Problem # 39 Partial Sawtooth Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T}\cdot\left[\int_0^{\frac{\tau}{2}} \frac{2\cdot A}{\tau}\cdot t\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] + \left(\frac{2}{T}\right)\cdot\left[\int_{\frac{\tau}{2}}^{T-\frac{\tau}{2}} 0\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] \ldots$$
$$+ \left(\frac{2}{T}\right)\cdot\left[\int_{T-\frac{\tau}{2}}^{T} \frac{2\cdot A}{\tau}\cdot(t-T)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

$$b(n) := \frac{2}{T}\cdot\left[\int_0^{\frac{\tau}{2}} \frac{2\cdot A}{\tau}\cdot t\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] + \left(\frac{2}{T}\right)\cdot\left[\int_{\frac{\tau}{2}}^{T-\frac{\tau}{2}} 0\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] \ldots$$
$$+ \left(\frac{2}{T}\right)\cdot\left[\int_{T-\frac{\tau}{2}}^{T} \frac{2\cdot A}{\tau}\cdot(t-T)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right]$$

Results:

$a(0) = 1.416 \cdot 10^{-15}$

$a(1) = 1.193 \cdot 10^{-15}$

$a(2) = 0$

$a(3) = 0$

$a(4) = 0$

$a(5) = 0$

$a(6) = 0$

$a(7) = 0$

$a(8) = 0$

$b(0) = 0$

$b(1) = 0.162$

$b(2) = 0.266$

$b(3) = 0.279$

$b(4) = 0.205$

$b(5) = 0.083$

$b(6) = -0.034$

$b(7) = -0.101$

$b(8) = -0.103$

Check using fourier series definition:

$$Y(h) := \frac{2 \cdot A}{\pi^2} \cdot \left(\frac{-\pi \cdot \cos\left(\frac{\pi \cdot h \cdot \tau}{T}\right)}{h} + T \cdot \frac{\sin\left(\frac{\pi \cdot h \cdot \tau}{T}\right)}{\tau \cdot h^2} \right)$$

$Y_0 := 0$

$Y(1) = 0.162$

$Y(2) = 0.266$

$Y(3) = 0.279$

$Y(4) = 0.205$

$Y(5) = 0.083$

$Y(6) = -0.034$

$Y(7) = -0.101$

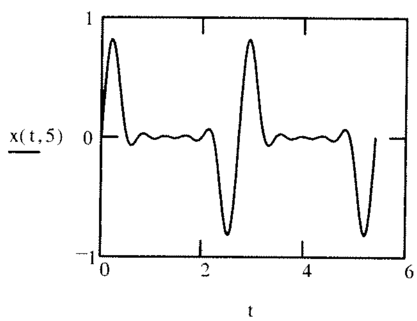
$Y(8) = -0.103$

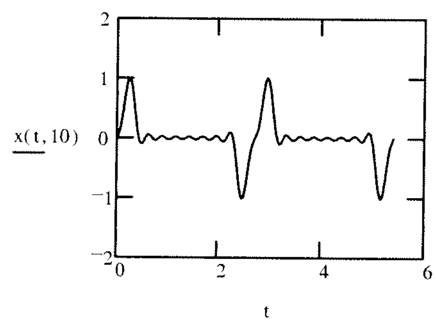
Conclusion: The results match the b coefficients.

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \frac{2\cdot A}{\pi^2}\cdot\sum_{n=1}^{c}\left(\frac{-\pi\cdot\cos\left(\frac{\pi\cdot n\cdot\tau}{T}\right)}{n} + T\cdot\frac{\sin\left(\frac{\pi\cdot n\cdot\tau}{T}\right)}{\tau\cdot n^2}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$





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Sawtooth Modulated Wave #1

$A := 1.3$ $T := 2.7$ $\tau := .68$ $\alpha := 9.4$ $f_r := 4.3\,\text{Hz}$ $a := .32$ $b := 2.1$ $V_{dc} := .47$ $\theta := \frac{\pi}{5}$

$$a(N) := \frac{2}{T}\cdot\int_0^T A\cdot\left(\frac{2}{T}\cdot t - 1\right)\cdot\sin\left[\frac{(2\cdot\pi)}{T}\cdot t\right]\cdot\cos\left[\frac{(2\cdot\pi\cdot N)}{T}\cdot t\right]dt$$

$$b(M) := \frac{2}{T}\cdot\int_0^T A\cdot\left(\frac{2}{T}\cdot t - 1\right)\cdot\sin\left[\frac{(2\cdot\pi)}{T}\cdot t\right]\cdot\sin\left[\frac{(2\cdot\pi\cdot M)}{T}\cdot t\right]dt$$

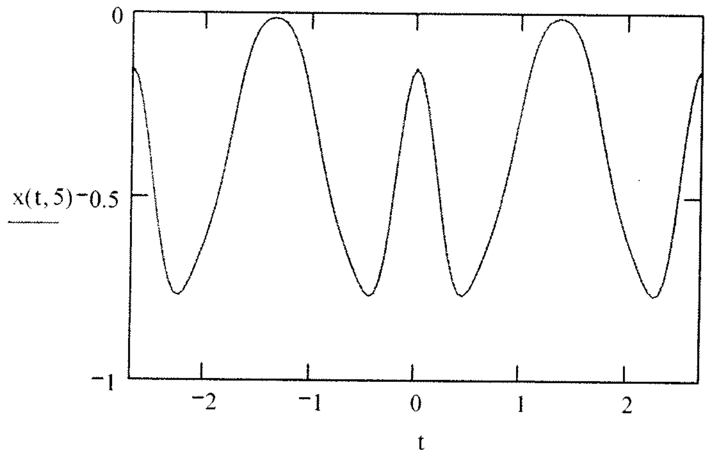
$$ap(n) := \frac{(2\cdot A)}{\pi}\cdot\frac{1}{(n-1)\cdot(n+1)}$$

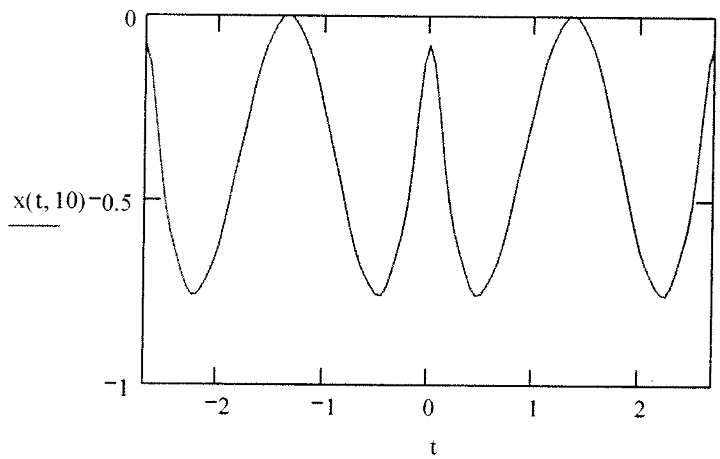
$$\frac{ap_o}{2} := \frac{-A}{\pi} \qquad ap_o := \frac{(-2\cdot A)}{\pi}$$

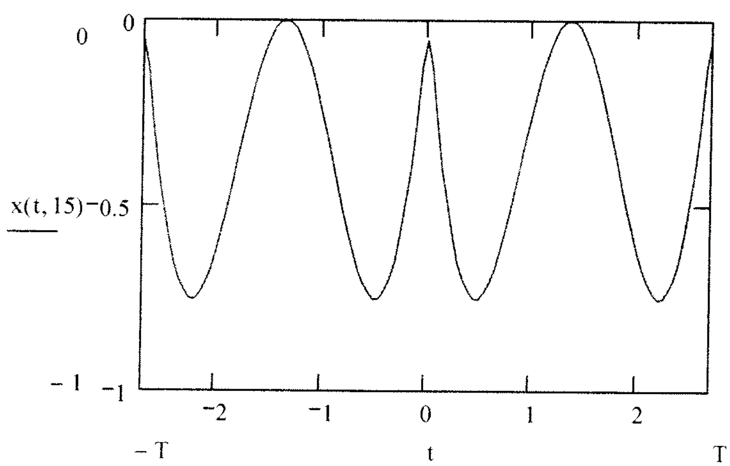
$$ap_1 := \frac{-A}{2\cdot\pi}$$

$a(0) = -0.82761$	$b(0) = 0$	$ap_o = -0.82761$
$a(1) = -0.2069$	$b(1) = 0$	$ap_1 = -0.2069$
$a(2) = 0.27587$	$b(2) = 0$	$ap(2) = 0.27587$
$a(3) = 0.10345$	$b(3) = 0$	$ap(3) = 0.10345$
$a(4) = 0.05517$	$b(4) = 0$	$ap(4) = 0.05517$
$a(5) = 0.03448$	$b(5) = 0$	$ap(5) = 0.03448$
$a(6) = 0.02365$	$b(6) = 0$	$ap(6) = 0.02365$
$a(7) = 0.01724$	$b(7) = 0$	$ap(7) = 0.01724$
$a(8) = 0.01314$	$b(8) = 0$	$ap(8) = 0.01314$

$$x(t,m) := \frac{a(0)}{2} + \sum_{N=1}^{m} a(N)\cdot\cos\left[\frac{(2\cdot\pi\cdot N)}{T}\cdot t\right] \quad t := -T, -T + .05 .. T$$







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2. Sawtooth Modulated Wave #2

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad i := \sqrt{(-1)}$

$$a_0(t) := -\frac{A}{\pi} \cdot sin\left(\frac{2 \cdot \pi t}{T}\right)$$

$$a(n) := \frac{2}{T} \cdot \int_0^T A \cdot \left(\frac{2 \cdot t}{T} - 1\right) \cdot cos\left(\frac{2 \cdot \pi t}{T}\right) \cdot cos\left(\frac{2 \cdot \pi n \cdot t}{T}\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_0^T A \cdot \left(\frac{2}{T} \cdot t - 1\right) \cdot cos\left(\frac{2 \cdot \pi}{T} \cdot t\right) \cdot sin\left(\frac{2 \cdot \pi n}{T} \cdot t\right) dt$$

$a(0) = 0$ $b(0) = 0$

$a(1) = 0$ $b(1) = -0.2069$

$a(2) = 0$ $b(2) = -0.55174$

$a(3) = 0$ $b(3) = -0.31035$

$a(4) = 0$ $b(4) = -0.22069$

$a(5) = 0$ $b(5) = -0.17242$

$a(6) = 0$ $b(6) = -0.14188$

$a(7) = 0$ $b(7) = -0.12069$

$a(8) = 0$ $b(8) = -0.10509$

$b(9) = -0.09311$

Check

$a_p(n) := 0$ $b_p(n) := \frac{-2 \cdot A}{\pi} \cdot \frac{n}{((n-1) \cdot (n+1))}$

$a_p(0) = 0$ $b_p(2) = -0.55174$

$a_p(1) = 0$ $b_p(3) = -0.31035$

$a_p(2) = 0$ $b_p(4) = -0.22069$

$a_p(3) = 0$ $b_p(5) = -0.17242$

$a_p(4) = 0$ $b_p(6) = -0.14188$

$a_p(5) = 0$ $b_p(7) = -0.12069$

$a_p(6) = 0$ $b_p(8) = -0.10509$

$a_p(7) = 0$ $b_p(9) = -0.09311$

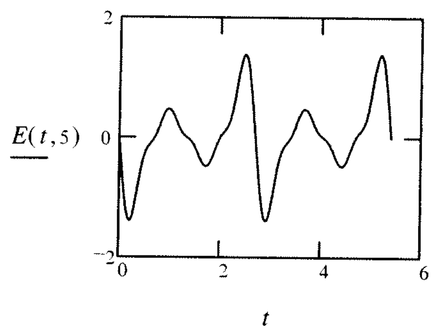
$a_p(8) = 0$ $b_p(0) = 0$

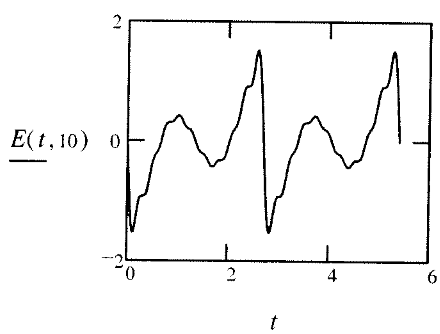
$$-\frac{A}{2 \cdot \pi} = -0.2069$$

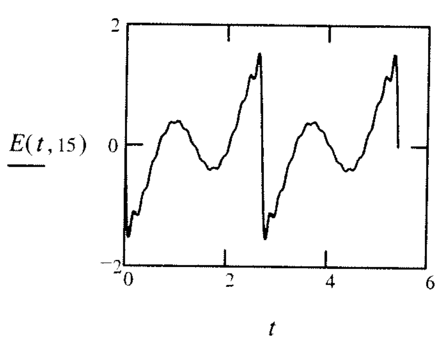
Exact Equation

$$E(t,m) := -\frac{A}{2\cdot\pi}\cdot \sin\left(\frac{2\cdot\pi t}{T}\right) - \frac{2\cdot A}{\pi}\cdot \sum_{n=2}^{m} \frac{n^3}{(n-1)^2\cdot(n+1)^2}\cdot \sin\left(\frac{2\cdot\pi n\cdot t}{T}\right) \qquad t := 0, \frac{T}{1000} .. 2\cdot T$$

Graphs







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2. Sawtooth Modulated Wave #2

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad i := \sqrt{(-1)}$

$$a_0(t) := -\frac{A}{\pi} \cdot sin\left(\frac{2 \cdot \pi t}{T}\right)$$

$$a(n) := \frac{2}{T} \cdot \int_0^T A \cdot \left(\frac{2 \cdot t}{T} - 1\right) \cdot cos\left(\frac{2 \cdot \pi t}{T}\right) \cdot cos\left(\frac{2 \cdot \pi n \cdot t}{T}\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_0^T A \cdot \left(\frac{2}{T} \cdot t - 1\right) \cdot cos\left(\frac{2 \cdot \pi}{T} \cdot t\right) \cdot sin\left(\frac{2 \cdot \pi n}{T} \cdot t\right) dt$$

$a(0) = 0$	$b(0) = 0$		
$a(1) = 0$	$b(1) = -0.2069$		
$a(2) = 0$	$b(2) = -0.55174$		
$a(3) = 0$	$b(3) = -0.31035$		
$a(4) = 0$	$b(4) = -0.22069$		
$a(5) = 0$	$b(5) = -0.17242$		
$a(6) = 0$	$b(6) = -0.14188$		
$a(7) = 0$	$b(7) = -0.12069$		
$a(8) = 0$	$b(8) = -0.10509$		
	$b(9) = -0.09311$		

Check

$$a_p(n) := 0 \qquad b_p(n) := \frac{-2\cdot A}{\pi}\cdot\frac{n}{((n-1)\cdot(n+1))}$$

$a_p(0) = 0$	$b_p(2) = -0.55174$
$a_p(1) = 0$	$b_p(3) = -0.31035$
$a_p(2) = 0$	$b_p(4) = -0.22069$
$a_p(3) = 0$	$b_p(5) = -0.17242$
$a_p(4) = 0$	$b_p(6) = -0.14188$
$a_p(5) = 0$	$b_p(7) = -0.12069$
$a_p(6) = 0$	$b_p(8) = -0.10509$
$a_p(7) = 0$	$b_p(9) = -0.09311$
$a_p(8) = 0$	$b_p(0) = 0$

$$\frac{-A}{2\pi} \simeq -.2069$$

Sawtooth Modulated Wave #3

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad F_r := 4.3 \quad a := 3.2 \quad b := 2.1 \quad w := \frac{T - 2\cdot\tau}{2} \qquad w = 0.67$

$$a(n) := \left[\frac{2}{T}\cdot\int_0^{\frac{T}{2}} \frac{2\cdot A}{T}\cdot t\cdot \sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \left[\frac{2}{T}\cdot\int_{\frac{T}{2}}^{T} 2\cdot A\cdot\left(\frac{t}{T} - 1\right)\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$$b(n) := \left[\frac{2}{T}\cdot\int_0^{\frac{T}{2}} \frac{2\cdot A}{T}\cdot t\cdot \sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \left[\frac{2}{T}\cdot\int_{\frac{T}{2}}^{T} 2\cdot A\cdot\left(\frac{t}{T} - 1\right)\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$ap_1 := \frac{-A}{2\cdot\pi} \qquad ap(n) := \frac{-2\cdot A}{\pi}\cdot\frac{(-1)^n}{(n-1)\cdot(n+1)} \qquad f_{avg} := \frac{A}{\pi}$

$\frac{a(0)}{2} = 0.4138 \qquad f_{avg} = 0.4138$

$a(1) = -0.2069$	$ap_1 = -0.2069$	$b(1) = 0$
$a(2) = -0.27587$	$ap(2) = -0.27587$	$b(2) = 0$
$a(3) = 0.10345$	$ap(3) = 0.10345$	$b(3) = 0$
$a(4) = -0.05517$	$ap(4) = -0.05517$	$b(4) = 0$
$a(5) = 0.03448$	$ap(5) = 0.03448$	$b(5) = 0$
$a(6) = -0.02365$	$ap(6) = -0.02365$	$b(6) = 0$
$a(7) = 0.01724$	$ap(7) = 0.01724$	$b(7) = 0$
$a(8) = -0.01314$	$ap(8) = -0.01314$	$b(8) = 0$

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Decloration of Coefficients

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad j := \sqrt{-1} \quad V_{dc} := .47$

Sawtooth Modulated Wave #4

$$a1(n) := \frac{2}{T}\cdot\int_0^{\left(\frac{T}{2}\right)} \left(\frac{2\cdot A}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \ldots$$
$$+\left(\frac{2}{T}\right)\cdot\int_{\left(\frac{T}{2}\right)}^{T} \left[\left[(2\cdot A)\cdot\left(\frac{t}{T}-1\right)\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\right]\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt$$

$$b1(n) := \frac{2}{T}\cdot\int_0^{\left(\frac{T}{2}\right)} \left(\frac{2\cdot A}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \ldots$$
$$+\left(\frac{2}{T}\right)\cdot\int_{\left(\frac{T}{2}\right)}^{T} \left[\left[(2\cdot A)\cdot\left(\frac{t}{T}-1\right)\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\right]\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt$$

Coefficient Solution

$a1(0) = 0.00000$	$a1(5) = 0.00000$	$b1(0) = 0.00000$	$b1(5) = -0.17242$
$a1(1) = 0.00000$	$a1(6) = 0.00000$	$b1(1) = -0.20690$	$b1(6) = 0.14188$
$a1(2) = 0.00000$	$a1(7) = 0.00000$	$b1(2) = 0.55174$	$b1(7) = -0.12069$
$a1(3) = 0.00000$	$a1(8) = 0.00000$	$b1(3) = -0.31035$	$b1(8) = 0.10509$
$a1(4) = 0.00000$		$b1(4) = 0.22069$	

b1(1) check? based on $\frac{-A}{2\pi}$ $\frac{-1.3}{2\pi} = -.206$ ✓

Check

$$b_p(n) := \left[\frac{2\cdot A}{\pi}\cdot\frac{\left[n\cdot(-1)^n\right]}{(n+1)\cdot(n-1)}\right]$$

$f_{avg} := 0$

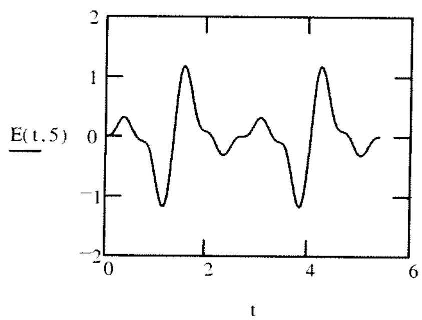
$b_p(2) = 0.55174$	$b_p(6) = 0.14188$
$b_p(3) = -0.31035$	$b_p(7) = -0.12069$
$b_p(4) = 0.22069$	$b_p(8) = 0.10509$
$b_p(5) = -0.17242$	

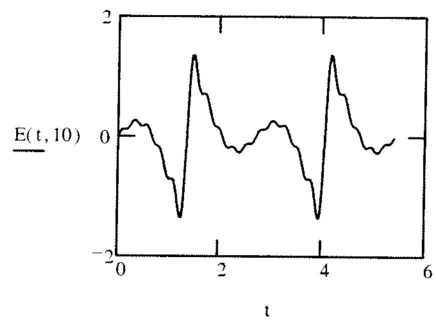
By examination it is clear that the average value of F is zero since the function does not have a DC off-set; the net area under the curve is zero.

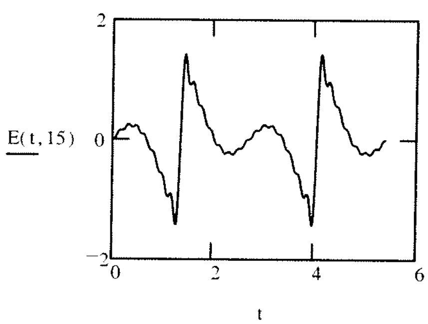
Exact Equation Solution

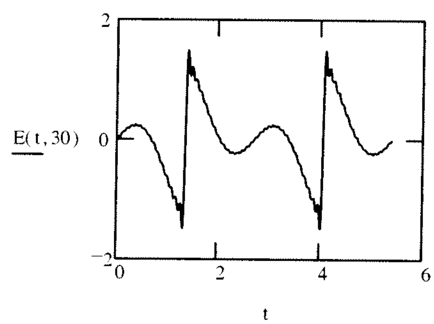
$$E(t,m) := -\frac{A}{2\cdot\pi}\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right) + \frac{2\cdot A}{\pi}\cdot\left[\sum_{n=2}^{m}\left[\frac{n\cdot(-1)^{n}}{((n-1)\cdot(n+1))}\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]\right]$$

$$t := 0, \frac{T}{10000} .. 2\cdot T$$









Ken Kaiser

Decloration of Coefficients

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad j := \sqrt{-1} \quad V_{dc} := .47$

Half-Rectified Triangular Wave #1

$$a1(n) := \left[\left(\frac{2}{T}\right)\cdot\int_0^{\left(\frac{T}{4}\right)}\left[\left[\left(\frac{-4\cdot A}{T}\cdot t\right)+A\right]\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt\right]+\left[\left(\frac{2}{T}\right)\cdot\int_{\left(\frac{T}{4}\right)}^{\left(\frac{3\cdot T}{4}\right)}\left[(0)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt\right]\ldots$$

$$+\left[\left(\frac{2}{T}\right)\cdot\int_{\left(\frac{3\cdot T}{4}\right)}^{T}\left[\left(\frac{4\cdot A}{T}\right)\cdot\left(t-\frac{3\cdot T}{4}\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt\right]$$

$$b1(n) := \left[\left(\frac{2}{T}\right)\cdot\int_0^{\left(\frac{T}{4}\right)}\left[\left[\left(\frac{-4\cdot A}{T}\cdot t\right)+A\right]\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt\right]+\left[\left(\frac{2}{T}\right)\cdot\int_{\left(\frac{T}{4}\right)}^{\left(\frac{3\cdot T}{4}\right)}\left[(0)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt\right]\ldots$$

$$+\left[\left(\frac{2}{T}\right)\cdot\int_{\left(\frac{3\cdot T}{4}\right)}^{T}\left[\left(\frac{4\cdot A}{T}\right)\cdot\left(t-\frac{3\cdot T}{4}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]dt\right]$$

Coefficient Solution

$a1(0) = 0.65000 \qquad b1(0) = 0.00000$

$a1(1) = 0.52687 \qquad b1(1) = 0.00000$

$a1(2) = 0.26344 \qquad b1(2) = 0.00000$

$a1(3) = 0.05854 \qquad b1(3) = 0.00000$

$a1(4) = -8.78912\cdot 10^{-10} \qquad b1(4) = 0.00000$

$a1(5) = 0.02107 \qquad b1(5) = 0.00000$

$a1(6) = 0.02927 \qquad b1(6) = 0.00000$

$a1(7) = 0.01075 \qquad b1(7) = 0.00000$

$a1(8) = -1.17167\cdot 10^{-9} \qquad b1(8) = 0.00000$

Check

$$a_p(N) := \frac{8\cdot A}{\pi^2}\cdot\left(\frac{1}{N^2}\right)\cdot\left(\sin\left(\frac{N\cdot\pi}{4}\right)\right)^2$$

$a_p(1) = 0.52687 \qquad a_p(5) = 0.02107$

$a_p(2) = 0.26344 \qquad a_p(6) = 0.02927$

$a_p(3) = 0.05854 \qquad a_p(7) = 0.01075$

$a_p(4) = 0.00000 \qquad a_p(8) = 0.00000$

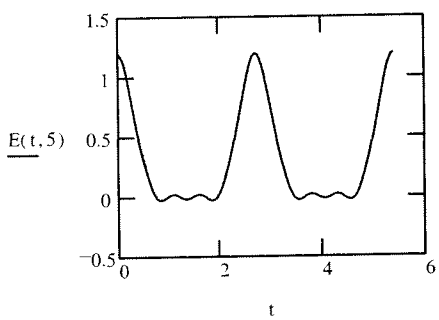
$F_{avg} := \left(\frac{A}{4}\right)$

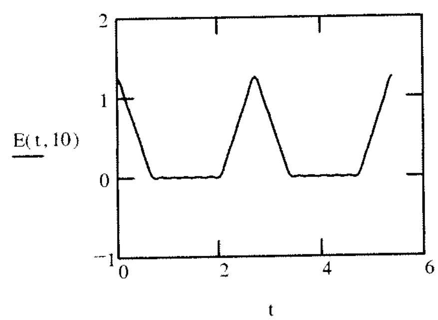
$F_{avg} = 0.32500 = \frac{a1(0)}{2}$

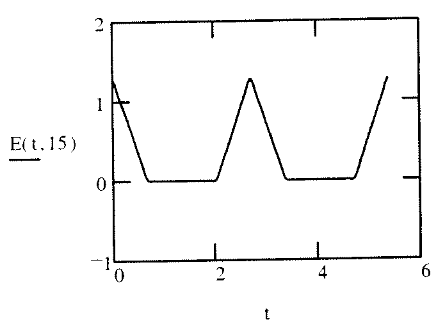
Exact Equation Solution

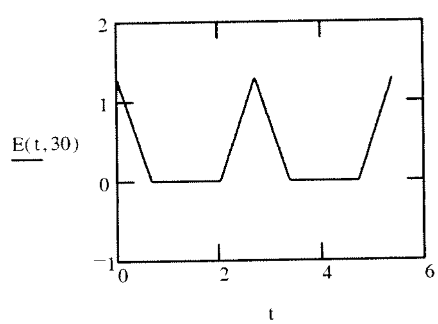
$$E(t,m) = \left(\frac{A}{4}\right) + \left(\frac{8\cdot A}{\pi^2}\right)\cdot\left[\sum_{n=1}^{m}\left[\frac{1}{n^2}\cdot\left(\sin\left(\frac{\pi\cdot n}{4}\right)\cdot\sin\left(\frac{\pi\cdot n}{4}\right)\right)\cdot\cos\left(\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right)\right]\right]$$

$$t := 0, \frac{T}{10000} .. 2\cdot T$$









Ken Kaiser

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Fourier Series Problem #2: Half-Rectified Triangular Wave

Coefficients

$A := 1.3 \quad T := 2.7 \quad \Theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad j := \sqrt{-1}$

Equations

$$F(n) := \frac{1}{T}\cdot\int_0^{\frac{T}{4}} \frac{4\cdot A}{T}\cdot t\cdot e^{\frac{-j\cdot 2\cdot\pi\cdot n}{T}\cdot t}\,dt + \frac{1}{T}\cdot\int_{\frac{T}{4}}^{\frac{T}{2}} \frac{4\cdot A}{T}\cdot\left(\frac{T}{2}-t\right)\cdot e^{\frac{-j\cdot 2\cdot\pi\cdot n}{T}\cdot t}\,dt + \frac{1}{T}\cdot\int_{\frac{T}{2}}^{T} 0\cdot\left(\frac{T}{2}-t\right)\cdot e^{\frac{-j\cdot 2\cdot\pi\cdot n}{T}\cdot t}\,dt$$

Determination of Series Harmonics

$F(0) = 0.325$

$F(1) = -0.263i \qquad F(-1) = 0.263i$

$F(2) = -0.132$

$F(3) = 0.029i \qquad F(-3) = -0.029i$

$F(4) = -4.395\cdot 10^{-10}$

$F(5) = -0.011i \qquad F(-5) = 0.011i$

$F(6) = -0.015$

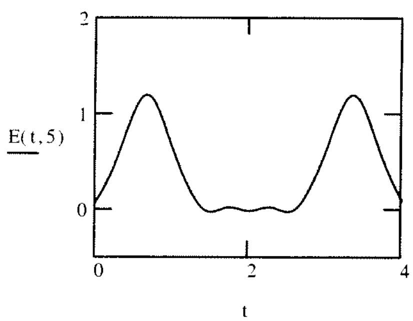
$F(7) = 5.376\cdot 10^{-3}i \qquad F(-7) = -5.376\cdot 10^{-3}i$

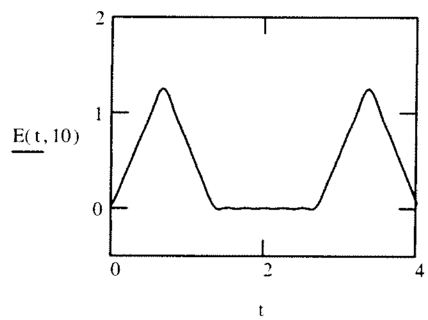
$F(8) = -5.858\cdot 10^{-10}$

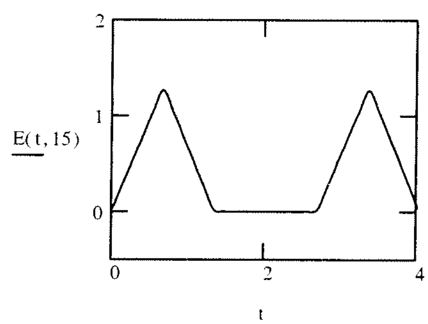
Graphing

$$E(t,m) := \frac{A}{4} + \frac{A}{\pi^2}\cdot\sum_{n=-m}^{-1}\frac{2\cdot e^{\frac{-j\cdot\pi\cdot n}{2}} - 1 + (-1)^{n-1}}{n^2}\cdot e^{\frac{j\cdot 2\cdot\pi\cdot n}{T}\cdot t} + \frac{A}{\pi^2}\cdot\sum_{n=1}^{m}\frac{2\cdot e^{\frac{-j\cdot\pi\cdot n}{2}} - 1 + (-1)^{n-1}}{n^2}\cdot e^{\frac{j\cdot 2\cdot\pi\cdot n}{T}\cdot t}$$

$$t := 0, \frac{T}{100} .. 2\cdot T$$







Verification of Coefficients

$$f(N) := \frac{A}{\pi^2}\cdot\frac{2\cdot e^{\frac{-j\cdot\pi\cdot N}{2}} - 1 + (-1)^{N-1}}{N^2}$$

? $\frac{A}{4} = \frac{1.3}{4} = .325$

$f(0) = 0$

$f(1) = -0.263i$	$f(-1) = 0.263i$
$f(2) = -0.132$	
$f(3) = 0.029i$	$f(-3) = -0.029i$
$f(4) = 0$	
$f(5) = -0.011i$	$f(-5) = 0.011i$
$f(6) = -0.015$	
$f(7) = 5.376\cdot 10^{-3}\,i$	$f(-7) = -5.376\cdot 10^{-3}\,i$
$f(8) = 0$	

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Problem # 2

Isosceles Triangular Wave

Given Constants:

$T := 2.7$

$\theta := \frac{\pi}{5}$

$\tau := .68$

$V := .47$

$A := 1.3$

$\alpha := 9.4$

$fr := 4.3$

Fourier Series-Integral Definitions

$$a(n) := \int_0^{\frac{\tau}{2}} \left(\frac{-2\cdot A}{\tau}\cdot t + A\right)\cdot \cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T} + \int_{T-\frac{\tau}{2}}^{T} \left[\frac{2\cdot A}{\tau}\cdot\left(t - T + \frac{\tau}{2}\right)\right]\cdot \cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T}$$

$$b(n) := \int_0^{\frac{\tau}{2}} \left(\frac{-2\cdot A}{\tau}\cdot t + A\right)\cdot \sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T} + \int_{T-\frac{\tau}{2}}^{T} \left[\frac{2\cdot A}{\tau}\cdot\left(t - T + \frac{\tau}{2}\right)\right]\cdot \sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T}$$

***Note: The height difference between the integrals is due to the limits that are used, meaning that if a fraction is present the integral is automatically lengthened.**

Results:

$a(0) = 0.32741$
$a(1) = 0.31068$
$a(2) = 0.26454$
$a(3) = 0.19982$
$a(4) = 0.13073$
$a(5) = 0.07055$
$a(6) = 0.02804$
$a(7) = 5.65030 \cdot 10^{-3}$
$a(8) = 1.76979 \cdot 10^{-5}$

$b(0) = 0$
$b(1) = 0$
$b(2) = 0$
$b(3) = 0$
$b(4) = 0$
$b(5) = 0$
$b(6) = 0$
$b(7) = 0$
$b(8) = 0$

Fourier Series-Summation Definition

$$Y(h) := \frac{4 \cdot A \cdot T}{\tau \cdot \pi^2} \cdot \frac{1}{h^2} \cdot \sin\left(\frac{\pi \cdot h \cdot \tau}{2 \cdot T}\right)^2$$

$$\left(\frac{A \cdot \tau}{2 \cdot T}\right) \cdot 2 = 0.32741 \quad \longleftarrow \quad Y(0)$$

Results:

$Y(1) = 0.31068$
$Y(2) = 0.26454$
$Y(3) = 0.19982$
$Y(4) = 0.13073$
$Y(5) = 0.07055$
$Y(6) = 0.02804$
$Y(7) = 5.65030 \cdot 10^{-3}$
$Y(8) = 1.76983 \cdot 10^{-5}$

Conclusion

Integral Definition Results

Harmonic	n	a_o	b_o
DC Term	0	0.32741	-
Fundamental	1	0.31068	0
Second	2	0.26454	0
Third	3	0.19982	0
Fourth	4	0.13073	0
Fifth	5	0.07055	0
Sixth	6	0.02804	0
Seventh	7	5.65030E-03	0
Eighth	8	1.76979E-05	0

Summation Definition Results

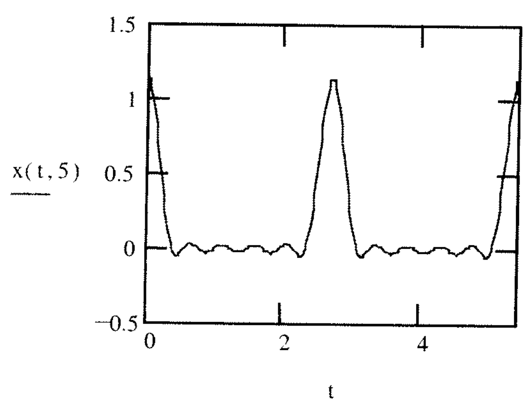
h	a_o	b_o
0	0.32741	-
1	0.31068	-
2	0.26454	-
3	0.19982	-
4	0.13073	-
5	0.07055	-
6	0.02804	-
7	5.65030E-03	-
8	1.76979E-05	-

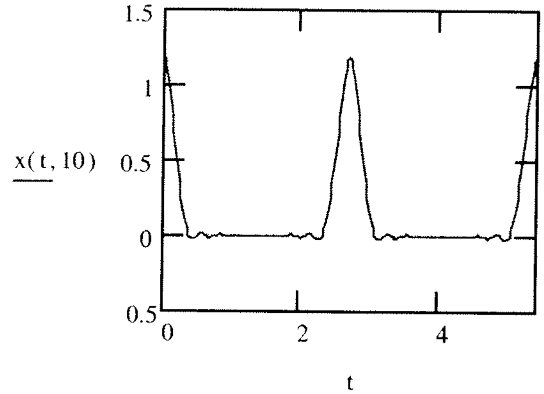
The results from the two methods are identical.

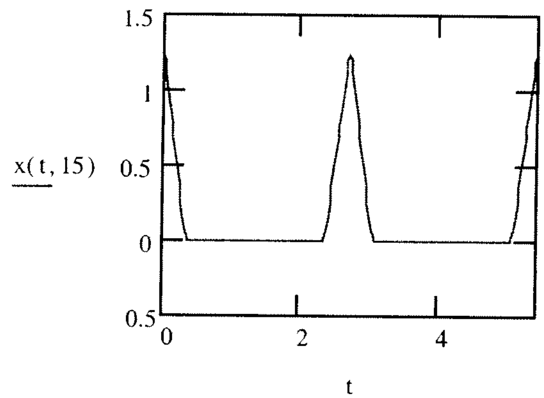
Plot of Function vs. Time

$t := 0, 0.027 .. 5.4$

$$x(t,h) := \frac{A\cdot\tau}{2\cdot T} + \frac{4\cdot A\cdot T}{\tau\cdot\pi^2}\cdot\sum_{n=1}^{h}\frac{1}{n^2}\cdot\sin\left(\frac{\pi\cdot n\cdot\tau}{2\cdot T}\right)^2\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$




1.5
1
x(t, 10)
0.5
0
0.5
0
2
4
t


1.5
1
x(t, 15)
0.5
0
0.5
0
2
4
t

Problem # 47 4th Order Approximate Cosine Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \left[\int_0^T \frac{8 \cdot A}{7} \cdot \left(1 - \frac{30}{T^2} \cdot t^2 + \frac{60}{T^3} \cdot t^3 - \frac{30}{T^4} \cdot t^4 \right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) dt \right]$$

$$b(n) := \frac{2}{T} \cdot \left[\int_0^T \frac{8 \cdot A}{7} \cdot \left(1 - \frac{30}{T^2} \cdot t^2 + \frac{60}{T^3} \cdot t^3 - \frac{30}{T^4} \cdot t^4 \right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) dt \right]$$

Results:

$a(0) = 1.162 \cdot 10^{-15}$

$a(1) = 1.373$

$a(2) = 0.086$

$a(3) = 0.017$

$a(4) = 5.362 \cdot 10^{-3}$

$a(5) = 2.196 \cdot 10^{-3}$

$a(6) = 1.059 \cdot 10^{-3}$

$a(7) = 5.717 \cdot 10^{-4}$

$a(8) = 3.44 \cdot 10^{-4}$

$b(0) = 0$

$b(1) = -1.179 \cdot 10^{-15}$

$b(2) = -1.234 \cdot 10^{-15}$

$b(3) = 0$

$b(4) = 0$

$b(5) = 0$

$b(6) = 0$

$b(7) = 0$

$b(8) = 0$

Check using fourier series definition:

$$Y(h) := \frac{720 \cdot A}{7 \cdot \pi^4} \cdot \frac{1}{h^4}$$

$Y(1) = 1.373$

$Y(2) = 0.086$

$Y(3) = 0.017$

$Y(4) = 5.362 \cdot 10^{-3}$

$Y(5) = 2.196 \cdot 10^{-3}$

$Y(6) = 1.059 \cdot 10^{-3}$

$Y(7) = 5.717 \cdot 10^{-4}$

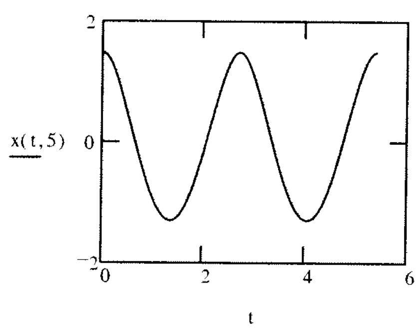
$Y(8) = 3.351 \cdot 10^{-4}$

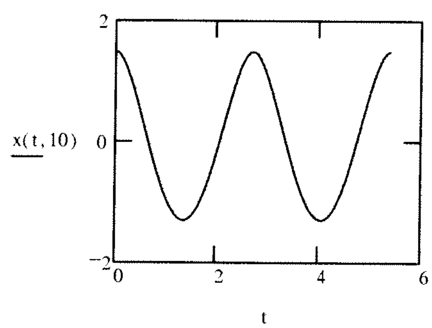
Conclusion: The results match a coefficients, and the average value is 0

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \frac{720 \cdot A}{7 \cdot \pi^4} \cdot \left[\sum_{n=1}^{c} \left(\frac{1}{n^4}\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) \right]$$





Problem # 47 4th Order Approximate Cosine Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T}\cdot\left[\int_0^T \frac{8\cdot A}{7}\cdot\left(1 - \frac{30}{T^2}\cdot t^2 + \frac{60}{T^3}\cdot t^3 - \frac{30}{T^4}\cdot t^4\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$$b(n) := \frac{2}{T}\cdot\left[\int_0^T \frac{8\cdot A}{7}\cdot\left(1 - \frac{30}{T^2}\cdot t^2 + \frac{60}{T^3}\cdot t^3 - \frac{30}{T^4}\cdot t^4\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

Results:

$a(0) = 1.162\cdot 10^{-15}$	$b(0) = 0$
$a(1) = 1.373$	$b(1) = -1.179\cdot 10^{-15}$
$a(2) = 0.086$	$b(2) = -1.234\cdot 10^{-15}$
$a(3) = 0.017$	$b(3) = 0$
$a(4) = 5.362\cdot 10^{-3}$	$b(4) = 0$
$a(5) = 2.196\cdot 10^{-3}$	$b(5) = 0$
$a(6) = 1.059\cdot 10^{-3}$	$b(6) = 0$
$a(7) = 5.717\cdot 10^{-4}$	$b(7) = 0$
$a(8) = 3.44\cdot 10^{-4}$	$b(8) = 0$

Check using fourier series definition:

$$Y(h) := \frac{720 \cdot A}{7 \cdot \pi^4} \cdot \frac{1}{h^4}$$

$$Y_0 := 0$$

$Y(1) = 1.373$

$Y(2) = 0.086$

$Y(3) = 0.017$

$Y(4) = 5.362 \cdot 10^{-3}$

$Y(5) = 2.196 \cdot 10^{-3}$

$Y(6) = 1.059 \cdot 10^{-3}$

$Y(7) = 5.717 \cdot 10^{-4}$

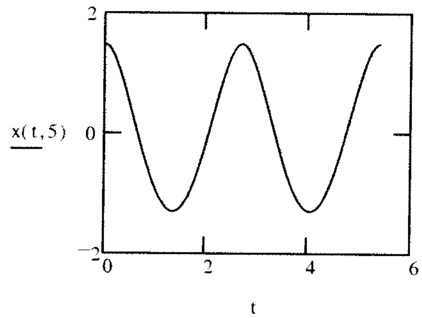
$Y(8) = 3.351 \cdot 10^{-4}$

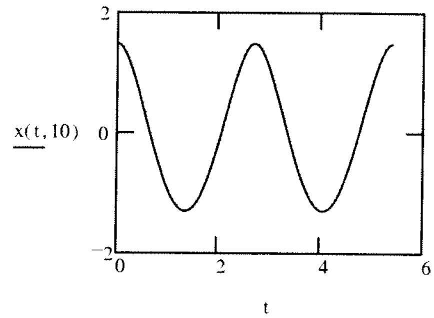
Conclusion: The results match a coefficients,

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \frac{720 \cdot A}{7 \cdot \pi^4} \cdot \left[\sum_{n=1}^{c} \left(\frac{1}{n^4} \right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) \right]$$


2
0
−2
x(t,5)
0
2
4
6
t


2
0
−2
x(t,10)
0
2
4
6
t

Problem # 49 3rd Order Approximate Sine Wave #2

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \left[\int_0^T \frac{12 \cdot \sqrt{3} \cdot A}{T^3} \cdot t \cdot \left(t - \frac{T}{2} \right) \cdot (t - T) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) dt \right]$$

$$b(n) := \frac{2}{T} \cdot \left[\int_0^T \frac{12 \cdot \sqrt{3} \cdot A}{T^3} \cdot t \cdot \left(t - \frac{T}{2} \right) \cdot (t - T) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t \right) dt \right]$$

Results:

$a(0) = 0$ $\quad b(0) = 0$

$a(1) = 0$ $\quad b(1) = 1.307$

$a(2) = 0$ $\quad b(2) = 0.163$

$a(3) = 0$ $\quad b(3) = 0.048$

$a(4) = 0$ $\quad b(4) = 0.02$

$a(5) = 0$ $\quad b(5) = 0.01$

$a(6) = 0$ $\quad b(6) = 6.052 \cdot 10^{-3}$

$a(7) = 0$ $\quad b(7) = 3.811 \cdot 10^{-3}$

$a(8) = 0$ $\quad b(8) = 2.553 \cdot 10^{-3}$

Check using fourier series definition:

$$Y(h) := \frac{18\cdot\sqrt{3}\cdot A}{\pi^3}\cdot\frac{1}{h^3}$$

$Y_0 := 0$

$Y(1) = 1.307$

$Y(2) = 0.163$

$Y(3) = 0.048$

$Y(4) = 0.02$

$Y(5) = 0.01$

$Y(6) = 6.052\cdot 10^{-3}$

$Y(7) = 3.811\cdot 10^{-3}$

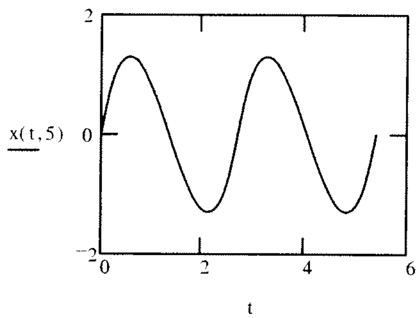
$Y(8) = 2.553\cdot 10^{-3}$

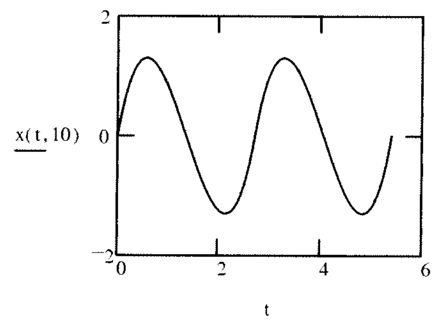
Conclusion: The results match the b coefficients.

$t := 0, 0.01 .. 5.4$

Graphically show how the function improves as the number of elements in the series is increased:

$$x(t,c) := \frac{18\cdot\sqrt{3}\cdot A}{\pi^3}\cdot\left[\sum_{n=1}^{c}\left(\frac{1}{n^3}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\right]$$





Ken Kaiser

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Problem 3: Determining Fourier Coefficients Using Integral Definitions for Third Order Approximate Sine Wave #1

Part I: Defining Constants

$A := 1.3$ $T := 2.7$ $\theta := \frac{\pi}{5}$ $\alpha := 9.4$ $f := 4.3$ $\tau := .68$ $a := .32$ $b := 2.1$ $j := \sqrt{-1}$ $V_{dc} := .47$

Part II: Defining Coefficient Equations

$$a1(n) := \frac{2}{T} \cdot \int_{\left(-\frac{T}{2}\right)}^{\left(\frac{T}{2}\right)} \left(\frac{12 \cdot A \cdot \sqrt{3}}{T^3} \cdot t\right) \cdot \left(\frac{T}{2} - t\right) \cdot \left(\frac{T}{2} + t\right) \cdot \cos\left(2 \cdot n \cdot \frac{\pi}{T} \cdot t\right) dt$$

$$b1(n) := \frac{2}{T} \cdot \left[\int_{\left(-\frac{T}{2}\right)}^{\left(\frac{T}{2}\right)} \left[\frac{12 \cdot \sqrt{3} \cdot (A)}{T^3} \cdot t\right] \cdot \left(\frac{T}{2} - t\right) \cdot \left(\frac{T}{2} + t\right) \cdot \sin\left(2 \cdot \pi \cdot \frac{n}{T} \cdot t\right) dt\right]$$

$$c1(n) := \left(a1(n)^2 + b1(n)^2\right)^{.5}$$

Part III: Defining Exact Equations

$$ap(n) := 18 \cdot \sqrt{3} \cdot \frac{A}{\pi^3} \cdot 0$$

ap(n) is the exact coefficient given in the exact Fourier Series equation given.

$$bp(n) := 18 \cdot \sqrt{3} \cdot \frac{A}{\pi^3} \cdot \frac{(-1)^{n-1}}{n^3}$$

bp(n) is the exact coefficient given in the exact Fourier Series equation given.
bp(n) cannot be used for n=0. bp(n) for n=0 is zero by inspection. There is no dc term.

$$cp(n) := \left(ap(n)^2 + bp(n)^2\right)^{.5}$$

cp(n) is the amplitude of the function.

Problem 3 (continued)

Part IV: Evaluating Integral Coefficient Equations to Determine Series Coefficients

$a1(0) = 0$	$ap(0) = 0$	$b1(0) = 0$	
$a1(1) = 0$	$ap(1) = 0$	$b1(1) = 1.30715$	$bp(1) = 1.30715$
$a1(2) = 0$	$ap(2) = 0$	$b1(2) = -0.16339$	$bp(2) = -0.16339$
$a1(3) = 0$	$ap(3) = 0$	$b1(3) = 0.04841$	$bp(3) = 0.04841$
$a1(4) = 0$	$ap(4) = 0$	$b1(4) = -0.02042$	$bp(4) = -0.02042$
$a1(5) = 0$	$ap(5) = 0$	$b1(5) = 0.01046$	$bp(5) = 0.01046$
$a1(6) = 0$	$ap(6) = 0$	$b1(6) = -6.05164 \cdot 10^{-3}$	$bp(6) = -6.05164 \cdot 10^{-3}$
$a1(7) = 0$	$ap(7) = 0$	$b1(7) = 3.81095 \cdot 10^{-3}$	$bp(7) = 3.81095 \cdot 10^{-3}$
$a1(8) = 0$	$ap(8) = 0$	$b1(8) = -2.55304 \cdot 10^{-3}$	$bp(8) = -2.55304 \cdot 10^{-3}$

Note: bp(0) is zero by inspection. The exact Fourier Series does not have a dc term which means that the dc value is 0.

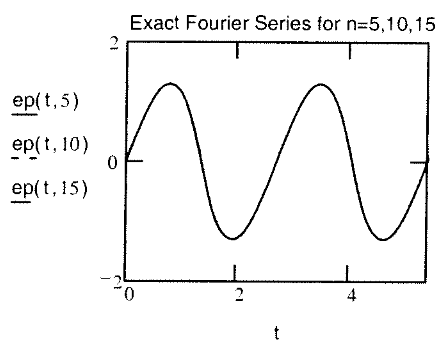
Problem 3 (continued)

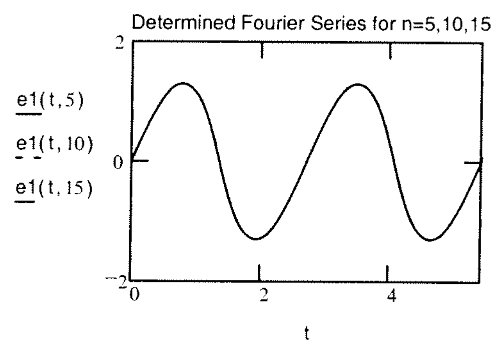
Part V: Comparing the Exact Fourier Series to Series Determined by Integral Coefficients

$$t := 0, \frac{T}{100} .. 100$$

$$ep(t, m) := 18 \cdot \sqrt{3} \cdot \frac{A}{\pi^3} \cdot \sum_{n=1}^{m} \frac{(-1)^{n-1}}{n^3} \cdot \sin\left(2 \cdot \pi \cdot n \cdot \frac{t}{T}\right)$$

$$e1(t, m) := \frac{a1(0)}{2} + \left[\sum_{n=1}^{m} \left(a1(n) \cdot \cos\left(2 \cdot \pi \cdot \frac{n}{T} \cdot t\right)\right)\right] + \left[\sum_{n=1}^{m} \left(b1(n) \cdot \sin\left(2 \cdot \pi \cdot \frac{n}{T} \cdot t\right)\right)\right]$$





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3rd Order Approximate Sine Wave #2

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47$

$$a(N) := \frac{2}{T} \cdot \int_0^T \left(\frac{12 \cdot \sqrt{3} \cdot A}{T^3}\right) \cdot t \cdot \left(t - \frac{T}{2}\right) \cdot (t - T) \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt$$

$$b(N) := \frac{2}{T} \cdot \int_0^T \left(\frac{12 \cdot \sqrt{3} \cdot A}{T^3}\right) \cdot t \cdot \left(t - \frac{T}{2}\right) \cdot (t - T) \cdot \sin\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt$$

$a(0) = 0$	$b(0) = 0$
$a(1) = 0$	$b(1) = 1.30715$
$a(2) = 0$	$b(2) = 0.16339$
$a(3) = 0$	$b(3) = 0.04841$
$a(4) = 0$	$b(4) = 0.02042$
$a(5) = 0$	$b(5) = 0.01046$
$a(6) = 0$	$b(6) = 0.00605$
$a(7) = 0$	$b(7) = 0.00381$
$a(8) = 0$	$b(8) = 0.00255$

$$E(m, M) := \frac{18 \cdot \sqrt{3} \cdot A}{\pi^3} \cdot \sum_{n=m}^{M} \frac{1}{n^3}$$

$E(1,1) = 1.30715$
$E(2,2) = 0.16339$
$E(3,3) = 0.04841$
$E(4,4) = 0.02042$
$E(5,5) = 0.01046$
$E(6,6) = 0.00605$
$E(7,7) = 0.00381$
$E(8,8) = 0.00255$

Exponential Wave #4

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

$$F(N) := \frac{1}{T} \cdot \int_0^T \frac{A}{T} \cdot t \cdot e^{\frac{-\alpha}{T} \cdot t} \cdot e^{j \cdot \frac{2 \cdot \pi \cdot N}{T} \cdot t} \, dt + A \cdot \left(\frac{1 - e^{-\alpha} - \alpha \cdot e^{-\alpha}}{\alpha^2} \right)$$

$F(0) = 0.0294$

$F(1) = 0.01858 + 0.00939i$	$F(-1) = 0.01858 - 0.00939i$
$F(2) = 0.01321 + 0.00506i$	$F(-2) = 0.01321 - 0.00506i$
$F(3) = 0.01293 + 0.00234i$	$F(-3) = 0.01293 - 0.00234i$
$F(4) = 0.01334 + 0.00118i$	$F(-4) = 0.01334 - 0.00118i$
$F(5) = 0.01369 + 0.00066i$	$F(-5) = 0.01369 - 0.00066i$
$F(6) = 0.01394 + 0.0004i$	$F(-6) = 0.01394 - 0.0004i$
$F(7) = 0.01411 + 0.00026i$	$F(-7) = 0.01411 - 0.00026i$
$F(8) = 0.01424 + 0.00018i$	$F(-8) = 0.01424 - 0.00018i$

$$E(m, M) := A \cdot \sum_{n=m}^{M} \frac{1 - j \cdot 2 \cdot \pi \cdot n \cdot e^{-\alpha} - e^{-\alpha} - \alpha \cdot e^{-\alpha}}{(\alpha + j \cdot 2 \cdot \pi \cdot n)^2} + A \cdot \left(\frac{1 - e^{-\alpha} - \alpha \cdot e^{-\alpha}}{\alpha^2} \right)$$

$E(0,0) = 0.0294$

$E(1,1) = 0.01858 - 0.00939i$	$E(-1,-1) = 0.01858 + 0.00939i$
$E(2,2) = 0.01321 - 0.00506i$	$E(-2,-2) = 0.01321 + 0.00506i$
$E(3,3) = 0.01293 - 0.00234i$	$E(-3,-3) = 0.01293 + 0.00234i$
$E(4,4) = 0.01334 - 0.00118i$	$E(-4,-4) = 0.01334 + 0.00118i$
$E(5,5) = 0.01369 - 0.00066i$	$E(-5,-5) = 0.01369 + 0.00066i$
$E(6,6) = 0.01394 - 0.0004i$	$E(-6,-6) = 0.01394 + 0.0004i$
$E(7,7) = 0.01411 - 0.00026i$	$E(-7,-7) = 0.01411 + 0.00026i$
$E(8,8) = 0.01424 - 0.00018i$	$E(-8,-8) = 0.01424 + 0.00018i$

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2nd Order Approximate Sine Wave

$A := 1.3 \quad T := 2.7 \quad \tau := .68 \quad \alpha := 9.4 \quad f_r := 4.3\,Hz \quad a := .32 \quad b := 2.1 \quad V_{dc} := .47 \quad \theta := \frac{\pi}{5}$

$$a(N) := \frac{2}{T}\cdot\int_0^{\frac{T}{2}} \frac{16\cdot A}{T^2}\cdot t\cdot\left(\frac{T}{2} - t\right)\cdot\cos\left[\frac{(2\cdot\pi\cdot N)}{T}\cdot t\right] dt + \frac{2}{T}\cdot\int_{\frac{-T}{2}}^{0} \frac{16\cdot A}{T^2}\cdot t\cdot\left(\frac{T}{2} + t\right)\cdot\cos\left[\frac{(2\cdot\pi\cdot N)}{T}\cdot t\right] dt$$

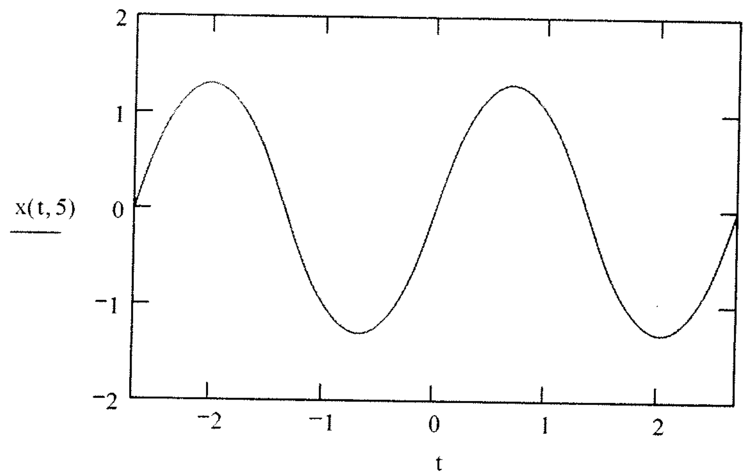
$$b(M) := \frac{2}{T}\cdot\int_0^{\frac{T}{2}} \frac{16\cdot A}{T^2}\cdot t\cdot\left(\frac{T}{2} - t\right)\cdot\sin\left[\frac{(2\cdot\pi\cdot M)}{T}\cdot t\right] dt + \frac{2}{T}\cdot\int_{\frac{-T}{2}}^{0} \frac{16\cdot A}{T^2}\cdot t\cdot\left(\frac{T}{2} + t\right)\cdot\sin\left[\frac{(2\cdot\pi\cdot M)}{T}\cdot t\right] dt$$

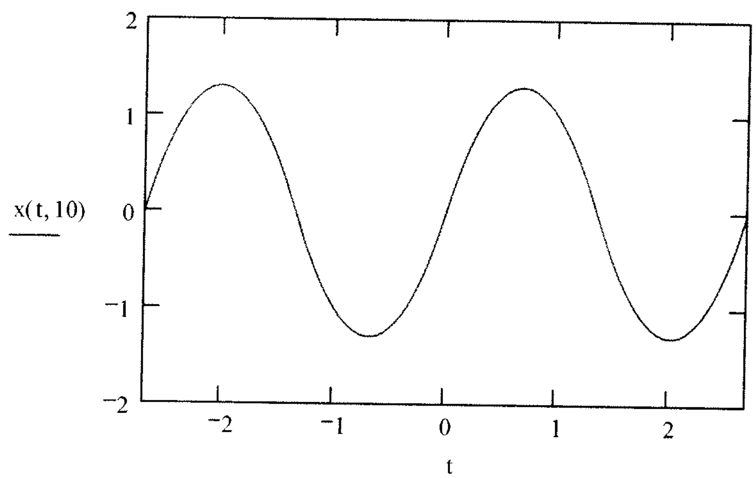
$$bp(n) := \frac{32\cdot A}{\pi^3}\cdot\frac{1}{(2\cdot n - 1)^3}$$

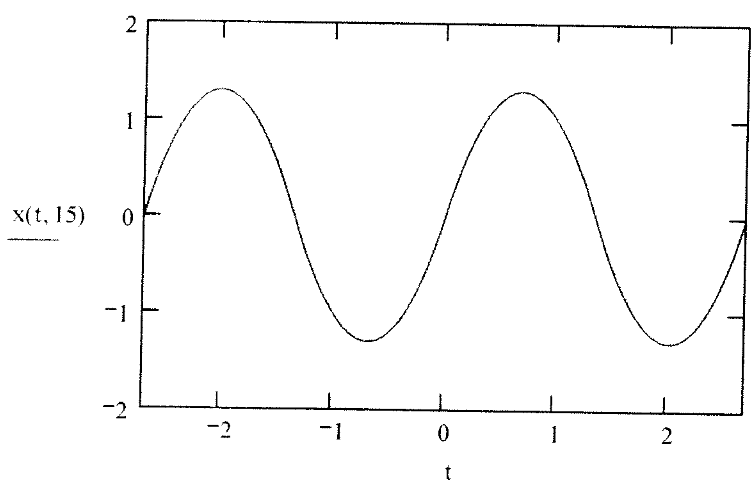
$a(0) = 0$	$b(0) = 0$	$bp_0 := 0$
$a(1) = 0$	$b(1) = 1.34166$	$bp(1) = 1.34166$
$a(2) = 0$	$b(2) = 0$	
$a(3) = 0$	$b(3) = 0.04969$	$bp(2) = 0.04969$
$a(4) = 0$	$b(4) = 0$	
$a(5) = 0$	$b(5) = 0.01073$	$bp(3) = 0.01073$
$a(6) = 0$	$b(6) = 0$	
$a(7) = 0$	$b(7) = 3.91156 \times 10^{-3}$	$bp(4) = 3.91156 \times 10^{-3}$
$a(8) = 0$	$b(8) = 0$	

Ken Kaiser

$$x(t,m) := \sum_{N=1}^{m} b(N)\cdot\sin\left(\frac{2\cdot\pi\cdot N}{T}\cdot t\right) \qquad t := -T, -T + .05 .. T$$







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2nd Order Approximate Full-Rectified Sinusoidal Wave

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47$

Equations:

$$a(N) := \frac{2}{T} \cdot \int_0^T \frac{(4A)}{T^2} \cdot t \cdot (T-t) \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt \qquad b(N) := \frac{2}{T} \cdot \int_0^T \frac{(4A)}{T^2} \cdot t \cdot (T-t) \cdot \sin\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right) dt$$

$$ap(n) := \left[\left[\frac{(-4 \cdot A)}{\pi^2}\right] \cdot \frac{1}{n^2}\right] \qquad Favg := \frac{(2A)}{3} \qquad Favg = 0.86667 \qquad \frac{a(0)}{2} = 0.86667$$

Solutions: a(N) and b(N)

$a(0) = 1.73333 \quad b(0) = 0$
$a(1) = -0.52687 \quad b(1) = 0$
$a(2) = -0.13172 \quad b(2) = 0$
$a(3) = -0.05854 \quad b(3) = 0$
$a(4) = -0.03293 \quad b(4) = 0$
$a(5) = -0.02107 \quad b(5) = 0$
$a(6) = -0.01464 \quad b(6) = 0$
$a(7) = -0.01075 \quad b(7) = 0$
$a(8) = -0.00823 \quad b(8) = 0$

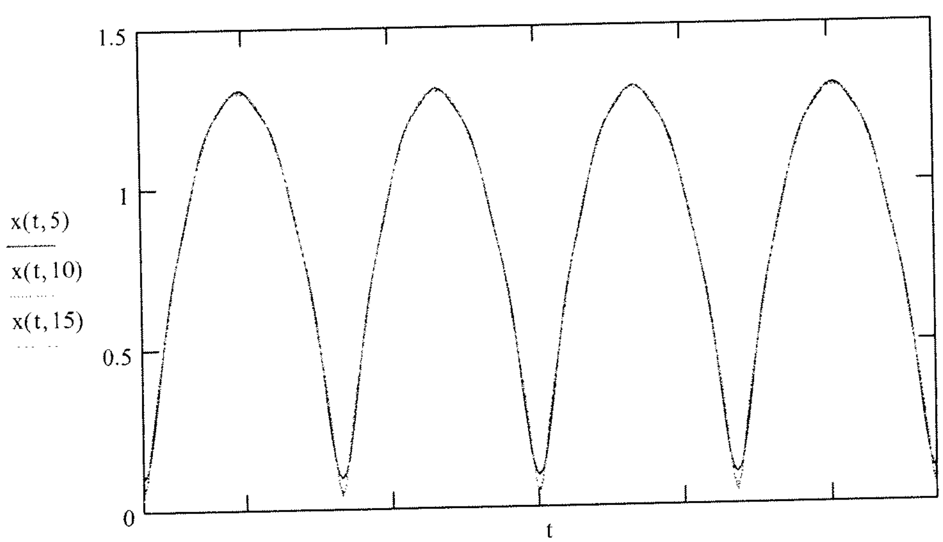
Solution: ap(n)

$ap(1) = -0.52687$
$ap(2) = -0.13172$
$ap(3) = -0.05854$
$ap(4) = -0.03293$
$ap(5) = -0.02107$
$ap(6) = -0.01464$
$ap(7) = -0.01075$
$ap(8) = -0.00823$

Graph of 2nd Order Approximate Full-Rectified Sinusoidal Wave

$$x(t,m) := \frac{2A}{3} + \left(\sum_{N=1}^{m} a(N) \cdot \cos\left(\frac{2 \cdot \pi \cdot N}{T} \cdot t\right)\right)$$

$t := -2T, -2T + .05 .. 4T$



Problem # 52 Half Rectified Sine Wave #1

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \left(\frac{2}{T}\right)\cdot\left[\int_0^{\frac{T}{2}} A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \left(\frac{2}{T}\right)\cdot\left[\int_{\frac{T}{2}}^{T} 0\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$$b(n) := \left(\frac{2}{T}\right)\cdot\left[\int_0^{\frac{T}{2}} A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \left(\frac{2}{T}\right)\cdot\left[\int_{\frac{T}{2}}^{T} 0\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

Results:

$a(0) = 0.828$

$a(1) = 0$

$a(2) = -0.276$

$a(3) = 0$

$a(4) = -0.055$

$a(5) = 0$

$a(6) = -0.024$

$a(7) = 0$

$a(8) = -0.013$

$b(0) = 0$

$b(1) = 0.65$

$b(2) = 0$

$b(3) = 1.172\cdot 10^{-9}$

$b(4) = 0$

$b(5) = -1.172\cdot 10^{-9}$

$b(6) = 0$

$b(7) = 1.25\cdot 10^{-9}$

$b(8) = 0$

Check using fourier series definition:

$$Y(h) := \frac{-2 \cdot A}{\pi} \cdot \frac{1}{4 \cdot h^2 - 1}$$

$$b_p := \frac{A}{2}$$

$Y(0) = 0.828$

$Y(1) = -0.276$

$b_p = 0.65$

$Y(2) = -0.055$

$Y(3) = -0.024$

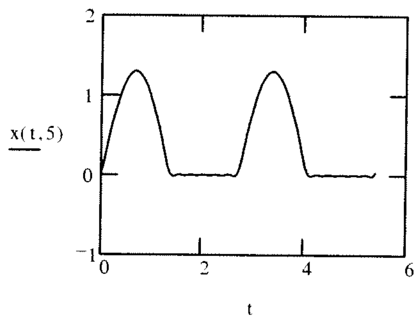
$Y(4) = -0.013$

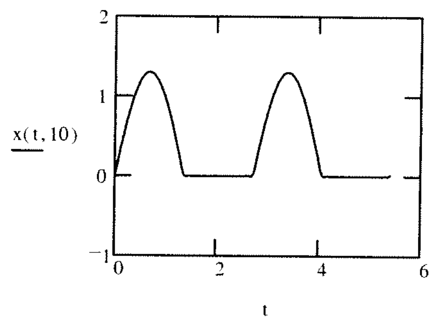
Conclusion: The results are all the a coefficients which are even harmonics. There is one b coeffecient which represents the fundamental frequency.

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \frac{A}{\pi} + \frac{A}{2}\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right) - \frac{2\cdot A}{\pi}\cdot\left[\sum_{n=1}^{c}\left(\frac{1}{4\cdot n^2 - 1}\right)\cdot\cos\left(\frac{4\cdot\pi\cdot n}{T}\cdot t\right)\right]$$





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Half-Rectified Sine Wave #1

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4$

$f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

$$a_1(n) := \frac{2}{T}\cdot\left[\int_0^{\frac{T}{2}} A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \frac{2}{T}\cdot\left[\int_{\frac{T}{2}}^{T} 0\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$$b_1(n) := \frac{2}{T}\cdot\left[\int_0^{\frac{T}{2}} A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \frac{2}{T}\cdot\left[\int_{\frac{T}{2}}^{T} 0\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$$c_1(n) := \sqrt{a_1(n)^2 + b_1(n)^2}$$

$a_1(0) = 0.828$	$b_1(0) = 0$	$c_1(0) = 0.828$
$a_1(1) = 0$	$b_1(1) = 0.65$	$c_1(1) = 0.65$
$a_1(2) = -0.276$	$b_1(2) = 0$	$c_1(2) = 0.276$
$a_1(3) = 0$	$b_1(3) = 1.172\cdot 10^{-9}$	$c_1(3) = 1.172\cdot 10^{-9}$
$a_1(4) = -0.055$	$b_1(4) = 0$	$c_1(4) = 0.055$
$a_1(5) = 0$	$b_1(5) = -1.172\cdot 10^{-9}$	$c_1(5) = 1.172\cdot 10^{-9}$
$a_1(6) = -0.024$	$b_1(6) = 0$	$c_1(6) = 0.024$
$a_1(7) = 0$	$b_1(7) = 1.25\cdot 10^{-9}$	$c_1(7) = 1.25\cdot 10^{-9}$
$a_1(8) = -0.013$	$b_1(8) = 0$	$c_1(8) = 0.013$

check:

$$a_2(n) := \frac{-2\cdot A}{\pi}\cdot\frac{1}{\left(4\cdot n^2 - 1\right)}$$

$a_2(0) = 0.828$

$a_2(1) = -0.276$

even harmonic function
$a_2(1)=a_1(2)$, $a_2(2)=a_1(4)$, etc.

$a_2(2) = -0.055$

$a_2(3) = -0.024$

$a_2(4) = -0.013$

$\frac{A}{\pi} = 0.414$

$A/\pi=a_0/2$

$\frac{a_1(0)}{2} = 0.414$

$\frac{A}{2} = 0.65$

$A/2=b_1$

$b_1(1) = 0.65$

Actual function:

$$E(t,m) := \frac{A}{\pi} + \frac{A}{2}\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right) - \frac{2\cdot A}{\pi}\cdot\sum_{n_1=1}^{m}\frac{1}{\left(4\cdot n_1{}^2 - 1\right)}\cdot\cos\left(\frac{4\cdot\pi\cdot n_1}{T}\cdot t\right)$$

Half-Rectified Sine Wave #2

$A := 1.3 \quad T := 2.7 \quad \tau := 0.68 \quad \alpha := 9.4 \quad fr := 4.3 \cdot Hz \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad \theta := \frac{\pi}{5}$

$$a(n) := \frac{2}{T} \cdot \int_0^{\frac{T}{2}} 0 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt + \frac{2}{T} \cdot \int_{\frac{T}{2}}^{T} \left(-A \cdot \sin\left(\frac{2 \cdot \pi}{T} \cdot t\right)\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

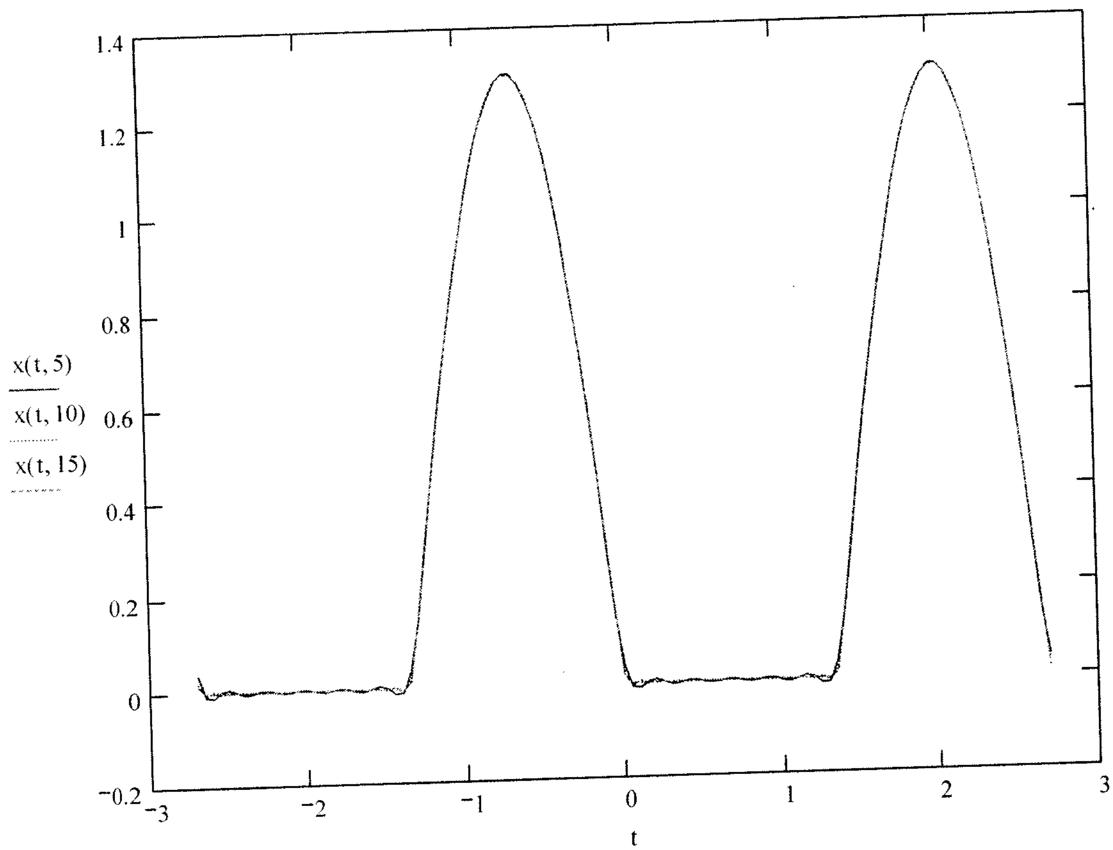
$$b(n) := \frac{2}{T} \cdot \int_0^{\frac{T}{2}} 0 \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt + \frac{2}{T} \cdot \int_{\frac{T}{2}}^{T} \left(-A \cdot \sin\left(\frac{2 \cdot \pi}{T} \cdot t\right)\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$X(n) := \frac{2A}{\pi} \cdot \frac{1}{1 - 4n^2}$$

$a(0) = 0.82761$	$b(0) = 0$	$X(0) = 0.82761$
$a(1) = 0$	$b(1) = -0.65$	$\frac{-A}{2} = -0.65$
$a(2) = -0.27587$	$b(2) = 0$	$X(1) = -0.27587$
$a(3) = 0$	$b(3) = 0$	$X(2) = -0.05517$
$a(4) = -0.05517$	$b(4) = 0$	$X(3) = -0.02365$
$a(5) = 0$	$b(5) = 0$	$X(4) = -0.01314$
$a(6) = -0.02365$	$b(6) = 0$	
$a(7) = 0$	$b(7) = 0$	
$a(8) = -0.01314$	$b(8) = 0$	

$$x(t,m) := \frac{A}{\pi} - \frac{A}{2}\cdot\sin\left(\frac{2\pi}{T}\cdot t\right) + \frac{2A}{\pi}\cdot\sum_{n=1}^{m}\frac{1}{1-4\cdot n^2}\cdot\cos\left(\frac{4\cdot\pi\cdot n}{T}\cdot t\right)$$

$$t := -T, -T + .05 .. T$$



Problem # 53 Half Rectified Sine Wave #2

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \left(\frac{2}{T}\right)\cdot\left[\int_0^{\frac{T}{2}} (0)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \left(\frac{2}{T}\right)\cdot\left[\int_{\frac{T}{2}}^{T} -A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

$$b(n) := \left(\frac{2}{T}\right)\cdot\left[\int_0^{\frac{T}{2}} (0)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right] + \left(\frac{2}{T}\right)\cdot\left[\int_{\frac{T}{2}}^{T} -A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\right]$$

Results:

$a(0) = 0.828$	$b(0) = 0$	$\frac{A}{\pi} = 0.414$
$a(1) = 0$	$b(1) = -0.65$	
$a(2) = -0.276$	$b(2) = 0$	
$a(3) = 0$	$b(3) = -1.172\cdot 10^{-9}$	
$a(4) = -0.055$	$b(4) = 0$	
$a(5) = 0$	$b(5) = 1.172\cdot 10^{-9}$	
$a(6) = -0.024$	$b(6) = 0$	
$a(7) = 0$	$b(7) = -1.25\cdot 10^{-9}$	
$a(8) = -0.013$	$b(8) = 0$	

Check using fourier series definition:

$$Y(h) := \frac{2 \cdot A}{\pi} \cdot \frac{1}{1 - 4 \cdot h^2}$$

$Y(0) = 0.828$

$Y(1) = -0.276$

$Y(2) = -0.055$

$Y(3) = -0.024$

$Y(4) = -0.013$

$$b_p := -\frac{A}{2}$$

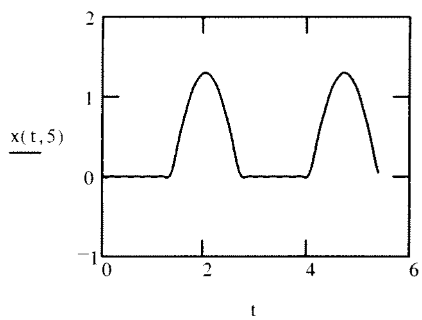
$b_p = -0.65$

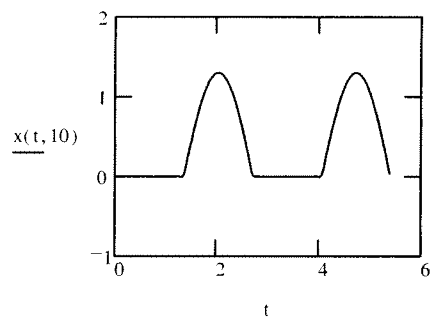
Conclusion: The solution matches the even harmonics of a with the fundamental being the only be coefficient b(1).

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \left(\frac{A}{\pi} - \frac{A}{2}\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\right) + \frac{2\cdot A}{\pi}\cdot\left[\sum_{n=1}^{c}\left(\frac{1}{1-4\cdot n^2}\right)\cdot\cos\left(\frac{4\cdot\pi\cdot n}{T}\cdot t\right)\right]$$





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Problem # 3

Half-Rectified Sine Wave with Added DC Offset

Given Constants:

$T := 2.7$

$\theta := \frac{\pi}{5}$

$\tau := .68$

$V := .47$

$A := 1.3$

$\alpha := 9.4$

$fr := 4.3$

Fourier Series-Integral Definitions

$$a(n) := \int_{0}^{\frac{T}{2}} \left(V + A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T} + \int_{\frac{T}{2}}^{T} V\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T}$$

$$b(n) := \int_{0}^{\frac{T}{2}} \left(V + A\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T} + \int_{\frac{T}{2}}^{T} V\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right) dt\cdot\frac{2}{T}$$

***Note: The height difference between the integrals is due to the limits that are used, meaning that if a fraction is present the integral is automatically lengthened.**

Results:

$a(0) = 1.76761$
$a(1) = 0$
$a(2) = -0.27587$
$a(3) = 0$
$a(4) = -0.05517$
$a(5) = 0$
$a(6) = -0.02365$
$a(7) = 0$
$a(8) = -0.01314$

$b(0) = 0$
$b(1) = 0.65$
$b(2) = 0$
$b(3) = 1.17188 \cdot 10^{-9}$
$b(4) = 0$
$b(5) = 6.44915 \cdot 10^{-9}$
$b(6) = 0$
$b(7) = 1.24972 \cdot 10^{-9}$
$b(8) = 0$

Fourier Series-Summation Definition

$$Y(h) := \frac{-2 \cdot A}{\pi} \cdot \frac{1}{(4 \cdot h^2 - 1)}$$

$\left(V + \frac{A}{\pi}\right) \cdot 2 = 1.76761$ ⟵ Y(0)

$\frac{A}{2} = 0.65$ ⟵ Y(0)

Results:

$Y(1) = -0.27587$
$Y(2) = -0.05517$
$Y(3) = -0.02365$
$Y(4) = -0.01314$

Conclusion

Integral Definition Results

Harmonic	n	a_o	b_o
DC Term	0	1.76761	0
Fundamental	1	0	0.65
Second	2	-0.27587	0
Third	3	0	1.17E-09
Fourth	4	-0.05517	0
Fifth	5	0	6.44E-09
Sixth	6	-0.02365	0
Seventh	7	0	1.24E-09
Eighth	8	-0.01314	0

Summation Definition Results

h	a_o	b_o
0	1.76761	0.65
1	-0.27587	-
2	-0.05517	-
3	-0.02365	-
4	-0.01314	-

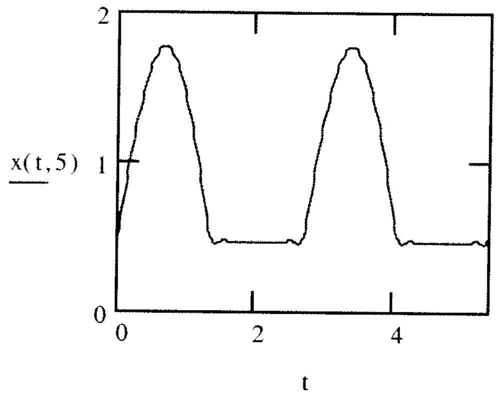
By viewing the above tables, the results from both definitions match. The results from the integral definition show that this function is an odd harmonic.

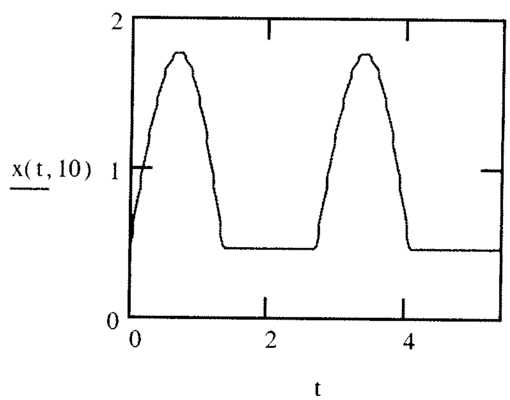
Plot of Function vs. Time

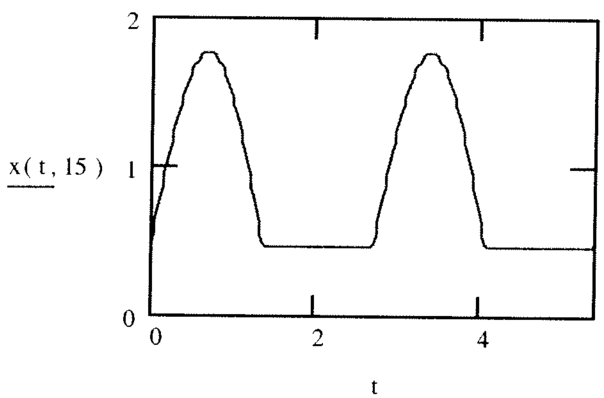
$t := 0, 0.027 .. 5.4$

$$x(t,h) := \left(V + \frac{A}{\pi} + \frac{A}{2}\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right)\right) - \frac{2\cdot A}{\pi}\cdot\sum_{n=1}^{h}\cos\left(\frac{4\cdot\pi\cdot n}{T}\cdot t\right)\cdot\frac{1}{\left(4\cdot n^2 - 1\right)}$$

Ken Kaiser







Problem # 55 Quarter Rectified Sine Wave #2

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{1}{T} \cdot \int_0^{\frac{T}{4}} A \cdot \sin\left(\frac{2 \cdot \pi}{T} \cdot t\right) \cdot e^{-j \cdot \left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)} dt + \frac{2}{T} \cdot \int_{\frac{T}{4}}^{T} 0 \cdot e^{-j \cdot \left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)} dt$$

Results:

$a(0) = 0.207$

$a(1) = 0.103 - 0.163i$

$a(2) = -0.069 - 0.138i$

$a(3) = -0.103 - 2.197 \cdot 10^{-10} i$

$a(4) = -0.014 + 0.055i$

$a(5) = 0.034$

$a(6) = -5.911 \cdot 10^{-3} - 0.035i$

$a(7) = -0.034 - 2.929 \cdot 10^{-10} i$

$a(8) = -3.284 \cdot 10^{-3} + 0.026i$

Check using fourier series definition:

$$Y(h) := \frac{A}{2\cdot\pi}\cdot\frac{-1 + j\cdot h\cdot e^{-j\cdot\frac{\pi\cdot h}{2}}}{h^2 - 1}$$

$Y(0) = 0.207$

$Y_1 := \frac{A}{8}\cdot(2\cdot\pi - j)$ $\qquad Y_1 = 1.021 - 0.163i$

$Y(2) = -0.069 - 0.138i$

$Y(3) = -0.103$

$Y(4) = -0.014 + 0.055i$

$Y(5) = 0.034$

$Y(6) = -5.911\cdot10^{-3} - 0.035i$

$Y(7) = -0.034$

$Y(8) = -3.284\cdot10^{-3} + 0.026i$

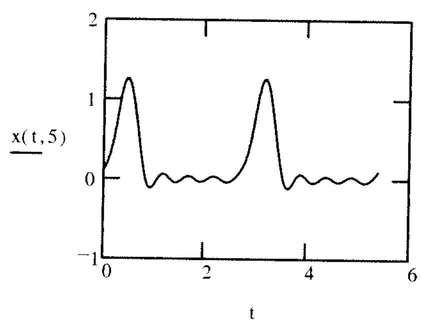
Conclusion: These values match the a coeffecients Fn Coeffecients

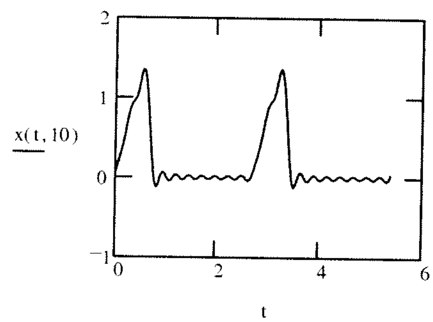
Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := \frac{A}{2\cdot\pi} + \frac{A}{8}\cdot\left(\frac{2}{\pi} + j\right)\cdot e^{-j\cdot\left(\frac{2\cdot\pi}{T}\cdot t\right)} + \frac{A}{8}\cdot\left(\frac{2}{\pi} - j\right)\cdot e^{j\cdot\left(\frac{2\cdot\pi}{T}\cdot t\right)} + \frac{A}{2\cdot\pi}\cdot\sum_{n=-c}^{-2} \frac{-1 + j\cdot n\cdot e^{-j\cdot\frac{n\cdot\pi}{2}}}{n^2 - 1}\cdot e^{j\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)} \ldots$$

$$+ \frac{A}{2\cdot\pi}\cdot\sum_{n=2}^{c} \frac{-1 + j\cdot n\cdot e^{-j\cdot\frac{n\cdot\pi}{2}}}{n^2 - 1}\cdot e^{j\cdot\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)}$$





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[redacted]

Fourier Series Problem 3
EE-310, Kaiser, Winter 04

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := .68 \quad \alpha := 9.4 \quad fr := 4.3 \quad a := .32 \quad b := 2.1 \quad j := \sqrt{-1} \quad Vdc := .74$

Half Rectified Cosine Wave # 1

1) Determination of Series Coefficients Using Integral Definitions

$$a(n) := \frac{2}{T}\cdot\int_0^{\frac{T}{4}} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt + \frac{2}{T}\cdot\int_{\frac{T}{4}}^{\frac{3\cdot T}{4}} 0\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt \ldots$$
$$+\frac{2}{T}\cdot\int_{\frac{3\cdot T}{4}}^{T} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$b(n) := \frac{2}{T}\cdot\int_0^{\frac{T}{4}} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt + \frac{2}{T}\cdot\int_{\frac{T}{4}}^{\frac{3\cdot T}{4}} 0\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt \ldots$$
$$+\frac{2}{T}\cdot\int_{\frac{3\cdot T}{4}}^{T} A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$a(0) = 0.82761$ $\quad a(5) = -8.78912\cdot10^{-10}$ $\quad b(0) = 0$ $\quad b(5) = 0$

$a(1) = 0.65$ $\quad a(6) = 0.02365$ $\quad b(1) = 0$ $\quad b(6) = 0$

$a(2) = 0.27587$ $\quad a(7) = -1.17167\cdot10^{-9}$ $\quad b(2) = 0$ $\quad b(7) = 0$

$a(3) = -8.78912\cdot10^{-10}$ $\quad a(8) = -0.01314$ $\quad b(3) = 0$ $\quad b(8) = 0$

$a(4) = -0.05517$ $\quad b(4) = 0$

2) Verification of Coefficients Using the Summation Forms

$\frac{A}{\pi} \cdot 2 = 0.82761$ = a(0)

$\frac{A}{2} = 0.65$ = a(1)

$$ap(n) := \frac{2 \cdot A}{\pi} \cdot \frac{(-1)^{(n-1)}}{4 \cdot n^2 - 1}$$

This equation will give me the even harmonics since cosine is an even function.

$ap(1) = 0.27587$

$ap(2) = -0.05517$

$ap(3) = 0.02365$

$ap(4) = -0.01314$

Half-Rectified Cosine Wave #2 page 57

$T := 1.4 \qquad A := 3.4$

$$a(n) := \frac{2}{T}\cdot\int_{\frac{T}{4}}^{3\cdot\frac{T}{4}} -A\cdot\cos\left(2\cdot\frac{\pi}{T}\cdot t\right)\cdot\cos\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\,dt$$

$$b(n) := \frac{2}{T}\cdot\int_{\frac{T}{4}}^{3\cdot\frac{T}{4}} -A\cdot\cos\left(2\cdot\frac{\pi}{T}\cdot t\right)\cdot\sin\left(2\cdot\pi\cdot\frac{n}{T}\cdot t\right)\,dt$$

$$ap(N) := \frac{2\cdot A}{\pi}\cdot\frac{(-1)^{N-1}}{4\cdot N^2 - 1}$$

even harmonics

$a(0) = 2.165$ $\qquad \frac{A}{\pi}\cdot 2 = 2.165$

$a(1) = -1.7$ $\qquad \frac{-A}{2} = -1.7$ $\qquad b(1) = 0$

$a(2) = 0.722$ $\qquad b(2) = 0$ $\qquad ap(1) = 0.722$

$a(3) = 0$ $\qquad b(3) = 0$ $\qquad ap(2) = -0.144$

$a(4) = -0.144$ $\qquad b(4) = 0$ $\qquad ap(3) = 0.062$

$a(5) = 0$ $\qquad b(5) = 0$ $\qquad ap(4) = -0.034$

$a(6) = 0.062$ $\qquad b(6) = 0$

$a(7) = 0$ $\qquad b(7) = 0$

$a(8) = -0.034$ $\qquad b(8) = 0$

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3. Half-Rectified Cosine Wave #2

$A := 1.3$ $T := 2.7$ $\theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := 9.4$ $f_r := 4.3$ $a := 0.32$ $b := 2.1$ $i := \sqrt{(-1)}$

$a_0 := \frac{2 \cdot A}{\pi}$ $a_1 := \frac{-A}{2}$

$$a(n) := \frac{2}{T} \cdot \left[\int_0^{\frac{T}{4}} 0 \cdot \cos\left(\frac{2 \cdot \pi \, n \cdot t}{T}\right) dt + \int_{\frac{T}{4}}^{\frac{3 \cdot T}{4}} -A \cdot \cos\left(\frac{2 \cdot \pi \, t}{T}\right) \cdot \cos\left(\frac{2 \cdot \pi \, n \cdot t}{T}\right) dt \ldots + \int_{\frac{3 \cdot T}{4}}^{T} 0 \cdot \cos\left(\frac{2 \cdot \pi \, n \cdot t}{T}\right) dt \right]$$

$$b(n) := \frac{2}{T} \cdot \left[\int_0^{\frac{T}{4}} 0 \cdot \sin\left(\frac{2 \cdot \pi \, n \cdot t}{T}\right) dt + \int_{\frac{T}{4}}^{\frac{3 \cdot T}{4}} -A \cdot \cos\left(\frac{2 \cdot \pi \, t}{T}\right) \cdot \sin\left(\frac{2 \cdot \pi \, n \cdot t}{T}\right) dt \ldots + \int_{\frac{3 \cdot T}{4}}^{T} 0 \cdot \sin\left(\frac{2 \cdot \pi \, n \cdot t}{T}\right) dt \right]$$

$a(0) = 0.82761$ $b(0) = 0$

$a(2) = 0.27587$ $b(1) = 0$

$a(4) = -0.05517$ $b(2) = 0$

$a(6) = 0.02365$ $b(3) = 0$

$a(8) = -0.01314$ $b(4) = 0$

$a(10) = 8.35677 \cdot 10^{-3}$ $b(5) = 0$

$a(12) = -5.78745 \cdot 10^{-3}$ $b(6) = 0$

$a(14) = 4.24413 \cdot 10^{-3}$ $b(7) = 0$

$a(16) = -3.24551 \cdot 10^{-3}$ $b(8) = 0$

*for the check ap(1) corresponds to a(2) because the frequency is 4*π instead of the standard 2*π.

Check

$$a_p(n) := \frac{2 \cdot A}{\pi} \cdot \frac{(-1)^{n-1}}{4 \cdot n^2 - 1} \qquad b_p(n) := 0$$

$a_p(0) = 0.82761$ $b_p(0) = 0$

$a_p(1) = 0.27587$ $b_p(1) = 0$

$a_p(2) = -0.05517$ $b_p(2) = 0$

$a_p(3) = 0.02365$ $b_p(3) = 0$

$a_p(4) = -0.01314$ $b_p(4) = 0$

$a_p(5) = 8.35965 \cdot 10^{-3}$ $b_p(5) = 0$

$a_p(6) = -5.78745 \cdot 10^{-3}$ $b_p(6) = 0$

$a_p(7) = 4.24413 \cdot 10^{-3}$ $b_p(7) = 0$

$a_p(8) = -3.24551 \cdot 10^{-3}$ $b_p(8) = 0$

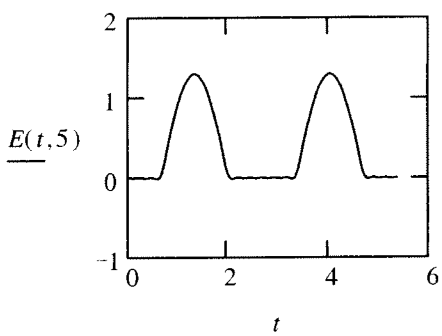
$a(1) = -0.65$

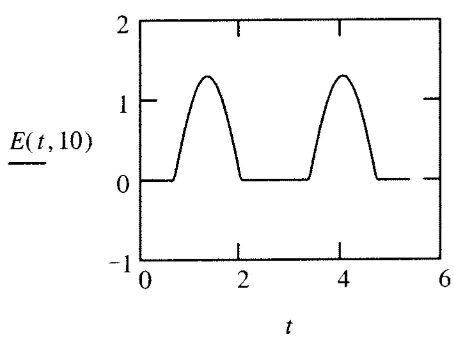
$$-\frac{A}{2} = -0.65$$

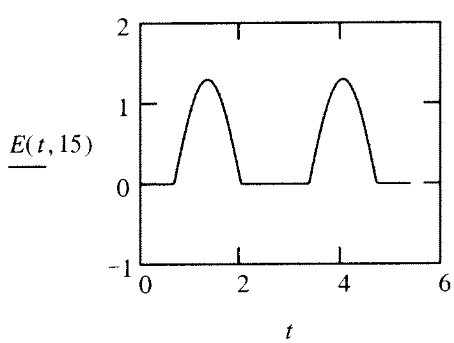
Exact Equation

$$E(t,m) := \frac{A}{\pi} - \frac{A}{2}\cdot cos\left(\frac{2\cdot\pi\,t}{T}\right) + \frac{2\cdot A}{\pi}\cdot\sum_{n=1}^{m} cos\left(\frac{4\cdot\pi\,n\cdot t}{T}\right)\cdot\frac{(-1)^{n-1}}{4\cdot n^2 - 1} \qquad t := 0, \frac{T}{1000} .. 2\cdot T$$

Graphs







Problem 58
"Quarter-Rectified Cosine Wave"

Initializing the Variables

$A := 1.3$ $T := 2.7$ $\theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := 9.4$

$fr := 4.3$ Hz $a := 0.32$ $b := 2.1$ $V_{DC} := 0.47$ V $j := \sqrt{-1}$

Evaluating F_n

$$F(n) := \frac{1}{T}\cdot\int_0^{\frac{T}{4}} A\cdot cos\left(\frac{2\cdot\pi}{T}\cdot t\right)\cdot e^{-j\cdot\frac{2\cdot\pi n}{T}\cdot t}\,dt + \frac{1}{T}\cdot\int_{\frac{T}{4}}^{T} 0\cdot e^{-j\cdot\frac{2\cdot\pi n}{T}\cdot t}\,dt$$

Evaluate Coefficients

$F(0) = 0.2069$

$F(1) = 0.1625 - 0.10345i$

$F(-1) = 0.1625 + 0.10345i$

$F(2) = 0.06897 - 0.13793i$

$F(-2) = 0.06897 + 0.13793i$

$F(3) = -2.19728\cdot 10^{-10} - 0.10345i$

$F(-3) = -2.19728\cdot 10^{-10} + 0.10345i$

$F(4) = -0.01379 - 0.05517i$

$F(-4) = -0.01379 + 0.05517i$

$F(5) = -0.03448i$

$F(-5) = 0.03448i$

$F(6) = 5.91147\cdot 10^{-3} - 0.03547i$

$F(-6) = 5.91147\cdot 10^{-3} + 0.03547i$

$F(7) = -2.92916\cdot 10^{-10} - 0.03448i$

$F(-7) = -2.92916\cdot 10^{-10} + 0.03448i$

$F(8) = -3.28415\cdot 10^{-3} - 0.02627i$

$F(-8) = -3.28415\cdot 10^{-3} + 0.02627i$

Exact Value Verification

DC = F_0 $F_o := \frac{A}{2\cdot\pi}$ $F_o = 0.2069$

1st Harmonic Frequency

$E_1 := \frac{A}{8}\cdot\left(1 - j\cdot\frac{2}{\pi}\right)$ $E_{1C} := \frac{A}{8}\cdot\left(1 + j\cdot\frac{2}{\pi}\right)$

$E_1 = 0.1625 - 0.10345i$ $E_{1C} = 0.1625 + 0.10345i$

$E_1 = F(1)$ $E_{1C} = F(-1)$

$$E(N) := \frac{A}{2\cdot\pi}\cdot\frac{j\cdot N + e^{-j\cdot\frac{\pi N}{2}}}{1 - N^2}$$

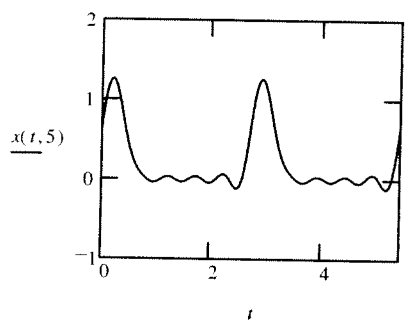
2nd Harmonic Frequency	$E(2) = 0.06897 - 0.13793i$
3rd Harmonic Frequency	$E(3) = -0.10345i$
4th Harmonic Frequency	$E(4) = -0.01379 - 0.05517i$
5th Harmonic Frequency	$E(5) = -0.03448i$
6th Harmonic Frequency	$E(6) = 5.91147\cdot 10^{-3} - 0.03547i$
7th Harmonic Frequency	$E(7) = -0.03448i$
8th Harmonic Frequency	$E(8) = -3.28415\cdot 10^{-3} - 0.02627i$

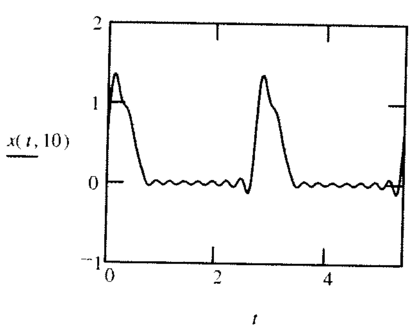
Graphing *x(t)*

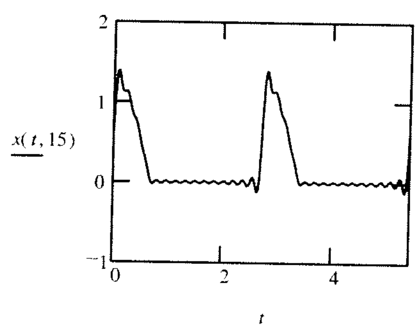
$$x(t,m) := \frac{A}{2\cdot\pi} + \frac{A}{8}\cdot\left(1 - j\cdot\frac{2}{\pi}\right)\cdot e^{j\cdot\left(\frac{2\cdot\pi}{T}\cdot t\right)} + \frac{A}{8}\cdot\left(1 + j\cdot\frac{2}{\pi}\right)\cdot e^{-j\cdot\left(\frac{2\cdot\pi}{T}\cdot t\right)} \ldots$$

$$+ \frac{A}{2\cdot\pi}\cdot\sum_{n=-m}^{-2}\frac{j\cdot n + e^{-j\cdot\left(\frac{\pi n}{2}\right)}}{1-n^2}\cdot e^{j\cdot\left(\frac{2\cdot\pi n}{T}\cdot t\right)} + \frac{A}{2\cdot\pi}\cdot\sum_{n=2}^{m}\frac{j\cdot n + e^{-j\cdot\left(\frac{\pi n}{2}\right)}}{1-n^2}\cdot e^{j\cdot\left(\frac{2\cdot\pi n}{T}\cdot t\right)}$$

$$t := 0, .01 .. 2\cdot T$$







Note: The above graphs match the figures provided in the Problem Statement, however, they are shown from 0 to 2T.

Ken Kaiser

$A := 1.3$ $T := 2.7$ $\theta := \frac{\pi}{5}$ $\tau := 0.68$ $\alpha := 9.4$ $f_r := 4.3$ $a := 0.32$ $b := 2.1$ $V_{dc} := 0.47$

$$a(N) := \frac{2}{T}\cdot\int_0^T A\cdot\cos\left(\frac{\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot N}{T}\cdot t\right)dt \qquad b(N) := \frac{2}{T}\cdot\int_0^T A\cdot\cos\left(\frac{\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot N}{T}\cdot t\right)dt$$

$a(0) = 0$
$a(1) = 0$
$a(2) = 0$
$a(3) = 0$
$a(4) = 0$
$a(5) = 0$
$a(6) = 0$
$a(7) = 0$
$a(8) = 0$

$b(0) = 0$
$b(1) = 1.10347$
$b(2) = 0.44139$
$b(3) = 0.28375$
$b(4) = 0.21019$
$b(5) = 0.16719$
$b(6) = 0.1389$
$b(7) = 0.11884$
$b(8) = 0.10386$

$$bp(n) := \frac{8\cdot A}{\pi}\cdot\frac{n}{4\cdot n^2 - 1}$$

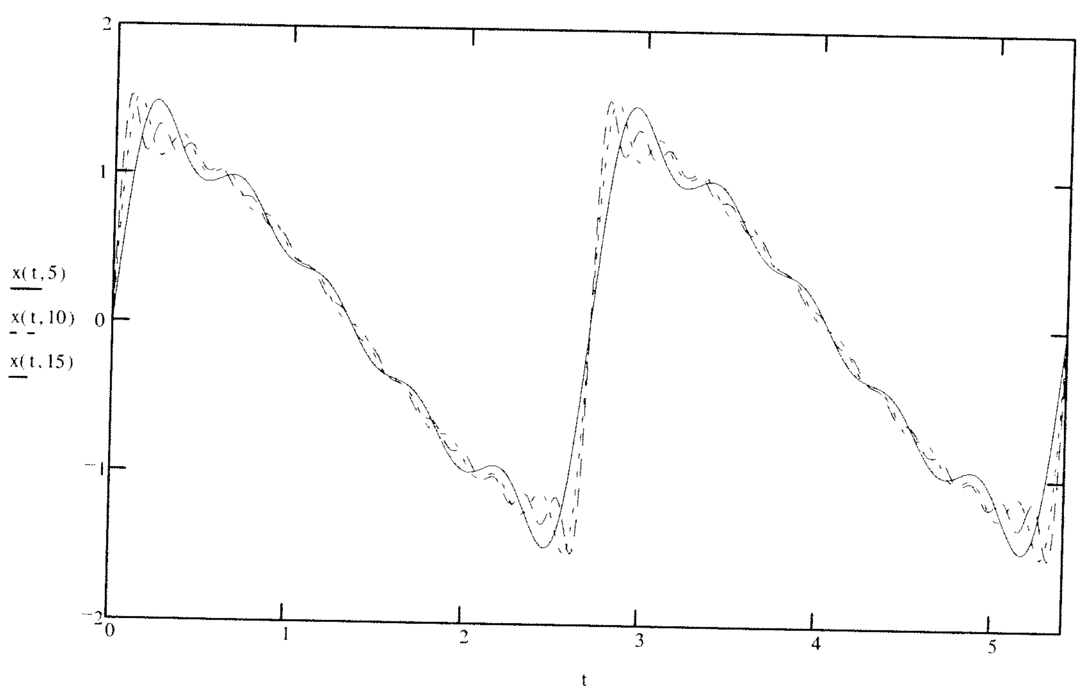
$bp(1) = 1.10347$
$bp(2) = 0.44139$
$bp(3) = 0.28375$
$bp(4) = 0.21019$
$bp(5) = 0.16719$
$bp(6) = 0.1389$
$bp(7) = 0.11884$
$bp(8) = 0.10386$

$Favg := 0$

$Favg = 0$

$\frac{a(0)}{2} = 0$

$$x(t,m) := \sum_{N=1}^{m} b(N)\cdot\sin\left(\frac{2\cdot\pi\cdot N}{T}\cdot t\right) \qquad t := 0, 0.01 .. 2\cdot T$$



Problem # 60 Fractional Rectified Cosine Wave

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1} \quad k := \frac{1}{8}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T}\cdot\left[\int_0^{k\cdot T}\left(\frac{A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right) - A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] + \frac{2}{T}\cdot\left[\int_{k\cdot T}^{T-k\cdot T}(0)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] \ldots$$
$$+\frac{2}{T}\cdot\int_{T-k\cdot T}^{T}\left(\frac{A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right) - A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

$$b(n) := \frac{2}{T}\cdot\left[\int_0^{k\cdot T}\left(\frac{A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right) - A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] + \frac{2}{T}\cdot\left[\int_{k\cdot T}^{T-k\cdot T}(0)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt\right] \ldots$$
$$+\frac{2}{T}\cdot\int_{T-k\cdot T}^{T}\left(\frac{A\cdot\cos\left(\frac{2\cdot\pi}{T}\cdot t\right) - A\cdot\cos(2\cdot\pi\cdot k)}{1-\cos(2\cdot\pi\cdot k)}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)dt$$

Results:

$a(0) = 0.429$

$a(1) = 0.403$

$a(2) = 0.333$

$a(3) = 0.235$

$a(4) = 0.133$

$a(5) = 0.047$

$a(6) = -9.514 \cdot 10^{-3}$

$a(7) = -0.034$

$a(8) = -0.032$

$b(0) = 0$

$b(1) = 0$

$b(2) = 0$

$b(3) = 0$

$b(4) = 0$

$b(5) = 0$

$b(6) = 0$

$b(7) = 0$

$b(8) = 0$

Check using fourier series definition:

$$C(h) := \frac{2 \cdot A}{\pi} \cdot \frac{(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))}{1 - \cos(2\cdot\pi\cdot k)} \cdot \frac{\sin(2\cdot\pi\cdot h\cdot k)\cdot\cos(2\cdot\pi\cdot k) - h\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot h\cdot k)}{h\cdot\left(h^2 - 1\right)\cdot(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))} *$$

$$C_0 := \lim_{h \to 0} \frac{2 \cdot A}{\pi} \cdot \frac{(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))}{1 - \cos(2\cdot\pi\cdot k)} \cdot \frac{\sin(2\cdot\pi\cdot h\cdot k)\cdot\cos(2\cdot\pi\cdot k) - h\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot h\cdot k)}{h\cdot\left(h^2 - 1\right)\cdot(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))} *$$

$C_0 = 0.429$

$$C_1 := \lim_{h \to 1} \frac{2 \cdot A}{\pi} \cdot \frac{(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))}{1 - \cos(2\cdot\pi\cdot k)} \cdot \frac{\sin(2\cdot\pi\cdot h\cdot k)\cdot\cos(2\cdot\pi\cdot k) - h\cdot\sin(2\cdot\pi\cdot k)\cdot\cos(2\cdot\pi\cdot h\cdot k)}{h\cdot\left(h^2 - 1\right)\cdot(\sin(2\cdot\pi\cdot k) - 2\cdot\pi\cdot k\cdot\cos(2\cdot\pi\cdot k))} *$$

$C_1 = 0.403$

$C(2) = 0.333$

$C(3) = 0.235$

$C(4) = 0.133$

$C(5) = 0.047$

$C(6) = -9.514 \cdot 10^{-3}$

$C(7) = -0.034$

$C(8) = -0.032$

Conclusion: These values match the a coeffecients, except for the first two The function returned 0 instead of a singularity error. It seems like 0 should be the correct answer for those two values.

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Cosine Pulse Train

Set up the given variables

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := .68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := .32 \quad b := 2.1 \quad V_{dc} := .47 \quad j := \sqrt{-1}$

Define the Function

$$x(t) := A\cdot\cos\left(\frac{\pi}{\tau}\cdot t\right)$$

Find the Fourier Series coefficients

$$a(n) := \frac{2}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt \qquad b(n) := \frac{2}{T}\cdot\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

Note that because the equation is zero elsewhere, the integral is zero elsewhere

Define the Coefficient Functions

$$a_0 := \frac{A\cdot\tau}{\pi\cdot T} \qquad a_p(n) := \frac{4\cdot A\cdot\tau}{\pi\cdot T}\cdot\frac{\cos\left(\frac{\pi\cdot n\cdot\tau}{T}\right)}{1-\left(\frac{2\cdot n\cdot\tau}{T}\right)^2} \qquad b_p := 0$$

Compare the Results

$a(0) = 0.41687$	$a_0 = 0.10422$	$b(0) = 0$
$a(1) = 0.39268$	$a_p(1) = 0.39268$	$b(1) = 0$
$a(2) = 0.32619$	$a_p(2) = 0.32619$	$b(2) = 0$
$a(3) = 0.23364$	$a_p(3) = 0.23364$	$b(3) = 0$
$a(4) = 0.13622$	$a_p(4) = 0.13622$	$b(4) = 0$
$a(5) = 0.05354$	$a_p(5) = 0.05354$	$b(5) = 0$

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Cosine Squared Pulse Train

Constants

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Equations for Determining Series Coefficients

$$a(n) := \frac{2}{T} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot \cos\left(\frac{\pi}{\tau} \cdot t\right)^2 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot \sin\left(\frac{\pi}{\tau} \cdot t\right)^2 \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$c(n) := \sqrt{a(n)^2 + b(n)^2}$$

$$F(n) := \frac{1}{2} \cdot (a(n) - j \cdot b(n)) \qquad F_o(n) := \frac{a(0)}{2}$$

Series Coefficients

$a(0) = 0.32741$

$a(1) = 0.314$

$a(2) = 0.277$

$a(3) = 0.223$

$a(4) = 0.162$

$a(5) = 0.103$

$a(6) = 0.054$

$a(7) = 0.019$

$a(8) = -7.866 \cdot 10^{-4}$

$b(0) = 0.327$

$b(1) = 0.274$

$b(2) = 0.137$

$b(3) = -0.032$

$b(4) = -0.167$

$b(5) = -0.223$

$b(6) = -0.192$

$b(7) = -0.099$

$b(8) = 5.6 \cdot 10^{-3}$

$c(0) = 0.463$

$c(1) = 0.417$

$c(2) = 0.309$

$c(3) = 0.226$

$c(4) = 0.232$

$c(5) = 0.246$

$c(6) = 0.199$

$c(7) = 0.101$

$c(8) = 5.655 \cdot 10^{-3}$

$F_0(0) = 0.164$

$F(1) = 0.157 - 0.137i$

$F(2) = 0.139 - 0.068i$

$F(3) = 0.112 + 0.016i$

$F(4) = 0.081 + 0.083i$

$F(5) = 0.051 + 0.112i$

$F(6) = 0.027 + 0.096i$

$F(7) = 9.503 \cdot 10^{-3} + 0.05i$

$F(8) = -3.933 \cdot 10^{-4} - 2.8 \cdot 10^{-3} i$

$F_0(-0) = 0.164$

$F(-1) = 0.157 - 0.137i$

$F(-2) = 0.139 - 0.068i$

$F(-3) = 0.112 + 0.016i$

$F(-4) = 0.081 + 0.083i$

$F(-5) = 0.051 + 0.112i$

$F(-6) = 0.027 + 0.096i$

$F(-7) = 9.503 \cdot 10^{-3} + 0.05i$

$F(-8) = -3.933 \cdot 10^{-4} - 2.8 \cdot 10^{-3} i$

Verification of Series Coefficients

$$a_p(N) := \frac{A}{\pi} \cdot \frac{\sin\left(\frac{\pi \cdot N \cdot \tau}{T}\right)}{N \cdot \left[1 - \left(\frac{N \cdot \tau}{T}\right)^2\right]}$$

$$a_o(n) := \frac{A \cdot \tau}{2 \cdot T} \cdot 2$$

$a_o(0) = 0.327$

$a_p(1) = 0.314$

$a_p(2) = 0.277$

$a_p(3) = 0.223$

$a_p(4) = 0.162$

$a_p(5) = 0.103$

$a_p(6) = 0.054$

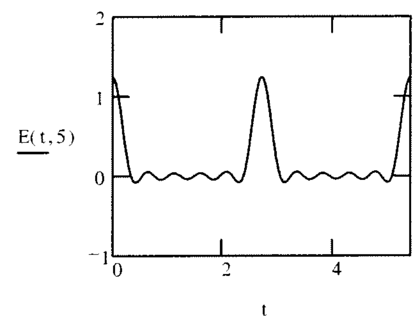
$a_p(7) = 0.019$

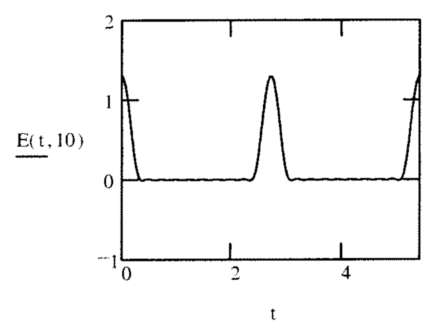
$a_p(8) = -7.866 \cdot 10^{-4}$

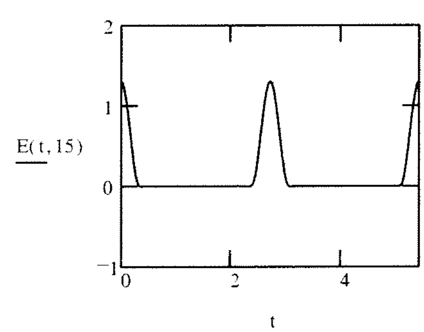
Time Function Plots

$$E(t,m) := \frac{A\cdot\tau}{2\cdot T} + \frac{A}{\pi}\cdot\sum_{n=1}^{m}\frac{\sin\left(\frac{\pi\cdot n\cdot\tau}{T}\right)}{n\cdot\left[1-\left(\frac{n\cdot\tau}{T}\right)^2\right]}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$

$$t := 0, \frac{T}{1000} .. 2\cdot T$$







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Full-Rectified Sine Wave

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad F_r := 4.3 \quad a := 3.2 \quad b := 2.1 \quad w := \frac{T - 2\cdot\tau}{2} \quad w = 0.67$

$$a(n) := \frac{2}{T}\cdot\int_0^T A\cdot\sin\left(\frac{\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

$$b(n) := \frac{2}{T}\cdot\int_0^T A\cdot\sin\left(\frac{\pi}{T}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)\,dt$$

$ap(n) := \frac{-4\cdot A}{\pi}\cdot\frac{1}{4\cdot n^2 - 1} \qquad F_{avg} := \frac{2\cdot A}{\pi}$

$\frac{a(0)}{2} = 0.82761 \qquad F_{avg} = 0.82761$

$a(1) = -0.55174 \qquad ap(1) = -0.55174 \qquad b(1) = 0$

$a(2) = -0.11035 \qquad ap(2) = -0.11035 \qquad b(2) = 0$

$a(3) = -0.04729 \qquad ap(3) = -0.04729 \qquad b(3) = 0$

$a(4) = -0.02627 \qquad ap(4) = -0.02627 \qquad b(4) = 0$

$a(5) = -0.01671 \qquad ap(5) = -0.01672 \qquad b(5) = 0$

$a(6) = -0.01157 \qquad ap(6) = -0.01157 \qquad b(6) = 0$

$a(7) = -0.00849 \qquad ap(7) = -0.00849 \qquad b(7) = 0$

$a(8) = -0.00649 \qquad ap(8) = -0.00649 \qquad b(8) = 0$

Problem # 64 Full Rectified Sine Wave with Added DC Offset

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \int_0^T \left(V_{dc} + A \cdot \sin\left(\frac{\pi}{T} \cdot t\right)\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_0^T \left(V_{dc} + A \cdot \sin\left(\frac{\pi}{T} \cdot t\right)\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Results:

$a(0) = 2.595$

$a(1) = -0.552$

$a(2) = -0.11$

$a(3) = -0.047$

$a(4) = -0.026$

$a(5) = -0.017$

$a(6) = -0.012$

$a(7) = -8.488 \cdot 10^{-3}$

$a(8) = -6.491 \cdot 10^{-3}$

$b(0) = 0$

$b(1) = 0$

$b(2) = 0$

$b(3) = 0$

$b(4) = 0$

$b(5) = 0$

$b(6) = 0$

$b(7) = 0$

$b(8) = 0$

Check using fourier series definition:

$$Y(h) := \frac{-4 \cdot A}{\pi} \cdot \frac{1}{4 \cdot h^2 - 1}$$

$Y(0) = 1.655$

$Y(1) = -0.552$

$Y(2) = -0.11$

$Y(3) = -0.047$

$Y(4) = -0.026$

$Y(5) = -0.017$

$Y(6) = -0.012$

$Y(7) = -8.488 \cdot 10^{-3}$

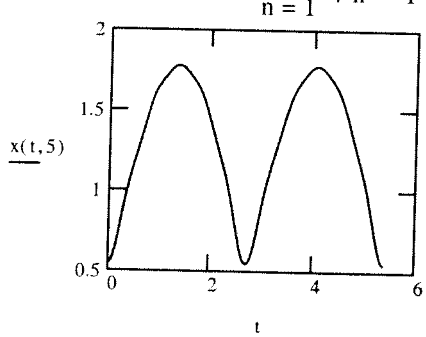
$Y(8) = -6.491 \cdot 10^{-3}$

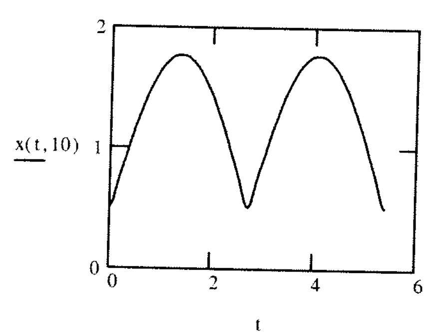
Conclusion: These values match the a coeffecients, and the 0 term is a0/2

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := V_{dc} + \frac{2 \cdot A}{\pi} - \frac{4 \cdot A}{\pi} \cdot \sum_{n=1}^{c} \frac{1}{4 \cdot n^2 - 1} \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right)$$





Problem # 64 Full Rectified Sine Wave with Added DC Offset

Constants:

$A := 1.3 \quad T := 2.7 \quad \theta := \frac{\pi}{5} \quad \tau := 0.68 \quad \alpha := 9.4 \quad f_r := 4.3 \quad a := 0.32 \quad b := 2.1 \quad V_{dc} := 0.47 \quad j := \sqrt{-1}$

Fourier Series Integral Definitions:

$$a(n) := \frac{2}{T} \cdot \int_0^T \left(V_{dc} + A \cdot \sin\left(\frac{\pi}{T} \cdot t\right)\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

$$b(n) := \frac{2}{T} \cdot \int_0^T \left(V_{dc} + A \cdot \sin\left(\frac{\pi}{T} \cdot t\right)\right) \cdot \sin\left(\frac{2 \cdot \pi \cdot n}{T} \cdot t\right) dt$$

Results:

$a(0) = 2.595$	$b(0) = 0$
$a(1) = -0.552$	$b(1) = 0$
$a(2) = -0.11$	$b(2) = 0$
$a(3) = -0.047$	$b(3) = 0$
$a(4) = -0.026$	$b(4) = 0$
$a(5) = -0.017$	$b(5) = 0$
$a(6) = -0.012$	$b(6) = 0$
$a(7) = -8.488 \cdot 10^{-3}$	$b(7) = 0$
$a(8) = -6.491 \cdot 10^{-3}$	$b(8) = 0$

Check using fourier series definition:

$$Y(h) := \frac{-4\cdot A}{\pi}\cdot\frac{1}{4\cdot h^2 - 1}$$

$$Y_0 := V_{dc} + \frac{2\cdot A}{\pi} \qquad Y_0 = 1.298$$

$Y(1) = -0.552$

$Y(2) = -0.11$

$Y(3) = -0.047$

$Y(4) = -0.026$

$Y(5) = -0.017$

$Y(6) = -0.012$

$Y(7) = -8.488\cdot 10^{-3}$

$Y(8) = -6.491\cdot 10^{-3}$

Conclusion: These values match the a coeffecients, except for the first term, which is a0/2

Graphically show how the function improves as the number of elements in the series is increased:

$t := 0, 0.01 .. 5.4$

$$x(t,c) := V_{dc} + \frac{2\cdot A}{\pi} - \frac{4\cdot A}{\pi}\cdot\sum_{n=1}^{c}\frac{1}{4\cdot n^2 - 1}\cdot\cos\left(\frac{2\cdot\pi\cdot n}{T}\cdot t\right)$$