Problem # 65 Full Rectified Cosine Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \cos\left(\frac{\pi}{T} \cdot t\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \cos\left(\frac{\pi}{T} \cdot t\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Results:

$a(0) = 1.655$ $\qquad b(0) = 0$

$a(1) = 0.552$ $\qquad b(1) = 0$

$a(2) = -0.11$ $\qquad b(2) = 0$

$a(3) = 0.047$ $\qquad b(3) = 0$

$a(4) = -0.026$ $\qquad b(4) = 0$

$a(5) = 0.017$ $\qquad b(5) = 0$

$a(6) = -0.012$ $\qquad b(6) = 0$

$a(7) = 8.488 \cdot 10^{-3}$ $\qquad b(7) = 0$

$a(8) = -6.491 \cdot 10^{-3}$ $\qquad b(8) = 0$

Check using fourier series definition:

$$Y(h) := \frac{4 \cdot A}{\pi} \cdot \frac{(-1)^{h-1}}{4 \cdot h^2 - 1}$$

$Y(0) = 1.655$

$Y(1) = 0.552$

$Y(2) = -0.11$

$Y(3) = 0.047$

$Y(4) = -0.026$

$Y(5) = 0.017$

$Y(6) = -0.012$

$Y(7) = 8.488 \cdot 10^{-3}$

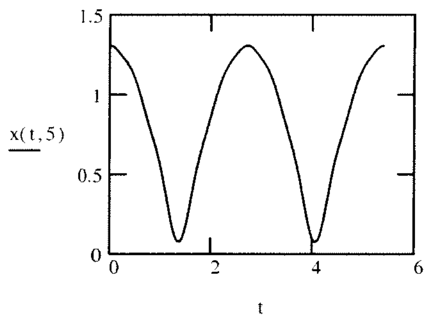
$Y(8) = -6.491 \cdot 10^{-3}$

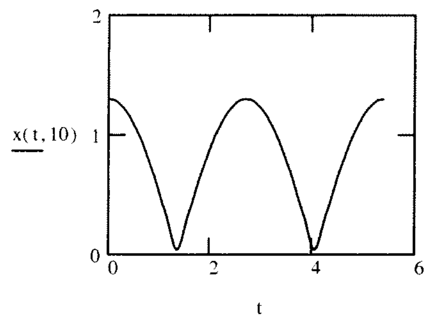
Conclusion: These values match the a coeffecients

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \frac{2\cdot A}{\pi} + \frac{4\cdot A}{\pi}\cdot \sum_{n=1}^{c} \frac{(-1)^{n-1}}{4\cdot n^2 - 1}\cdot \cos\left(\frac{2\cdot \pi\cdot n}{T}\cdot t\right)$$





Problem 65
Full-Rectified Cosine Wave

Initial Values

$A := 1.3$ $\quad T := 2.7$ $\quad \theta := \frac{\pi}{5}$ $\quad \tau := 0.68$ $\quad \alpha := 9.4$ $\quad fr := 4.3$ Hz

$a := 0.32$ $\quad b := 2.1$ $\quad VDC := 0.47V$ $\quad j := \sqrt{-1}$

Coefficient Equations

$$a(n) := \frac{2}{T} \cdot \int_{\frac{-T}{2}}^{\frac{T}{2}} A \cdot \cos\left(\frac{\pi}{T} t\right) \cdot \cos\left(\frac{2\pi n}{T} t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{\frac{-T}{2}}^{\frac{T}{2}} A \cdot \cos\left(\frac{\pi}{T} t\right) \cdot \sin\left(\frac{2\pi n}{T} t\right) dt$$

Coefficient Values

$a(0) = 1.65521$	$b(0) = 0$
$a(1) = 0.55174$	$b(1) = 0$
$a(2) = -0.11035$	$b(2) = 0$
$a(3) = 0.04729$	$b(3) = 0$
$a(4) = -0.02627$	$b(4) = 0$
$a(5) = 0.01672$	$b(5) = 0$
$a(6) = -0.01157$	$b(6) = 0$
$a(7) = 8.48826 \times 10^{-3}$	$b(7) = 0$
$a(8) = -6.49103 \times 10^{-3}$	$b(8) = 0$

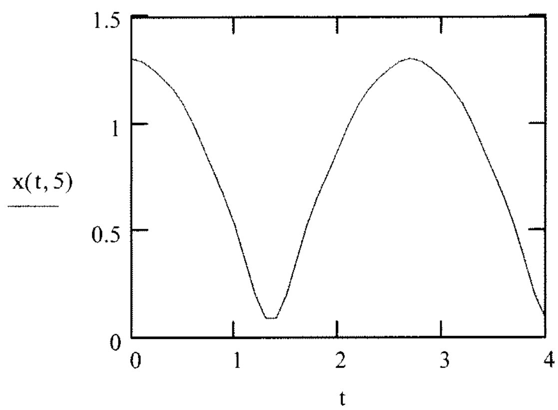
Exact Value Check

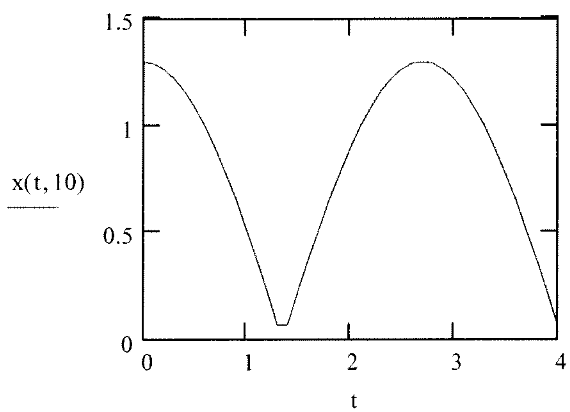
$$E(N) := \frac{4A}{\pi} \cdot \frac{(-1)^{N-1}}{4N^2 - 1}$$

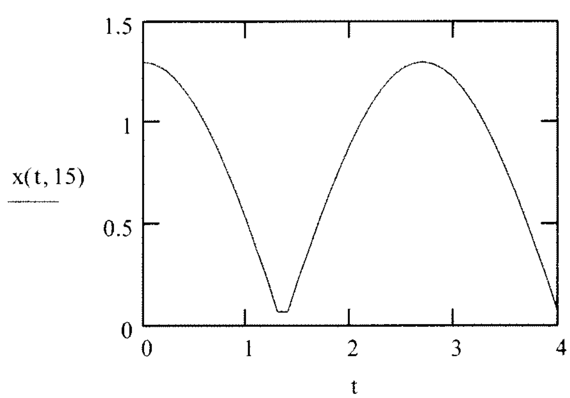
DC Offset	$E(0) = 1.65521$
1st Harmonic Frequency	$E(1) = 0.55174$
2nd Harmonic Frequency	$E(2) = -0.11035$
3rd Harmonic Frequency	$E(3) = 0.04729$
4th Harmonic Frequency	$E(4) = -0.02627$
5th Harmonic Frequency	$E(5) = 0.01672$
6th Harmonic Frequency	$E(6) = -0.01157$
7th Harmonic Frequency	$E(7) = 8.48826 \times 10^{-3}$
8th Harmonic Frequency	$E(8) = -6.49103 \times 10^{-3}$

Graphing

$$x(t,m) := \frac{2A}{\pi} + \frac{4A}{\pi} \cdot \sum_{n=1}^{m} \frac{(-1)^{n-1}}{4n^2 - 1} \cdot \cos\left(\frac{2\pi n}{T} t\right) \qquad t := 0, .1 .. \frac{3T}{2}$$







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Exponential Wave #1

Constants

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 0.94 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Equations for Determining Series Coefficients

$$a(n) := \frac{2}{T} \cdot \int_0^T A \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_0^T A \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$c(n) := \sqrt{a(n)^2 + b(n)^2}$$

$$F(n) := \frac{1}{2} \cdot (a(n) - j \cdot b(n)) \qquad F_o(n) := \frac{a(0)}{2}$$

Series Coefficients

$a(0) = 1.68550$	$b(0) = 0$	$c(0) = 1.685$
$a(1) = 0.037$	$b(1) = 0.247$	$c(1) = 0.249$
$a(2) = 9.379\cdot 10^{-3}$	$b(2) = 0.125$	$c(2) = 0.126$
$a(3) = 4.181\cdot 10^{-3}$	$b(3) = 0.084$	$c(3) = 0.084$
$a(4) = 2.354\cdot 10^{-3}$	$b(4) = 0.063$	$c(4) = 0.063$
$a(5) = 1.507\cdot 10^{-3}$	$b(5) = 0.05$	$c(5) = 0.05$
$a(6) = 1.047\cdot 10^{-3}$	$b(6) = 0.042$	$c(6) = 0.042$
$a(7) = 7.695\cdot 10^{-4}$	$b(7) = 0.036$	$c(7) = 0.036$
$a(8) = 5.892\cdot 10^{-4}$	$b(8) = 0.032$	$c(8) = 0.032$

$F_0(0) = 0.843$	$F_0(-0) = 0.843$
$F(1) = 0.018 - 0.123i$	$F(-1) = 0.018 + 0.123i$
$F(2) = 4.689\cdot 10^{-3} - 0.063i$	$F(-2) = 4.689\cdot 10^{-3} + 0.063i$
$F(3) = 2.091\cdot 10^{-3} - 0.042i$	$F(-3) = 2.091\cdot 10^{-3} + 0.042i$
$F(4) = 1.177\cdot 10^{-3} - 0.031i$	$F(-4) = 1.177\cdot 10^{-3} + 0.031i$
$F(5) = 7.536\cdot 10^{-4} - 0.025i$	$F(-5) = 7.536\cdot 10^{-4} + 0.025i$
$F(6) = 5.236\cdot 10^{-4} - 0.021i$	$F(-6) = 5.236\cdot 10^{-4} + 0.021i$
$F(7) = 3.848\cdot 10^{-4} - 0.018i$	$F(-7) = 3.848\cdot 10^{-4} + 0.018i$
$F(8) = 2.946\cdot 10^{-4} - 0.016i$	$F(-8) = 2.946\cdot 10^{-4} + 0.016i$

Verification of Series Coefficients

$$F_p(N) := A\cdot\frac{1-e^{-\alpha}}{\alpha + j\cdot 2\cdot\pi\cdot N}$$

$F_p(0) = 0.843$

$F_p(1) = 0.018 - 0.123i$

$F_p(2) = 4.689\cdot 10^{-3} - 0.063i$

$F_p(3) = 2.091\cdot 10^{-3} - 0.042i$

$F_p(4) = 1.177\cdot 10^{-3} - 0.031i$

$F_p(5) = 7.538\cdot 10^{-4} - 0.025i$

$F_p(6) = 5.236\cdot 10^{-4} - 0.021i$

$F_p(7) = 3.848\cdot 10^{-4} - 0.018i$

$F_p(8) = 2.946\cdot 10^{-4} - 0.016i$

$F_p(-0) = 0.843$

$F_p(-1) = 0.018 + 0.123i$

$F_p(-2) = 4.689\cdot 10^{-3} + 0.063i$

$F_p(-3) = 2.091\cdot 10^{-3} + 0.042i$

$F_p(-4) = 1.177\cdot 10^{-3} + 0.031i$

$F_p(-5) = 7.538\cdot 10^{-4} + 0.025i$

$F_p(-6) = 5.236\cdot 10^{-4} + 0.021i$

$F_p(-7) = 3.848\cdot 10^{-4} + 0.018i$

$F_p(-8) = 2.946\cdot 10^{-4} + 0.016i$

Ken Kaiser

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Problem 4: Determining Fourier Coefficients Using Integral Definitions for Exponential Wave #1

Part I: Defining Constants

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \alpha := 9.4 \quad f := 4.3 \quad \tau := .68 \quad a := .32 \quad b := 2.1 \quad j := \sqrt{-1} \quad V_{dc} := .47$

Part II: Defining Coefficient Equations

$$f1(n) := \frac{1}{T} \cdot \left[\int_0^a A \cdot e^{\left(-\frac{\alpha}{T} \cdot t\right)} \cdot e^{\left(-j \cdot 2 \cdot \pi \cdot \frac{n}{T} \cdot t\right)} dt \right] + \frac{1}{T} \cdot \int_a^T e^{\left(-j \cdot 2 \cdot \pi \cdot \frac{n}{T} \cdot t\right)} \cdot 0 \, dt$$

Part III: Defining Exact Equations

$$fp(n) := A \cdot \left[\frac{1 - \exp\left[-a \cdot \left(\frac{\alpha + j \cdot 2 \cdot \pi \cdot n}{T}\right)\right]}{\alpha + j \cdot 2 \cdot \pi \cdot n} \right]$$

fp(n) is the exact coefficient given in the exact Fourier Series equation given.

Part IV: Evaluating Integral Coefficient Equations to Determine Series Coefficients

$f1(0) = 0.09291$	$f1(0) = 0.09291$
$f1(1) = 0.08673 - 0.02721i$	$f1(-1) = 0.08673 + 0.02721i$
$f1(2) = 0.06999 - 0.04833i$	$f1(-2) = 0.06999 + 0.04833i$
$f1(3) = 0.04739 - 0.05927i$	$f1(-3) = 0.04739 + 0.05927i$
$f1(4) = 0.02488 - 0.05917i$	$f1(-4) = 0.02488 + 0.05917i$
$f1(5) = 7.63029 \cdot 10^{-3} - 0.05044i$	$f1(-5) = 7.63029 \cdot 10^{-3} + 0.05044i$
$f1(6) = -1.60145 \cdot 10^{-3} - 0.03762i$	$f1(-6) = -1.60145 \cdot 10^{-3} + 0.03762i$
$f1(7) = -3.05038 \cdot 10^{-3} - 0.02556i$	$f1(-7) = -3.05038 \cdot 10^{-3} + 0.02556i$
$f1(8) = 5.94886 \cdot 10^{-4} - 0.01771i$	$f1(-8) = 5.94886 \cdot 10^{-4} + 0.01771i$

Problem 4: Part IV: (cont.)

$fp(0) = 0.09291$

$fp(1) = 0.08673 - 0.02721i$

$fp(2) = 0.06999 - 0.04833i$

$fp(3) = 0.04739 - 0.05927i$

$fp(4) = 0.02488 - 0.05917i$

$fp(5) = 7.63029 \cdot 10^{-3} - 0.05044i$

$fp(6) = -1.60145 \cdot 10^{-3} - 0.03762i$

$fp(7) = -3.05038 \cdot 10^{-3} - 0.02556i$

$fp(8) = 5.94886 \cdot 10^{-4} - 0.01771i$

$fp(0) = 0.09291$

$fp(-1) = 0.08673 + 0.02721i$

$fp(-2) = 0.06999 + 0.04833i$

$fp(-3) = 0.04739 + 0.05927i$

$fp(-4) = 0.02488 + 0.05917i$

$fp(-5) = 7.63029 \cdot 10^{-3} + 0.05044i$

$fp(-6) = -1.60145 \cdot 10^{-3} + 0.03762i$

$fp(-7) = -3.05038 \cdot 10^{-3} + 0.02556i$

$fp(-8) = 5.94886 \cdot 10^{-4} + 0.01771i$

$f1(0) = 0.09291$

$fp(0) = 0.09291$

$$A \cdot \left[\frac{1 - \exp\left[-a \cdot \left(\frac{\alpha}{T} \right) \right]}{\alpha} \right] = 0.09291$$

The values of f1(0), fp(0) and the exact value of the dc coefficient agree.

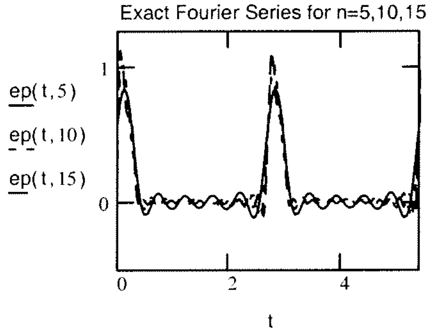
Problem 4: Part V: Comparing the Exact Fourier Series to Series Determined by Integral Coefficients

$$t := 0, \frac{T}{100} .. 100$$

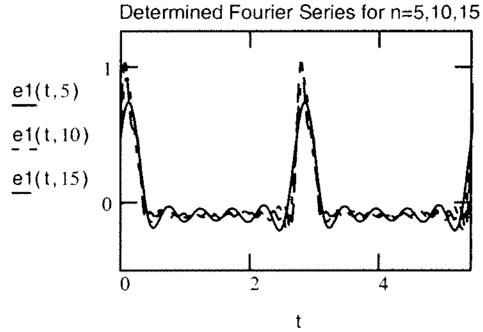
$$ep(t,m) := A\cdot\left[\frac{1-\exp\left[-a\cdot\left(\frac{\alpha}{T}\right)\right]}{\alpha}\right] + \left[A\cdot\sum_{n=-m}^{-1}\left[\frac{1-\exp\left[-a\cdot\left(\frac{\alpha+j\cdot 2\cdot\pi\cdot n}{T}\right)\right]}{\alpha+j\cdot 2\cdot\pi\cdot n}\right]\cdot\exp\left(j\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t\right) + A\cdot\sum_{n=1}^{m}\left[\frac{1-\exp\left[-a\cdot\left(\frac{\alpha+j\cdot 2\cdot\pi\cdot n}{T}\right)\right]}{\alpha+j\cdot 2\cdot\pi\cdot n}\right]\cdot\exp\left(j\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\right]$$

$$e1(t,m) := \sum_{n=-m}^{-1} f1(n)\cdot\exp\left(j\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t\right) + \sum_{n=1}^{m} f1(n)\cdot\exp\left(j\cdot 2\cdot\pi\cdot\frac{n}{T}\cdot t\right)$$

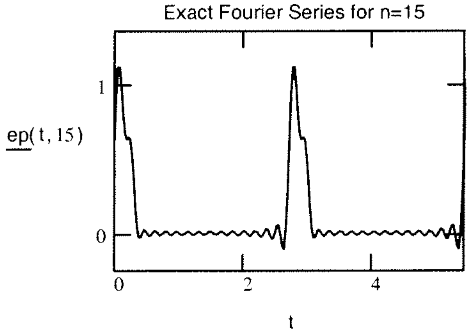
Exact Fourier Series for n=5,10,15



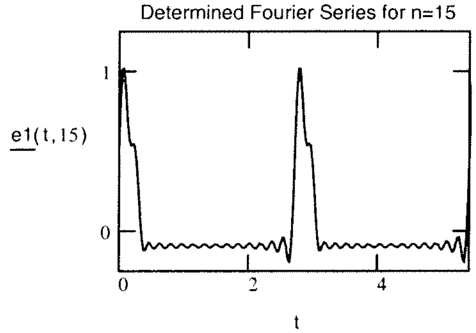
Determined Fourier Series for n=5,10,15



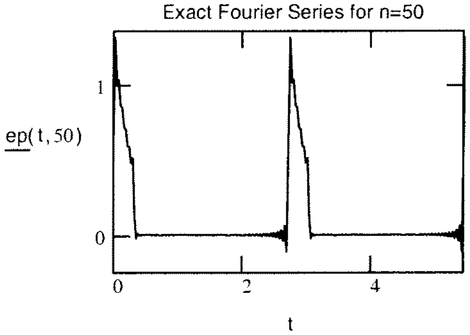
Exact Fourier Series for n=15



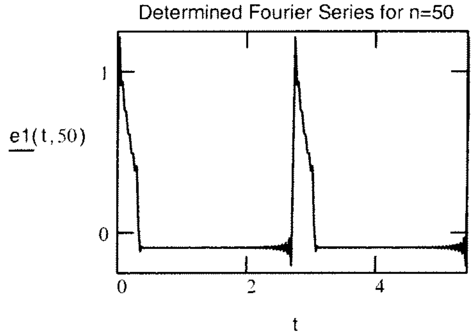
Determined Fourier Series for n=15



Exact Fourier Series for n=50



Determined Fourier Series for n=50



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$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47$

$$a(N) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt$$

$$b(N) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot \sin\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt$$

$a(0) = 30.4084$	$b(0) = 0$
$a(1) = -21.01783$	$b(1) = -14.04882$
$a(2) = 10.91017$	$b(2) = 14.58523$
$a(3) = -6.05611$	$b(3) = -12.14414$
$a(4) = 3.73171$	$b(4) = 9.97746$
$a(5) = -2.49868$	$b(5) = -8.3509$
$a(6) = 1.77989$	$b(6) = 7.13831$
$a(7) = -1.3283$	$b(7) = -6.21507$
$a(8) = 1.0275$	$b(8) = 5.49444$

$$ap(n) := 2 \cdot A \cdot \frac{e^{\frac{\alpha}{2}} - e^{-\frac{\alpha}{2}}}{1} \cdot \frac{(-1)^n}{\alpha^2 + 4 \cdot n^2 \cdot \pi^2} \cdot \alpha \qquad bp(n) := 2 \cdot A \cdot \frac{e^{\frac{\alpha}{2}} - e^{-\frac{\alpha}{2}}}{1} \cdot \frac{(-1)^n}{\alpha^2 + 4 \cdot n^2 \cdot \pi^2} \cdot 2 \cdot n \cdot \pi$$

$ap(1) = -21.01783$	$bp(1) = -14.04882$
$ap(2) = 10.91017$	$bp(2) = 14.58523$
$ap(3) = -6.05611$	$bp(3) = -12.14414$
$ap(4) = 3.73171$	$bp(4) = 9.97746$
$ap(5) = -2.49868$	$bp(5) = -8.3509$
$ap(6) = 1.77989$	$bp(6) = 7.13831$
$ap(7) = -1.3283$	$bp(7) = -6.21507$
$ap(8) = 1.0275$	$bp(8) = 5.49444$

$$Favg := A \cdot \frac{e^{\frac{\alpha}{2}} - e^{-\frac{\alpha}{2}}}{\alpha}$$

$Favg = 15.2042$

$\frac{a(0)}{2} = 15.2042$

Problem # 69 (To obtain an answer the value of α had to be changed to a lower value. I picked 2): Exponential Wave #4

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 2 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$F(n) := \frac{1}{T} \cdot \int_{(0)}^{T} \frac{A}{T} \cdot t \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot e^{-j \cdot \frac{2 \cdot \pi \cdot n}{T} \cdot t} \, dt$$

Results:

$F(0) = 0.193$

$F(1) = -0.029 + 0.01i$

$F(2) = -8.773 \cdot 10^{-3} + 0.011i$

$F(3) = -4.038 \cdot 10^{-3} + 8.573 \cdot 10^{-3} i$

$F(4) = -2.3 \cdot 10^{-3} + 6.677 \cdot 10^{-3} i$

$F(5) = -1.48 \cdot 10^{-3} + 5.434 \cdot 10^{-3} i$

$F(6) = -1.031 \cdot 10^{-3} + 4.57 \cdot 10^{-3} i$

$F(7) = -7.59 \cdot 10^{-4} + 3.939 \cdot 10^{-3} i$

$F(8) = -5.818 \cdot 10^{-4} + 3.459 \cdot 10^{-3} i$

Check using fourier series definition:

$$Y(h) := A \cdot \frac{1 - j \cdot 2 \cdot \pi \cdot h \cdot e^{-\alpha} - e^{-\alpha} - \alpha \cdot e^{-\alpha}}{(\alpha + j \cdot 2 \cdot \pi \cdot h)^2}$$

$Y(0) = 0.193$

$Y(1) = -0.029 + 0.01i$

$Y(2) = -8.773 \cdot 10^{-3} + 0.011i$

$Y(3) = -4.038 \cdot 10^{-3} + 8.573 \cdot 10^{-3}i$

$Y(4) = -2.3 \cdot 10^{-3} + 6.677 \cdot 10^{-3}i$

$Y(5) = -1.48 \cdot 10^{-3} + 5.434 \cdot 10^{-3}i$

$Y(6) = -1.031 \cdot 10^{-3} + 4.57 \cdot 10^{-3}i$

$Y(7) = -7.59 \cdot 10^{-4} + 3.939 \cdot 10^{-3}i$

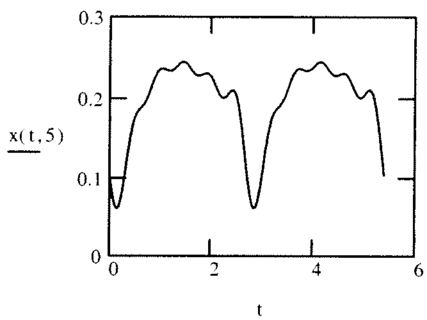
$Y(8) = -5.818 \cdot 10^{-4} + 3.459 \cdot 10^{-3}i$

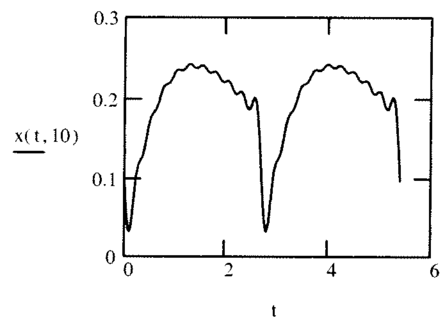
Conclusion: These values match the a coeffecients

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := A \cdot \left[\sum_{n=-c}^{c} \frac{1 - j \cdot 2 \cdot \pi \cdot n \cdot \exp(-\alpha) - \exp(-\alpha) - \alpha \cdot \exp(-\alpha)}{(\alpha + j \cdot 2 \cdot \pi \cdot n)^2} \cdot \exp\left(j \cdot \frac{2 \cdot \pi \cdot n}{T} \cdot t\right) \right]$$





Problem # 70: Alternating Exponential Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$F(n) := \frac{1}{T}\cdot\int_0^{\frac{T}{2}} A\cdot e^{-\frac{\alpha}{T}\cdot t}\cdot e^{-j\cdot\frac{2\cdot\pi\cdot n}{T}\cdot t}\,dt + \frac{1}{T}\cdot\int_{\frac{T}{2}}^{T} -A\cdot e^{\left[-\frac{\alpha}{T}\cdot\left(t-\frac{T}{2}\right)\right]}\cdot e^{-j\cdot\frac{2\cdot\pi\cdot n}{T}\cdot t}\,dt$$

Results:

$F(0) = 0$

$F(1) = 0.193 - 0.129i$

$F(2) = 0$

$F(3) = 0.056 - 0.111i$

$F(4) = 0$

$F(5) = 0.023 - 0.077i$

$F(6) = 0$

$F(7) = 0.012 - 0.057i$

$F(8) = 0$

Check using fourier series definition:

$$Y(h) := 2\cdot A\cdot\frac{1+\exp\left(-\frac{\alpha}{2}\right)}{\alpha + j\cdot 2\cdot\pi\cdot(2\cdot h - 1)}$$

$Y(1) = 0.193 - 0.129i$

$Y(2) = 0.056 - 0.111i$

$Y(3) = 0.023 - 0.077i$

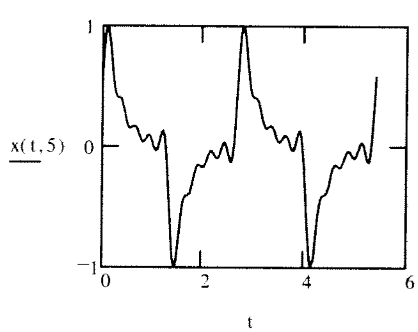
$Y(4) = 0.012 - 0.057i$

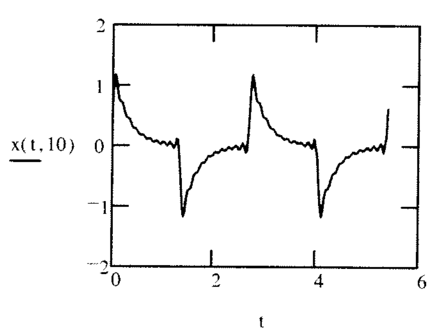
Conclusion: These values match the odd harmonics of the Fn function

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.027 .. 5.4$

$$x(t,c) := 2\cdot A\cdot\left[\sum_{n=-c}^{-1}\frac{1+\exp\left(-\frac{\alpha}{2}\right)}{\alpha + j\cdot 2\cdot\pi\cdot(2\cdot n + 1)}\cdot\exp\left[\frac{j\cdot 2\cdot\pi\cdot(2\cdot n+1)}{T}\cdot t\right]\right] \dots$$
$$+ 2\cdot A\cdot\left[\sum_{n=1}^{c}\frac{1+\exp\left(-\frac{\alpha}{2}\right)}{\alpha + j\cdot 2\cdot\pi\cdot(2\cdot n - 1)}\cdot\exp\left[\frac{j\cdot 2\cdot\pi\cdot(2\cdot n-1)}{T}\cdot t\right]\right]$$





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Alternating Exponential Wave

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

$$F_1(N) := \frac{1}{T}\cdot\int_0^{\frac{T}{2}} A\cdot e^{-\frac{\alpha}{T}\cdot t}\cdot e^{j\cdot\frac{2\cdot\pi\cdot(2\cdot N+1)}{T}\cdot t}\,dt + \frac{1}{T}\cdot\int_{\frac{T}{2}}^{T} -A\cdot e^{-\frac{\alpha}{T}\cdot\left(t-\frac{T}{2}\right)}\cdot e^{j\cdot\frac{2\cdot\pi\cdot(2\cdot N+1)}{T}\cdot t}\,dt$$

$$F_2(N) := \frac{1}{T}\cdot\int_0^{\frac{T}{2}} A\cdot e^{-\frac{\alpha}{T}\cdot t}\cdot e^{j\cdot\frac{2\cdot\pi\cdot(2\cdot N-1)}{T}\cdot t}\,dt + \frac{1}{T}\cdot\int_{\frac{T}{2}}^{T} -A\cdot e^{-\frac{\alpha}{T}\cdot\left(t-\frac{T}{2}\right)}\cdot e^{j\cdot\frac{2\cdot\pi\cdot(2\cdot N-1)}{T}\cdot t}\,dt$$

$F_1(0) = 0.19292 + 0.12895i$	$F_2(0) = 0.19292 - 0.12895i$
$F_1(-1) = 0.19292 - 0.12895i$	$F_2(1) = 0.19292 + 0.12895i$
$F_1(-2) = 0.05559 - 0.11147i$	$F_2(2) = 0.05559 + 0.11147i$
$F_1(-3) = 0.02293 - 0.07665i$	$F_2(3) = 0.02293 + 0.07665i$
$F_1(-4) = 0.01219 - 0.05705i$	$F_2(4) = 0.01219 + 0.05705i$
$F_1(-5) = 0.00751 - 0.04515i$	$F_2(5) = 0.00751 + 0.04515i$
$F_1(-6) = 0.00507 - 0.03727i$	$F_2(6) = 0.00507 + 0.03727i$
$F_1(-7) = 0.00365 - 0.0317i$	$F_2(7) = 0.00365 + 0.0317i$
$F_1(-8) = 0.00275 - 0.02756i$	$F_2(8) = 0.00275 + 0.02756i$

$$E_1(m,M) := 2\cdot A\cdot \sum_{n=m}^{M} \frac{1+e^{\frac{-\alpha}{2}}}{\alpha + j\cdot 2\cdot\pi\cdot(2\cdot n+1)} \qquad E_2(m,M) := 2\cdot A\cdot \sum_{n=m}^{M} \frac{1+e^{\frac{-\alpha}{2}}}{\alpha + j\cdot 2\cdot\pi\cdot(2\cdot n-1)}$$

$E_1(0,0) = 0.19292 - 0.12895i$ $\quad$ $E_2(0,0) = 0.19292 + 0.12895i$

$E_1(-1,-1) = 0.19292 + 0.12895i$ $\quad$ $E_2(1,1) = 0.19292 - 0.12895i$

$E_1(-2,-2) = 0.05559 + 0.11147i$ $\quad$ $E_2(2,2) = 0.05559 - 0.11147i$

$E_1(-3,-3) = 0.02293 + 0.07665i$ $\quad$ $E_2(3,3) = 0.02293 - 0.07665i$

$E_1(-4,-4) = 0.01219 + 0.05705i$ $\quad$ $E_2(4,4) = 0.01219 - 0.05705i$

$E_1(-5,-5) = 0.00751 + 0.04515i$ $\quad$ $E_2(5,5) = 0.00751 - 0.04515i$

$E_1(-6,-6) = 0.00507 + 0.03727i$ $\quad$ $E_2(6,6) = 0.00507 - 0.03727i$

$E_1(-7,-7) = 0.00365 + 0.0317i$ $\quad$ $E_2(7,7) = 0.00365 - 0.0317i$

$E_1(-8,-8) = 0.00275 + 0.02756i$ $\quad$ $E_2(8,8) = 0.00275 - 0.02756i$

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Decloration of Coefficients

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad j := \sqrt{-1} \quad V_{dc} := .47$

Decaying Exponential Sinsoidal Wave

$$F1(n) := \left(\frac{1}{T}\right)\cdot\int_0^T \left[\left(A\cdot e^{-\frac{\alpha}{T}\cdot t}\cdot\cos(2\cdot\pi\cdot f_r\cdot t+\theta)\right)\cdot\left[e^{-j\cdot\left(\frac{2\cdot\pi\cdot n}{T}\right)\cdot t}\right]\right]dt$$

Coefficient Solution

$F1(-8) = -0.00894 - 0.02127i$ $\quad F1(-2) = -0.00860 - 0.00290i$ $\quad F1(4) = -0.00896 + 0.00642i$

$F1(-7) = -0.00949 - 0.01557i$ $\quad F1(-1) = -0.00851 - 0.00142i$ $\quad F1(5) = -0.00921 + 0.00870i$

$F1(-6) = -0.00943 - 0.01162i$ $\quad F1(0) = -0.00848$ $\quad F1(6) = -0.00943 + 0.01162i$

$F1(-5) = -0.00921 - 0.00870i$ $\quad F1(1) = -0.00851 + 0.00142i$ $\quad F1(7) = -0.00949 + 0.01557i$

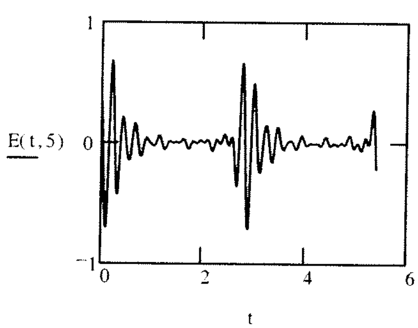
$F1(-4) = -0.00896 - 0.00642i$ $\quad F1(2) = -0.00860 + 0.00290i$ $\quad F1(8) = -0.00894 + 0.02127i$

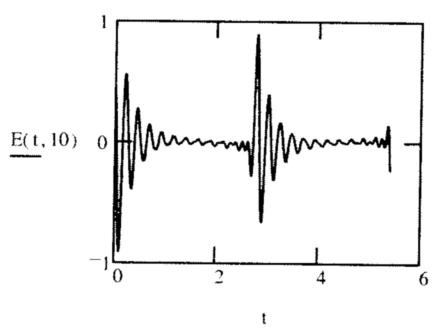
$F1(-3) = -0.00876 - 0.00454i$ $\quad F1(3) = -0.00876 + 0.00454i$

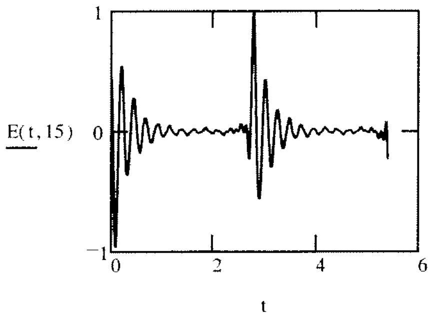
Exact Equation Solution

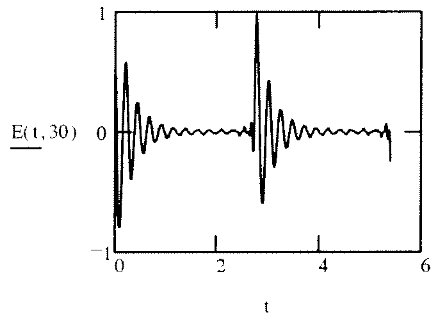
$$E(t,m) := \left[A\cdot\left(\frac{1-e^{-\alpha}}{\alpha}\right) + A\cdot\sum_{n=-m}^{-1}\frac{1-e^{-\alpha}}{\alpha+j\cdot 2\cdot\pi\cdot n}\cdot e^{j\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)} \ldots + \sum_{n=1}^{m}\frac{1-e^{-\alpha}}{\alpha+j\cdot 2\cdot\pi\cdot n}\cdot e^{j\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}\right]\cdot\left[\frac{1}{2}\cdot e^{j\cdot(2\cdot\pi\cdot f_r\cdot t+\theta)} + \frac{1}{2}\cdot e^{-j\cdot(2\cdot\pi\cdot f_r\cdot t+\theta)}\right]$$

$$t := 0, \frac{T}{10000} .. 2\cdot T$$









Check

$T := 1$ $\quad F_0 := 1$

$\alpha := 2 \cdot T$ $\quad F_r := 5$

$$F1(n) := \left(\frac{1}{T}\right) \cdot \int_0^T \left[\left(A \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot \cos(2 \cdot \pi \cdot f_r \cdot t + \theta)\right) \cdot \left[e^{-j \cdot \left(\frac{2 \cdot \pi \cdot n}{T}\right) \cdot t}\right]\right] dt$$

$\lvert F1(-8)\rvert = 0.03436$	$\lvert F1(-2)\rvert = 0.03726$	$\lvert F1(4)\rvert = 0.23537$
$\lvert F1(-7)\rvert = 0.04512$	$\lvert F1(-1)\rvert = 0.02473$	$\lvert F1(5)\rvert = 0.14363$
$\lvert F1(-6)\rvert = 0.06765$	$\lvert F1(0)\rvert = 0.02107$	$\lvert F1(6)\rvert = 0.06765$
$\lvert F1(-5)\rvert = 0.14363$	$\lvert F1(1)\rvert = 0.02473$	$\lvert F1(7)\rvert = 0.04512$
$\lvert F1(-4)\rvert = 0.23537$	$\lvert F1(2)\rvert = 0.03726$	$\lvert F1(8)\rvert = 0.03436$
$\lvert F1(-3)\rvert = 0.07073$	$\lvert F1(3)\rvert = 0.07073$	

$T := 1$ $\quad F_0 := 1$

$\alpha := 8 \cdot T$ $\quad F_r := 5$

$$F1(n) := \left(\frac{1}{T}\right) \cdot \int_0^T \left[\left(A \cdot e^{-\frac{\alpha}{T} \cdot t} \cdot \cos(2 \cdot \pi \cdot f_r \cdot t + \theta)\right) \cdot \left[e^{-j \cdot \left(\frac{2 \cdot \pi \cdot n}{T}\right) \cdot t}\right]\right] dt$$

$\lvert F1(-8)\rvert = 0.02841$	$\lvert F1(-2)\rvert = 0.02699$	$\lvert F1(4)\rvert = 0.06717$
$\lvert F1(-7)\rvert = 0.03571$	$\lvert F1(-1)\rvert = 0.01825$	$\lvert F1(5)\rvert = 0.06545$
$\lvert F1(-6)\rvert = 0.04769$	$\lvert F1(0)\rvert = 0.01539$	$\lvert F1(6)\rvert = 0.04769$
$\lvert F1(-5)\rvert = 0.06545$	$\lvert F1(1)\rvert = 0.01825$	$\lvert F1(7)\rvert = 0.03571$
$\lvert F1(-4)\rvert = 0.06717$	$\lvert F1(2)\rvert = 0.02699$	$\lvert F1(8)\rvert = 0.02841$
$\lvert F1(-3)\rvert = 0.04376$	$\lvert F1(3)\rvert = 0.04376$	

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Fourier Series Problem #3: Critically Damped Exponential Wave

Coefficients

$A := 1.3$ $T := 2.7$ $\Theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := 9.4$ $f_r := 4.3$ $a := 0.32$ $b := 2.1$

Equations

$$a(n) := \frac{2}{T} \cdot \int_0^T \left(A \cdot e \cdot t \cdot \alpha \cdot e^{-\alpha \cdot t}\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_0^T \left(A \cdot e \cdot t \cdot \alpha \cdot e^{-\alpha \cdot t}\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Series Coefficients for Zero through Eighth Harmonics

$a(0) = 0.278$	$b(0) = 0$
$a(1) = 0.232$	$b(1) = 0.122$
$a(2) = 0.136$	$b(2) = 0.178$
$a(3) = 0.052$	$b(3) = 0.172$
$a(4) = 1.376 \cdot 10^{-3}$	$b(4) = 0.141$
$a(5) = -0.023$	$b(5) = 0.108$
$a(6) = -0.033$	$b(6) = 0.08$
$a(7) = -0.035$	$b(7) = 0.06$
$a(8) = -0.034$	$b(8) = 0.046$

$$c_n(n) := \sqrt{\left(a(n)^2 + b(n)^2\right)}$$

$c_n(0) = 0.278$	$c_n(6) = 0.087$
$c_n(1) = 0.262$	$c_n(7) = 0.07$
$c_n(2) = 0.224$	$c_n(8) = 0.057$
$c_n(3) = 0.179$	
$c_n(4) = 0.141$	
$c_n(5) = 0.11$	

Verification of Harmonics

$$c_N(N) := \frac{2 \cdot A \cdot e}{\alpha \cdot T} \cdot \frac{1}{1 + \left(\frac{2 \cdot \pi \cdot N}{\alpha \cdot T}\right)^2}$$

$c_N(0) = 0.278$

$c_N(1) = 0.262$

$c_N(2) = 0.224$

$c_N(3) = 0.179$

$c_N(4) = 0.141$

$c_N(5) = 0.11$

$c_N(6) = 0.087$

$c_N(7) = 0.07$

$c_N(8) = 0.057$

Problem # 73: Double-Sided Exponential Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{\alpha}{T} \cdot |t|} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt \right]$$

$$b(n) := \frac{2}{T} \cdot \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{\alpha}{T} \cdot |t|} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt \right]$$

Results:

$a(0) = 0.54816$	$b(0) = 0$
$a(1) = 0.38584$	$b(1) = 0$
$a(2) = 0.19667$	$b(2) = 0$
$a(3) = 0.11118$	$b(3) = 0$
$a(4) = 0.06727$	$b(4) = 0$
$a(5) = 0.04587$	$b(5) = 0$
$a(6) = 0.03209$	$b(6) = 0$
$a(7) = 0.02438$	$b(7) = 0$
$a(8) = 0.01852$	$b(8) = 0$

Check using fourier series definition:

$$Y(h) := 4\cdot\alpha\cdot A\cdot\frac{1+(-1)^{h-1}\cdot e^{-\frac{\alpha}{2}}}{\left(\alpha^2+4\cdot\pi^2\cdot h^2\right)}$$

$Y(0) = 0.54816$

$Y(1) = 0.38584$

$Y(2) = 0.19667$

$Y(3) = 0.11118$

$Y(4) = 0.06727$

$Y(5) = 0.04587$

$Y(6) = 0.03209$

$Y(7) = 0.02438$

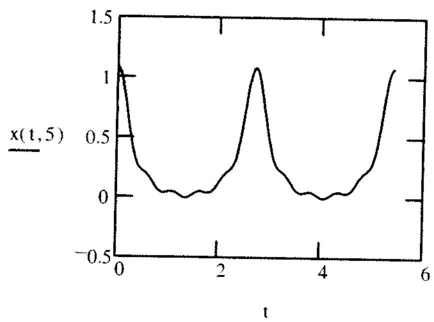
$Y(8) = 0.01852$

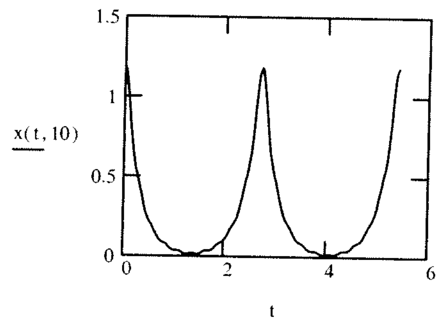
Conclusion: These values match the a coefficients

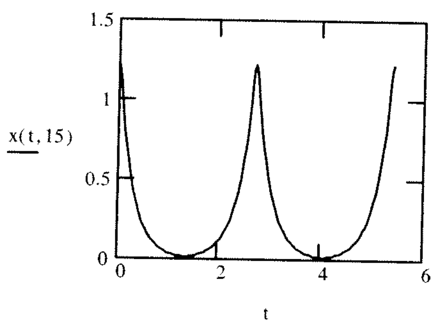
Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.027 .. 5.4$

$$x(t,c) := 2 \cdot A \cdot \left(\frac{1 - e^{-\frac{\alpha}{2}}}{\alpha} \right) + 4 \cdot \alpha \cdot A \cdot \sum_{n=1}^{c} \frac{1 + (-1)^{n-1} \cdot e^{-\frac{\alpha}{2}}}{\left(\alpha^2 + 4 \cdot \pi^2 \cdot n^2\right)} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right)$$







Problem # 73: Double-Sided Exponential Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{\alpha}{T} \cdot |t|} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) dt \right]$$

$$b(n) := \frac{2}{T} \cdot \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-\frac{\alpha}{T} \cdot |t|} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) dt \right]$$

Results:

$a(0) = 0.548$ $\quad b(0) = 0$

$a(1) = 0.386$ $\quad b(1) = 0$

$a(2) = 0.197$ $\quad b(2) = 0$

$a(3) = 0.111$ $\quad b(3) = 0$

$a(4) = 0.067$ $\quad b(4) = 0$

$a(5) = 0.046$ $\quad b(5) = 0$

$a(6) = 0.032$ $\quad b(6) = 0$

$a(7) = 0.024$ $\quad b(7) = 0$

$a(8) = 0.019$ $\quad b(8) = 0$

Check using fourier series definition:

$$Y(h) := 4 \cdot \alpha \cdot A \cdot \frac{1 + (-1)^{h-1} \cdot e^{-\frac{\alpha}{2}}}{\left(\alpha^2 + 4 \cdot \pi^2 \cdot h^2\right)}$$

$Y(0) = 0.548$

$Y(1) = 0.386$

$Y(2) = 0.197$

$Y(3) = 0.111$

$Y(4) = 0.067$

$Y(5) = 0.046$

$Y(6) = 0.032$

$Y(7) = 0.024$

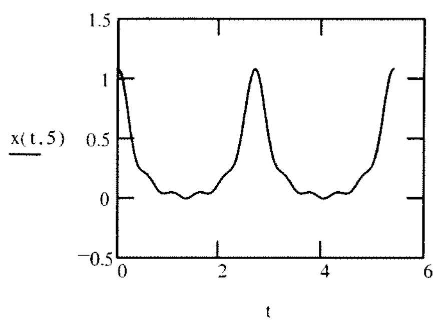
$Y(8) = 0.019$

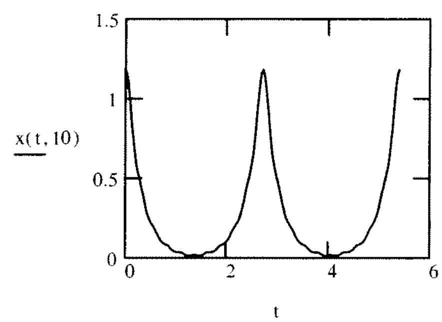
Conclusion: These values match the a coefficients

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.027 .. 5.4$

$$x(t,c) := 2 \cdot A \cdot \left(\frac{1 - e^{-\frac{\alpha}{2}}}{\alpha} \right) + 4 \cdot \alpha \cdot A \cdot \sum_{n=1}^{c} \frac{1 + (-1)^{n-1} \cdot e^{-\frac{\alpha}{2}}}{\left(\alpha^2 + 4 \cdot \pi^2 \cdot n^2\right)} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right)$$





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Problem # 4

Parabolic Wave

Given Constants:

$T := 2.7$

$\theta := \frac{\pi}{5}$

$\tau := .68$

$V := .47$

$A := 1.3$

$\alpha := 9.4$

$fr := 4.3$

Fourier Series-Integral Definitions

$$a(n) := \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot t^2 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt \cdot \frac{2}{T}$$

$$b(n) := \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot t^2 \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt \cdot \frac{2}{T}$$

Results:

$a(0) = 1.5795$	$b(0) = 0$
$a(1) = -0.96022$	$b(1) = 0$
$a(2) = 0.24006$	$b(2) = 0$
$a(3) = -0.10669$	$b(3) = 0$
$a(4) = 0.06001$	$b(4) = 0$
$a(5) = -0.03841$	$b(5) = 0$
$a(6) = 0.02667$	$b(6) = 0$
$a(7) = -0.0196$	$b(7) = 0$
$a(8) = 0.015$	$b(8) = 0$

Fourier Series-Summation Definition

$$Y(h) := \frac{A \cdot T^2}{\pi^2} \cdot \frac{(-1)^h}{h^2}$$

$$\left(\frac{A \cdot T^2}{12}\right) \cdot 2 = 1.5795 \quad \longleftarrow \quad Y(0)$$

Results:

$Y(1) = -0.96022$
$Y(2) = 0.24006$
$Y(3) = -0.10669$
$Y(4) = 0.06001$
$Y(5) = -0.03841$
$Y(6) = 0.02667$
$Y(7) = -0.0196$
$Y(8) = 0.015$

Conclusion

Integral Definition Results

Harmonic	n	a_o	b_o
DC Term	0	1.5795	-
Fundamental	1	-0.96022	-
Second	2	0.24006	-
Third	3	-0.10669	-
Fourth	4	0.06001	-
Fifth	5	-0.03841	-
Sixth	6	0.02667	-
Seventh	7	-0.01960	-
Eighth	8	0.01500	-

Summation Definition Results

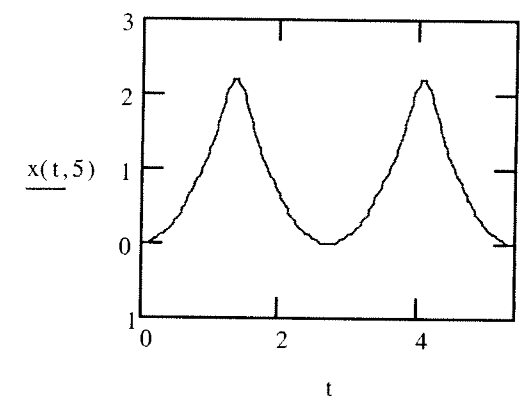
h	a_o	b_o
0	1.5795	-
1	-0.96022	-
2	0.24006	-
3	-0.10669	-
4	0.06001	-
5	-0.03841	-
6	0.02667	-
7	-0.01960	-
8	0.01500	-

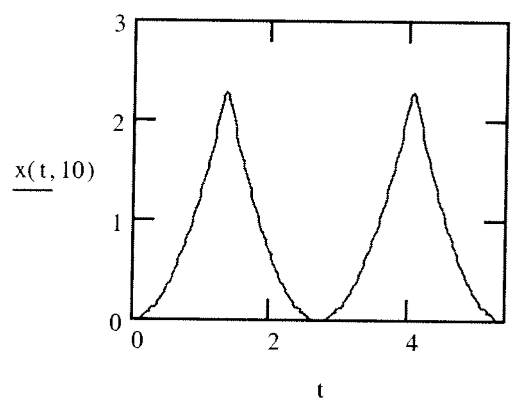
The results from the two methods are identical.

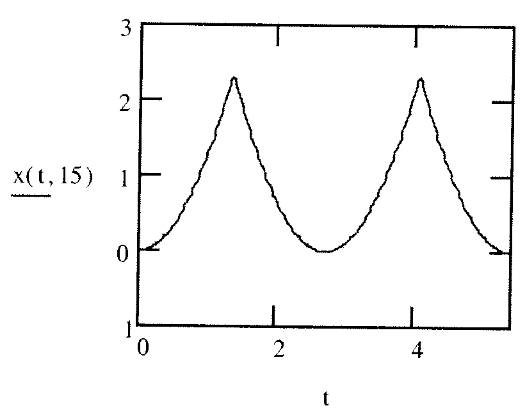
Plot of Function vs. Time

$t := 0, 0.027 .. 5.4$

$$x(t,h) := \frac{A \cdot T^2}{12} + \frac{A \cdot T^2}{\pi^2} \cdot \sum_{n=1}^{h} \frac{(-1)^n}{n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$







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Half Parabolic Wave

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad F_r := 4.3 \quad a := 3.2 \quad b := 2.1 \quad w := \frac{T - 2\cdot\tau}{2} \quad w = 0.67$

$$a(n) := \frac{2}{T}\cdot\int_0^T A\cdot t^2\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt \qquad ap(n) := \frac{A\cdot T^2}{\pi^2}\cdot\frac{1}{n^2}$$

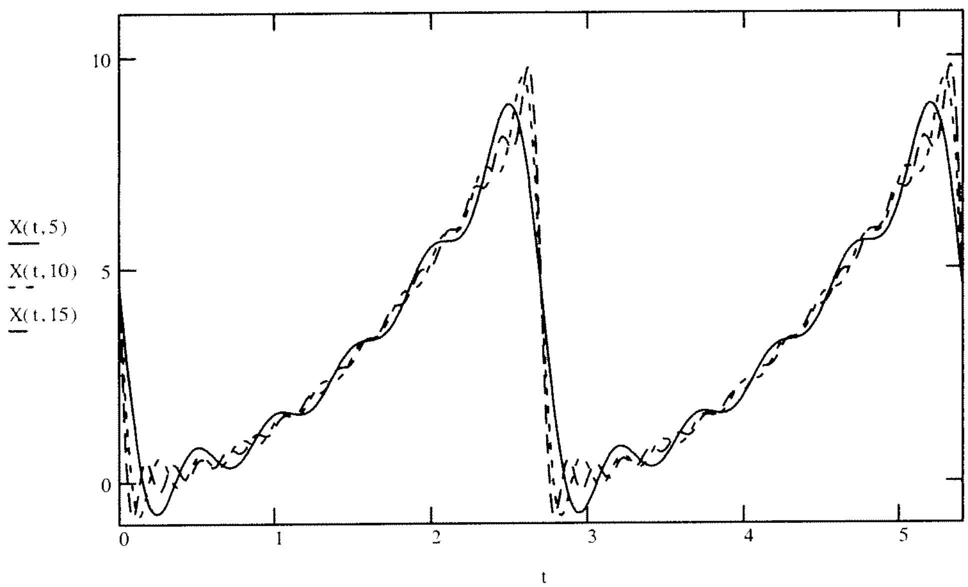
$$b(n) := \frac{2}{T}\cdot\int_0^T A\cdot t^2\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt \qquad bp(n) := \frac{A\cdot T^2}{\pi^2}\cdot -\frac{\pi}{n} \qquad f_{avg} := \frac{A\cdot T^2}{3}$$

$\frac{a(0)}{2} = 3.159 \qquad f_{avg} = 3.159$

$a(1) = 0.96022$	$ap(1) = 0.96022$	$b(1) = -3.01662$	$bp(1) = -3.01662$
$a(2) = 0.24006$	$ap(2) = 0.24006$	$b(2) = -1.50831$	$bp(2) = -1.50831$
$a(3) = 0.10669$	$ap(3) = 0.10669$	$b(3) = -1.00554$	$bp(3) = -1.00554$
$a(4) = 0.06001$	$ap(4) = 0.06001$	$b(4) = -0.75416$	$bp(4) = -0.75416$
$a(5) = 0.03841$	$ap(5) = 0.03841$	$b(5) = -0.60332$	$bp(5) = -0.60332$
$a(6) = 0.02667$	$ap(6) = 0.02667$	$b(6) = -0.50277$	$bp(6) = -0.50277$
$a(7) = 0.0196$	$ap(7) = 0.0196$	$b(7) = -0.43095$	$bp(7) = -0.43095$
$a(8) = 0.015$	$ap(8) = 0.015$	$b(8) = -0.37708$	$bp(8) = -0.37708$

Plot of Half Parabolic Wave

$$X(t,m) := \frac{a(0)}{2} + \left[\sum_{n=1}^{m} a(n)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) + b(n)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right] \qquad t := 0, 0.01 .. 2\cdot T$$



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Cubic Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := .68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad V_{dc} := .47$

$$a(n) := \frac{2}{T} \cdot \int_{\left(\frac{-1 \cdot T}{2}\right)}^{\frac{T}{2}} \left(A \cdot t^3\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{\left(\frac{-1 \cdot T}{2}\right)}^{\frac{T}{2}} \left(A \cdot t^3\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$f_{avg} := 0 \qquad f_{avg} = 0 \qquad \frac{a(0)}{2} = 0$

$$bp(N) := \frac{A \cdot T^3}{4 \cdot \pi} \cdot \frac{(-1)^{N+1}}{N} \cdot \left(1 - \frac{6}{\pi^2 \cdot N^2}\right)$$

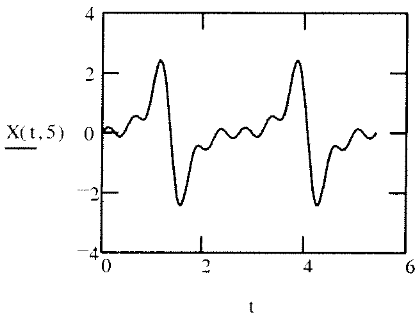
		Check:
$a(0) = 0$	$b(0) = 0$	
$a(1) = 0$	$b(1) = 0.79835$	$bp(1) = 0.79835$
$a(2) = 0$	$b(2) = -0.86338$	$bp(2) = -0.86338$
$a(3) = 0$	$b(3) = 0.63289$	$bp(3) = 0.63289$
$a(4) = 0$	$b(4) = -0.48971$	$bp(4) = -0.48971$
$a(5) = 0$	$b(5) = 0.39734$	$bp(5) = 0.39734$
$a(6) = 0$	$b(6) = -0.33364$	$bp(6) = -0.33364$
$a(7) = 0$	$b(7) = 0.28728$	$bp(7) = 0.28728$
$a(8) = 0$	$b(8) = -0.25211$	$bp(8) = -0.25211$

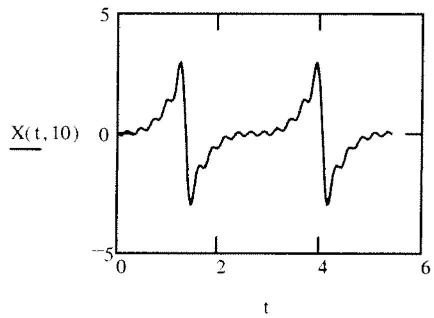
a coefficients are all zero which is expected

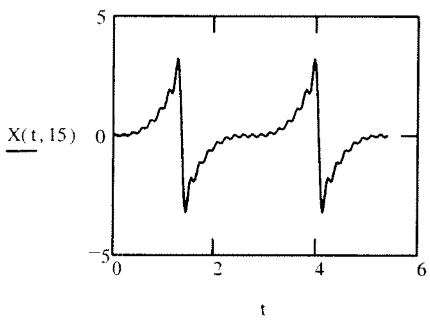
b coefficients match the given values

Plot of Cubic Wave

$$X(t,m) := \frac{a(0)}{2} + \left[\sum_{n=1}^{m} a(n)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) + b(n)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right] \qquad t := 0, \frac{T}{100} .. 2\cdot T$$







Problem # 77: Quadratic Wave #1

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad B := .32 \quad C := 2.1V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A \cdot t^2 + B \cdot t + C\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt \right]$$

$$b(n) := \frac{2}{T} \cdot \left[\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A \cdot t^2 + B \cdot t + C\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt \right]$$

Results:

$a(0) = 5.78$ | $b(0) = 0$

$a(1) = -0.96$ | $b(1) = 0.275$

$a(2) = 0.24$ | $b(2) = -0.138$

$a(3) = -0.107$ | $b(3) = 0.092$

$a(4) = 0.06$ | $b(4) = -0.069$

$a(5) = -0.038$ | $b(5) = 0.055$

$a(6) = 0.027$ | $b(6) = -0.046$

$a(7) = -0.02$ | $b(7) = 0.039$

$a(8) = 0.015$ | $b(8) = -0.034$

Check using fourier series definition:

$$a_p(h) := \frac{(-1)^h \cdot A \cdot T^2}{\pi^2 \cdot h^2} \qquad b_p(h) := -\frac{(-1)^h \cdot B \cdot T}{\pi \cdot h}$$

$$a_{p0} := \frac{A \cdot T^2}{12} + C \qquad a_{p0} = 2.89$$

$a_p(1) = -0.96$

$a_p(2) = 0.24$

$a_p(3) = -0.107$

$a_p(4) = 0.06$

$a_p(5) = -0.038$

$a_p(6) = 0.027$

$a_p(7) = -0.02$

$a_p(8) = 0.015$

$b_p(1) = 0.275$

$b_p(2) = -0.138$

$b_p(3) = 0.092$

$b_p(4) = -0.069$

$b_p(5) = 0.055$

$b_p(6) = -0.046$

$b_p(7) = 0.039$

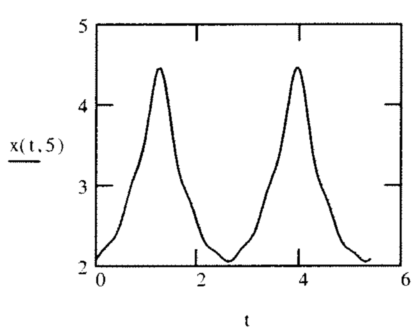
$b_p(8) = -0.034$

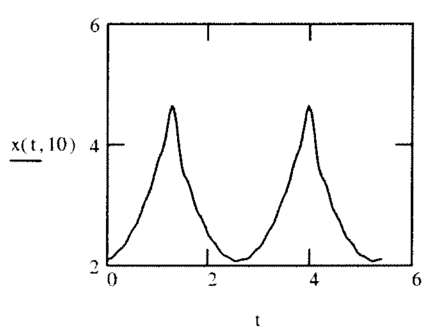
Conclusion: The b and a coefficients match, the average value ap0 is a0/2

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.027 .. 5.4$

$$x(t,c) := \frac{A \cdot T^2}{12} + C + \left[\sum_{n=1}^{c} \frac{(-1)^n \cdot A \cdot T^2}{\pi^2 \cdot n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) - \frac{(-1)^n \cdot B \cdot T}{\pi \cdot n} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)\right]$$





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Quadratic Wave #1

$A_1 := 2 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4$

$f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

$B_1 := -2 \quad C_1 := -1$

$$a_1(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A_1 \cdot t^2 + B_1 \cdot t + C_1\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b_1(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A_1 \cdot t^2 + B_1 \cdot t + C_1\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$c_1(n) := \sqrt{a_1(n)^2 + b_1(n)^2}$$

$a_1(0) = 0.43$	$b_1(0) = 0$	$c_1(0) = 0.43$
$a_1(1) = -1.477$	$b_1(1) = -1.719$	$c_1(1) = 2.266$
$a_1(2) = 0.369$	$b_1(2) = 0.859$	$c_1(2) = 0.935$
$a_1(3) = -0.164$	$b_1(3) = -0.573$	$c_1(3) = 0.596$
$a_1(4) = 0.092$	$b_1(4) = 0.43$	$c_1(4) = 0.44$
$a_1(5) = -0.059$	$b_1(5) = -0.344$	$c_1(5) = 0.349$
$a_1(6) = 0.041$	$b_1(6) = 0.286$	$c_1(6) = 0.289$
$a_1(7) = -0.03$	$b_1(7) = -0.246$	$c_1(7) = 0.247$
$a_1(8) = 0.023$	$b_1(8) = 0.215$	$c_1(8) = 0.216$

check:

$$a_2(n) := \frac{(-1)^n \cdot A_1 \cdot T^2}{\pi^2 \cdot n^2} \qquad b_2(n) := -\left[\frac{(-1)^n \cdot B_1 \cdot T}{\pi \cdot n}\right]$$

$a_2(1) = -1.477$ $\qquad b_2(1) = -1.719$

$a_2(2) = 0.369$ $\qquad b_2(2) = 0.859$

$a_2(3) = -0.164$ $\qquad b_2(3) = -0.573$

$a_2(4) = 0.092$ $\qquad b_2(4) = 0.43$

$a_2(5) = -0.059$ $\qquad b_2(5) = -0.344$

$a_2(6) = 0.041$ $\qquad b_2(6) = 0.286$

$a_2(7) = -0.03$ $\qquad b_2(7) = -0.246$

$a_2(8) = 0.023$ $\qquad b_2(8) = 0.215$

$$\frac{A_1 \cdot T^2}{12} + C_1 = 0.215$$

$$\frac{a_1(0)}{2} = 0.215$$

$A_1 * T^2/12 = a_0/2$

Actual function:

$$E(t) := \frac{A_1 \cdot T^2}{12} + C_1 + \left[\sum_{n=1}^{\infty} \left[\frac{(-1)^n \cdot A_1 \cdot T^2}{\pi^2 + n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) - \frac{(-1)^n \cdot B_1 \cdot T}{\pi \cdot n} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)\right]\right]$$

change constants:

$$A_2 := 8 \quad B_2 := -2 \quad C_2 := -1$$

$$a_1(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A_2 \cdot t^2 + B_2 \cdot t + C_2\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b_1(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(A_2 \cdot t^2 + B_2 \cdot t + C_2\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$c_1(n) := \sqrt{a_1(n)^2 + b_1(n)^2}$$

$a_1(0) = 7.72$	$b_1(0) = 0$	$c_1(0) = 7.72$
$a_1(1) = -5.909$	$b_1(1) = -1.719$	$c_1(1) = 6.154$
$a_1(2) = 1.477$	$b_1(2) = 0.859$	$c_1(2) = 1.709$
$a_1(3) = -0.657$	$b_1(3) = -0.573$	$c_1(3) = 0.871$
$a_1(4) = 0.369$	$b_1(4) = 0.43$	$c_1(4) = 0.567$
$a_1(5) = -0.236$	$b_1(5) = -0.344$	$c_1(5) = 0.417$
$a_1(6) = 0.164$	$b_1(6) = 0.286$	$c_1(6) = 0.33$
$a_1(7) = -0.121$	$b_1(7) = -0.246$	$c_1(7) = 0.274$
$a_1(8) = 0.092$	$b_1(8) = 0.215$	$c_1(8) = 0.234$

check:

$$a_2(n) := \frac{(-1)^n \cdot A_2 \cdot T^2}{\pi^2 \cdot n^2} \qquad b_2(n) := -\left[\frac{(-1)^n \cdot B_2 \cdot T}{\pi \cdot n}\right]$$

$a_2(1) = -5.909$ $\quad$ $b_2(1) = -1.719$

$a_2(2) = 1.477$ $\quad$ $b_2(2) = 0.859$

$a_2(3) = -0.657$ $\quad$ $b_2(3) = -0.573$

$a_2(4) = 0.369$ $\quad$ $b_2(4) = 0.43$

$a_2(5) = -0.236$ $\quad$ $b_2(5) = -0.344$

$a_2(6) = 0.164$ $\quad$ $b_2(6) = 0.286$

$a_2(7) = -0.121$ $\quad$ $b_2(7) = -0.246$

$a_2(8) = 0.092$ $\quad$ $b_2(8) = 0.215$

$$\frac{A_2 \cdot T^2}{12} + C_1 = 3.86$$

$$\frac{a_1(0)}{2} = 3.86$$

$A_1*T^2/12=a_0/2$

Actual function:

$$E(t) := \frac{A_2 \cdot T^2}{12} + C_2 + \left[\sum_{n=1}^{\infty} \left[\frac{(-1)^n \cdot A_2 \cdot T^2}{\pi^2 + n^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) - \frac{(-1)^n \cdot B_2 \cdot T}{\pi \cdot n} \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)\right]\right]$$

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Quadratic Wave #2

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := .68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad V_{dc} := .47$

$B := .6 \quad C := 1.8$

$$a(n) := \frac{2}{T} \cdot \int_{(0)}^{T} \left(A \cdot t^2 + B \cdot t + C\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{(0)}^{T} \left(A \cdot t^2 + B \cdot t + C\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$f_{avg} := \frac{A \cdot T^2}{3} + \frac{B \cdot T}{2} + C \qquad f_{avg} = 5.769 \qquad \frac{a(0)}{2} = 5.769$

$ap(N) := \frac{A \cdot T^2}{\pi^2 \cdot N^2} \qquad bp(N) := \frac{A \cdot T^2 + B \cdot T}{\pi \cdot N} \cdot (-1)$

$a(0) = 11.538$	Check:	$b(0) = 0$	Check:
$a(1) = 0.96022$	$ap(1) = 0.96022$	$b(1) = -3.53228$	$bp(1) = -3.53228$
$a(2) = 0.24006$	$ap(2) = 0.24006$	$b(2) = -1.76614$	$bp(2) = -1.76614$
$a(3) = 0.10669$	$ap(3) = 0.10669$	$b(3) = -1.17743$	$bp(3) = -1.17743$
$a(4) = 0.06001$	$ap(4) = 0.06001$	$b(4) = -0.88307$	$bp(4) = -0.88307$
$a(5) = 0.03841$	$ap(5) = 0.03841$	$b(5) = -0.70646$	$bp(5) = -0.70646$
$a(6) = 0.02667$	$ap(6) = 0.02667$	$b(6) = -0.58871$	$bp(6) = -0.58871$
$a(7) = 0.0196$	$ap(7) = 0.0196$	$b(7) = -0.50461$	$bp(7) = -0.50461$
$a(8) = 0.015$	$ap(8) = 0.015$	$b(8) = -0.44154$	$bp(8) = -0.44154$

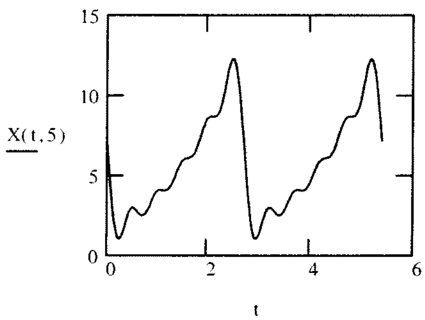
a coefficients match the given values

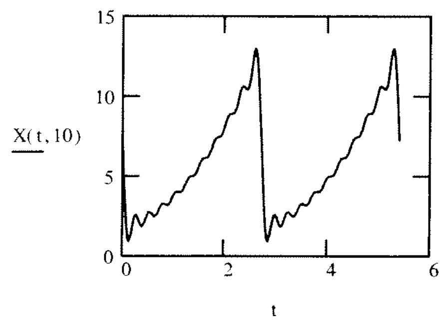
b coefficients match the given values

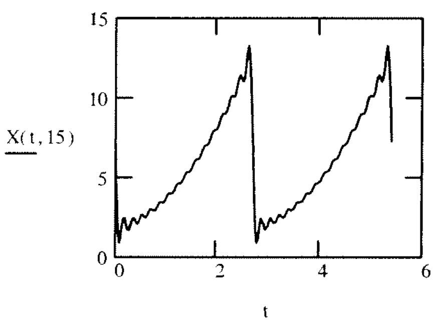
Ken Kaiser

Plot of Quadratic Wave #2

$$X(t,m) := \frac{a(0)}{2} + \left[\sum_{n=1}^{m} a(n)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) + b(n)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right] \qquad t := 0, \frac{T}{100} .. 2\cdot T$$







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Following is problem #3: Noninteger Cycles Sine Wave Coefficients:

$$a3(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \sin(\alpha \cdot t) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b3(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \sin(\alpha \cdot t) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Solution:

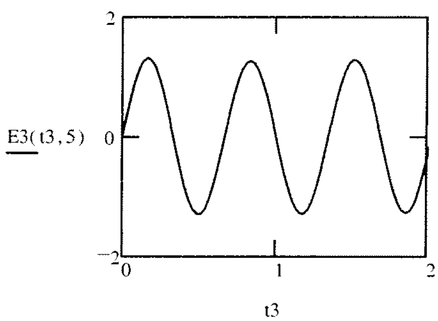
$a3(0) = 0$
$a3(1) = 0$
$a3(2) = 0$
$a3(3) = 0$
$a3(4) = 0$
$a3(5) = 0$
$a3(6) = 0$
$a3(7) = 0$
$a3(8) = 0$

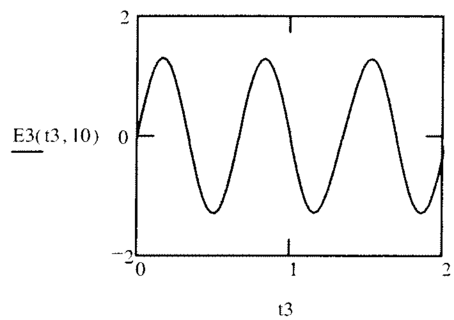
$b3(1) = -6.663 \cdot 10^{-3}$
$b3(2) = 0.017$
$b3(3) = -0.042$
$b3(4) = 1.29$
$b3(5) = 0.059$
$b3(6) = -0.031$
$b3(7) = 0.022$
$b3(8) = -0.017$

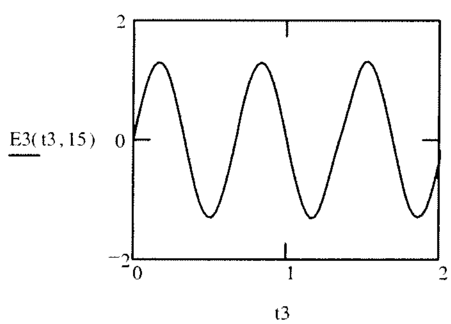
Graphs for Problem #3 using exact equation

$$E3(t3,m) := 8\cdot A\cdot\pi\cdot\sum_{n=1}^{m}\frac{(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2-\alpha^2\cdot T^2}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t3\right)$$

$$t3 := 0, \frac{T}{100} .. 2\cdot T$$







Check:

$a_{30}(n) := 0$

$a_{3p}(n) := 0$

$$b_{3p}(n) := \frac{8\cdot A\cdot\pi\cdot(-1)^{n+1}\cdot n\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2-\alpha^2\cdot T^2}$$

$b_{3p}(1) = -6.663\cdot 10^{-3}$

$b_{3p}(2) = 0.017$

$b_{3p}(3) = -0.042$

$b_{3p}(4) = 1.29$

$b_{3p}(5) = 0.059$

$b_{3p}(6) = -0.031$

$b_{3p}(7) = 0.022$

$b_{3p}(8) = -0.017$

80

[redacted]
Fourier Series Problem 4
EE-310, Kaiser, Winter 04

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := .68 \quad \alpha := 9.4 \quad fr := 4.3 \quad a := .32 \quad b := 2.1 \quad j := \sqrt{-1} \quad Vdc := .74$

Noninteger Cycles Cosine Wave

1) Determination of Series Coefficients Using Integral Definitions

$$a(n) := \frac{2}{T}\cdot\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\cos(\alpha\cdot t)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

$$b(n) := \frac{2}{T}\cdot\left[\int_{-\frac{T}{2}}^{\frac{T}{2}} A\cdot\cos(\alpha\cdot t)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

$a(0) = 0.02527$	$b(0) = 0$
$a(1) = -0.02691$	$b(1) = 0$
$a(2) = 0.03347$	$b(2) = 0$
$a(3) = -0.05634$	$b(3) = 0$
$a(4) = 1.30304$	$b(4) = 0$
$a(5) = 0.04747$	$b(5) = 0$
$a(6) = -0.02094$	$b(6) = 0$
$a(7) = 0.01261$	$b(7) = 0$
$a(8) = -8.64532\cdot10^{-3}$	$b(8) = 0$

2) Verification of Coefficients Using the Summation Forms

$$ap(n) := (4\cdot A\cdot\alpha\cdot T)\cdot\left[\frac{(-1)^{(n+1)}\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)}{4\cdot n^2\cdot\pi^2 - \alpha^2\cdot T^2}\right] \qquad \frac{2\cdot A}{\alpha\cdot T}\cdot\sin\left(\frac{\alpha\cdot T}{2}\right)\cdot 2 = 0.02527 \quad = a(0)$$

$ap(1) = -0.02691$

$ap(2) = 0.03347$

$ap(3) = -0.05634$

$ap(4) = 1.30304$

$ap(5) = 0.04747$

$ap(6) = -0.02094$

$ap(7) = 0.01261$

$ap(8) = -8.64531 \cdot 10^{-3}$

Problem # 81 Impulse Train

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$F(n) := \frac{A \cdot e^{\frac{-j \cdot 2 \cdot \pi \cdot n}{T} \cdot a}}{T}$$

Results:

$F(0) = 0.481$

$F(1) = 0.354 - 0.326i$

$F(2) = 0.039 - 0.48i$

$F(3) = -0.296 - 0.379i$

$F(4) = -0.475 - 0.078i$

$F(5) = -0.402 + 0.265i$

$F(6) = -0.116 + 0.467i$

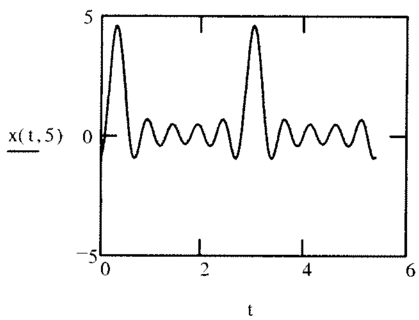
$F(7) = 0.231 + 0.422i$

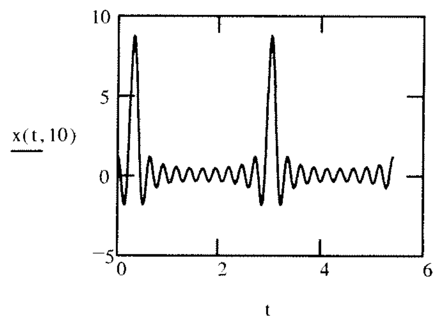
$F(8) = 0.456 + 0.154i$

Graphically show how the function improves as the number of elements in the series is increased, and that the two function are equal!

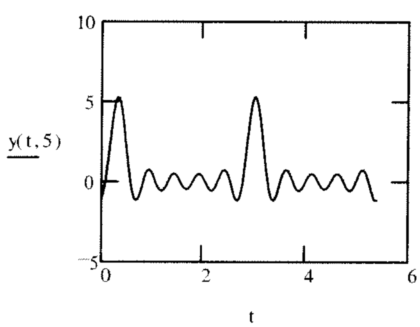
$t := 0, 0.01 .. 5.4$

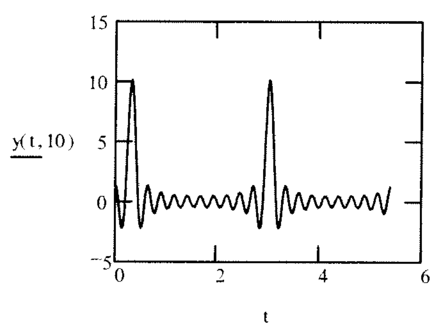
$$x(t,c) := \left(\frac{A}{T}\right) + \frac{2 \cdot A}{\pi} \cdot \sum_{n=1}^{c} \cos\left[\frac{2 \cdot \pi \cdot n}{T} \cdot (t-a)\right]$$





$$y(t,c) := \frac{A}{T} \cdot \left[\sum_{n=-c}^{c} e^{\frac{j \cdot 2 \cdot \pi \cdot n}{T} \cdot (t-a)}\right]$$





8 2

4. Alternating Impulse Train

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad i := \sqrt{(-1)}$

$$F(n) := \frac{A}{T} \cdot e^{-i \cdot 2 \cdot \pi \cdot \frac{a}{T}} - \frac{A}{T} \cdot e^{-i \cdot 2 \cdot \pi \cdot \frac{b}{T}}$$

Please do not forget that the hand written work is
part of this problem.

$F(0) = 0.27043 - 0.80048i$

$F(1) = 0.27043 - 0.80048i$

$F(2) = 0.27043 - 0.80048i$

$F(3) = 0.27043 - 0.80048i$

$F(4) = 0.27043 - 0.80048i$

$F(5) = 0.27043 - 0.80048i$

$F(6) = 0.27043 - 0.80048i$

$F(7) = 0.27043 - 0.80048i$

$F(8) = 0.27043 - 0.80048i$

Exact Equation

$$x(t,m) := 2\cdot\frac{A}{T}\cdot\left[\sum_{n=1}^{m}\left(\cos\left(2\cdot\pi n\cdot\frac{t-a}{T}\right)-\cos\left(2\cdot\pi n\cdot\frac{t-b}{T}\right)\right)\right]$$

$$y(t,m) := \frac{A}{T}\cdot\sum_{n=-m}^{-1}\left(e^{-i\cdot 2\cdot\pi\cdot n\cdot\frac{a}{T}}-e^{-i\cdot 2\cdot\pi\cdot n\cdot\frac{b}{T}}\right)\cdot e^{i\cdot 2\cdot\pi\cdot n\cdot\frac{t}{T}}\ \ldots$$
$$+\frac{A}{T}\cdot\left[\sum_{n=1}^{m}\left(e^{-i\cdot 2\cdot\pi\cdot n\cdot\frac{a}{T}}-e^{-i\cdot 2\cdot\pi\cdot n\cdot\frac{b}{T}}\right)\cdot e^{i\cdot 2\cdot\pi\cdot n\cdot\frac{t}{T}}\right]$$

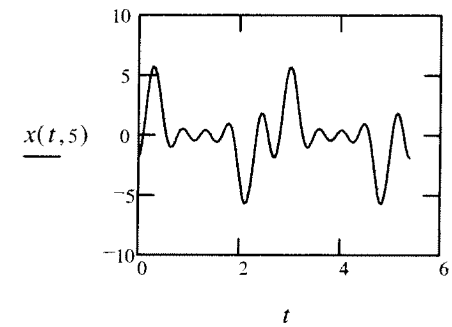
$x(0.3,20) = 17.36205$ $\quad x(0.3,200) = 2.4048$

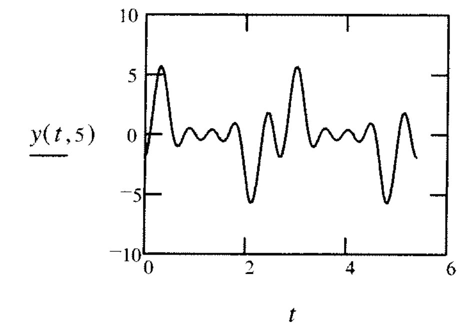
$y(0.3,20) = 17.36205$ $\quad y(0.3,200) = 2.4048$

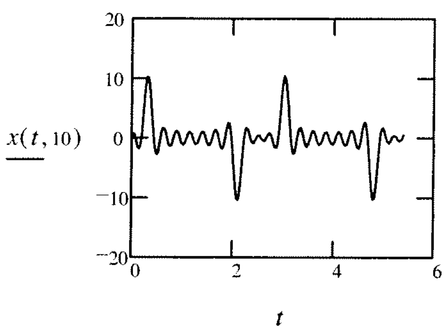
I checked to make sure the cosine term was equal to the expanded exponential term by getting values for both functions and by comparing the graphs side by side..

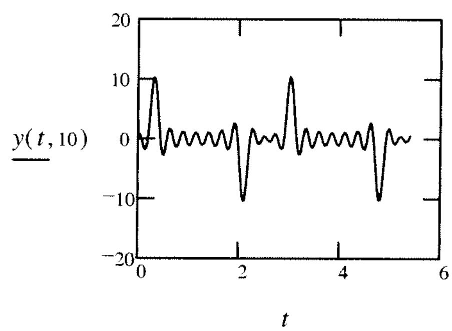
Graphs

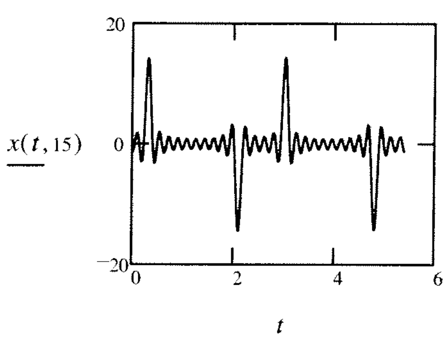
$$t := 0, \frac{T}{1000} .. 2 \cdot T$$

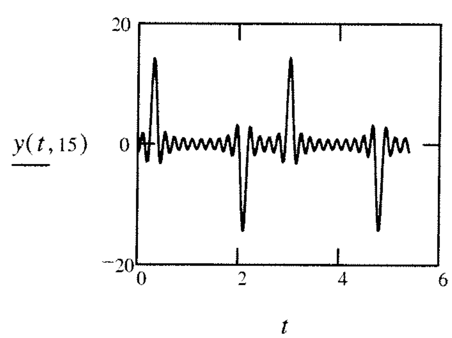










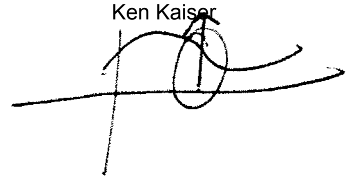


$$F_N = \frac{1}{T}\int_0^T \left[A\delta(t-a) - A\delta(t-b)\right] e^{-j\frac{2\pi N}{T}t}\,dt$$

$$= \frac{1}{T}\int_0^T \left[A\delta(t-a)\,e^{-\frac{2\pi N}{T}t} - A\delta(t-b)\,e^{-\frac{2\pi N}{T}t}\right] dt$$

$$= \frac{A}{T} e^{-\frac{2\pi N}{T}a} - \frac{A}{T} e^{-\frac{2\pi N}{T}b}$$

$$= \frac{A}{T}\left(e^{-\frac{2\pi N}{T}a} - e^{-\frac{2\pi N}{T}b}\right)$$



$$a_N = \frac{2}{T}\int_0^T \left[A\delta(t-a) - A\delta(t-b)\right]\cos\left(\frac{2\pi N}{T}t\right)dt$$

$$= \frac{2}{T}\int_0^T A\delta(t-a)\cos\left(\frac{2\pi N}{T}t\right)dt - \frac{2}{T}\int_0^T A\delta(t-b)\cos\left(\frac{2\pi N}{T}t\right)dt$$

$$= \frac{2}{T}\int_0^T A\delta(t-a)\cos\left(\frac{2\pi N}{T}a\right)dt - \frac{2}{T}\int_0^T A\delta(t-b)\cos\left(\frac{2\pi N}{T}b\right)dt$$

$$= \frac{2}{T}A\cos\left(\frac{2\pi N}{T}a\right)\int_0^T \delta(t-a)dt - \frac{2}{T}A\cos\left(\frac{2\pi N}{T}b\right)\int_0^T \delta(t-b)dt$$

$$a_N = \frac{2A}{T}\left[\cos\left(\frac{2\pi N}{T}a\right) - \cos\left(\frac{2\pi N}{T}b\right)\right]$$

$$b_N = \frac{2A}{T}\left[\sin\left(\frac{2\pi N}{T}a\right) - \sin\left(\frac{2\pi N}{T}b\right)\right]$$

Ken Kaiser

$$\frac{2A}{T}\cos\left(\frac{2\pi N}{T}(t-a)\right) - \frac{2A}{T}\cos\left(\frac{2\pi N}{T}(t-b)\right)$$

$$\cos x = \frac{e^{jx}+e^{-jx}}{2}$$

$$\frac{2A}{T}\left[\frac{e^{j\frac{2\pi N}{T}(t-a)} + e^{-\frac{2\pi N}{T}(t-a)}}{2} - \frac{e^{+j\frac{2\pi N}{T}(t-b)} - e^{-j\frac{2\pi N}{T}(t-b)}}{2}\right]$$

$$\frac{A}{T}\left[e^{j\frac{2\pi N}{T}t}e^{-j\frac{2\pi N}{T}a} + e^{-j\frac{2\pi N}{T}t}e^{+\frac{2\pi N}{T}a} - e^{j\frac{2\pi N}{T}t}e^{-j\frac{2\pi N}{T}b} - e^{-j\frac{2\pi N}{T}t}e^{j\frac{2\pi N}{T}b}\right]$$

$$\frac{A}{T}\left[\left(e^{-j\frac{2\pi N}{T}a} - e^{-j\frac{2\pi N}{T}b}\right)e^{j\frac{2\pi N}{T}t} + \left(e^{\frac{2\pi N}{T}a} - e^{\frac{2\pi N}{T}b}\right)e^{-j\frac{2\pi N}{T}t}\right]$$

$N = 1, \infty$

$A := 1.3 \quad a := 0.32 \quad b := 2.1 \quad j := \sqrt{-1} \quad T := 2.7$

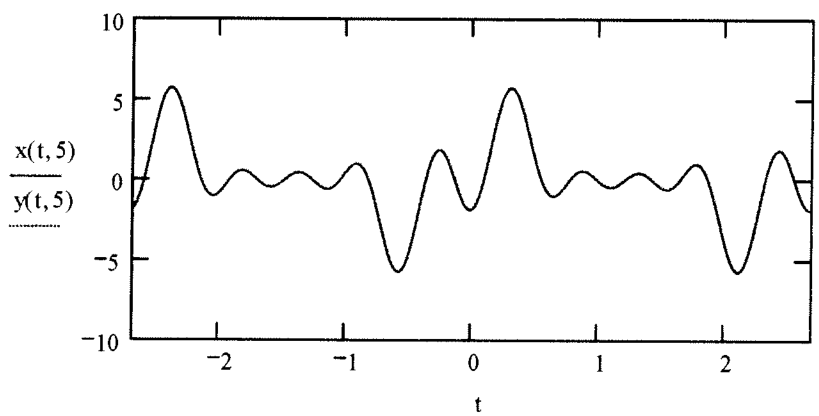
$$x(t,M) := 2\cdot\frac{A}{T}\cdot\sum_{n=1}^{M}\left[\cos\left[2\cdot\pi\cdot n\cdot\frac{(t-a)}{T}\right]-\cos\left[2\cdot\pi\cdot n\cdot\frac{(t-b)}{T}\right]\right]$$

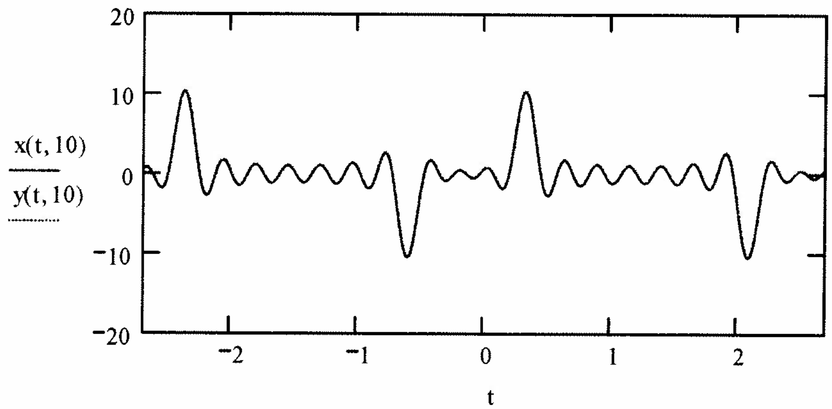
$$y(t,M) := \frac{A}{T}\cdot\sum_{n=-M}^{-1}\left(e^{-j\cdot 2\cdot\pi\cdot n\cdot\frac{a}{T}}-e^{-j\cdot 2\cdot\pi\cdot n\cdot\frac{b}{T}}\right)\cdot e^{j\cdot 2\cdot\pi\cdot n\cdot\frac{t}{T}}+\frac{A}{T}\cdot\sum_{n=1}^{M}\left(e^{-j\cdot 2\cdot\pi\cdot n\cdot\frac{a}{T}}-e^{-j\cdot 2\cdot\pi\cdot n\cdot\frac{b}{T}}\right)\cdot$$

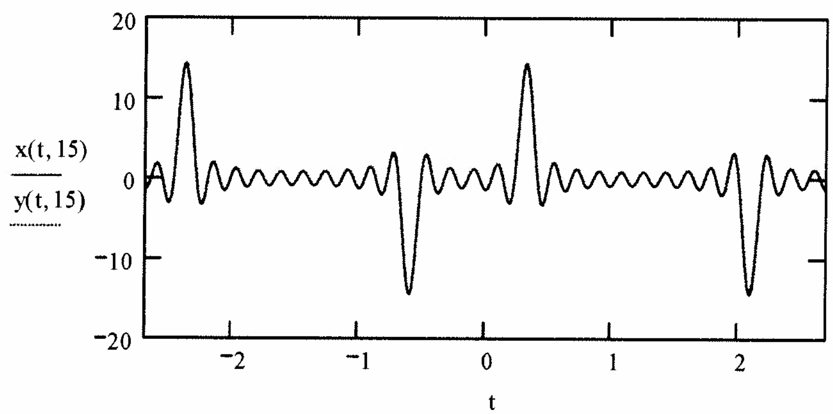
$x(0.3,20) = 17.362$

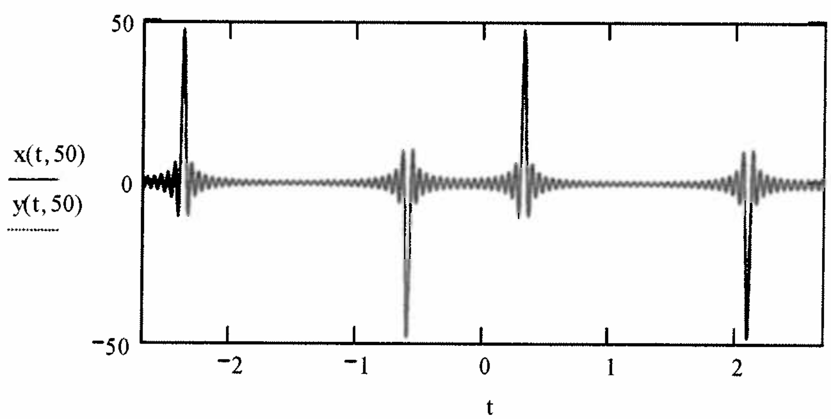
$y(0.3,20) = 17.362$

$t := -T,-T+\frac{T}{1000}\,..\,T$




20
10
0
−10
−20
x(t, 10)
y(t, 10)
−2
−1
0
1
2
t


20
10
0
−10
−20
x(t, 15)
y(t, 15)
−2
−1
0
1
2
t


50
0
−50
x(t, 50)
y(t, 50)
−2
−1
0
1
2
t

Hyperbolic Sine Wave

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47$

Integral Equations:

$$a(N) := \frac{2}{T} \cdot \left[\int_{\frac{-T}{2}}^{\frac{T}{2}} (A \cdot \sinh(\alpha \cdot t)) \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt \right]$$

$$b(N) := \frac{2}{T} \cdot \left[\int_{\frac{-T}{2}}^{\frac{T}{2}} (A \cdot \sinh(\alpha \cdot t)) \cdot \sin\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt \right]$$

Coefficient Equation:

$$bp(n) := (8 \cdot A \cdot \pi) \cdot \left[\frac{\left[(-1)^{n+1} \cdot n \cdot \sinh\left(\alpha \cdot \frac{T}{2}\right) \right]}{\left(4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2\right)} \right]$$

Integral Coefficient Solutions: a(N) and b(N)

$a(0) = 0$ | $b(0) = 0$
$a(1) = 0$ | $b(1) = 7754.14007$
$a(2) = 0$ | $b(2) = -13218.26256$
$a(3) = 0$ | $b(3) = 15911.47003$
$a(4) = 0$ | $b(4) = -16619.88054$
$a(5) = 0$ | $b(5) = 16249.43704$
$a(6) = 0$ | $b(6) = -15399.41125$
$a(7) = 0$ | $b(7) = 14390.18794$
$a(8) = 0$ | $b(8) = -13374.4638$

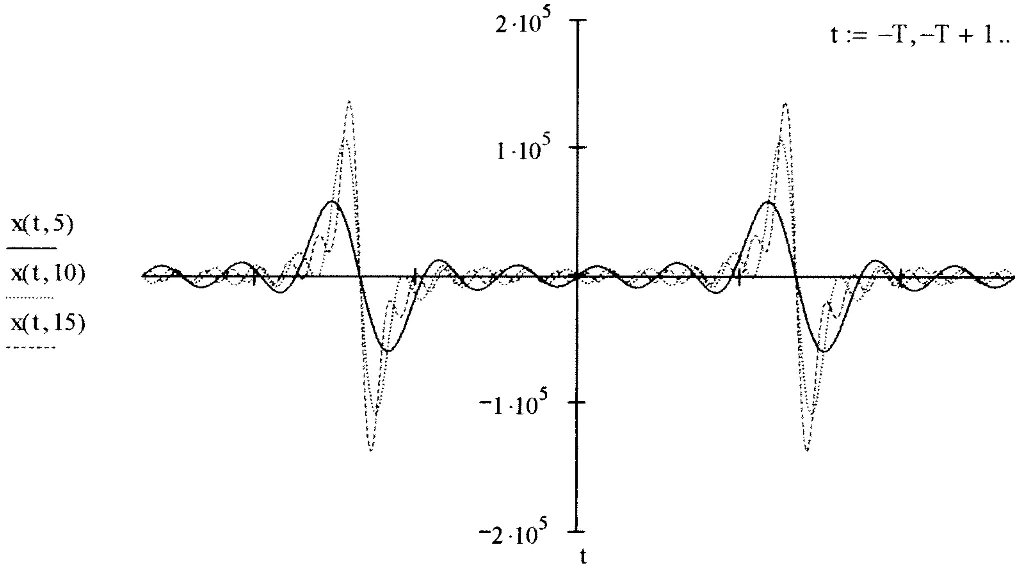
Coefficients: bp(n)

$bp(1) = 7754.14007$
$bp(2) = -13218.26256$
$bp(3) = 15911.47003$
$bp(4) = -16619.88054$
$bp(5) = 16249.43704$
$bp(6) = -15399.41125$
$bp(7) = 14390.18794$
$bp(8) = -13374.4638$

Hyperbolic Sine Wave

$$x(t,m) := (8\cdot A\cdot\pi)\cdot\sum_{N=1}^{m}\frac{\left[(-1)^{N+1}\cdot N\cdot\sinh\left(\frac{T\cdot\alpha}{2}\right)\right]}{\left(4\cdot N^2\cdot\pi^2+\alpha^2\cdot T^2\right)}\cdot\sin\left(\frac{2\cdot\pi\cdot N}{T}\cdot t\right)$$

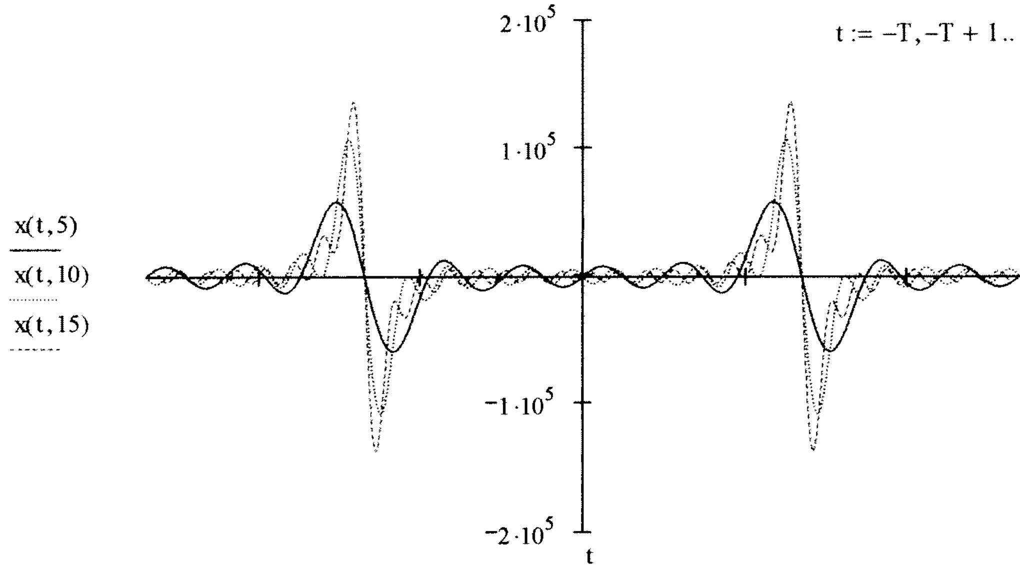
$t := -T, -T+1 .. T$



Hyperbolic Sine Wave

$$x(t,m) := \frac{a(0)}{2} + \sum_{N=1}^{m} b(N)\cdot\sin\left(\frac{2\cdot\pi N}{T}\cdot t\right)$$

$t := -T, -T + 1 .. T$



Problem # 83 (α was changed to 1.2 to obtain a more practical result): Hyperbolic Sine Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 1.2 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \sinh(\alpha \cdot t) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \sinh(\alpha \cdot t) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Results:

$a(0) = 0$ | $b(0) = 0$

$a(1) = 0$ | $b(1) = 1.587$

$a(2) = 0$ | $b(2) = -0.942$

$a(3) = 0$ | $b(3) = 0.65$

$a(4) = 0$ | $b(4) = -0.494$

$a(5) = 0$ | $b(5) = 0.398$

$a(6) = 0$ | $b(6) = -0.332$

$a(7) = 0$ | $b(7) = 0.285$

$a(8) = 0$ | $b(8) = -0.25$

Check using fourier series definition:

$$Y(h) := 8 \cdot A \cdot \pi \cdot \frac{(-1)^{h+1} \cdot h \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot h^2 \cdot \pi^2 + \alpha^2 \cdot T^2}$$

$Y(0) = 0$

$Y(1) = 1.587$

$Y(2) = -0.942$

$Y(3) = 0.65$

$Y(4) = -0.494$

$Y(5) = 0.398$

$Y(6) = -0.332$

$Y(7) = 0.285$

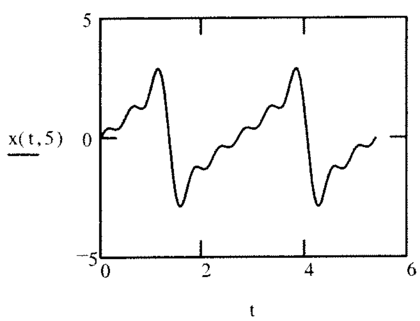
$Y(8) = -0.25$

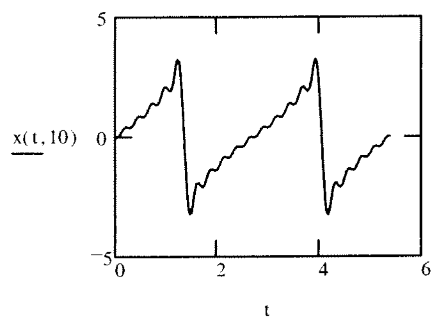
Conclusion: These values match the b coeffecients

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := 8 \cdot A \cdot \pi \cdot \left[\sum_{n=1}^{c} \left[\frac{(-1)^{n+1} \cdot n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \right] \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) \right]$$





Problem # 84: Hyperbolic Cosine Wave (α was changed to 1.2 to obtain a more practical result)

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 1.2 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \cosh(\alpha \cdot t) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot \cosh(\alpha \cdot t) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Results:

$a(0) = 3.896$ $\qquad$ $b(0) = 0$

$a(1) = -0.818$ $\qquad$ $b(1) = 0$

$a(2) = 0.243$ $\qquad$ $b(2) = 0$

$a(3) = -0.112$ $\qquad$ $b(3) = 0$

$a(4) = 0.064$ $\qquad$ $b(4) = 0$

$a(5) = -0.041$ $\qquad$ $b(5) = 0$

$a(6) = 0.029$ $\qquad$ $b(6) = 0$

$a(7) = -0.021$ $\qquad$ $b(7) = 0$

$a(8) = 0.016$ $\qquad$ $b(8) = 0$

Check using fourier series definition:

$$Y(h) := 4 \cdot A \cdot \alpha \cdot T \cdot \frac{(-1)^h \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot h^2 \cdot \pi^2 + \alpha^2 \cdot T^2}$$

$Y(0) = 3.896$

$$\frac{2A}{\alpha T} \sinh\left(\frac{\alpha T}{2}\right) = 1.948$$

$x = 3.896$ ✓

$Y(1) = -0.818$

$Y(2) = 0.243$

$Y(3) = -0.112$

$Y(4) = 0.064$

$Y(5) = -0.041$

$Y(6) = 0.029$

$Y(7) = -0.021$

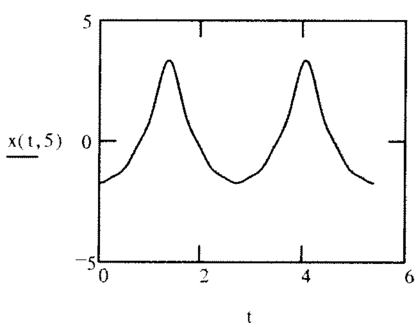
$Y(8) = 0.016$

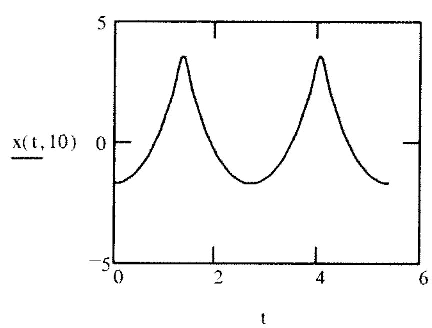
Conclusion: These values match the a coeffecients

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := 4 \cdot A \cdot \pi \cdot T \cdot \left[\sum_{n=1}^{c} \left[\frac{(-1)^n \cdot \sinh\left(\frac{\alpha \cdot T}{2}\right)}{4 \cdot n^2 \cdot \pi^2 + \alpha^2 \cdot T^2} \right] \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) \right] + \text{dc offset}$$





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Natural Log of Sine

~~Cosine Pulse Train~~

Set up the given variables

$A := 1.3 \quad T := 2\cdot\pi \quad \theta := \frac{\pi}{5} \quad \tau := .68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad V_{dc} := .47 \quad j := \sqrt{-1}$

Define the Function

$$x(t) := -A\cdot\ln\left(2\cdot\sin\left(\frac{t}{2}\right)\right)$$

Find the Fourier Series coefficients

$$a(n) := \frac{2}{T}\cdot\int_0^T x(t)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt \qquad b(n) := \frac{2}{T}\cdot\int_0^T x(t)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

Note that here T is defined to be 2π, not 2.7.

Define the Coefficient Functions

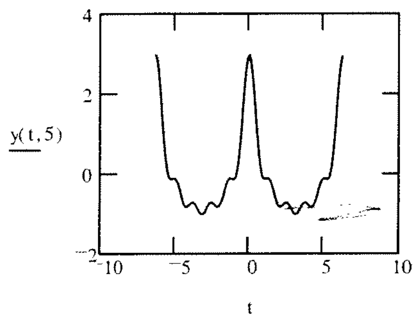
$a_0 := 0 \qquad a_p(n) := \frac{A}{n} \qquad b_p := 0$

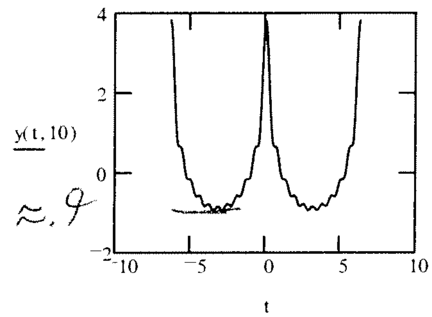
Compare the Results

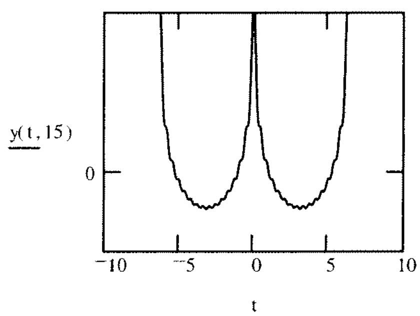
$a(0) = 4.85189\cdot10^{-5}$	$a_0 = 0$	$b(0) = 0$
$a(1) = 1.30005$	$a_p(1) = 1.3$	$b(1) = 0$
$a(2) = 0.65044$	$a_p(2) = 0.65$	$b(2) = 0$
$a(3) = 0.43338$	$a_p(3) = 0.43333$	$b(3) = 0$
$a(4) = 0.32505$	$a_p(4) = 0.325$	$b(4) = 1.47642\cdot10^{-15}$
$a(5) = 0.26005$	$a_p(5) = 0.26$	$b(5) = 1.23434\cdot10^{-15}$

Plots of the Fourier Series of the Natural Log of a Sine Wave

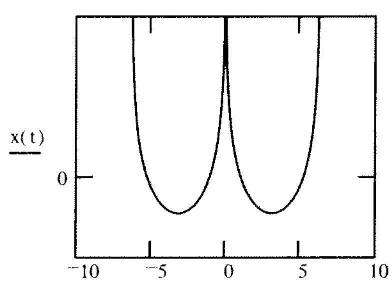
$$y(t,m) := A\cdot\sum_{n=1}^{m}\frac{1}{n}\cdot\cos(n\cdot t) \qquad t := -T, -T+\frac{T}{200}..T$$







Plot of Actual Function



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Natural Log of a Cosine Wave #1

$A := 1.3$ $T := 2\cdot\pi$ $\tau := .68$ $\alpha := 9.4$ $f_r := 4.3\,Hz$ $a := .32$ $b := 2.1$ $V_{dc} := .47$ $\theta := \frac{\pi}{5}$

$$a(N) := \frac{2}{T}\cdot\int_{-\pi}^{\pi} A\cdot\ln\left(2\cdot\cos\left(\frac{t}{2}\right)\right)\cdot\cos\left[\frac{(2\cdot\pi\cdot N)}{T}\cdot t\right]dt$$

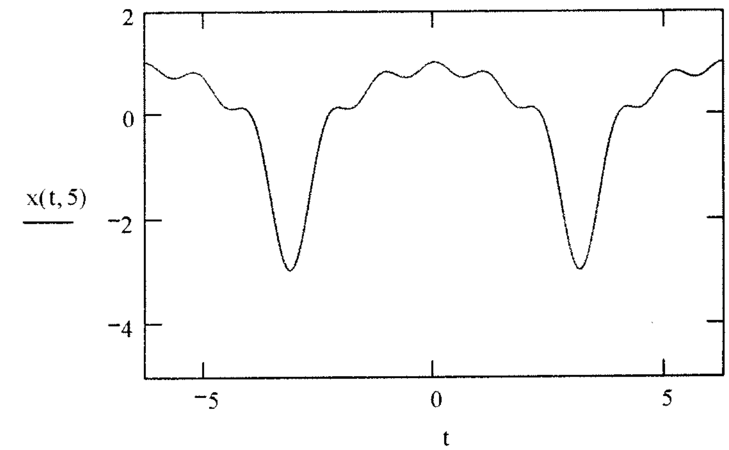
$$b(M) := \frac{2}{T}\cdot\int_{-\pi}^{\pi} A\cdot\ln\left(2\cdot\cos\left(\frac{t}{2}\right)\right)\cdot\sin\left[\frac{(2\cdot\pi\cdot M)}{T}\cdot t\right]dt$$

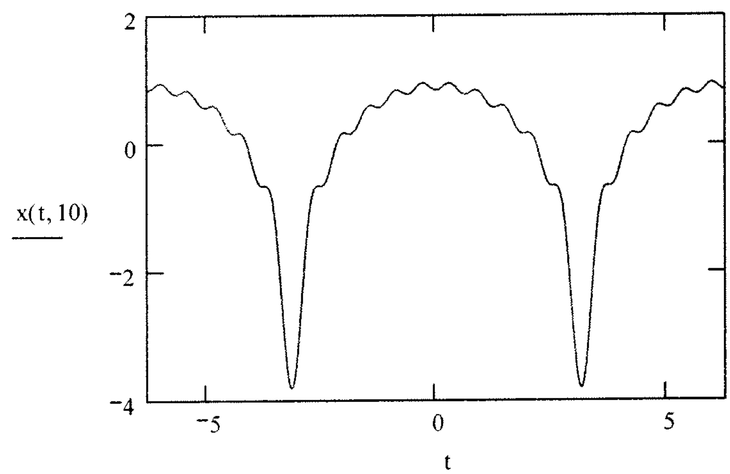
$$ap(n) := A\cdot\frac{(-1)^{n+1}}{n}$$

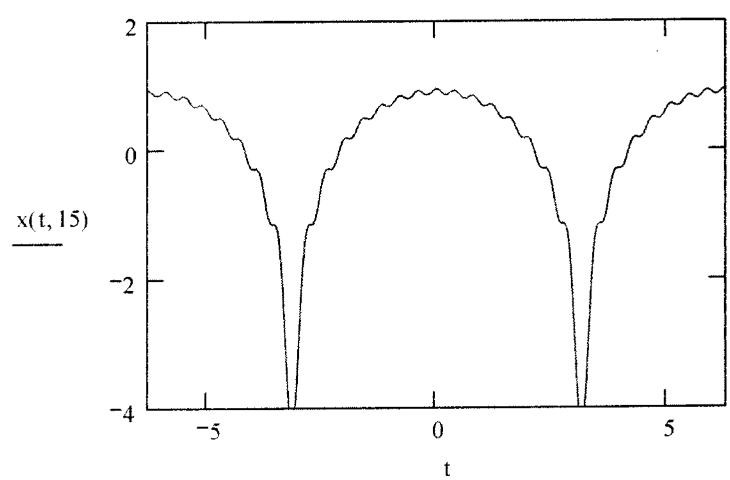
$a(0) = 3.95519\times 10^{-15}$	$b(0) = 0$	$ap_0 := 0$
$a(1) = 1.3$	$b(1) = 0$	$ap(1) = 1.3$
$a(2) = -0.65$	$b(2) = 0$	$ap(2) = -0.65$
$a(3) = 0.43333$	$b(3) = 0$	$ap(3) = 0.43333$
$a(4) = -0.325$	$b(4) = 0$	$ap(4) = -0.325$
$a(5) = 0.26$	$b(5) = 0$	$ap(5) = 0.26$
$a(6) = -0.21667$	$b(6) = 0$	$ap(6) = -0.21667$
$a(7) = 0.18571$	$b(7) = 0$	$ap(7) = 0.18571$
$a(8) = -0.1625$	$b(8) = 0$	$ap(8) = -0.1625$

Ken Kaiser

$$x(t,m) := \sum_{N=1}^{m} a(N)\cdot\cos\left(\frac{2\cdot\pi\cdot N}{T}\cdot t\right) \qquad t := -T, -T+.01\,..\,T$$







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Natural Log of Cosine Wave #2

$A := 1.3$ $T := 2\pi$ $\theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := .54$ $f_r := 4.3$ $a := 0.32$ $b := 2.1$ $V_{dc} := 0.47$

Equations:

$$a(N) := \frac{2}{T} \cdot \int_{-\pi}^{\pi} A \cdot \ln\left(1 - 2\alpha \cdot \cos(t) + \alpha^2\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt$$

$Favg := 0$

$Favg = 0$

$$b(N) := \frac{2}{T} \cdot \int_{-\pi}^{\pi} A \cdot \ln\left(1 - 2\alpha \cdot \cos(t) + \alpha^2\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt \qquad ap(n) := (-2 \cdot A) \cdot \frac{\alpha^n}{n}$$

$\frac{a(0)}{2} = 0$

Solutions: a(N) and b(N)

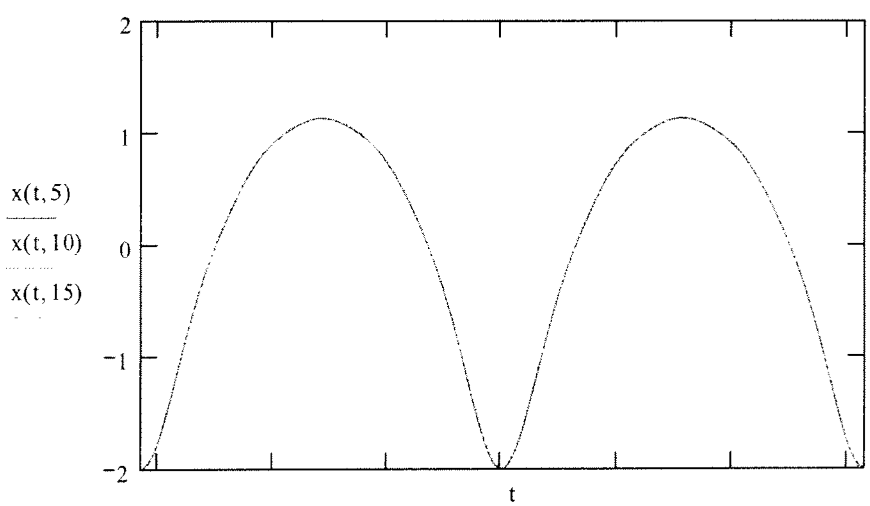
$a(0) = 0$ $b(0) = 0$
$a(1) = -1.404$ $b(1) = 0$
$a(2) = -0.37908$ $b(2) = 0$
$a(3) = -0.13647$ $b(3) = 0$
$a(4) = -0.05527$ $b(4) = 0$
$a(5) = -0.02388$ $b(5) = 0$
$a(6) = -0.01074$ $b(6) = 0$
$a(7) = -0.00497$ $b(7) = 0$
$a(8) = -0.00235$ $b(8) = 0$

Solution: ap(n)

$ap(1) = -1.404$
$ap(2) = -0.37908$
$ap(3) = -0.13647$
$ap(4) = -0.05527$
$ap(5) = -0.02388$
$ap(6) = -0.01074$
$ap(7) = -0.00497$
$ap(8) = -0.00235$

Graph of Natural Log of Cosine Wave #2

$$x(t, m) := \sum^{m} a(N) \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) \qquad t := -T, -T + .05 .. T$$



Problem 88
"Natural Log of a Tangent Wave #1"

Initializing the Variables

$A := 1.3$ $T := 2\cdot\pi$ $\theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := 9.4$

$fr := 4.3\text{Hz}$ $a := 0.32$ $b := 2.1$ $V_{DC} := 0.47\,\text{V}$ $j := \sqrt{-1}$

Evaluating a_n and b_n

$$a(n) := \frac{2}{T}\cdot\int_0^{\pi} -\frac{A}{2}\cdot ln\left(tan\left(\frac{t}{2}\right)\right)\cdot cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt + \frac{2}{T}\cdot\int_{-\pi}^{0} -\frac{A}{2}\cdot ln\left(-tan\left(\frac{t}{2}\right)\right)\cdot cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$b(n) := \frac{2}{T}\cdot\int_0^{\pi} -\frac{A}{2}\cdot ln\left(tan\left(\frac{t}{2}\right)\right)\cdot sin\left(\frac{2\cdot\pi\cdot n}{T}\right)dt + \frac{2}{T}\cdot\int_{-\pi}^{0} -\frac{A}{2}\cdot ln\left(-tan\left(\frac{t}{2}\right)\right)\cdot sin\left(\frac{2\cdot\pi\cdot n}{T}\right)dt$$

Evaluate Coefficients

$a(0) = 0$	$b(0) = 0$
$a(1) = 1.3$	$b(1) = 0$
$a(2) = 0$	$b(2) = 0$
$a(3) = 0.4333$	$b(3) = 0$
$a(4) = 0$	$b(4) = 0$
$a(5) = 0.26$	$b(5) = 0$
$a(6) = 0$	$b(6) = 0$
$a(7) = 0.18571$	$b(7) = 0$
$a(8) = 0$	$b(8) = 0$

Exact Value Verification

The function is even, therefore $f_{avg} = 0$

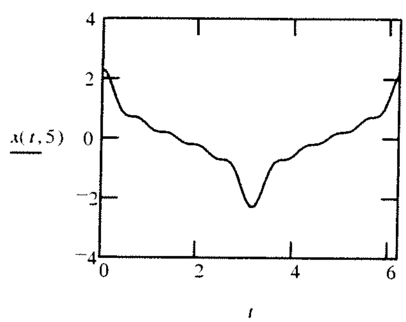
$$E(N) := \frac{A}{2\cdot N - 1}$$

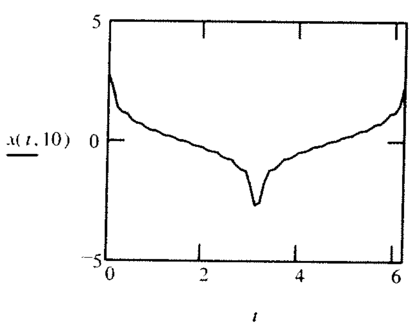
1st Harmonic Frequency	$E(1) = 1.3$
3rd Harmonic Frequency	$E(2) = 0.4333$
5th Harmonic Frequency	$E(3) = 0.26$
7th Harmonic Frequency	$E(4) = 0.18571$

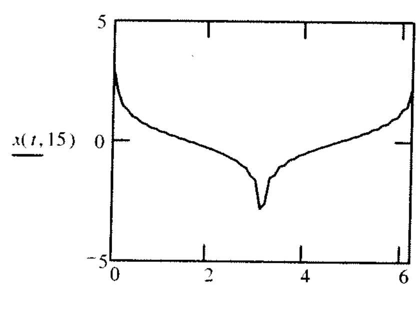
Graphing *x(t)*

$$x(t,m) := A\cdot\sum_{n=1}^{m}\frac{1}{2\cdot n-1}\cdot cos((2\cdot n-1)\cdot t)$$

$$t := 0,.1..2\cdot\pi$$







Note: The above graphs match the figures provided in the Problem Statement, however, they are shown from 0 to 2π.

$A := 1.3 \quad T := 2\cdot\pi \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{DC} := .47 \quad i := \sqrt{-1}$

$\Delta := .000001$

Natural Log of a Tangent Wave #2

$$a_4(n) := \frac{2}{T}\cdot\int_{-\pi}^{-\Delta-\frac{\pi}{2}} -\frac{A}{2}\cdot\ln\left(-\tan\left(\frac{\pi}{4}-\frac{1}{2}\cdot t\right)\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \ldots$$
$$+\left[\frac{2}{T}\cdot\int_{-\frac{\pi}{2}+\Delta}^{\frac{\pi}{2}-\Delta} -\frac{A}{2}\cdot\ln\left(\tan\left(\frac{\pi}{4}-\frac{1}{2}\cdot t\right)\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \ldots + \frac{2}{T}\cdot\int_{\frac{\pi}{2}+\Delta}^{\pi} -\frac{A}{2}\cdot\ln\left(-\tan\left(\frac{\pi}{4}-\frac{1}{2}\cdot t\right)\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt\right]$$

$$b_4(n) := \frac{2}{T}\cdot\int_{-\pi}^{-\frac{\pi}{2}-\Delta} -\frac{A}{2}\cdot\ln\left(-\tan\left(\frac{\pi}{4}-\frac{1}{2}\cdot t\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \ldots$$
$$+\left[\frac{2}{T}\cdot\int_{-\frac{\pi}{2}+\Delta}^{\frac{\pi}{2}-\Delta} -\frac{A}{2}\cdot\ln\left(\tan\left(\frac{\pi}{4}-\frac{1}{2}\cdot t\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \ldots + \frac{2}{T}\cdot\int_{\frac{\pi}{2}+\Delta}^{\pi} -\frac{A}{2}\cdot\ln\left(-\tan\left(\frac{\pi}{4}-\frac{1}{2}\cdot t\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt\right]$$

$$c_4(n) := \sqrt{a_4(n)^2 + b_4(n)^2}$$

Coefficient Solution

$a_4(0) = 2.40596 \cdot 10^{-12}$	$b_4(0) = 0.00000$	$c_4(0) = 2.40596 \cdot 10^{-12}$
$a_4(1) = 0.00000$	$b_4(1) = 1.30031$	$c_4(1) = 1.30031$
$a_4(2) = -2.40569 \cdot 10^{-12}$	$b_4(2) = 5.26761 \cdot 10^{-9}$	$c_4(2) = 5.26761 \cdot 10^{-9}$
$a_4(3) = 0.00000$	$b_4(3) = -0.43355$	$c_4(3) = 0.43355$
$a_4(4) = 2.40624 \cdot 10^{-12}$	$b_4(4) = -6.57902 \cdot 10^{-10}$	$c_4(4) = 6.57907 \cdot 10^{-10}$
$a_4(5) = 0.00000$	$b_4(5) = 0.26021$	$c_4(5) = 0.26021$
$a_4(6) = -2.40599 \cdot 10^{-12}$	$b_4(6) = 9.86117 \cdot 10^{-10}$	$c_4(6) = 9.86120 \cdot 10^{-10}$
$a_4(7) = 0.00000$	$b_4(7) = -0.18593$	$c_4(7) = 0.18593$
$a_4(8) = 2.40634 \cdot 10^{-12}$	$b_4(8) = -2.15342 \cdot 10^{-5}$	$c_4(8) = 2.15342 \cdot 10^{-5}$

Check

NOTE: N represents the odd harmonics, replacing n

$$b_{p4}(N) := A \cdot \frac{(-1)^{(N-1)}}{2 \cdot N - 1}$$

$b_{p4}(1) = 1.30000$

$b_{p4}(2) = -0.43333$

$b_{p4}(3) = 0.26000$

$b_{p4}(4) = -0.18571$

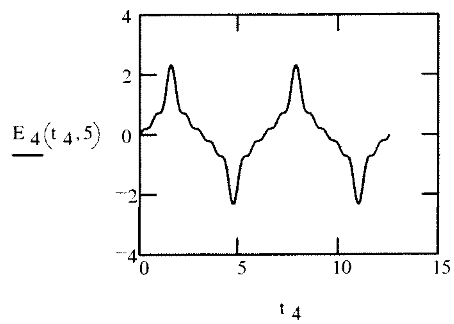
$$f_{avg} := \frac{2}{T} \cdot \int_{-\pi}^{-\frac{\pi}{2} - \Delta} -\frac{A}{2} \cdot \ln\left(-\tan\left(\frac{\pi}{4} - \frac{1}{2} \cdot t\right)\right) dt + \left[\frac{2}{T} \cdot \left[\int_{-\frac{\pi}{2} + \Delta}^{\frac{\pi}{2} - \Delta} -\frac{A}{2} \cdot \ln\left(\tan\left(\frac{\pi}{4} - \frac{1}{2} \cdot t\right)\right) dt \right] \ldots \atop + \frac{2}{T} \cdot \int_{\frac{\pi}{2} + \Delta}^{\pi} -\frac{A}{2} \cdot \ln\left(-\tan\left(\frac{\pi}{4} - \frac{1}{2} \cdot t\right)\right) dt \right]$$

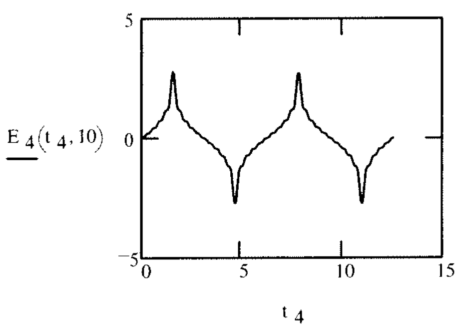
$|f_{avg}| = 2.40596 \cdot 10^{-12}$

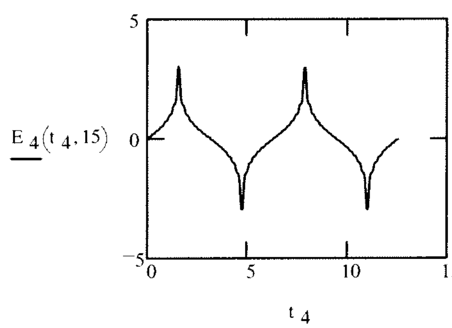
Graph Correlation

$$t_4 := 0, \frac{T}{10000} .. 2\cdot T$$

$$E_4(t,m) := A\cdot\left[\sum_{n=1}^{m}\left[\frac{(-1)^{(n-1)}}{2\cdot n-1}\cdot\sin((2\cdot n-1)\cdot t)\right]\right]$$







Problem # 90: Arctan of Trigonometric Function Wave #1 (α was changed to .25, it must be less than 1 in this problem)

Constants:

$A := 1.3$ $T := 2.7$ $\theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := .7$ $f_r := 4.3$ $a := 0.32$ $b := 2.1$ $V_{dc} := 0.47$ $j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} A \cdot \mathrm{atan}\left(\frac{\alpha \cdot \sin(t)}{1 - \alpha \cdot \cos(t)}\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{2 \cdot \pi} \cdot t\right) dt$$

$$b(n) := \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} A \cdot \mathrm{atan}\left(\frac{\alpha \cdot \sin(t)}{1 - \alpha \cdot \cos(t)}\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{2 \cdot \pi} \cdot t\right) dt$$

Results:

$a(0) = 0$

$a(1) = 0$

$a(2) = 0$

$a(3) = 0$

$a(4) = 0$

$a(5) = 0$

$a(6) = 0$

$a(7) = 0$

$a(8) = 0$

$b(0) = 0$

$b(1) = 0.91$

$b(2) = 0.319$

$b(3) = 0.149$

$b(4) = 0.078$

$b(5) = 0.044$

$b(6) = 0.025$

$b(7) = 0.015$

$b(8) = 9.368 \cdot 10^{-3}$

Check using fourier series definition:

$$Y(h) = A\cdot\frac{\alpha^h}{h}$$

$$Y_0 := 0$$

$Y(1) = 0.91$

$Y(2) = 0.318$

$Y(3) = 0.149$

$Y(4) = 0.078$

$Y(5) = 0.044$

$Y(6) = 0.025$

$Y(7) = 0.015$

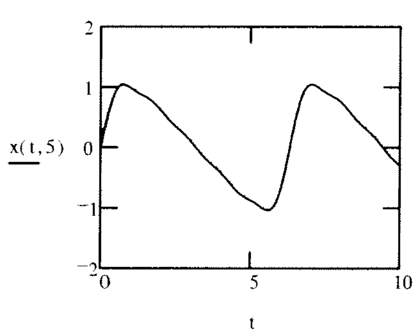
$Y(8) = 9.368\cdot 10^{-3}$

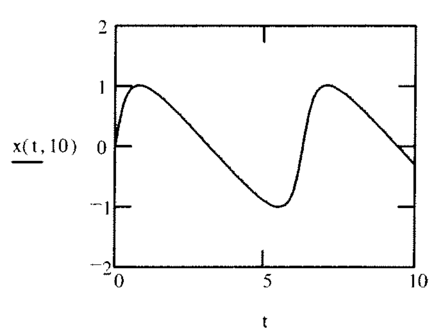
Conclusion: These values match the b coeffecients

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 10$

$$x(t,c) := A\cdot\left[\sum_{n=1}^{c}\left(\frac{\alpha^n}{n}\right)\cdot\sin(n\cdot t)\right]$$





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Arctangent of Trigonmetric Function Wave #1

$A := 1.3$ $T := 2\cdot\pi$ $\tau := 0.68$ $\alpha := 0.7$ $fr := 4.3\cdot Hz$ $a := 0.32$ $b := 2.1$ $V_{dc} := 0.47$ $\theta := \frac{\pi}{5}$

Coefficient Equations for a(n) and b(n)

$$a(n) := \frac{2}{T}\cdot\int_{-\pi}^{\pi} A\cdot atan\left(\frac{\alpha\cdot\sin(t)}{1-\alpha\cdot\cos(t)}\right)\cdot\cos\left[\frac{(2\cdot\pi\cdot n)\cdot t}{T}\right] dt$$

$$b(n) := \frac{2}{T}\cdot\int_{-\pi}^{\pi} A\cdot atan\left(\frac{\alpha\cdot\sin(t)}{1-\alpha\cdot\cos(t)}\right)\cdot\sin\left[\frac{(2\cdot\pi\cdot n\cdot t)}{T}\right] dt$$

$$X(n) := A\cdot\frac{\alpha^n}{n}$$

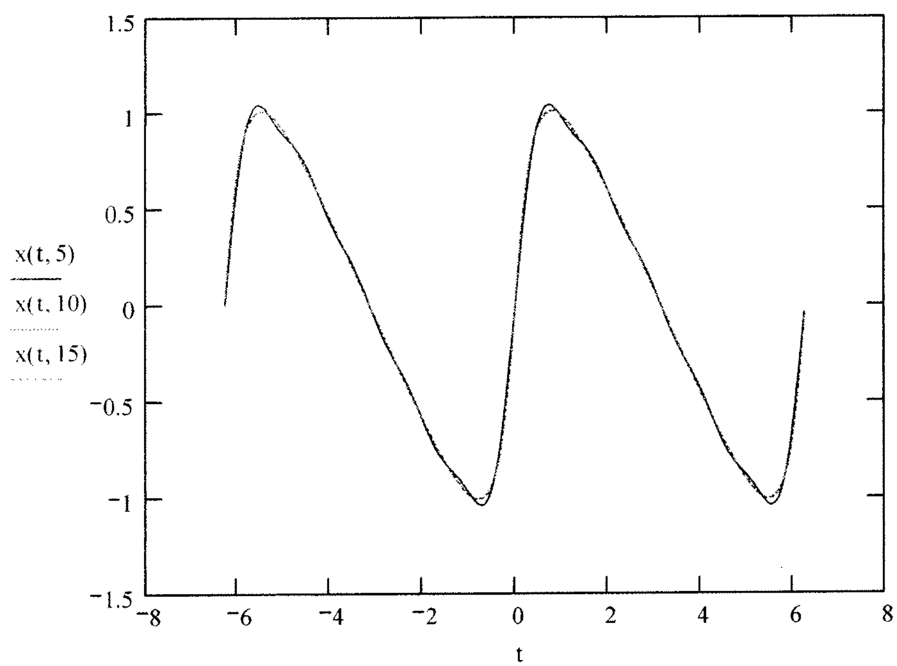
Solutions of Coefficients a(n) and b(n)

$a(0) = 0$	$b(0) = 0$	
$a(1) = 0$	$b(1) = 0.91$	$X(1) = 0.91$
$a(2) = 0$	$b(2) = 0.3185$	$X(2) = 0.3185$
$a(3) = 0$	$b(3) = 0.14863$	$X(3) = 0.14863$
$a(4) = 0$	$b(4) = 0.07803$	$X(4) = 0.07803$
$a(5) = 0$	$b(5) = 0.0437$	$X(5) = 0.0437$
$a(6) = 0$	$b(6) = 0.02549$	$X(6) = 0.02549$
$a(7) = 0$	$b(7) = 0.01529$	$X(7) = 0.01529$
$a(8) = 0$	$b(8) = 9.3678 \times 10^{-3}$	$X(8) = 9.3678 \times 10^{-3}$

Graph of Arctangent of Trigonometric Function Wave #1

$$x(t,m) := A\cdot\sum_{n=1}^{m}\frac{\alpha^{n}}{n}\cdot\sin(n\cdot t)$$

$$t := -2\cdot\pi, -2\cdot\pi + .05 .. 2\cdot\pi$$



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Fourier Problem #4: Arctangent of Trigonometric Function

Coefficients

$A := 1.3 \quad T := 2\cdot\pi \quad \Theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 0.42 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad j := \sqrt{-1}$

Equations

$$a(n) := \frac{2}{T}\cdot\int_{-\pi}^{\pi} \frac{A}{2}\cdot\operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^2}\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

$$b(n) := \frac{2}{T}\cdot\int_{-\pi}^{\pi} \frac{A}{2}\cdot\operatorname{atan}\left(\frac{2\cdot\alpha\cdot\sin(t)}{1-\alpha^2}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

Series Coefficients for First through Eighth Harmonics

$a(0) = 0$ $\qquad b(0) = 0$

$a(1) = 0$ $\qquad b(1) = 0.546$

$a(2) = 0$ $\qquad b(2) = 0$

$a(3) = 0$ $\qquad b(3) = 0.032$

$a(4) = 0$ $\qquad b(4) = 0$

$a(5) = 0$ $\qquad b(5) = 3.398\cdot 10^{-3}$

$a(6) = 0$ $\qquad b(6) = 0$

$a(7) = 0$ $\qquad b(7) = 4.281\cdot 10^{-4}$

$a(8) = 0$ $\qquad b(8) = 0$

Verification of Coefficients

$$b_p(N) := A \cdot \frac{\alpha^{2 \cdot N - 1}}{2 \cdot N - 1}$$

$$b_p(1) = 0.546$$

$$b_p(2) = 0.032$$

$$b_p(3) = 3.398 \cdot 10^{-3}$$

$$b_p(4) = 4.281 \cdot 10^{-4}$$

9v

Following is Problem #4: Arctangent of Trigonmetric Function Wave #3

Updated Constant:

$\alpha_4 := 0.7$ $\quad T_4 := 2\cdot\pi$

Coefficients:

$$a_4(n) := \frac{2}{T_4}\cdot\int_{-\pi}^{\pi} \frac{A}{2}\cdot \operatorname{atan}\left(\frac{2\cdot\alpha_4\cdot\cos(t)}{1-\alpha_4^{\,2}}\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T_4}\cdot t\right) dt$$

$$b_4(n) := \frac{2}{T_4}\cdot\int_{-\pi}^{\pi} \frac{A}{2}\cdot \operatorname{atan}\left(\frac{2\cdot\alpha_4\cdot\cos(t)}{1-\alpha_4^{\,2}}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T_4}\cdot t\right) dt$$

Solution:

*Note: There is no dc term for a(n), and for all other even functions n blows up.

$a_4(0) = -5.242\cdot 10^{-13}$

$a_4(1) = 0.91$

$a_4(2) = -5.246\cdot 10^{-13}$

$a_4(3) = -0.149$

$a_4(4) = -5.248\cdot 10^{-13}$

$a_4(5) = 0.044$

$a_4(6) = -5.247\cdot 10^{-13}$

$a_4(7) = -0.015$

$a_4(8) = -5.247\cdot 10^{-13}$

$b_4(0) = 0$

$b_4(1) = 0$

$b_4(2) = 0$

$b_4(3) = 0$

$b_4(4) = 0$

$b_4(5) = 0$

$b_4(6) = 0$

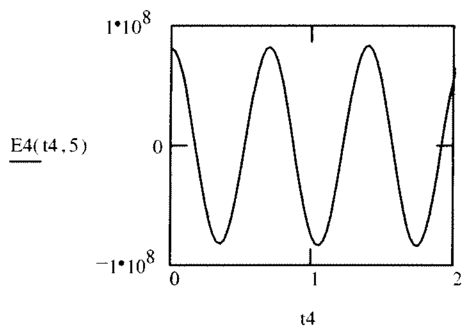
$b_4(7) = 0$

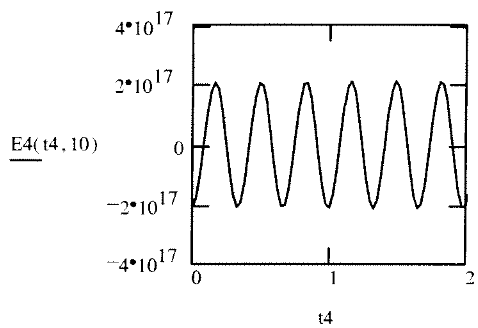
$b_4(8) = 0$

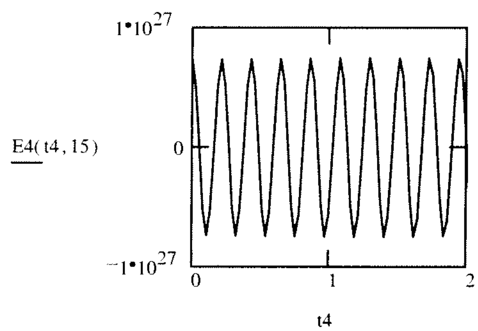
Graphs for Problem #4 using exact equation

$$E4(t4, m) := A \cdot \sum_{n=1}^{m} \frac{(-1)^{n-1} \cdot \alpha^{2 \cdot n - 1}}{2 \cdot n - 1} \cdot \cos((2 \cdot n - 1) \cdot t4)$$

$$t4 := 0, \frac{T}{100} .. 2 \cdot T$$







Check for Problem #4:

Note: N corresponds to odd hamonics

$$a_{4p}(N) := \frac{A \cdot (-1)^{N-1} \cdot \alpha_4^{2 \cdot N - 1}}{2 \cdot N - 1}$$

$a_{4p}(1) = 0.91$

$a_{4p}(2) = -0.149$

$a_{4p}(3) = 0.044$

$a_{4p}(4) = -0.015$

$b_{4p}(n) := 0$