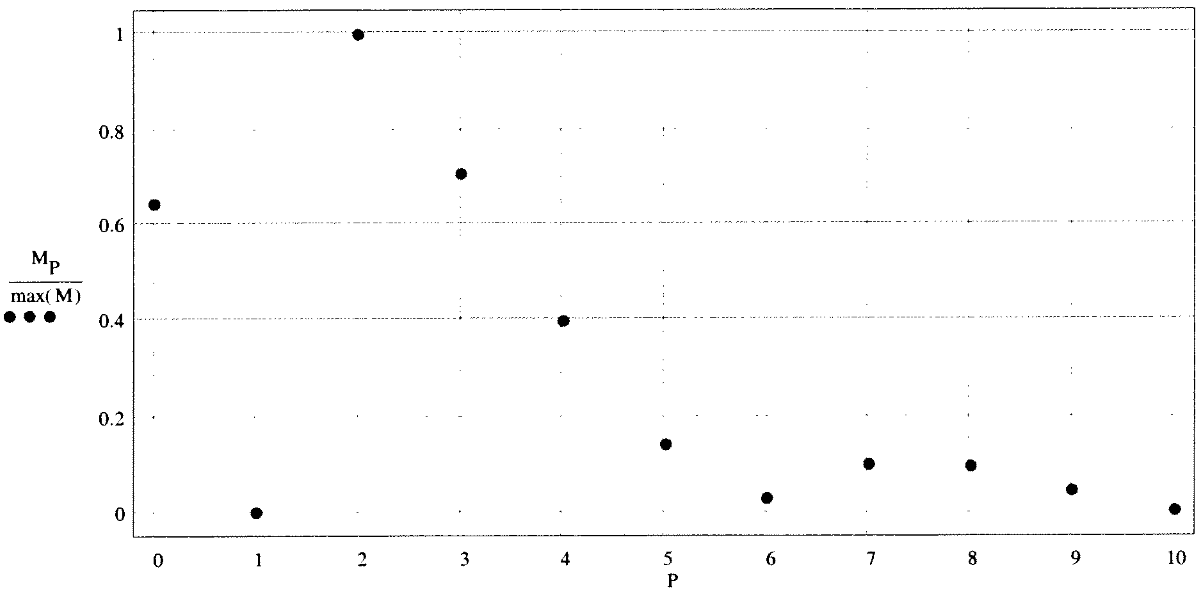
$$T := 2 \qquad A := 2.98 \qquad \tau := \frac{T}{3} \qquad t := -\pi, -\pi + 0.01 .. \pi$$

$$N := 1 .. 10 \qquad k := \frac{1}{8} \qquad P := 0 .. 10$$

$$f(n) := \frac{2 \cdot A}{\pi} \cdot \frac{(\sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k))}{1 - \cos(2 \cdot \pi \cdot k)} \cdot \left[\frac{(\sin(2 \cdot \pi \cdot n \cdot k) \cdot \cos(2 \cdot \pi \cdot k)) - n \cdot \sin(2 \cdot \pi \cdot k) \cdot \cos(2 \cdot \pi \cdot n \cdot k)}{n \cdot \left(n^2 - 1\right) \cdot (\sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k))} \right]$$

$$M_0 := \left| \frac{A}{\pi} \cdot \frac{(\sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k))}{1 - \cos(2 \cdot \pi \cdot k)} \right|$$

$$M_N := | f(N) |$$



P 402

plots of graphs for rectified cosine wave

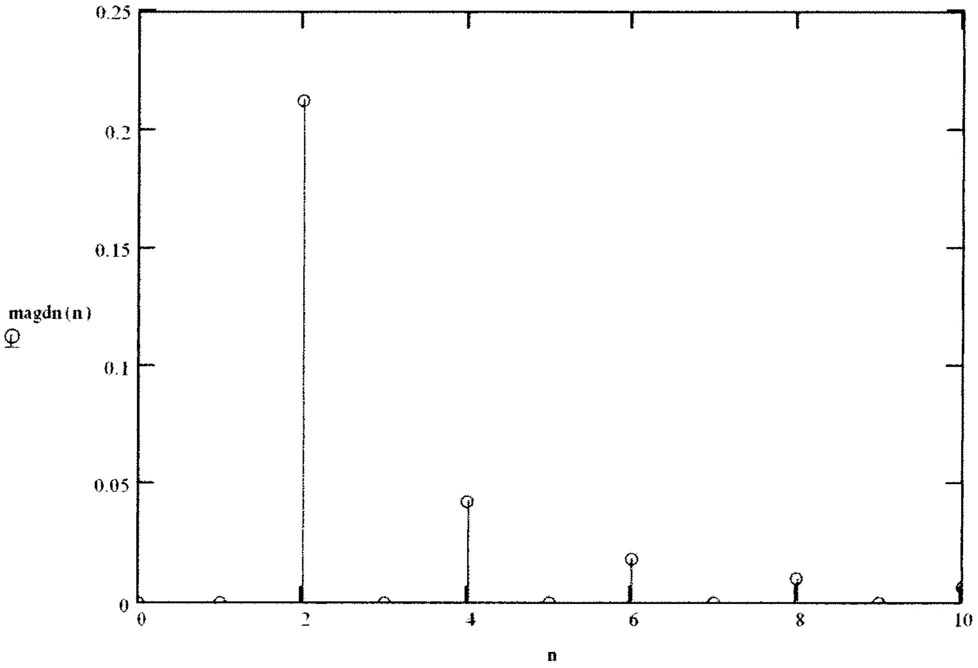
$n := 0, 1 .. 10$

$a := 1$

$k := .25$

$$dn(n) := cn \cdot \frac{[(\sin(2 \cdot \pi \cdot k \cdot n) \cdot \cos(2 \cdot \pi \cdot k)) - (n \cdot \sin(2 \cdot \pi \cdot k) \cdot \cos(2 \cdot \pi \cdot n \cdot k))]}{n \cdot (n^2 - 1) \cdot \sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k)}$$

$magdn(n) := |dn(n)|$



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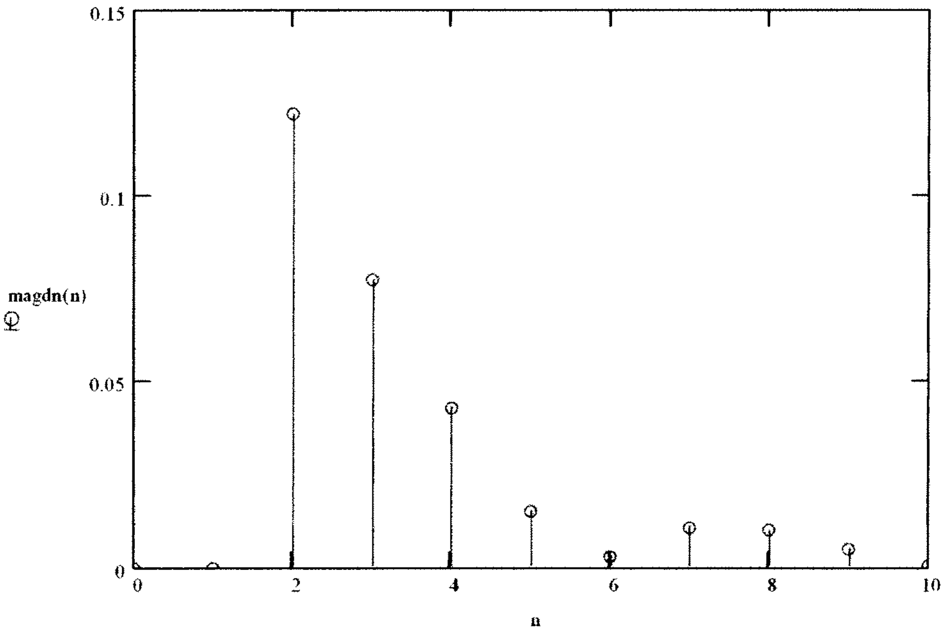
$n := 0, 1 .. 10$

$a := 1$

$k := .125$

$$dn(n) := cn \cdot \frac{[(\sin(2 \cdot \pi \cdot k \cdot n) \cdot \cos(2 \cdot \pi \cdot k)) - (n \cdot \sin(2 \cdot \pi \cdot k) \cdot \cos(2 \cdot \pi \cdot n \cdot k))]}{n \cdot (n^2 - 1) \cdot \sin(2 \cdot \pi \cdot k) - 2 \cdot \pi \cdot k \cdot \cos(2 \cdot \pi \cdot k)}$$

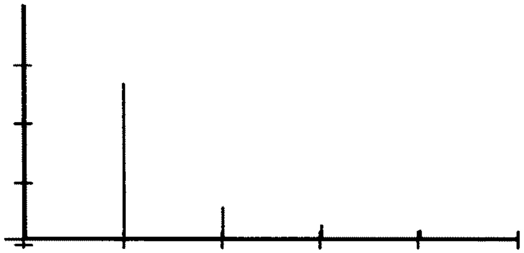
$magdn(n) := |dn(n)|$



$m := -1,0..11 \qquad T := 1 \qquad \tau := \frac{T}{2} \qquad A := 1 \qquad k := \frac{T}{4}$

$n := 0,1..10$

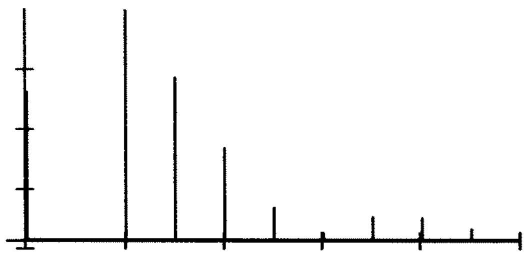
$$g(n) := \text{if}\left[n < 1, \frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1 - \cos(2\cdot\pi\cdot k)}\cdot\frac{A}{\pi}, \left|\frac{2\cdot A}{\pi}\cdot\frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1 - \cos(2\cdot\pi\cdot k)}\cdot\frac{\sin(2\cdot\pi\cdot k\cdot n)\cdot\cos(2\cdot\pi\cdot k) - n\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot k\cdot n)}{\left(n^2 - 1\right)\cdot n\cdot\left(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)\right)}\right|\right]$$



$m := -1, 0 .. 11$ $\qquad T := 1 \qquad \tau := \frac{T}{2} \qquad A := 1 \qquad k := \frac{T}{8}$

$n := 0, 1 .. 10$

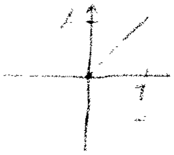
$$g(n) := \mathrm{if}\left[n < 1, \frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1 - \cos(2\cdot\pi\cdot k)}\cdot\frac{A}{\pi}, \left|\frac{2\cdot A}{\pi}\cdot\frac{\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)}{1 - \cos(2\cdot\pi\cdot k)}\cdot\frac{\sin(2\cdot\pi\cdot k\cdot n)\cdot\cos(2\cdot\pi\cdot k) - n\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot k\cdot n)}{\left(n^2 - 1\right)\cdot n\cdot\left(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k)\right)}\right|\right]$$

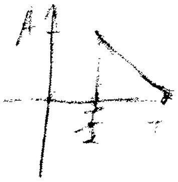


Function 1

$f(t) = \frac{2A}{T} t \quad 0 < t < \frac{T}{2}$

f is linear with a slope of $\frac{A}{\frac{T}{2}}$ and intercept of 0





$f(t) = -\frac{2A}{T} t + 2A \quad \frac{T}{2} < t < T$

slope $= -\frac{A}{\frac{T}{2}}$

intercept = 2A

Approximation: $\frac{A}{\sqrt{3}} = \frac{1}{\sqrt{3}} A \approx .577A$ ✓

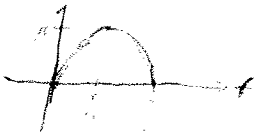
Function 2

$f(t) = \frac{16A}{T^2} t\left(\frac{T}{2} - t\right) \quad 0 < t < \frac{T}{2}$

$= \frac{16A}{T^2}\left(\frac{T}{2}t - t^2\right)$

$= -\frac{16A}{T^2}\left[\left(t - \frac{T}{4}\right)^2 - \frac{T^2}{16}\right]$

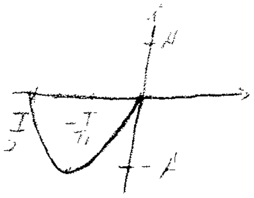
parabola w/ vertex shifted $\frac{T}{4}$ to the right opening down and shifted A upwards



$f(t) = \frac{16A}{T^2} t\left(\frac{T}{2} + t\right) \quad -\frac{T}{2} < t < 0$

$= \frac{16A}{T^2}\left(\frac{T}{2}t + t^2\right)$

$= \frac{16A}{T^2}\left[\left(t + \frac{T}{4}\right)^2 - \frac{T^2}{16}\right]$



approximation: $\frac{2\sqrt{2}}{\sqrt{15}} A \approx .730A$

Function 2

f_{avg}

$$\frac{1}{T}\cdot\left[\int_{\frac{-T}{2}}^{0}\left(\frac{16\cdot A}{T^2}\cdot t\right)\cdot\left(\frac{T}{2}+t\right)dt\right]+\frac{1}{T}\cdot\int_{0}^{\frac{T}{2}}\left(\frac{16\cdot A}{T^2}\cdot t\right)\cdot\left(\frac{T}{2}-t\right)dt$$

$$0$$

f_{rms}

$$\sqrt{\frac{1}{T}\cdot\int_{\frac{-T}{2}}^{0}\left[\left(\frac{16\cdot A}{T^2}\cdot t\right)\cdot\left(\frac{T}{2}+t\right)\right]^2 dt+\frac{1}{T}\cdot\int_{0}^{\frac{T}{2}}\left[\left(\frac{16\cdot A}{T^2}\cdot t\right)\cdot\left(\frac{T}{2}-t\right)\right]^2 dt}$$

$$\frac{2}{15}\cdot\sqrt{2}\cdot\sqrt{15}\cdot A \;=\; \frac{2\sqrt{2}}{\sqrt{15}}A$$

Ken Kaiser

[redacted]

Topic # 16

Cosine Pulse Train

$A := 2 \qquad T := 3 \qquad \tau := 0.5$

a)

$F_{avg} := \frac{2 \cdot A \cdot \tau}{\pi \cdot T} \qquad F_{rms} := A \cdot \sqrt{\left[\frac{\tau}{(2 \cdot T)}\right]}$

$F_{avg} = 0.212 \qquad F_{rms} = 0.577$

b)

$$F_{avg2} := \left(\frac{1}{T}\right) \cdot \int_{\left(\frac{-\tau}{2}\right)}^{\left(\frac{\tau}{2}\right)} A \cdot \cos\left[\left(\frac{\pi}{\tau}\right) \cdot t\right] dt + \left(\frac{1}{T}\right) \cdot \int_{\left(\frac{\tau}{2}\right)}^{\left(\frac{3 \cdot \tau}{2}\right)} 0 \, dt$$

$F_{avg2} = 0.212$

$$F_{rms2} := \sqrt{\left[\left(\frac{1}{T}\right) \cdot \int_{\left(\frac{-\tau}{2}\right)}^{\left(\frac{\tau}{2}\right)} \left[A \cdot \cos\left[\left(\frac{\pi}{\tau}\right) \cdot t\right]\right]^2 dt + \left(\frac{1}{T}\right) \cdot \int_{\left(\frac{\tau}{2}\right)}^{\left(\frac{3 \cdot \tau}{2}\right)} 0^2 \, dt\right]}$$

$F_{rms2} = 0.577$

c) Both parts a & b match 100%

Cosine Pulse Train
p.403

Ken Kaiser

Solving for b_n and a_n.

$$\frac{2}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

0

$$\frac{2}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\times t\right)\,dt$$

forgot the t

~~I don't get~~ this result?

$$\frac{4}{(T\cdot\pi)}\cdot\tau\cdot A\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\right)$$

– different than my result?

Plugging in n=0 for a_n.

$$\frac{2}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot 0}{T}\right)\,dt$$

$$\frac{4}{(T\cdot\pi)}\cdot\tau\cdot A$$

I checked

$$\frac{-\frac{\pi\tau}{\cancel{T}}\sin\left(\frac{\pi n\tau}{T}\right)}{-2\left(\frac{2n\tau}{T}\right)\frac{2\pi}{\cancel{T}}}$$

$$\tau\frac{\pi}{4}\left[\frac{\sin\frac{\pi n}{2\cancel{n}}}{\frac{\cancel{2n}}{\cancel{2n}}}\right]$$

why $\tau\frac{\pi}{4}$

why not just use a_n and $n=0$ instead of c_n?

$$\frac{\cancel{\pi}}{4}\,\frac{4A}{\cancel{\pi}}\,\frac{1}{2n}$$

$$\tau\frac{A}{2n}$$

m := -1, 0 .. 11

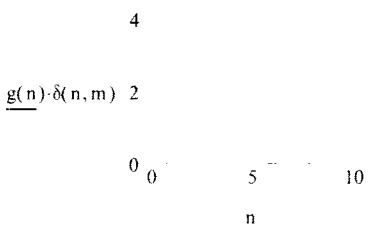
n := 0, 1 .. 10

A := 10

T := 1

$\tau := \frac{T}{2}$

$$g(n) := \text{if}\left[n<2,\frac{2\cdot A\cdot\tau}{\pi\cdot T},\frac{4\cdot A\cdot\tau}{\pi\cdot T}\cdot\left|\frac{\cos\left(\frac{\pi\cdot\tau\cdot n}{T}\right)}{1-\left(\frac{2\cdot\tau\cdot n}{T}\right)^2}\right|\right]$$



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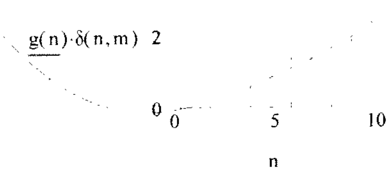
$m := -1, 0 .. 11$ $\quad A := 10$ $\quad T := 1$

$n := 0, 1 .. 10$ $\quad \tau := \frac{T}{4}$

$$g(n) := \text{if}\left[n<2, \left|\frac{\cos\left(\frac{\pi\cdot n\cdot \tau}{T}\right)}{1-\left(\frac{2\cdot \tau\cdot n}{T}\right)^2}\right|, \frac{A\cdot \tau}{T}\right]$$

"blows-up" for certain values of $\frac{N}{T}$



#403

$$\frac{2}{(T\cdot\pi)}\cdot\tau\cdot A$$

$$\frac{2}{T}\cdot\int_{\frac{\tau}{2}}^{\frac{\tau}{2}}\left(A\cdot\cos\left(\frac{\pi\cdot t}{\tau}\right)\right)\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\,dt$$

✓

$$2\cdot\tau\cdot\left[\sin\left[\frac{1}{2}\cdot\pi\cdot\frac{(-T+2\cdot n\cdot\tau)}{T}\right]\cdot T+2\cdot\sin\left[\frac{1}{2}\cdot\pi\cdot\frac{(-T+2\cdot n\cdot\tau)}{T}\right]\cdot n\cdot\tau-\sin\left[\frac{1}{2}\cdot\pi\cdot\frac{(T+2\cdot n\cdot\tau)}{T}\right]\cdot T+2\cdot\sin\left[\frac{1}{2}\cdot\pi\cdot\frac{(T+2\cdot n\cdot\tau)}{T}\right]\cdot n\cdot\tau\right]\cdot\frac{A}{\left[\pi\cdot\left(-T^2+4\cdot n^2\cdot\tau^2\right)\right]}$$

$$2\tau\left[T\sin\left[\frac{-\pi}{2}+\frac{\pi n\tau}{T}\right]+2n\tau\sin\left[\frac{-\pi}{2}+\frac{n\pi\tau}{T}\right]-T\sin\left[\frac{\pi}{2}+\frac{n\pi\tau}{T}\right]+2n\tau\sin\left[\frac{\pi}{2}+\frac{\pi n\tau}{T}\right]\right]$$

use expand command →

$$-4\tau\frac{A}{\pi\left[-T^2+4n^2\tau^2\right]}\;T\cos\frac{\pi}{T}n\tau$$

$$=\frac{-4\cancel{\pi}A\tau}{\pi T^2\left[-1+\frac{4n^2\tau^2}{T^2}\right]}\cos\frac{n\pi\tau}{T}=\frac{\cancel{-}4\tau A}{\pi T\left[-1+\frac{4n^2\tau^2}{T^2}\right]}\cos\frac{n\pi\tau}{T}$$

$$\left(\frac{2n\tau}{T}\right)^2$$ ✓

P. 403

$n=0$ $$\frac{2At}{\pi T} = \frac{2A}{2\pi 0} = 0$$

$n=\cancel{4}$ any $$\frac{4A}{2\pi}\left(\frac{\overset{=0}{\cos\frac{\pi}{2}}}{\underset{=0}{1-1^2}}\right)\cos\pi =$$

$n=1$ $\frac{t}{T}=\frac{1}{2n}$

$$\frac{d\cos\frac{\pi nt}{T}}{dn} = \frac{-\sin\left(\frac{n+\pi}{T}\right)t\,\pi}{T} = \frac{-\sin\left(\frac{\pi}{2}\right)\pi}{2}$$

$$\frac{d\,1-\left(\frac{2nt}{\cancel{T}}\right)^2}{dn} = \frac{-8nt^2}{\cancel{T}^2} = \frac{-8}{4} - -2$$

$$\frac{\frac{-\overset{=1}{\sin\left(\frac{\pi}{2}\right)}\pi}{2}}{-2} = \frac{-\pi}{-4} > \frac{\cancel{\pi}}{\cancel{4}}\cdot\frac{\cancel{4}At}{\cancel{\pi}T} = \frac{At}{T}$$

Graph 1 P. 403

$A=1 \quad t=\frac{T}{2} \rightarrow \frac{t}{T}=\frac{1}{2} \rightarrow \underline{\underline{n=1}} \quad t=\overset{1}{\cancel{2}} \quad T=2$

$dc = \frac{2At}{\pi T} = .318$

n=1

$f_0 = \frac{At}{T} = .5$

$2f_0 = \frac{4At}{\pi T} \frac{\cos \frac{\pi n t}{T}}{1-\left(\frac{2nt}{T}\right)^2} = .212$

$3f_0 = \downarrow = 0$

$4f_0 = \downarrow = .042$

$5f_0 = \downarrow = 0$

$6f_0 = \downarrow = .018$

$7f_0 = \downarrow = 0$

Graph 2 on back

403

[redacted]

Ken Kaiser

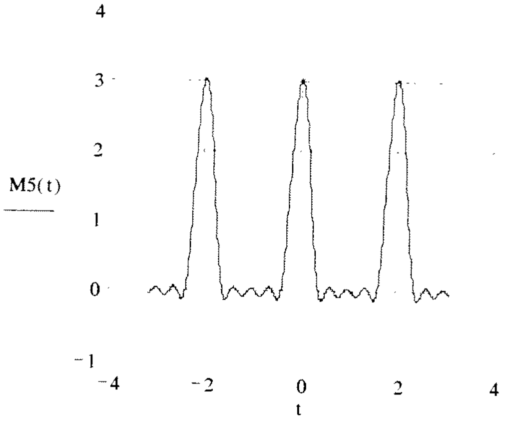
$T := 2 \qquad A := 2.98 \qquad \tau := \frac{T}{3}$

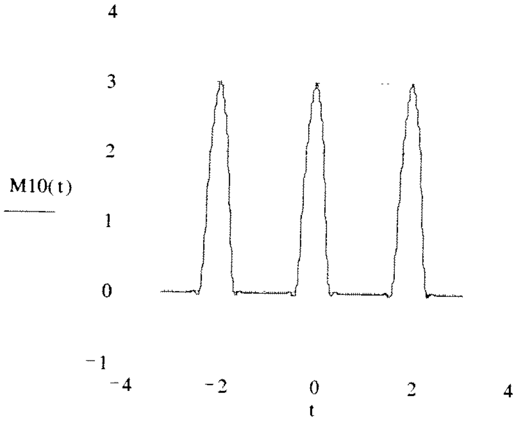
$$f(n,t) := \frac{4 \cdot A \cdot \tau}{\pi \cdot T} \cdot \cos\left(\frac{\pi \cdot n \cdot \tau}{T}\right) \cdot \frac{1}{1 - \left(\frac{2 \cdot n \cdot \tau}{T}\right)^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$

$$M_0 := \frac{2 \cdot A \cdot \tau}{\pi \cdot T}$$

$$M5(t) := \sum_{g=1}^{5} f(g,t) + M_0 \qquad\qquad t := -\pi, -\pi + 0.01 .. \pi$$

$$M10(t) := \sum_{g=1}^{10} f(g,t) + M_0$$





Ken Kaiser

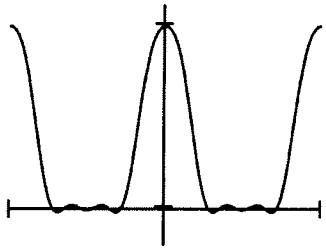
COSINE PULSE TRAIN

$T := 1 \qquad A := 1$

$t := -1\cdot T, -1\cdot T + \frac{T}{1000} .. 1\cdot T \qquad \tau := \frac{T}{2.001}$

$$g(t,m) := \frac{4\cdot A\cdot \tau}{\pi\cdot T}\cdot\left[\sum_{n=1}^{m}\frac{\cos\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1-\left(2\cdot n\cdot\frac{\tau}{T}\right)^{2}}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right] + 2\cdot\frac{A\cdot\tau}{\pi\cdot T}$$

Kaiser



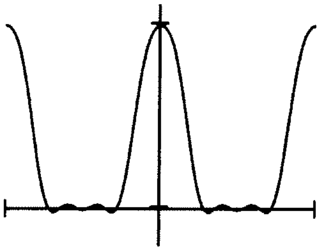
Ken Kaiser

$T := 1 \qquad A := 1$

$t := -1 \cdot T, -1 \cdot T + \frac{T}{1000} .. 1 \cdot T \qquad \tau := \frac{T}{2}$

$$h(n) := \text{if}\left[\left|\frac{\tau}{T} - \frac{1}{2 \cdot n}\right| < 10^{-12}, \frac{A \cdot \tau}{T}, \left[\frac{4 \cdot A \cdot \tau}{\pi \cdot T} \cdot \frac{\cos\left(\pi \cdot n \cdot \frac{\tau}{T}\right)}{1 - \left(2 \cdot n \cdot \frac{\tau}{T}\right)^2}\right]\right]$$

$$gm(t, m) := \left(\sum_{n=1}^{m} h(n) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)\right) + 2 \cdot \frac{A \cdot \tau}{\pi \cdot T}$$



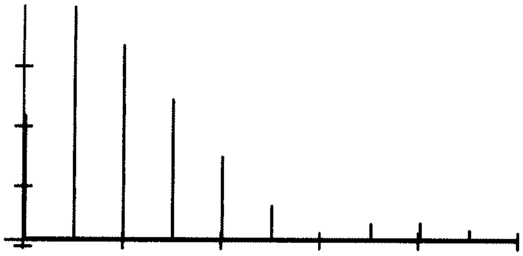
Ken Kaiser

$m := -1, 0 .. 11$ $\quad T := 2 \quad \tau := \frac{T}{4} \quad A := 1$

$n := 0, 1 .. 10$

$$g(n) := \text{if}\left[n < 1, 2\cdot A\cdot\frac{\tau}{\pi\cdot T}, \text{if}\left[\left|\frac{\tau}{T} - \frac{1}{2\cdot n}\right| < 0.01, A\cdot\frac{\tau}{T}, \left|2\cdot\left[2\cdot A\cdot\frac{\tau}{\pi\cdot T}\cdot\frac{\cos\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1 - \left(2\cdot n\cdot\frac{\tau}{T}\right)^2}\right]\right|\right]\right].$$

$g(1) = 0.3$



$n := 1.9990001$ $\quad \tau := \frac{T}{4}$

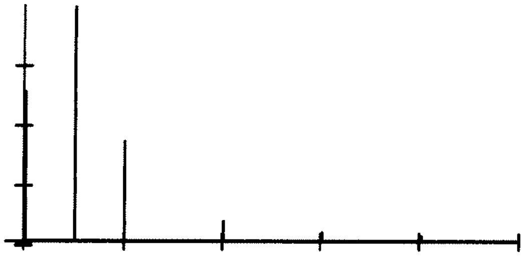
$$\frac{2\cdot\left[2\cdot A\cdot\frac{\tau}{\pi\cdot T}\cdot\frac{\cos\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1 - \left(2\cdot n\cdot\frac{\tau}{T}\right)^2}\right]}{A\cdot\frac{\tau}{T}} = 1$$

$m := -1, 0 .. 11$ $\quad T := 2 \quad \tau := \frac{T}{2} \quad A := 1$

$n := 0, 1 .. 10$

$$g(n) := \text{if}\left[n < 1, 2\cdot A\cdot\frac{\tau}{\pi\cdot T}, \text{if}\left[\left|\frac{\tau}{T} - \frac{1}{2\cdot n}\right| < 0.01, A\cdot\frac{\tau}{T}, \left|2\cdot\left[2\cdot A\cdot\frac{\tau}{\pi\cdot T}\cdot\frac{\cos\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1 - \left(2\cdot n\cdot\frac{\tau}{T}\right)^2}\right]\right|\right]\right]$$

$g(1) = 0.5$



$n := 1.9990001 \qquad \tau := \frac{T}{4}$

$$\frac{2\cdot\left[2\cdot A\cdot\frac{\tau}{\pi\cdot T}\cdot\frac{\cos\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1 - \left(2\cdot n\cdot\frac{\tau}{T}\right)^2}\right]}{A\cdot\frac{\tau}{T}} = 1$$

Ken Kaiser

$$n := 2, 3 .. 10 \qquad \tau := \frac{1}{2} \quad T := 1 \quad \mathbf{A} := 3$$

$$f1(t) := A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right) \qquad f2(t) := 0$$

$$a_n := \frac{2}{T}\cdot\left(\int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} f1(t)\cos\left(\frac{2\cdot\pi n\cdot t}{T}\right)dt + \int_{\frac{\tau}{2}}^{T-\frac{\tau}{2}} f2(t)\cdot\cos\left(\frac{2\cdot\pi\cdot n\cdot t}{T}\right)dt\right)$$

$$b_n := \frac{2}{T}\cdot\left(\int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} f1(t)\sin\left(\frac{2\cdot\pi n\cdot t}{T}\right)dt + \int_{\frac{\tau}{2}}^{T-\frac{\tau}{2}} f2(t)\cdot\sin\left(\frac{2\cdot\pi\cdot n\cdot t}{T}\right)dt\right)$$

$$c_n := \frac{4\cdot A\cdot\tau}{\pi\cdot T}\cdot\left|\frac{\cos\left(\frac{\pi\cdot n\cdot\tau}{T}\right)}{1-\left(\frac{2\cdot n\cdot\tau}{T}\right)^2}\right|$$

$$\mathrm{error}_n := c_n - \sqrt{\left(a_n\right)^2 + \left(b_n\right)^2}$$

$$\max(|\mathrm{error}|) = 0$$

Ken Kaiser

$$A\cos\left(\frac{\Pi}{\tau}t\right) \qquad \frac{-\Pi}{2} < t < \frac{\tau}{2}$$

$$0 \qquad \text{elsewhere over } T$$

$$f_{avg} = \frac{\int_{-\left(\frac{\tau}{2}\right)}^{\frac{\tau}{2}} A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)\,dt}{T} + \frac{\int_{\frac{\tau}{2}}^{\left[T-\left(\frac{\tau}{2}\right)\right]} 0\,dt}{T}$$

$$f_{avg} = \frac{2}{\pi}\cdot\tau\cdot\frac{A}{T}$$

$$f_{rms} = \sqrt{\frac{\int_{-\left(\frac{\tau}{2}\right)}^{\frac{\tau}{2}} \left(A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)\right)^2\,dt}{T} + \frac{\int_{\frac{\tau}{2}}^{\left[T-\left(\frac{\tau}{2}\right)\right]} 0^2\,dt}{T}}$$

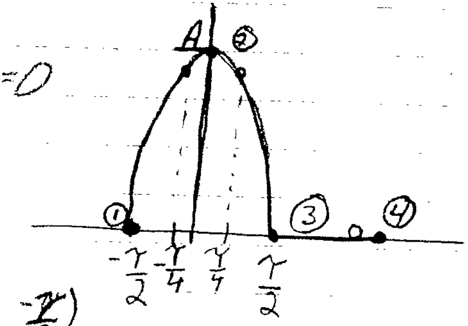
$$f_{rms} = \frac{1}{2}\cdot\sqrt{2}\cdot A\cdot\frac{\sqrt{\tau}}{\sqrt{T}} \qquad A\sqrt{\frac{\tau}{2T}}$$

Ken Kaiser

③ ① $A\cos\left(\frac{\pi}{\tau}\left(-\frac{\tau}{2}\right)\right) = A\cos\left(-\frac{\pi}{2}\right) = 0$

② $A\cos\left(\frac{\pi}{\tau}(0)\right) = 1$

③ $A\cos\left(\frac{\pi}{\tau}\left(\frac{\tau}{2}\right)\right) = A(0) = 0$



④ 0 elsewhere $\left(\frac{\tau}{2} < t < -\frac{\tau}{2}\right)$

Ken Kaiser

Cosine-Squared Pulse Train

$$\frac{1}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)^2\,dt$$

$$\boxed{\frac{1}{(2\cdot T)}\cdot\tau\cdot A = f_{avg}}$$

$$\sqrt{\frac{1}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\left(A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)^2\right)^2 dt}$$

$$\frac{1}{4}\cdot\sqrt{3}\cdot\sqrt{\frac{2}{T}}\cdot\sqrt{\tau}\cdot A = \frac{A\cdot\sqrt{6\tau}}{4\sqrt{T}} = A\cdot\sqrt{\frac{6\tau}{16T}} = A\cdot\sqrt{\frac{3\tau}{8\cdot T}} = \boxed{\frac{A}{2}\cdot\sqrt{\frac{3\tau}{2T}} = f_{rms}}$$

[redacted]

Circuits-1 Kaiser

Test Topic #16

```
>
```

Parameters:

A = 2, T = 3, tau = .5

```
>
```

Cosine Squared Pulse Train

a) Favg = 0.1667

Frms = 0.50

```
>
```

b) Favg =

```
> (1/3)*int(2*(cos(2*3.14*t)^2), t=(-1/4)..(1/4))+(1/3)*int(0,
  t=(3)..(inf));
```

.1667512024

frms =

```
> sqrt((1/3)*int((2*((cos(2*3.14*t))^2)^2),
  t=(-1/4)..(1/4))+(1/3)*int(0, t=(3)..(inf)));
```

~~.3536430429~~

```
>
```

different?

My work yield - via Mathcad

$$\frac{\sqrt{6}}{4}\left(T\frac{A^4}{T}\right)^{1/2} = \frac{\overset{=\sqrt{2.3}}{\sqrt{6}}A}{4}\left(\frac{T}{T}\right)^{1/2}$$

$$= \frac{\sqrt{2}A}{4}\left(\frac{3T}{T}\right)^{1/2} \quad ✓$$

$$= \frac{2A}{4\sqrt{2}}\left(\frac{3T}{T}\right)^{1/2} = \frac{A}{2\text{[illegible]}}\left(\frac{3T}{2T}\right)^{1/2}$$

$\boxed{.5}$ ✓ My result

Ken Kaiser

[redacted]

Topic # 16

Cosine-Squared Pulse Train

$A := 2$ $\qquad T := 3$ $\qquad \tau := 0.5$

a)

$$F_{avg} := \frac{A \cdot \tau}{2 \cdot T}$$

$$F_{rms} := \left(\frac{A}{2}\right) \cdot \sqrt{\left(\frac{3 \cdot \tau}{2 \cdot T}\right)}$$

$F_{avg} = 0.167$

$F_{rms} = 0.5$

b)

$$F_{avg2} := \left(\frac{1}{T}\right) \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot \left[\cos\left[\left(\frac{\pi}{\tau}\right) \cdot t\right]\right]^2 dt + \left(\frac{1}{T}\right) \cdot \int_{\frac{\tau}{2}}^{\frac{3 \cdot \tau}{2}} 0 \, dt$$

$F_{avg2} = 0.167$

$$F_{rms2} := \sqrt{\left[\left(\frac{1}{T}\right) \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \left[A \cdot \left[\cos\left[\left(\frac{\pi}{\tau}\right) \cdot t\right]\right]^2\right]^2 dt + \left(\frac{1}{T}\right) \cdot \int_{\frac{\tau}{2}}^{\frac{3 \cdot \tau}{2}} 0^2 \, dt\right]}$$

$F_{rms2} = 0.5$

c) Both parts a & b match 100%

Page 404, Cosine-Squared Pulse Train

Because this function has even symmetry, $b_n = 0$ ✓

$$\frac{2}{T}\cdot\left[\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\cdot\left(\cos\left(\pi\cdot\frac{t}{\tau}\right)\right)^2\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T}\right)dt\right]$$

$$\frac{1}{2}\cdot\left[\sin\left[\pi\cdot\frac{(n\cdot\tau-T)}{T}\right]\cdot\tau^2\cdot n^2+\sin\left[\pi\cdot\frac{(n\cdot\tau-T)}{T}\right]\cdot\tau\cdot n\cdot T+\sin\left[\pi\cdot\frac{(n\cdot\tau+T)}{T}\right]\cdot\tau^2\cdot n^2-\sin\left[\pi\cdot\frac{(n\cdot\tau+T)}{T}\right]\cdot\tau\cdot n\cdot T+2\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)\cdot n^2\cdot\tau^2-2\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)\cdot T^2\right]\cdot\frac{A}{\left(\pi\cdot n^3\cdot\tau^2-\pi\cdot n\cdot T^2\right)}$$

$$\frac{-A}{\left(\pi\cdot n^3\cdot\tau^2-\pi\cdot n\cdot T^2\right)}\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)\cdot T^2 \longleftarrow \text{expand}$$

Verified ✓

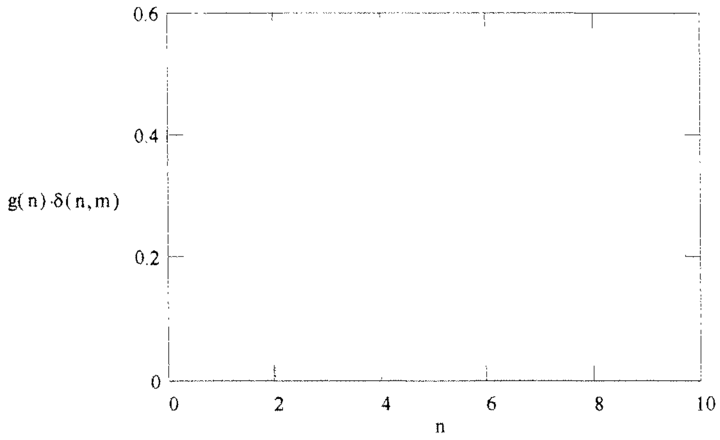
$$-A\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)\cdot\frac{T^2}{\left(\pi\cdot n^3\cdot\tau^2-\pi\cdot n\cdot T^2\right)} = \frac{-A\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{\pi\cdot n}\cdot\frac{T^2}{\left(n^2\cdot\tau^2-T^2\right)} = \frac{-A\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{\pi\cdot n}\cdot\frac{1}{\left(\frac{n^2\cdot\tau^2}{T^2}-1\right)} = \frac{-A\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{\pi\cdot n}\cdot\frac{1}{\left[\left(\frac{n\cdot\tau}{T}\right)^2-1\right]}$$

$$\frac{A\cdot\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{\pi\cdot n}\cdot\frac{1}{\left[1-\left(\frac{n\cdot\tau}{T}\right)^2\right]} = \frac{A}{\pi\cdot n}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{\left[1-\left(\frac{n\cdot\tau}{T}\right)^2\right]}$$

Ken Kaiser

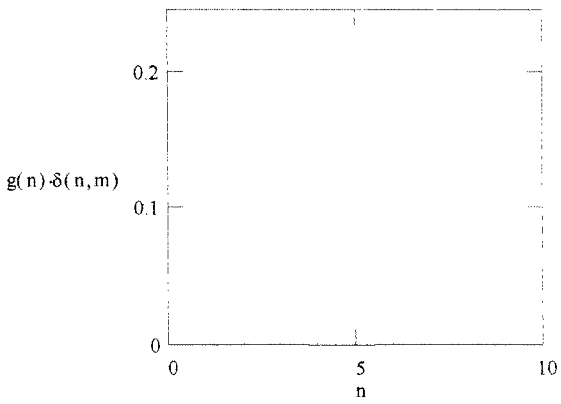
$m = -1, 0 .. 11 \quad n = 0, 1 .. 10 \quad t = 1, 1.01 .. 5 \quad \tau = \frac{T}{2}$

$$g(n) = \mathrm{if}\left[n<1, \frac{A\cdot\tau}{2\cdot T}, \mathrm{if}\left[n=2, \frac{A\cdot\tau}{2\cdot T}, \frac{A}{\pi\cdot n}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{\left[1-\left(\frac{n\cdot\tau}{T}\right)^2\right]}\right]\right]$$



$m = -1, 0 .. 11 \quad n = 0, 1 .. 10 \quad t = 1, 1.01 .. 5 \quad \tau = \frac{T}{4}$

$$g(n) = \mathrm{if}\left[n<1, \frac{A\cdot\tau}{2\cdot T}, \mathrm{if}\left[n=4, \frac{A\cdot\tau}{2\cdot T}, \frac{A}{\pi\cdot n}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1-\left(\frac{n\cdot\tau}{T}\right)^2}\right]\right]$$



p.404

$n=0$ $\quad \frac{A}{2\cdot 0} = 0$

$n=1$ $\quad \frac{A}{\pi} \frac{\overset{=0}{\sin \pi}}{1(1-1^2)} \cos \overset{1}{2\pi} = 0$

$= 0$

$\downarrow$

$n=1 \quad \frac{t}{T} = \frac{1}{n}$

$$\frac{d \sin \frac{\pi n t}{T}}{dn} = \frac{\cos\left(\frac{n t \pi}{T}\right) t \pi}{T} \Rightarrow \cos\left(\frac{1 \pi}{1}\right) \pi = -\pi$$

$\frac{t}{T} = \frac{1}{n} \qquad n=1$

$$\frac{d\; n\left[1-\left(\frac{nt}{T}\right)^2\right]}{dn} = \frac{-(3n^2t^2 - \cancel{T}^2)}{T^2} = \frac{-3n^2}{n^2} - 1 = -3 - 1 = -2$$

$$\frac{-\pi}{-2} = \frac{\pi}{\cancel{2}} \cdot \frac{A}{\pi} = \frac{A}{2}$$

$$\frac{A}{2\cancel{4}} = \frac{A}{2}\frac{t}{T} \text{ when } \frac{t}{T} = \frac{1}{\cancel{2}n}$$

P. 404

Graph 1

A 1 $t = 2$ $T = 4$ $t = \frac{T}{\cancel{4}2}$

$$dC = \frac{At}{2T} = \frac{2}{8} = .25$$

$$f_0 = \frac{A}{\pi} \frac{\sin \frac{\pi n t}{T}}{n\left(1 - \left(\frac{nt}{T}\right)^2\right)} = .424$$

$$2f_0 = \frac{At}{2T} = .25$$

$$3f_0 = \quad = .085$$

$4f_0 =$ = tiny

$5f_0 =$ = tiny

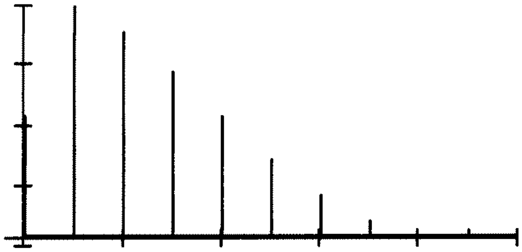
Graph 2 on back

$m := -1, 0 .. 11 \qquad T := 2 \qquad \tau := \frac{T}{4} \qquad A := 1$

$n := 0, 1 .. 10$

$$g(n) := \mathrm{if}\left[n < 1, A\cdot\frac{\tau}{2\cdot T}, \mathrm{if}\left[\left|\frac{\tau}{T} - \frac{1}{n}\right| < 0.01, \frac{A}{2\cdot n}, \left[\left[A\cdot\frac{1}{n\cdot\pi}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1-\left(n\cdot\frac{\tau}{T}\right)^2}\right]\right]\right]\right]$$

$g(1) = 0.24$



$n := 4.0001 \qquad \tau := \frac{T}{4}$

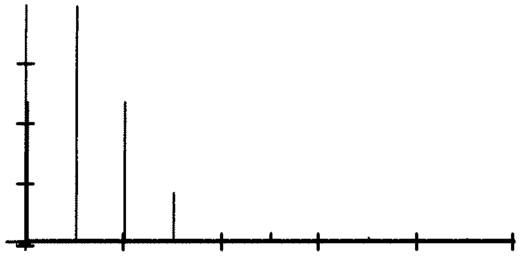
$$\frac{A\cdot\frac{1}{n\cdot\pi}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1-\left(n\cdot\frac{\tau}{T}\right)^2}}{\left(\frac{A\cdot\tau}{2\cdot T}\right)} = 1$$

$m := -1, 0 .. 11 \qquad T := 2 \qquad \tau := \frac{T}{2} \qquad A := 1$

$n := 0, 1 .. 10$

$$g(n) := \mathrm{if}\left[n < 1, A\cdot\frac{\tau}{2\cdot T}, \mathrm{if}\left[\left|\frac{\tau}{T} - \frac{1}{n}\right| < 0.01, \frac{A}{2\cdot n}, \left\|\left[A\cdot\frac{1}{n\cdot\pi}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1 - \left(n\cdot\frac{\tau}{T}\right)^2}\right]\right\|\right]\right]$$

$g(1) = 0.424$



$n := 4.0001 \qquad \tau := \frac{T}{4}$

$$\frac{A\cdot\frac{1}{n\cdot\pi}\cdot\frac{\sin\left(\pi\cdot n\cdot\frac{\tau}{T}\right)}{1 - \left(n\cdot\frac{\tau}{T}\right)^2}}{\left(\frac{A\cdot\tau}{2\cdot T}\right)} = 1$$

Ken Kaiser

[redacted]

Topic # 16

Full-Rectified Sine Wave

$A := 2 \qquad T := 3$

a)

$$F_{avg} := \frac{2 \cdot A}{\pi} \qquad\qquad F_{rms} := \frac{A}{\sqrt{2}}$$

$$F_{avg} = 1.273 \qquad\qquad F_{rms} = 1.414$$

b)

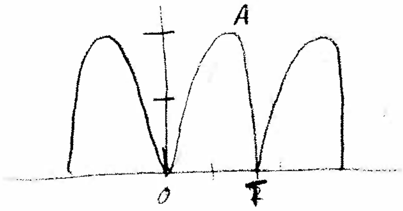
$$F_{avg2} := \left(\frac{1}{T}\right) \cdot \int_0^T \left[A \cdot \sin\left[\left(\frac{\pi}{T}\right) \cdot t\right]\right] dt \qquad\qquad F_{rms2} := \sqrt{\left[\left(\frac{1}{T}\right) \cdot \int_0^T \left[\left[A \cdot \sin\left[\left(\frac{\pi}{T}\right) \cdot t\right]\right]^2\right] dt\right]}$$

$$F_{avg2} = 1.273 \qquad\qquad F_{rms2} = 1.414$$

c) Both parts a & b match 100%

[redacted]

405



$A \sin\left(\frac{\pi}{T} t\right) \quad 0 < t < T$

when $t = 0$ $\quad A \sin\left(\frac{\pi}{T} 0\right) = 0$

when $t = T$ $\quad A \sin\left(\frac{\pi}{\cancel{T}} \cancel{T}\right) = 0$

max at $\frac{T}{2} \Rightarrow A \sin\left(\frac{\pi}{\cancel{T}} \frac{\cancel{T}}{2}\right) = A \sin\left(\frac{\pi}{2}\right) = A$

All given equations + time periods for this equation are correct

$$\frac{d}{dt} A \sin\left(\frac{\pi}{T} t\right) = \frac{A\pi}{T} \cos\left(\frac{\pi}{T} t\right)$$

$\frac{A\pi}{T} \cos\left(\frac{\pi}{T} t\right) = 0$ when $t = \frac{T}{2}$

↑ So that is a maximum

[redacted] Ken Kaiser

405

$$\frac{1}{T}\cdot\int_{0}^{T} A\cdot\sin\left(\frac{\pi}{T}\cdot t\right)dt$$

$$\frac{2}{\pi}\cdot A \Rightarrow \boxed{f_{avg} = \frac{2A}{\pi}}$$

$$\sqrt{\frac{1}{T}\cdot\int_{0}^{T} \left(A\cdot\sin\left(\frac{\pi}{T}\cdot t\right)\right)^{2}dt}$$

$$\frac{1}{2}\cdot\sqrt{2}\cdot A$$

$$\frac{1}{2} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} \text{ so } \frac{1}{\sqrt{2}}\cdot\frac{1}{\cancel{\sqrt{2}}}\cdot\cancel{\sqrt{2}}\cdot A =$$

$$\frac{1}{\sqrt{2}}\cdot A \Rightarrow \boxed{\frac{A}{\sqrt{2}} = f_{rms}}$$

Ken Kaiser

Pg. 405 11:

An $\frac{2}{T}\cdot\int_0^T A\cdot\sin\left(\frac{\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$

$\frac{2}{T}\cdot\left[\frac{-1}{2}\cdot T\cdot(-\cos(\pi\cdot(1+2\cdot n))+2\cdot\cos(\pi\cdot(1+2\cdot n))\cdot n-\cos(\pi\cdot(-1+2\cdot n))-2\cdot\cos(\pi\cdot(-1+2\cdot n))\cdot n)\cdot\frac{A}{(\pi\cdot((1+2\cdot n)\cdot(-1+2}\right.$

$-2\cdot A\cdot\frac{(\cos(\cancelto{1}{2\cdot\pi\cdot n})+1)}{(\pi\cdot(-1+4\cdot n^2))}$

$=\frac{-4\cdot A}{\pi\cdot(4\cdot n^2-1)}$ ✓

$N=0$

$\frac{-4A}{-\pi}=\frac{4A}{\pi}$ ✓ ✓

$\frac{a_0}{2}=\frac{4A}{\pi}/2$

Bn $\frac{2}{T}\cdot\int_0^T A\cdot\sin\left(\frac{\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$

$-(-\sin(\pi\cdot(-1+2\cdot n))-2\cdot\sin(\pi\cdot(-1+2\cdot n))\cdot n-\sin(\pi\cdot(1+2\cdot n))+2\cdot\sin(\pi\cdot(1+2\cdot n))\cdot n)\cdot\frac{A}{(\pi\cdot((-1+2\cdot n)\cdot(1+2\cdot n)))}$

$-2\cdot\sin(\cancelto{0}{2\cdot\pi\cdot n})\cdot\frac{A}{(\pi\cdot(-1+4\cdot n^2))}$ ✓

0

$$\frac{2\cdot A}{\pi}-\sum_{n=1}^{3}\frac{4\cdot A}{\pi\cdot(4\cdot n^2-1)}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$ ✓

$$\boxed{2\cdot\frac{A}{\pi}-\frac{4}{3}\cdot\frac{A}{\pi}\cdot\cos\left(2\cdot\frac{\pi}{T}\cdot t\right)-\frac{4}{15}\cdot\frac{A}{\pi}\cdot\cos\left(4\cdot\frac{\pi}{T}\cdot t\right)-\frac{4}{35}\cdot\frac{A}{\pi}\cdot\cos\left(6\cdot\frac{\pi}{T}\cdot t\right)}$$ ✓

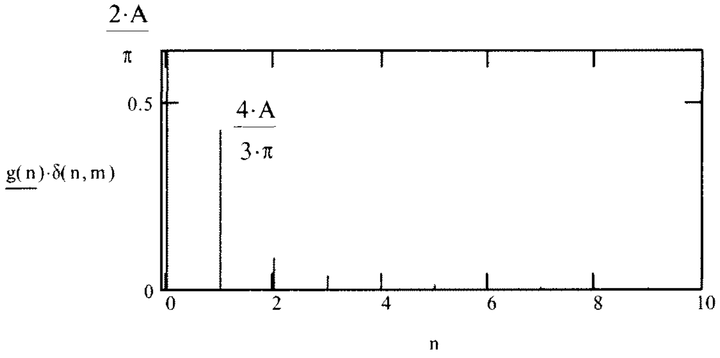
Ken Kaiser

Pg 405 (2)

$m := -1, 0 .. 11 \qquad A := 1$

$n := 0, 1 .. 10$

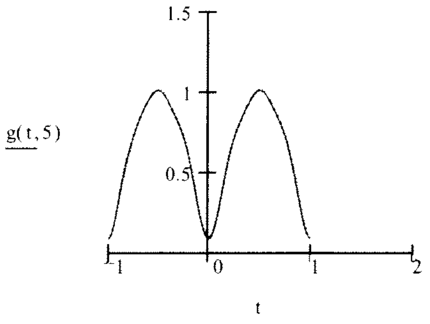
$$g(n) := \mathrm{if}\left[n<1, \frac{2\cdot A}{\pi}, \left|\frac{-4\cdot A}{\pi\cdot(4\cdot n^2-1)}\right|\right]$$



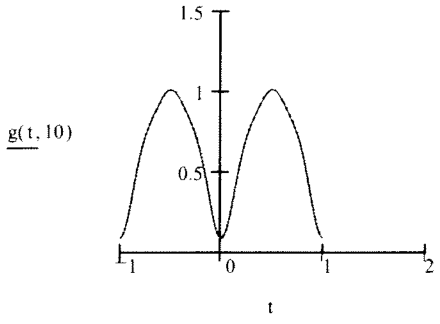
$T := 1 \qquad t := -T, -T + \frac{T}{1000} .. T$

$$g(t,m) := \frac{2\cdot A}{\pi} - \sum_{n=1}^{3} \frac{4\cdot A}{\pi\cdot(4\cdot n^2-1)}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$

n=5



n=10



Pg. 405 (1A)

$$\frac{}{\cdot n)))} - T\cdot\frac{A}{(\pi\cdot((1+2\cdot n)\cdot(-1+2\cdot n)))}\Bigg]$$

P.405

n=0 $\frac{2A}{\pi}$ √ed

n=1 $-\frac{4A}{\pi}\left(\frac{1}{4-1}\right)\cos\left(\frac{2\pi}{T}t\right) = \frac{-4A}{3\pi}\cos\left(\frac{2\pi}{T}t\right)$ √ed

n=2 $\frac{-4A}{\pi}\left(\frac{1}{15}\right)\cos\left(\frac{4\pi}{T}t\right) = \frac{-4A}{15\pi}\cos\left(\frac{4\pi}{T}t\right)$ √ed

n=3 $\frac{-4A}{\pi}\left(\frac{1}{35}\right)\cos\left(\frac{6\pi}{T}t\right) = \frac{-4A}{35\pi}\cos\left(\frac{6\pi}{T}t\right)$ √ed

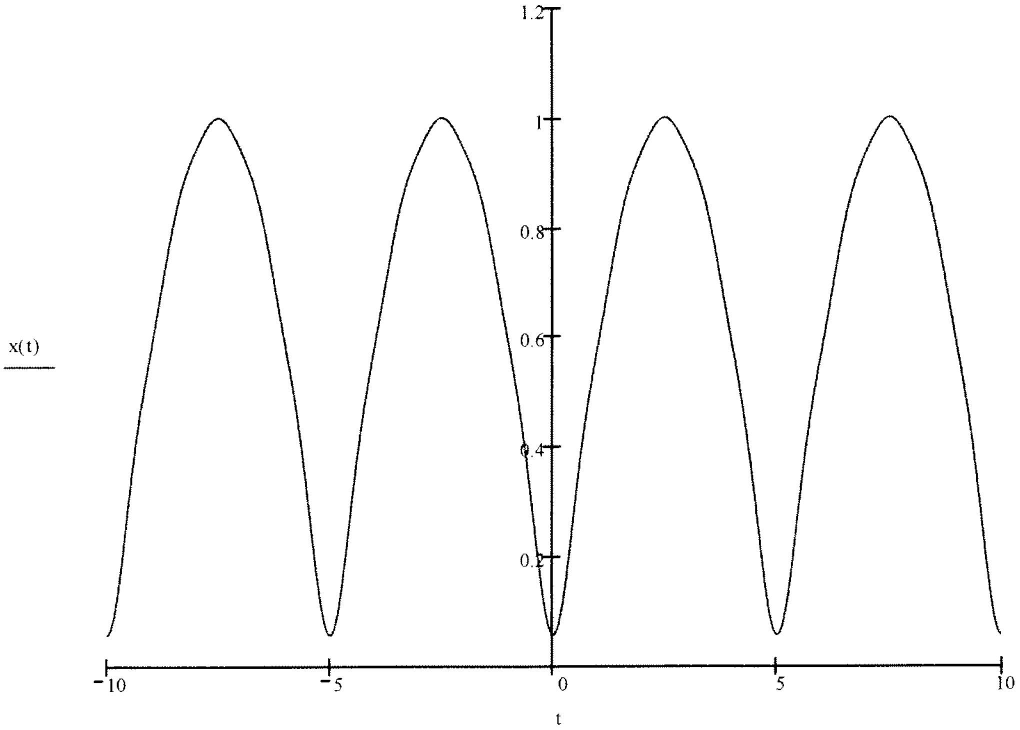
Ken Kaiser

$A := 1$

$f_0 := \frac{1}{5}$

$N := 5 \quad T := \frac{1}{f_0}$

$$x(t) := \frac{2 \cdot A}{\pi} - 4 \cdot \frac{A}{\pi} \cdot \sum_{n=1}^{N} \frac{1}{4 \cdot n^2 - 1} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$

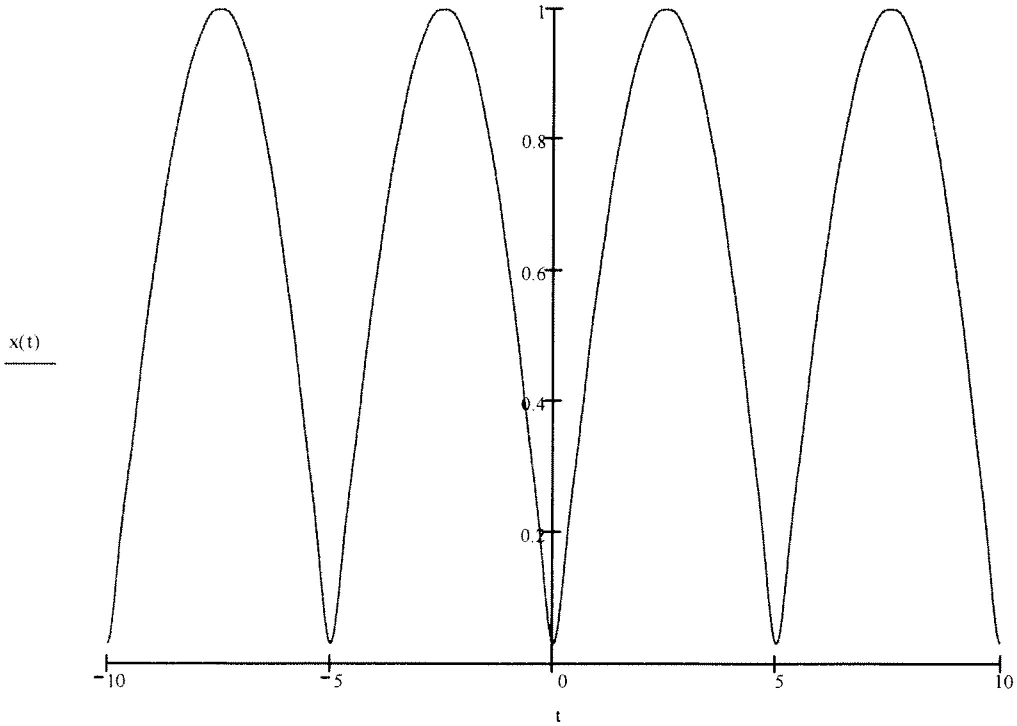


$A := 1$

$f_0 := \frac{1}{5}$

$N := 10 \quad T := \frac{1}{f_0}$

$$x(t) := \frac{2 \cdot A}{\pi} - 4 \cdot \frac{A}{\pi} \cdot \sum_{n=1}^{N} \frac{1}{4 \cdot n^2 - 1} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$



$m := -1, 0 .. 11$

$n := 0, 1 .. 10$ $\quad$ $A := 1$

$$g(n) := \mathrm{if}\left(n < 1, \frac{2 \cdot A}{\pi}, 4 \cdot \frac{A}{\pi} \cdot \frac{1}{4 \cdot n^2 - 1}\right)$$