p 542

Ken Kaiser

$$h(t) = e^{\frac{-t^2}{2a^2}} \Longleftrightarrow H(\omega) = a\sqrt{2\pi}\, e^{\frac{-a^2\omega^2}{2}}$$

Proof

$$a\cdot\sqrt{2\cdot\pi}\cdot e^{\frac{-a^2\cdot\omega^2}{2}}$$

$$\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)$$

Table 6

PS 3

Ken Kaiser

$h(t) = e^{-at} u(t)$

$H(s) = \frac{1}{s+a}$ Single pole

$10\text{-}90 = \frac{2.2}{a} = T_r$

slope $= T_r = \frac{2}{a}$

Std dev $\approx T_r = \frac{3.5}{a}$

	3dB	eq	75%	99%
10-90	$\frac{2.2}{a} \cdot a = 2.2$	$\frac{2.2}{a} \cdot \frac{\pi a}{2} = 3.45$	$\frac{2.2}{a} \cdot 2.4a = 5.28$	$\frac{2.2}{a} \cdot 60a = 132$
slope	$\frac{2}{a} \cdot a = 2$	$\frac{2}{a} \cdot \frac{\pi a}{2} = \pi$	$\frac{2}{a} \cdot 2.4a = 4.8$	$\frac{2}{a} \cdot 60a = 120$
Std dev	$\frac{3.5}{a} \cdot a = 3.5$	$\frac{3.5}{a} \cdot \frac{\pi a}{2} = 5.5$	$\frac{3.5}{a} \cdot 2.4a = 8.4$	$\frac{3.5}{a} \cdot 60a = 210$
Std dev 2	$\frac{1.3}{a} \cdot a = 1.3$	$\frac{1.3}{a} \cdot \frac{\pi a}{2} = 2.04$	$\frac{1.3}{a} \cdot 2.4a = 3.12$	$\frac{1.3}{a} \cdot 60a = 78$

Table 7
p544

$$h(t) = e^{-\frac{t^2}{2a^2}}$$

Ken Kaiser

$$H(\omega) = a\sqrt{2\pi}\, e^{\frac{-a^2\omega^2}{2}}$$

Non Cause Gaussian

10-90 $\quad T_r = 2.6a$

slope $\quad T_r = a\sqrt{2\pi} \simeq 2.5a$

std $\quad T_r = a\sqrt{2\pi} \simeq 2.5a$

std^2 $\quad \tau_r = a\sqrt{2\pi - 4},\ t_r = 2.5$

$= \cancel{1.511a}\ 2.5a$

$\frac{1.72}{a}$

	3dB	eq	75%	99%	RMS
10-90	2.2	2.3	2.1	4.7	1.8
slope	2.1	2.2	2.0	4.5	1.8
std dev	2.1	2.2	2.0	4.5	1.8
std^2	1.5	1.6	1.4	3.2	1.3

Table 1

Average delay time (same as 50% delay time)

$$w(t) = \int_0^t h(t)\,dt = \int_0^t e^{-at}\,dt = \frac{-1}{a} e^{-at}\Big|_0^t = \frac{1 - e^{-at}}{a}$$

$$w(\tau_d) = \frac{1}{2a}$$

$$\frac{1 - e^{-a\tau_d}}{\cancel{a}} = \frac{1}{2\cancel{a}}$$

$$\cancel{-}e^{-a\tau_d} = \cancel{-}1/2$$

$$\ln e^{-a\tau_d} = \ln 1 - \ln 2$$

$$\cancel{-}a\tau_d = \cancel{-}\ln 2$$

$$\tau_d = \frac{\ln 2}{a} \approx \frac{0.69}{a}$$

10-90% Rise Time

$$\tau_r = t_{90\%} - t_{10\%}$$

$$0.1\left(\tfrac{1}{\cancel{a}}\right) = \frac{1 - e^{-at_{10\%}}}{\cancel{a}}$$

$$0.1 - 1 = -e^{-at_{10\%}}$$

$$\cancel{-}0.9 = \cancel{-}e^{-at_{10\%}}$$

$$\ln 0.9 = \cancel{\ln e}^{-at_{10\%}}$$

$$t_{10\%} = \frac{\ln 0.9}{-a} \approx \frac{0.105}{a}$$

$$0.9\left(\tfrac{1}{\cancel{a}}\right) = \frac{1 - e^{-at_{90\%}}}{\cancel{a}}$$

$$0.9 - 1 = -e^{-at_{90\%}}$$

$$\ln 0.1 = \cancel{\ln e}^{-at_{90\%}}$$

$$t_{90\%} = \frac{\ln 0.1}{-a} \approx \frac{2.3}{a}$$

$$\tau_r = t_{90\%} - t_{10\%} = \frac{2.3}{a} - \frac{0.105}{a} \approx \frac{2.2}{a}$$

Centroid delay time Ken Kaiser Table 1

$a = 3$

$$\frac{\int_0^{\infty} (t)\cdot e^{-3\cdot t}\,dt}{\int_0^{\infty} e^{-3\cdot t}\,dt}$$

$$\frac{1}{3} = \frac{1}{a}$$

$a = 5$

$$\frac{\int_0^{\infty} (t)\cdot e^{-5\cdot t}\,dt}{\int_0^{\infty} e^{-5\cdot t}\,dt}$$

$$\frac{1}{5} = \frac{1}{a}$$

Standard deviation rise time

Ken Kaiser

Table 1

$a = 3$

$$\sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^{\infty} t^2\cdot e^{-3\cdot t}\,dt}{\int_0^{\infty} e^{-3\cdot t}\,dt}}$$

$$\frac{2}{3}\cdot\sqrt{\pi}$$

$$= \frac{2}{a}\cdot\sqrt{\pi}$$

$$= \frac{2\sqrt{\pi}}{a}$$

$a = 5$

$$\sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^{\infty} t^2\cdot e^{-5\cdot t}\,dt}{\int_0^{\infty} e^{-5\cdot t}\,dt}}$$

$$\frac{2}{5}\cdot\sqrt{\pi}$$

$$= \frac{2}{a}\cdot\sqrt{\pi}$$

$$= \frac{2\sqrt{\pi}}{a}$$

these assume that $T_d = 0$ which it is not!

My work

$a = 2$

$\frac{1}{2}\sqrt{2\pi}$

$a = 3$

$\frac{1}{3}\sqrt{2\pi}$

$$T_r = \sqrt{2\pi}\sqrt{\frac{\int_0^{\infty}(t - T_d)^2 h(t)\,dt}{\int_0^{\infty} h(t)\,dt}}$$

$$T_d = \frac{\int_0^{\infty} t\,h(t)\,dt}{H(0)}$$

centroid delay time

$$= \frac{1}{a}$$

Ken Kaiser Table 1

E) determination of Slope Based rise time @50% Delay time

Here we are using $t_o = .69/a$

Therefore rise time is obtained by the expression:

$$\tau_r = \frac{A}{h(t_o)}$$

From graph $A = \frac{1}{a}$

and $h(t_o) = e^{-a \cdot \frac{.69}{a}}$

$$= .501$$

$$\tau_r = \frac{1}{a \times 0.501} \approx \frac{1}{a \cdot 1/2}$$

$$\tau_r \simeq \frac{2}{a}$$

Hence Proved

$$\cancel{\int e^{-a \cdot t}\, dt}$$

$$\cancel{\frac{-1}{a} \cdot \exp(-a \cdot t)}$$

G) Standard Deviation Rise time for h^2(t):

The definition used to find the rise time is:

$$\tau_r = \sqrt{2 \cdot \pi} \cdot \sqrt{\frac{\int_0^\infty t^2 \cdot h^2(t)\, dt}{\int_0^\infty h^2(t)\, dt} - \left[\frac{\int_0^\infty t \cdot h^2(t)\, dt}{\int_0^\infty h^2(t)\, dt}\right]^2}$$

where $h(t) = e^{-at}$

Now evaluating the integrals individually and substituting in the above equation:

$$\int_0^{\infty} t^2\cdot h^2\cdot(t)\,dt \quad = \quad \int_0^{\infty} t^2\cdot e^{-2\cdot a\cdot t}\,dt$$

$$\text{limit}\left[\frac{-1}{8}\cdot\frac{\left(4\cdot a^2\cdot t^2\cdot \exp(-2\cdot a\cdot t)+4\cdot a\cdot t\cdot \exp(-2\cdot a\cdot t)+2\cdot \exp(-2\cdot a\cdot t)\right)}{a^3}+\frac{1}{\left(4\cdot a^3\right)}, t=\infty, \text{left}\right]$$

as e^-∞ = 0

$$\int_0^{\infty} t^2\cdot e^{-2\cdot a\cdot t}\,dt \quad = \quad \frac{-1}{8}\cdot\frac{\left(4\cdot a^2\cdot\infty\cdot 0+4\cdot a\cdot\infty\cdot 0+2\cdot 0\right)}{a^3}+\frac{1}{\left(4\cdot a^3\right)}$$

$$= \quad \frac{1}{\left(4\cdot a^3\right)}$$

$$\int_0^{\infty} t\cdot h^2\cdot(t)\,dt \quad = \quad \int_0^{\infty} t\cdot e^{\;2\cdot a\cdot t}\,dt$$

$$\text{limit}\left[\frac{-1}{4}\cdot\frac{(2\cdot a\cdot t\cdot \exp(-2\cdot a\cdot t)+\exp(-2\cdot a\cdot t)-1)}{a^2}, t=\infty, \text{left}\right]$$

as e^-∞ = 0

$$\frac{-1}{4}\cdot\frac{\left(2\cdot a\cdot\infty\cdot 0+0-1\right)}{a^2}$$

$$\frac{1}{\left(4\cdot a^2\right)}$$

$$\int_0^{\infty} h^2\cdot(t)\,dt \quad = \quad \int_0^{\infty} e^{-2\cdot a\cdot t}\,dt$$

$$\text{limit}\left[\frac{-1}{2}\cdot\frac{(\exp(-2\cdot a\cdot t)-1)}{a}, t=\infty, \text{left}\right]$$

as e^-∞ = 0

$$\frac{-1}{2}\cdot\frac{(0-1)}{a}$$

$$\frac{1}{(2\cdot a)}$$

Therefore on substituting these values in the main equation we get,

$$\tau_r \quad = \quad \sqrt{2\cdot\pi}\cdot\sqrt{\frac{\frac{1}{(4\cdot a^3)}}{\frac{1}{(2\cdot a)}} - \left[\frac{\frac{1}{(4\cdot a^2)}}{\frac{1}{(2\cdot a)}}\right]^2}$$

$$\frac{1}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{\pi}}{a}$$

This can be re-written as:

$$\frac{1}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{\pi}}{a}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{1}{a}\cdot\sqrt{\frac{\pi}{2}}$$

Which is equivalent to:

$$\frac{1.253}{a}$$ Hence Proved.

Table 5
#1

Ken Kaiser

$\omega := 0, 0.01 .. 3$ $\qquad j := \sqrt{-1}$

$a := 2$

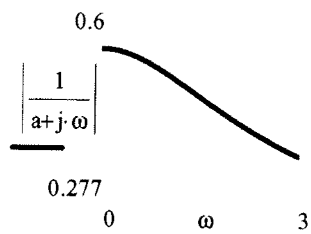


Table 5
#2

Ken Kaiser

$\omega := 0, 0.01 .. 3$ $\qquad$ $j := \sqrt{-1}$

$a := 2$

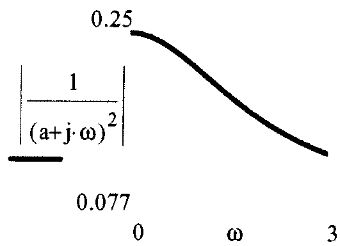


Table 5

#3

Ken Kaiser

$\omega := 0, 0.01 .. 3$ $\qquad j := \sqrt{-1}$

$a := 2$

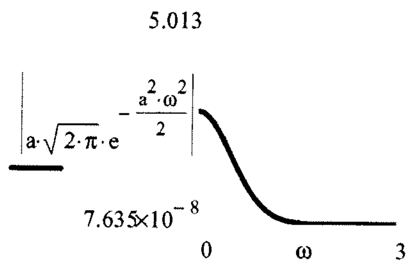


Table #5
#5

Ken Kaiser

$\omega := 0.01, 0.02 .. 5 \qquad j := \sqrt{-1}$

$T := 6$

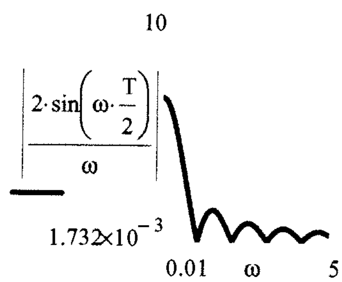


Table 5

#1 BW_{3dB}

$$\frac{\left|\frac{1}{(i\cdot\omega)+a}\right|}{\left|\frac{1}{a}\right|} = \frac{1}{\sqrt{2}}$$

$$\begin{bmatrix} -\sqrt{2\cdot(|a|)^2 - a^2} \\ \sqrt{2\cdot(|a|)^2 - a^2} \end{bmatrix} \leftarrow$$

$$\sqrt{2(|a|)^2 - a^2}$$

$$= \sqrt{2a^2 - a^2}$$

$$= \sqrt{a^2}$$

$$= a$$

Table 5
#2 BW3dB

$$\frac{\left|\frac{1}{((i\cdot\omega)+a)^2}\right|}{\left|\frac{1}{a^2}\right|} = \frac{1}{\sqrt{2}}$$

$$\begin{bmatrix} \frac{-1}{2}\cdot 2^{\left(\frac{3}{4}\right)}\cdot\sqrt{-a^2\cdot\left(-2+\sqrt{2}\right)} \\ \frac{1}{2}\cdot 2^{\left(\frac{3}{4}\right)}\cdot\sqrt{-a^2\cdot\left(-2+\sqrt{2}\right)} \end{bmatrix}$$

$$\begin{pmatrix} -.645\cdot a \\ .645\cdot a \end{pmatrix}$$

5:39 Table 5 # 3 BW_{3dB}

Ken Kaiser

3dB $H(\omega) = a\sqrt{2\pi}\, e^{-\frac{a^2\omega^2}{2}}$

$$\frac{\left|a\cdot\sqrt{2\cdot\pi}\cdot e^{\frac{-a^2\cdot\omega^2}{2}}\right|}{\left|a\cdot\sqrt{2\cdot\pi}\right|} = \frac{1}{\sqrt{2}}$$

$$\begin{bmatrix} \frac{1}{a^2}\cdot\sqrt{2}\cdot\sqrt{-a^2\cdot\ln\left(\frac{1}{2}\cdot\sqrt{2}\right)} \\ \frac{-1}{a^2}\cdot\sqrt{2}\cdot\sqrt{-a^2\cdot\ln\left(\frac{1}{2}\cdot\sqrt{2}\right)} \end{bmatrix}$$

$$\frac{1}{a^2}\cdot\sqrt{2}\cdot\sqrt{-a^2\cdot\ln\left(\frac{1}{2}\cdot\sqrt{2}\right)}$$

$$.83255461115769775636\cdot\frac{i}{a^2}\cdot\sqrt{-1.\cdot a^2}$$

$$\frac{-1}{a^2}\cdot\sqrt{2}\cdot\sqrt{-a^2\cdot\ln\left(\frac{1}{2}\cdot\sqrt{2}\right)}$$

$$-.83255461115769775636\cdot\frac{i}{a^2}\cdot\sqrt{-1.\cdot a^2}$$

$$= -.832\cdot\frac{i\cdot i}{a^2}\sqrt{a^2}$$

$$= \frac{.832}{a}$$

Table 5
\# 3 BWeq

Ken Kaiser

$$\frac{\displaystyle\int_0^{\infty}\left(\left|a\cdot\sqrt{2\cdot\pi}\cdot e^{\frac{-a^2\cdot\omega^2}{2}}\right|\right)^2 d\omega}{\left(\left|a\cdot\sqrt{2\cdot\pi}\cdot e^{\frac{-a^2\cdot\omega^2}{2}}\right|\right)^2}$$

$$\frac{1}{2}\cdot\frac{\sqrt{\pi}}{a}\cdot\exp\left(a^2\cdot\omega^2\right)$$

$$\omega := 0$$

$$\frac{1}{2}\cdot\frac{\sqrt{\pi}}{a}\cdot\exp\left(a^2\cdot 0^2\right)$$

$$\frac{1}{2}\cdot\frac{\sqrt{\pi}}{a}$$

$$\frac{.885}{a}$$

Table 5
#1 BWeq

$$\frac{\displaystyle\int_0^{\infty} \left[\left|\frac{1}{(i \cdot \omega) + a}\right|\right]^2 d\omega}{\left[\left|\frac{1}{(i \cdot \omega) + a}\right|\right]^2}$$

$$\frac{1}{2} \cdot \pi \cdot \frac{|a|}{a^2} \cdot \left(a^2 + \omega^2\right)$$

$$\omega := 0$$

$$\frac{1}{2} \cdot \pi \cdot \frac{|a|}{a^2} \cdot \left(a^2 + 0^2\right)$$

$$\frac{1}{2} \cdot \pi \cdot |a|$$

$$1.57 \cdot |a|$$

Table 5
#2
BWeq

$$\frac{\int_0^\infty \left[\left|\frac{1}{((i\cdot\omega)+a)^2}\right|\right]^2 d\omega}{\left[\left|\frac{1}{((i\cdot\omega)+a)^2}\right|\right]^2}$$

$$\frac{1}{4}\cdot\pi\cdot\frac{|a|}{a^4}\cdot\left(a^2+\omega^2\right)^2$$

$$\omega := 0$$

$$\frac{1}{4}\cdot\pi\cdot\frac{|a|}{a^4}\cdot\left(a^2+0^2\right)^2$$

$$\frac{1}{4}\cdot\pi\cdot|a|$$

$$.785\cdot|a|$$

My Method

BWeq for $H(w) = \dfrac{2\sin\left(\frac{wT}{2}\right)}{w}$

$$\frac{\int_0^\infty \left|\frac{2\sin\left(\frac{wT}{2}\right)}{w}\right|^2 dw}{T^2} =$$

If $T=3$
$1.0475 \approx \frac{\pi}{3}$

If $T=7$
$.44855 \approx \frac{\pi}{7}$

If $T=\pi$
$\approx 1.0002 \approx \frac{\pi}{\pi}$

$W = .01, .02, \ldots, 3$

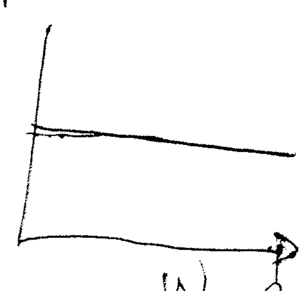


Table 5

Verifications of Table Entries for $BW_{75\%}$ and $BW_{99\%}$

$BW := 0, .01 .. 10$
$\omega := 0, 6.34 .. 2\cdot\pi$
$a := 1$

$$z\%(\omega) := 100\cdot\frac{\frac{1}{\pi}\cdot\int_0^{2.3\cdot a}\left(\left|\frac{1}{\omega\cdot\sqrt{-1}+a}\right|\right)^2 d\omega}{\frac{1}{\pi}\cdot\int_0^{\infty}\left(\left|\frac{1}{\omega\cdot\sqrt{-1}+a}\right|\right)^2 d\omega}$$

73.890482582275448917

#1

$$z\%(\omega) := 100\cdot\frac{\frac{1}{\pi}\cdot\int_0^{2.4\cdot a}\left(\left|\frac{1}{\omega\cdot\sqrt{-1}+a}\right|\right)^2 d\omega}{\frac{1}{\pi}\cdot\int_0^{\infty}\left(\left|\frac{1}{\omega\cdot\sqrt{-1}+a}\right|\right)^2 d\omega}$$

74.866816724399526473

$$z\%(\omega) := 100\cdot\frac{\frac{1}{\pi}\cdot\int_0^{60\cdot a}\left(\left|\frac{1}{\omega\cdot\sqrt{-1}+a}\right|\right)^2 d\omega}{\frac{1}{\pi}\cdot\int_0^{\infty}\left(\left|\frac{1}{\omega\cdot\sqrt{-1}+a}\right|\right)^2 d\omega}$$

$\frac{200}{\pi}\cdot\operatorname{atan}(60)$ which simplifies to 98.94

#2

$$z\%(\omega) := 100\cdot\frac{\frac{1}{\pi}\cdot\int_0^{.82\cdot a}\left[\left|\frac{1}{\left(\omega\cdot\sqrt{-1}+a\right)^2}\right|\right]^2 d\omega}{\frac{1}{\pi}\cdot\int_0^{\infty}\left[\left|\frac{1}{\left(\omega\cdot\sqrt{-1}+a\right)^2}\right|\right]^2 d\omega}$$

74.938485139714481481

$$z\%(\omega) := 100\cdot\frac{\frac{1}{\pi}\cdot\int_0^{3.4\cdot a}\left[\left|\frac{1}{\left(\omega\cdot\sqrt{-1}+a\right)^2}\right|\right]^2 d\omega}{\frac{1}{\pi}\cdot\int_0^{\infty}\left[\left|\frac{1}{\left(\omega\cdot\sqrt{-1}+a\right)^2}\right|\right]^2 d\omega}$$

99.022737415870755129

#3

$$z\%(\omega) := 100 \cdot \frac{\frac{1}{\pi} \cdot \int_0^{\frac{.81}{a}} \left[\left| a \cdot \sqrt{2 \cdot \pi} \cdot e^{\frac{(-a^2 \cdot \omega^2)}{2}} \right| \right]^2 d\omega}{\frac{1}{\pi} \cdot \int_0^{\infty} \left[\left| a \cdot \sqrt{2 \cdot \pi} \cdot e^{\frac{(-a^2 \cdot \omega^2)}{2}} \right| \right]^2 d\omega}$$

74.800328059778955924

$$z\%(\omega) := 100 \cdot \frac{\frac{1}{\pi} \cdot \int_0^{\frac{1.8}{a}} \left[\left| a \cdot \sqrt{2 \cdot \pi} \cdot e^{\frac{(-a^2 \cdot \omega^2)}{2}} \right| \right]^2 d\omega}{\frac{1}{\pi} \cdot \int_0^{\infty} \left[\left| a \cdot \sqrt{2 \cdot \pi} \cdot e^{\frac{(-a^2 \cdot \omega^2)}{2}} \right| \right]^2 d\omega}$$

98.909050163573071419

#4

$$z\%(\omega) := 100 \cdot \frac{\frac{1}{\pi} \cdot \int_0^{\frac{2.9}{T}} \left(\left| \frac{2 \cdot \sin\left(\frac{\omega \cdot T}{2}\right)}{\omega} \right| \right)^2 d\omega}{\frac{1}{\pi} \cdot \int_0^{\infty} \left(\left| \frac{2 \cdot \sin\left(\frac{\omega \cdot T}{2}\right)}{\omega} \right| \right)^2 d\omega}$$

$$z\%(\omega) := 100 \cdot \frac{\frac{1}{\pi} \cdot \int_0^{\frac{35}{T}} \left(\left| \frac{2 \cdot \sin\left(\frac{\omega \cdot T}{2}\right)}{\omega} \right| \right)^2 d\omega}{\frac{1}{\pi} \cdot \int_0^{\infty} \left(\left| \frac{2 \cdot \sin\left(\frac{\omega \cdot T}{2}\right)}{\omega} \right| \right)^2 d\omega}$$

- no closed form found for integral
- I tried changing to finite limits and got the same error message

so did I

Since blow-up @ 0 I mentioned this in class and in my book (if you can find it!)

however It does work if you eliminate the | | signs!!

#4

$$\frac{\int_0^{\frac{34}{T}} \left(\frac{2\cdot\sin\left(\omega\cdot\frac{T}{2}\right)}{\omega}\right)^2 d\omega}{\int_0^{\infty} \left(\frac{2\cdot\sin\left(\omega\cdot\frac{T}{2}\right)}{\omega}\right)^2 d\omega}$$

$$\frac{1}{17}\cdot\frac{(-1+\cos(34)+34\cdot\mathrm{Si}(34))}{(\mathrm{signum}(T)\cdot\pi)}$$

$$\frac{1}{17}\cdot\frac{(-1+\cos(34)+34\cdot\mathrm{Si}(34))}{(\mathrm{signum}(T)\cdot\pi)}$$

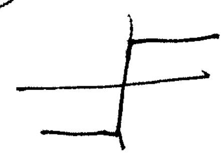
$$\frac{.98095879455122605235}{\mathrm{signum}(T)}$$

← a little low

$$\frac{\int_0^{\frac{3}{T}} \left(\frac{2\cdot\sin\left(\omega\cdot\frac{T}{2}\right)}{\omega}\right)^2 d\omega}{\int_0^{\infty} \left(\frac{2\cdot\sin\left(\omega\cdot\frac{T}{2}\right)}{\omega}\right)^2 d\omega}$$

$$\frac{.75459922816211845107}{\mathrm{signum}(T)}$$

sig T



$$\cancel{\frac{.74010196497890650231}{\mathrm{signum}(T)}}$$

$\frac{65}{T}$ close to .9900

3dB bandwidth

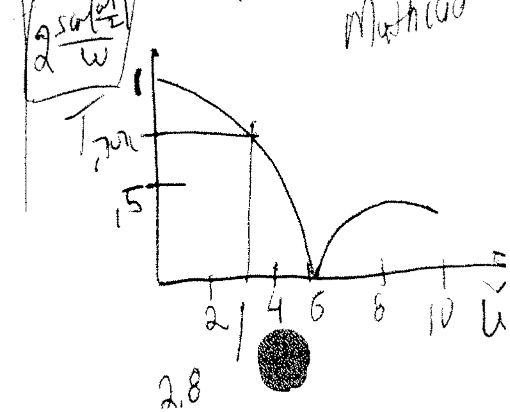
$$\left|\frac{X(\omega_1)}{X(0)}\right| = \frac{1}{\sqrt{2}}$$

$H(0) = \frac{0}{0}$ L'Hospital's

$$\frac{\frac{T}{2}\times 2\cos\frac{\omega T}{2}}{1} = T$$

$$\left|\frac{\frac{2\sin\left(\frac{\omega T}{2}\right)}{\omega}}{T}\right| = \frac{1}{\sqrt{2}}$$

Mathcad



$$\omega = \frac{2.8}{T}$$

Table 5 #4 75% & 99%

$$\frac{2\cdot\sin\left(\frac{\omega\cdot T}{2}\right)}{\omega}$$

Proven by plugging in value to the integral

$$100\cdot\frac{\left[\frac{1}{\pi}\cdot\int_0^{\frac{2.9}{T}}\left[\left(\frac{2\cdot\sin\left(\frac{\omega\cdot T}{2}\right)}{\omega}\right)^2\right]d\omega\right]}{\frac{1}{\pi}\cdot\int_0^{\infty}\left[\left(\frac{2\cdot\sin\left(\frac{\omega\cdot T}{2}\right)}{\omega}\right)^2\right]d\omega}$$

$$\frac{232.50988960851031024}{\pi\cdot\underbrace{\mathrm{signum}(T)}_{\text{input}}}$$

$$\frac{232.5099}{\pi}$$

74.010199805604550658

This is approximately 75%

3/7 better

Si function shown on next page

$$100\cdot\frac{\left[\frac{1}{\pi}\cdot\int_0^{\frac{35}{T}}\left[\left(\frac{2\cdot\sin\left(\frac{\omega\cdot T}{2}\right)}{\omega}\right)^2\right]d\omega\right]}{\frac{1}{\pi}\cdot\int_0^{\infty}\left[\left(\frac{2\cdot\sin\left(\frac{\omega\cdot T}{2}\right)}{\omega}\right)^2\right]d\omega}$$

$$\frac{40}{7\cdot\pi}\cdot\frac{(-1+\cos(35)+35\cdot\mathrm{Si}(35))}{\mathrm{signum}(T)}$$

input

$$\frac{40}{7\cdot\pi}\cdot(-1+\cos(35)+35\cdot1.597)$$

98.205525937779043319

This is approximately 99%

85/7 better

Could be improved 62/7

Table 5
#1 BWrms

Ken Kaiser

$$\sqrt{\frac{\int_{-\infty}^{\infty}\left[\left|\frac{1}{(i\cdot\omega)+a}\right|\right]^2\cdot\omega^2\,d\omega}{\int_{-\infty}^{\infty}\left[\left|\frac{1}{(i\cdot\omega)+a}\right|\right]^2\,d\omega}}$$

$$\frac{1}{\sqrt{|a|}}\cdot a\cdot\infty$$

Table 5
#2 BW_{rms}

My Mathcad
$a = 3$ ✓
$\sqrt{9}$
$a = 7$
7 ✓

$$\sqrt{\frac{\int_{-\infty}^{\infty}\left[\left|\frac{1}{((i\cdot\omega)+a)^2}\right|\right]^2\cdot\omega^2\,d\omega}{\int_{-\infty}^{\infty}\left[\left|\frac{1}{((i\cdot\omega)+a)^2}\right|\right]^2 d\omega}}$$

a

$$BW_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty}\omega^2|X(\omega)|^2 d\omega}{\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega}}$$

$|X(\omega)|^2$ must drop off faster than $1/\omega^2$

$$\frac{1}{(s+a)^2} = \frac{1}{(j\omega+a)^2} = \frac{1}{-\omega^2+2aj\omega+a^2}$$

$$|X(\omega)| = \sqrt{\frac{1}{(a^2-\omega^2)^2+(2a\omega)^2}}$$

$$|X(\omega)|^2 = \frac{1}{(a^2-\omega^2)^2+(2a\omega)^2}$$ drops off at ω^4 ✓

If $X(\omega) = \frac{1}{j\omega+a}$ $|X(\omega)|^2 = \frac{1}{\omega^2+a^2}$ drops off @ ω^2

Table 5

#.3 BW_{rms}

Ken Kaiser

$$\sqrt{\frac{\displaystyle\int_{-\infty}^{\infty}\left(\left|a\cdot\sqrt{2\cdot\pi}\cdot e^{\frac{-a^2\cdot\omega^2}{2}}\right|\right)^2\cdot\omega^2\,d\omega}{\displaystyle\int_{-\infty}^{\infty}\left(\left|a\cdot\sqrt{2\cdot\pi}\cdot e^{\frac{-a^2\cdot\omega^2}{2}}\right|\right)^2\,d\omega}}$$

$$\frac{1}{2}\cdot\frac{\sqrt{2}}{a}$$

$$= \frac{.707}{a} \approx \frac{.71}{a}$$

Table #5
#4 BW_{rms}

Ken Kaiser

$$\sqrt{\frac{\displaystyle\int_{-\infty}^{\infty}\left(\frac{2\cdot\sin\left(\frac{\omega\cdot a}{2}\right)}{\omega}\right)^2\cdot\omega^2\,d\omega}{\displaystyle\int_{-\infty}^{\infty}\left(\frac{2\cdot\sin\left(\frac{\omega\cdot a}{2}\right)}{\omega}\right)^2\,d\omega}}$$

$$\frac{1}{\left(\sqrt{\mathrm{signum}(a)}\cdot\sqrt{a}\right)}\cdot\infty$$

Table 5
4 BW3d

$$\frac{\left|\dfrac{2\cdot\sin\left(\dfrac{\omega\cdot T}{2}\right)}{\omega}\right|}{T} = \frac{1}{\sqrt{2}}$$

$$\omega := \frac{2.2}{\blacksquare} \qquad \omega := \frac{2.3}{\blacksquare}$$

$$\frac{\left|\dfrac{2\cdot\sin\left(\dfrac{\frac{2.2}{T}\cdot T}{2}\right)}{\dfrac{2.2}{T}}\right|}{T} = \frac{1}{\sqrt{2}} \qquad \frac{\left|\dfrac{2\cdot\sin\left(\dfrac{\frac{2.3}{T}\cdot T}{2}\right)}{\dfrac{2.3}{T}}\right|}{T} = \frac{1}{\sqrt{2}}$$

$$.81018850914675939995\cdot\frac{|T|}{T} = \frac{1}{2}\cdot\sqrt{2} \qquad .79370777413958354865\cdot\frac{|T|}{T} = \frac{1}{2}\cdot\sqrt{2}$$

$$.81018850914675939995\cdot\frac{|T|}{T} = .705 \qquad .794\cdot\frac{|T|}{T} = .705$$

$$\omega := \frac{2.5}{\blacksquare} \qquad \omega := \frac{2.8}{\blacksquare}$$

shows 2.8 will get closer than 2.2 or 2.3

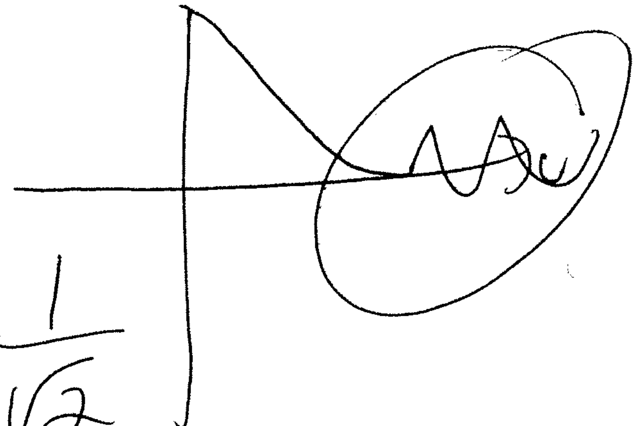
$$\frac{\left|\dfrac{2\cdot\sin\left(\dfrac{\frac{2.5}{T}\cdot T}{2}\right)}{\dfrac{2.5}{T}}\right|}{T} = \frac{1}{\sqrt{2}} \qquad \frac{\left|\dfrac{2\cdot\sin\left(\dfrac{\frac{2.8}{T}\cdot T}{2}\right)}{\dfrac{2.8}{T}}\right|}{T} = \frac{1}{\sqrt{2}}$$

$$.75918769548446897148\cdot\frac{|T|}{T} = \frac{1}{2}\cdot\sqrt{2} \qquad .70389266427747155761\cdot\frac{|T|}{T} = \frac{1}{2}\cdot\sqrt{2}$$

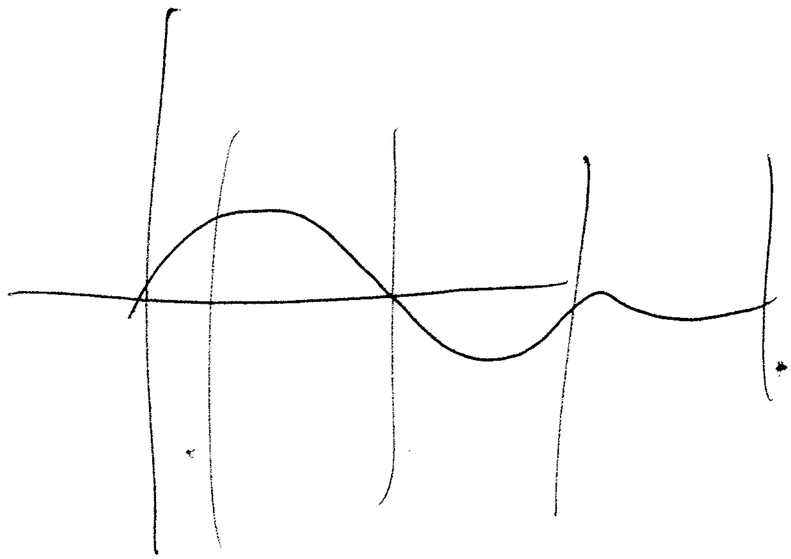
$$.759\cdot\frac{|T|}{T} = .705 \qquad .704\cdot\frac{|T|}{T} = .705$$

$$\frac{2\frac{T}{2}}{1}$$

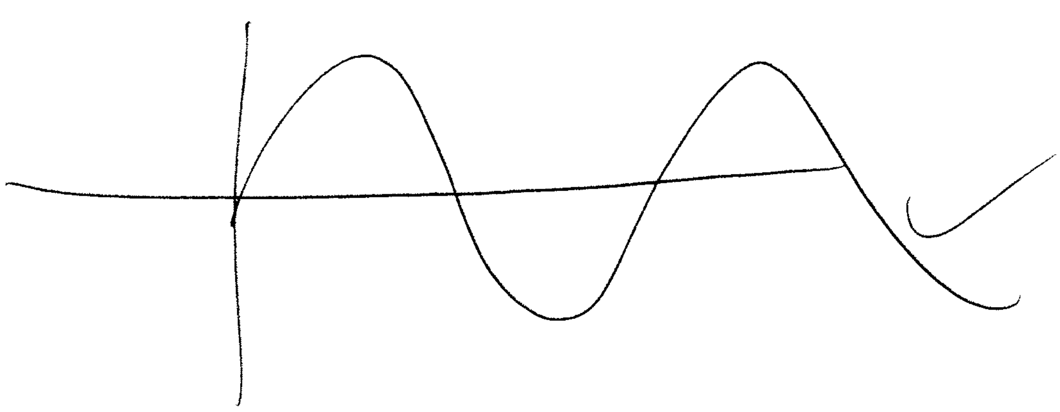
$$\frac{\left|\frac{2\sin\left(\omega\frac{T}{2}\right)}{\omega}\right|}{T} = \frac{1}{\sqrt{2}}$$



$$\lambda = \frac{2\pi}{T}$$



$$\frac{\int_0^\infty \left|\frac{2\sin\left(\frac{\omega T}{2}\right)}{\omega}\right|^2 d\omega}{T^2}$$



p446 ~~3rd~~ 4th on left side

$$\frac{\dfrac{n!}{(i\cdot\omega)^{n}}}{n!\cdot\left(-\dfrac{i}{\omega}\right)^{n+1}}$$

Comparing your table with ours on p C-128 of the transform and applying the final one yields

$i\cdot\omega$, not 1.

It should be $n!\left(-\frac{i}{\omega}\right)^{n+1}$ instead of $\frac{n!}{(i\omega)^{n}}$

p 446

$$\frac{x}{3^2 + x^2}$$

$$i \cdot \exp(3 \cdot \omega) \cdot \pi \cdot \Phi(-\omega) - i \cdot \exp(-3 \cdot \omega) \cdot \pi \cdot \Phi(\omega) = j e^{3w} \pi \, \Phi(-w) - j e^{-3w} \pi \, \Phi w$$

$$= j \pi \left[e^{3w} \Phi(w) - e^{-3w} \Phi(w) \right]$$

not +

$$\frac{x}{11^2 + x^2}$$

$$i \cdot \exp(11 \cdot \omega) \cdot \pi \cdot \Phi(-\omega) - i \cdot \exp(-11 \cdot \omega) \cdot \pi \cdot \Phi(\omega)$$

r(t) yields 2a

$$\int_{-a}^{a} 1^2\,dt$$

$$2\cdot a$$

x(t) yields 2a via piece wise solution

$$\int_{-3\cdot a}^{0} \left(\frac{t}{3\cdot a}+1\right)^2 dt + \int_{0}^{3\cdot a} \left(\frac{-t}{3\cdot a}+1\right)^2 dt$$

$$2\cdot a$$

y(t) gives 2a

$$\int_{\frac{-8\cdot a}{3}}^{\frac{8\cdot a}{3}} \left[\frac{1}{2}\cdot\left(1+\cos\left(\frac{3\cdot\pi}{8\cdot a}\cdot t\right)\right)\right]^2 dt$$

$$2\cdot a$$

z(t) is a number close to 2a, but it was not using ∞, just a large num because it gets minutely small

$$\int_{0}^{1000000} \left(e^{\frac{t}{2\cdot a}}\right)^2 dt + \int_{-1000000}^{0} e^{\frac{t}{2\cdot a}}\,dt$$

$$\exp\left(\frac{500000}{a}\right)^2\cdot a + a - 2\cdot\exp\left(\frac{-500000}{a}\right)\cdot a$$

Should be 2a not 4a

p 454

Quadratic

$$(s+a)^2 + b^2 = 0$$

$$s^2 + 2\cdot s\cdot a + a^2 + b^2 = 0$$

$$\begin{pmatrix} -a + \sqrt{-b^2} \\ -a - \sqrt{-b^2} \end{pmatrix}$$

$$\begin{pmatrix} -a + i \cdot b \\ -a - i \cdot b \end{pmatrix}$$

$$= \left(j\frac{A}{T}\pi\frac{d\delta(\omega)}{d\omega}\right)\boxed{-\frac{A}{T\omega^2}}\left(-j\frac{A}{T}\pi\frac{d\delta(\omega)}{d\omega}\right) + j\frac{A}{T}(jT)\pi\delta(\omega)\boxed{+\frac{A}{T\omega^2}e^{-j\omega T}} - A\pi\delta(\omega)\boxed{-\frac{A}{j\omega}e^{-j\omega T}}$$

$$= \boxed{-\frac{A}{T\omega^2}}\boxed{+\frac{A}{T\omega^2}e^{-j\omega T}}\boxed{-\frac{A}{j\omega}e^{-j\omega T}}$$

$$-j\frac{A}{T}(jT)\pi\delta(\omega) - A\pi\delta(\omega)$$

$$-\frac{\overbrace{j\cdot A\cdot j}^{-1}\cdot \cancel{T}}{\cancel{T}}$$

$$+A\pi\delta(\omega) - A\pi\delta(\omega)$$

$$-2A\pi\delta(\omega)$$

p 468

$\cos(w \cdot t)$

$\pi \cdot (\mathrm{Dirac}(\omega - w) + \mathrm{Dirac}(\omega + w))$

$A \cdot \cos(w \cdot t)$

$A \cdot \pi \cdot (\mathrm{Dirac}(\omega - w) + \mathrm{Dirac}(\omega + w))$

p468

$$\int_{-\infty}^{\infty} \mathrm{Dirac}\left(\omega+\frac{2\cdot\pi}{T}\right)\,d\omega$$

$$1$$

$$\int_{-\infty}^{\infty} \mathrm{Dirac}\left(\omega-\frac{2\cdot\pi}{T}\right)\,d\omega$$

$$1$$

$$\frac{A^2}{4}\cdot\int_{-\infty}^{\infty} \mathrm{Dirac}\left(\omega-\frac{2\cdot\pi}{T}\right)\,d\omega+\frac{A^2}{4}\cdot\int_{-\infty}^{\infty} \mathrm{Dirac}\left(\omega+\frac{2\cdot\pi}{T}\right)\,d\omega$$

$$\frac{1}{2}\cdot A^2$$

p 46a
Figure 15

$$a := 1$$

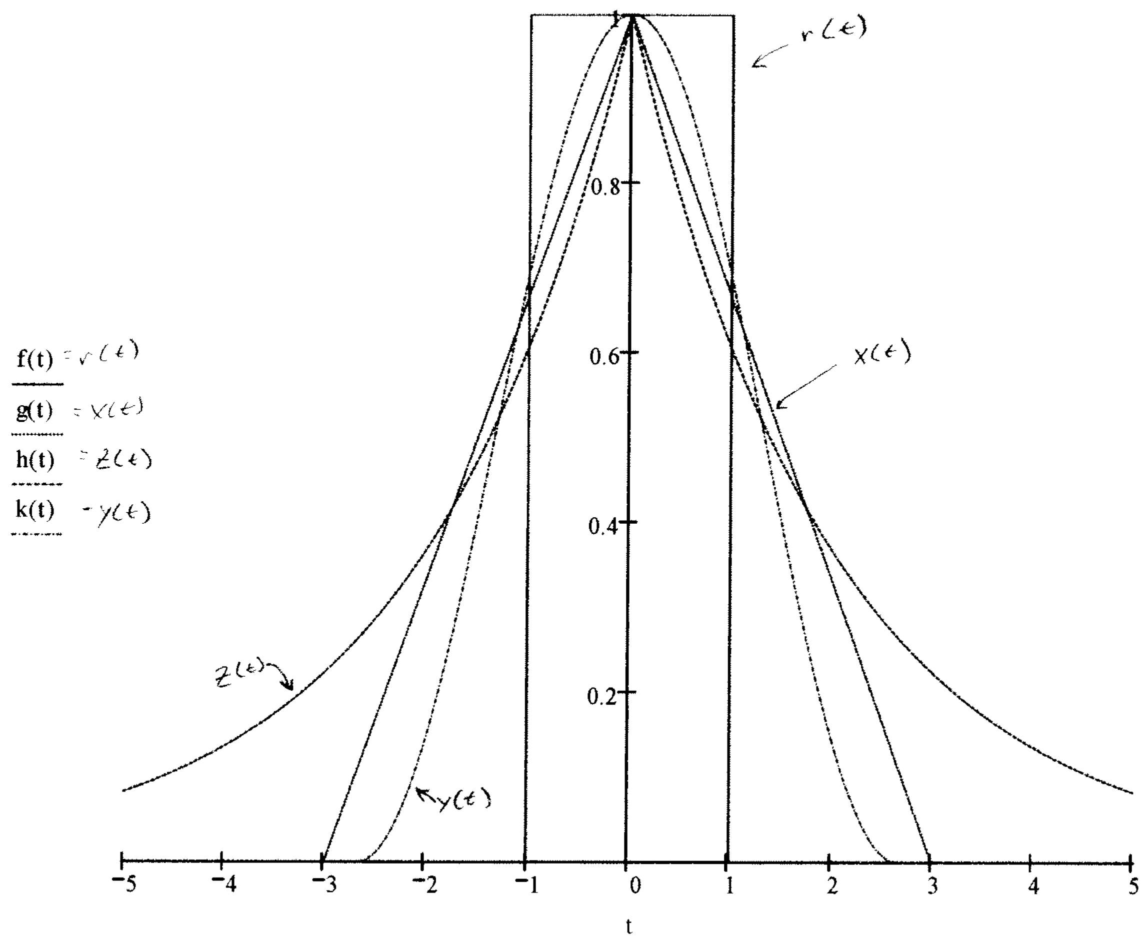
$$t := -5\cdot a, -5\cdot a + \frac{a}{1000} .. 5\cdot a$$

$$f(t) := \Phi(t+a) - \Phi(t-a)$$

$$g(t) := \left(\frac{t}{3\cdot a} + 1\right)\cdot\left(\Phi(t+3\cdot a) - \Phi(t)\right) + \left(\frac{-t}{3\cdot a} + 1\right)\cdot\left(\Phi(t) - \Phi(t-3\cdot a)\right)$$

$$h(t) := e^{\frac{t}{2\cdot a}}\cdot\Phi(-t) + e^{\frac{-t}{2\cdot a}}\cdot\Phi(t)$$

$$k(t) := \left(\frac{1}{2} + \frac{1}{2}\cdot\cos\left(\frac{3\cdot\pi}{8\cdot a}\cdot t\right)\right)\cdot\left(\Phi\left(t + \frac{8\cdot a}{3}\right) - \Phi\left(t - \frac{8\cdot a}{3}\right)\right)$$



pg. 469

$Y(t)$

$$\int_0^{3\cdot a} \left(\frac{-t}{3\cdot a}+1\right)^2 dt + \int_{-3\cdot a}^{0} \left(\frac{t}{3\cdot a}+1\right)^2 dt$$

$$a + a = 2a$$

$\Pi(t)$ $X(t)$

$$\int_{-a}^{a} 1^2\, dt \qquad \int_{\frac{-8\cdot a}{3}}^{\frac{8\cdot a}{3}} \left[\frac{1}{2}\cdot\left(1+\cos\left(\frac{3\cdot\pi}{8\cdot a}\cdot t\right)\right)\right]^2 dt$$

$2\cdot a\cdot t$ $2\cdot a$

p470
Figure 16

Ken Kaiser

$a := 1.5$

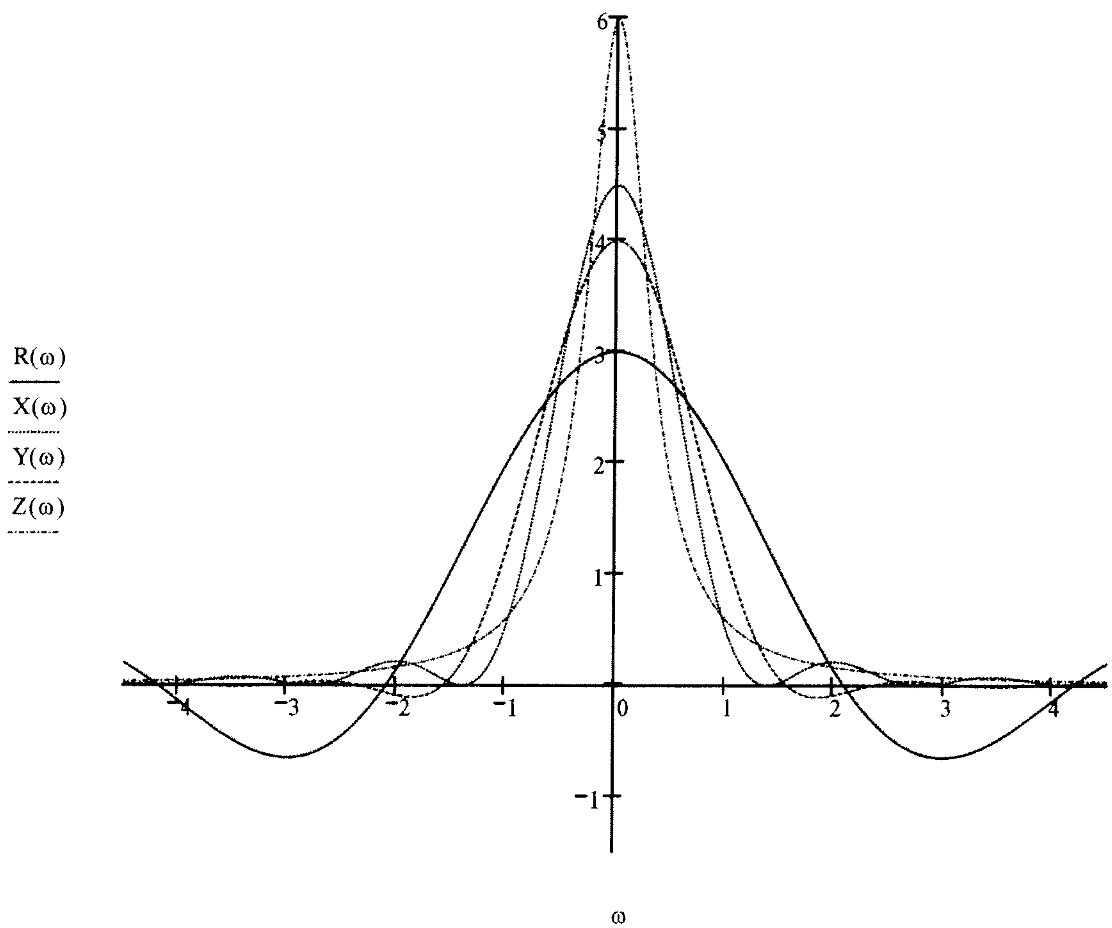
$\omega := -5 \cdot a, -5 \cdot a + \frac{a}{1000} .. 5 \cdot a$

$R(\omega) := 2 \cdot \frac{\sin(a \cdot \omega)}{\omega}$

$X(\omega) := 3 \cdot a \cdot \left(\frac{\sin\left(3 \cdot a \cdot \frac{\omega}{2}\right)}{3 \cdot a \cdot \frac{\omega}{2}} \right)^2$

$Y(\omega) := 9 \cdot \pi^2 \cdot \frac{\sin\left(8 \cdot a \cdot \frac{\omega}{3}\right)}{\omega \cdot \left[(9) \cdot \pi^2 - 64 \cdot a^2 \cdot \omega^2\right]}$

$Z(\omega) := 4 \cdot \frac{a}{1 + 4 \cdot a^2 \cdot \omega^2}$



p 472
Figure 1a

$a := 0.1$

$L := 10^{-4}$

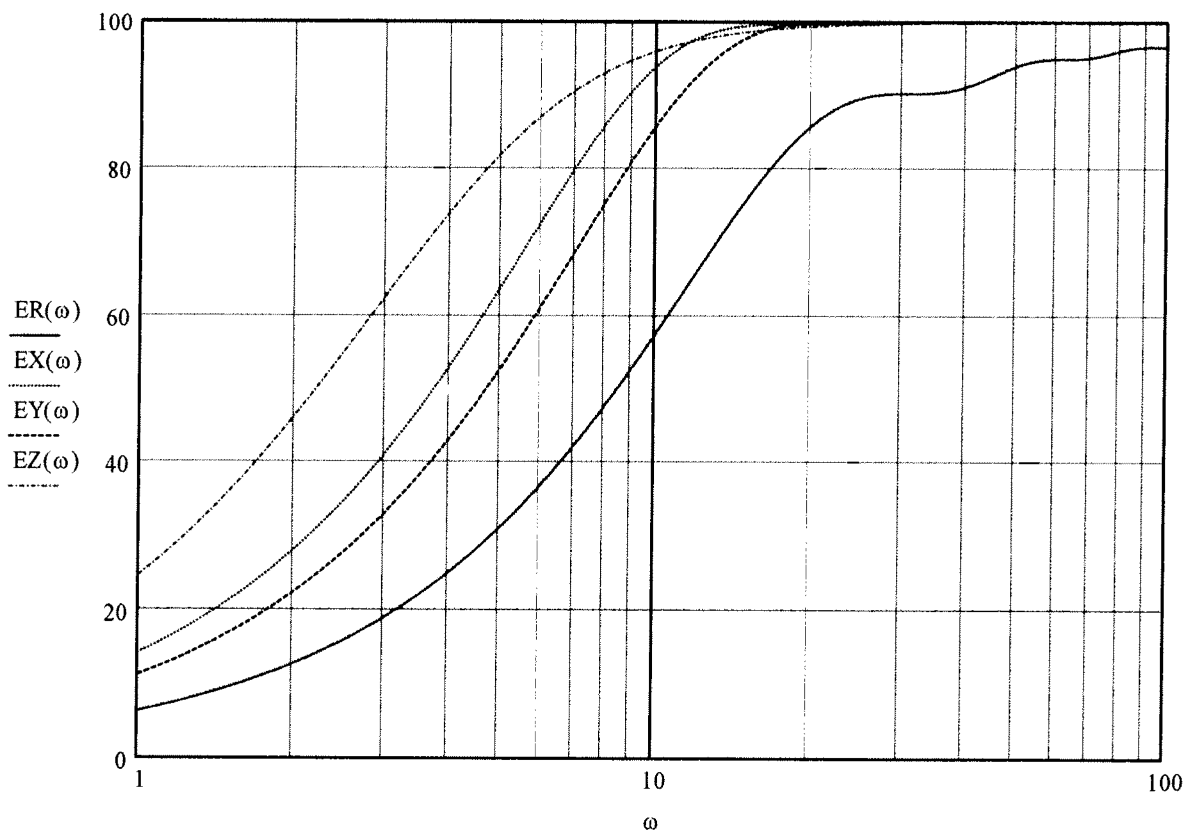
$\omega := \frac{1}{10 \cdot a}, \frac{1.1}{10 \cdot a} .. \frac{10}{a}$

$$ER(\omega) := \frac{\dfrac{\int_L^{\omega} \left(\left| 2 \cdot \frac{\sin(a \cdot \omega)}{\omega} \right| \right)^2 d\omega}{\dfrac{\pi}{2 \cdot a}}} {} \cdot 100$$

$$EX(\omega) := \frac{\int_L^{\omega} \left[\left| 3 \cdot a \left(\frac{\sin\left(3 \cdot a \cdot \frac{\omega}{2}\right)}{3 \cdot a \cdot \frac{\omega}{2}} \right)^2 \right| \right]^2 d\omega}{\frac{\pi}{2 \cdot a}} \cdot 100$$

$$EY(\omega) := \frac{\int_L^{\omega} \left[\left| 9 \cdot \pi^2 \cdot \frac{\sin\left(8 \cdot a \cdot \frac{\omega}{3}\right)}{\omega \cdot \left(9 \cdot \pi^2 - 64 \cdot a^2 \cdot \omega^2\right)} \right| \right]^2 d\omega}{\frac{\pi}{2 \cdot a}} \cdot 100$$

$$EZ(\omega) := \frac{\int_L^{\omega} \left(\left| 4 \cdot \frac{a}{1 + 4 \cdot a^2 \cdot \omega^2} \right| \right)^2 d\omega}{\frac{\pi}{2 \cdot a}} \cdot 100$$



Pg 471
Figure 18

$a := 1$

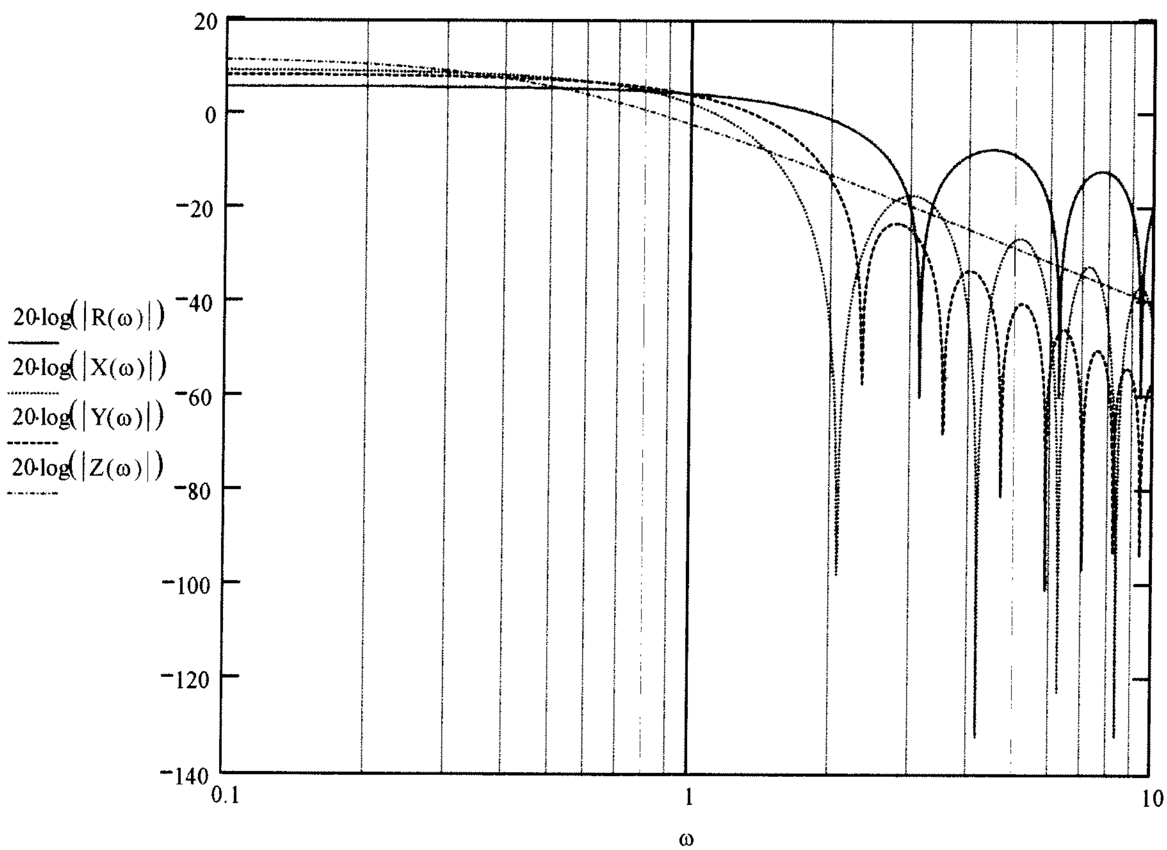
$\omega := 0.1, 0.11 .. 10$

$$R(\omega) := 2 \cdot \frac{\sin(a \cdot \omega)}{\omega}$$

$$X(\omega) := 3 \cdot a \cdot \left(\frac{\sin\left(3 \cdot a \cdot \frac{\omega}{2}\right)}{3 \cdot a \cdot \frac{\omega}{2}} \right)^2$$

$$Y(\omega) := 9 \cdot \pi^2 \cdot \frac{\sin\left(8 \cdot a \cdot \frac{\omega}{3}\right)}{\omega \cdot \left[(9) \cdot \pi^2 - 64 \cdot a^2 \cdot \omega^2\right]}$$

$$Z(\omega) := 4 \cdot \frac{a}{1 + 4 \cdot a^2 \cdot \omega^2}$$



p 470

Ken Kaiser

As x gets larger the result blows up

$$20\cdot\log\left(\frac{\sin(2)}{2}\right) = -6.846$$

$$20\cdot\log\left(\frac{\sin(20)}{20}\right) = -26.812$$

$$20\cdot\log\left(\frac{\sin(2000)}{2000}\right) = -66.651$$

$$20\cdot\log\left(\frac{\sin(20000)}{20000}\right) = -90.722$$

As x gets smaller the result tends to zero

$$20\cdot\log\left(\frac{\sin(.2)}{.2}\right) = -0.058 \qquad -20\cdot\log(.2) = 13.979$$

$$20\cdot\log\left(\frac{\sin(.02)}{.02}\right) = -5.791\cdot 10^{-4} \qquad -20\cdot\log(.02) = 33.979$$

$$20\cdot\log\left(\frac{\sin(.0002)}{.0002}\right) = -5.791\cdot 10^{-8} \qquad -20\cdot\log(.0002) = 73.979$$

$$20\cdot\log\left(\frac{\sin(.000002)}{.000002}\right) = -5.79\cdot 10^{-12} \qquad -20\cdot\log(.000002) = 113.979$$

$$20\cdot\log\left(\frac{\sin(.00000002)}{.00000002}\right) = 0 \qquad -20\cdot\log(.00000002) = 153.979$$

pg. 476

Ken Kaiser

$$i := \sqrt{-1}$$

$$|\cos(45) + i\cdot\sin(45)| = 1$$

$$\sqrt{\cos(45)^2 + (\sin(45)\cdot i)^2} = 0.669i$$

$$\sqrt{\cos(45)^2 + (\sin(45))^2} = 1$$

$$\sqrt{\cos(45)^2 + (\sin(45))^2\cdot -1} = 0.669i$$

$$\sqrt{\cos(90)^2 + \sin(90)^2} = 1$$

$$\sqrt{\cos(125)^2 + \sin(125)^2} = 1$$

Ken Kaiser

p47a
Proof
of
Mathcad 1

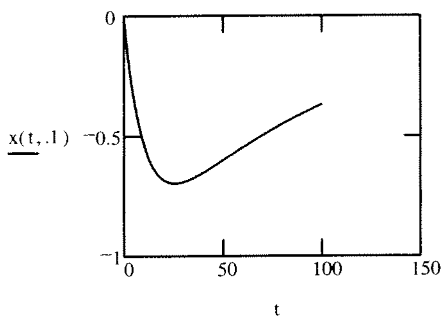
$C := 1$

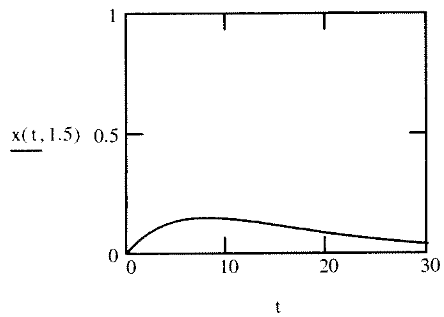
$\alpha := 0.1$

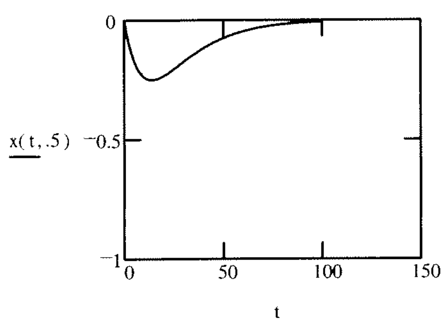
$t := 0, 0.2 .. 100$

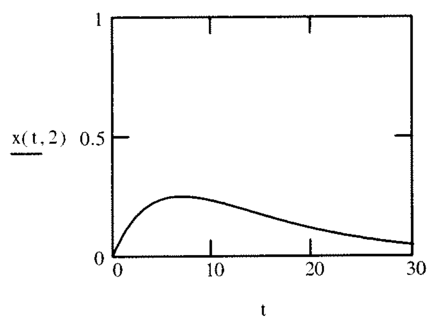
$x(t,k) := C \cdot \left(e^{-\alpha \cdot t} - e^{-\alpha \cdot k \cdot t}\right)$

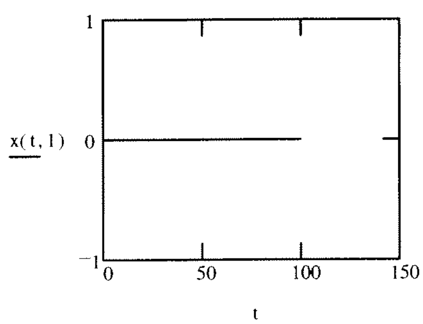
$y(t) := C \cdot e^{-\alpha \cdot t}$

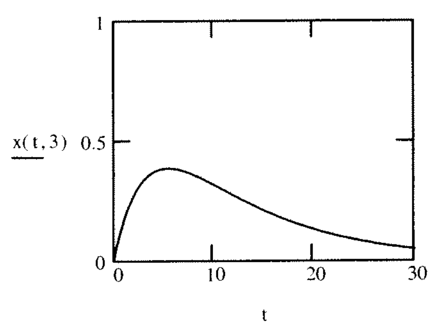


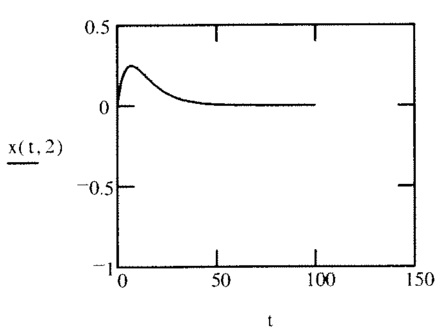


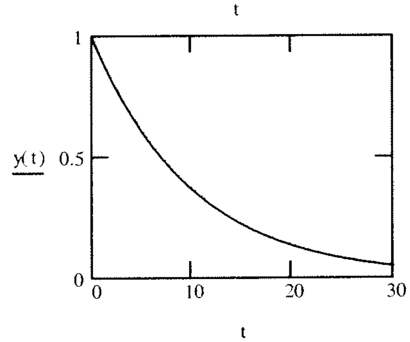












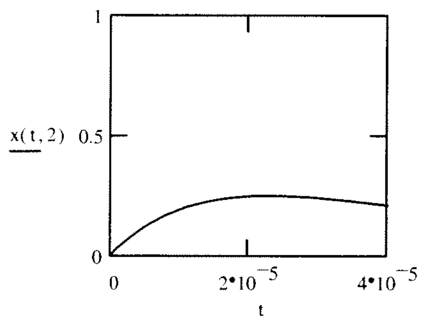
Mathcad #2

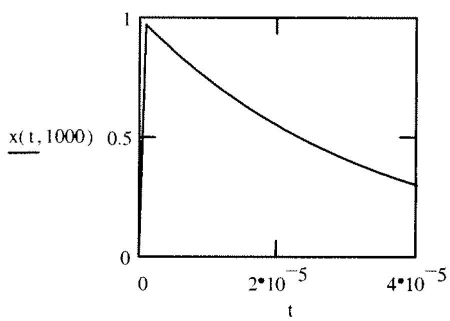
Ken Kaiser

$C := 1 \qquad \alpha := 30000$

$t := 0, 1\cdot 10^{-6} .. 2\cdot 10^{-4}$

$x(t,k) := C\cdot\left(e^{-\alpha\cdot t} - e^{-\alpha\cdot k\cdot t}\right)$





$h := 1\cdot 10^{-10}$

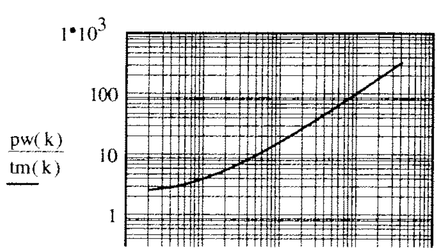
$g := 5\cdot 10^{-5}$

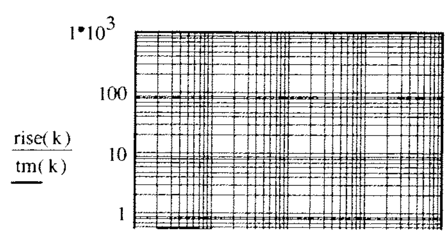
$k := 2, 4 .. 4000$

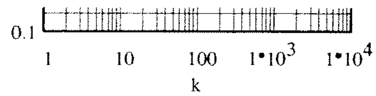
$A(k) := \frac{-\ln(k)}{k-1} \qquad tm(k) := \frac{-A(k)}{\alpha} \qquad d(t,k) := \frac{t}{tm(k)} \qquad xr(t,k) := \frac{e^{A(k)\cdot d(t,k)} - e^{k\cdot A(k)\cdot d(t,k)}}{e^{A(k)} - e^{k\cdot A(k)}}$

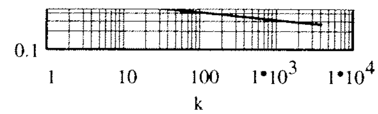
$pw1(k) := root(xr(g,k) - .5, g) \qquad pw2(k) := root(xr(h,k) - .5, h)$

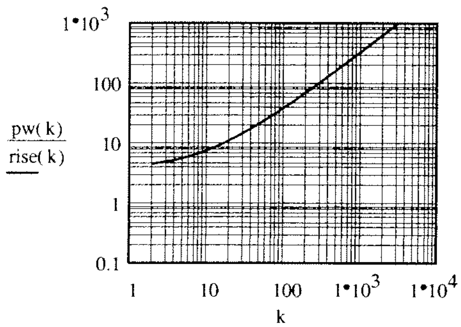
$pw(k) := pw1(k) - pw2(k) \qquad rise(k) := root(xr(h,k) - .9, h) - root(xr(h,k) - .1, h)$






0.1
1
10
100
1•10^3
1•10^4
k


0.1
1
10
100
1•10^3
1•10^4
k


1•10^3
100
10
1
0.1
pw(k)
rise(k)
1
10
100
1•10^3
1•10^4
k

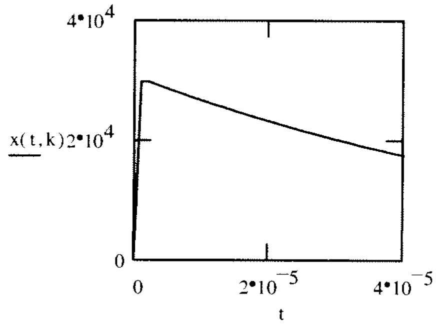
Midterm #3

Ken Kaiser

$C := 30680 \quad \alpha := 14000 \quad k := 300$

$t := 0, 1\cdot10^{-6} .. 2\cdot10^{-4}$

$x(t,k) := C\cdot\left(e^{-\alpha\cdot t} - e^{-\alpha\cdot k\cdot t}\right)$



$h := 1\cdot10^{-10}$

$g := 5\cdot10^{-5}$

$A(k) := \dfrac{-\ln(k)}{k-1} \quad tm(k) := \dfrac{-A(k)}{\alpha} \quad d(t,k) := \dfrac{t}{tm(k)} \quad xr(t,k) := \dfrac{e^{A(k)\cdot d(t,k)} - e^{k\cdot A(k)\cdot d(t,k)}}{e^{A(k)} - e^{k\cdot A(k)}}$

$pw1(k) := root(xr(g,k) - .5, g) \quad pw2(k) := root(xr(h,k) - .5, h)$

$pw(k) := pw1(k) - pw2(k) \quad rise(k) := root(xr(h,k) - .9, h) - root(xr(h,k) - .1, h)$

$pw(k) = 5.094\cdot10^{-5} \quad rise(k) = 4.948\cdot10^{-7} \quad x(tm(k),k) = 3\cdot10^{4}$

py
495

$$\frac{C\cdot\left(0+0+\frac{1}{\alpha+i\cdot\omega}-\frac{1}{\beta+i\cdot\omega}\right)}{\left[\frac{C\cdot(\beta-\alpha)}{(\alpha+i\cdot\omega)\cdot(\beta+i\cdot\omega)}\right]}$$

1

$$\left[\frac{C\cdot(\beta-\alpha)}{(\alpha+i\cdot\omega)\cdot(\beta+i\cdot\omega)}\right]$$

$$(\alpha+i\cdot\omega)\cdot(\beta+i\cdot\omega)$$

$$(\alpha+i\cdot\omega)\cdot(\beta+i\cdot\omega)$$

$$\alpha\cdot\beta+i\cdot\omega\cdot\alpha+i\cdot\omega\cdot\beta-\omega^2$$

$$\frac{\frac{C\cdot(\beta-\alpha)}{(\alpha+i\cdot\omega)\cdot(\beta+i\cdot\omega)}}{\left[\frac{C\cdot(\beta-\alpha)}{\alpha\cdot\beta\cdot\left(1+\frac{i\cdot\omega}{\alpha}\right)\cdot\left(1+\frac{i\cdot\omega}{\beta}\right)}\right]}$$

1

Section 11.10 – 11.13 Check

Ken Kaiser

Page 105

$N := 3.25 \quad V_s := 5.42 \quad R_1 := 3 \quad R_2 := 4.5 \quad C_1 := 2.65 \quad C_2 := 1.38 \quad s := (1.7 + i \cdot 2.6)$

$$\frac{N \cdot V_s}{s} \cdot \frac{\dfrac{R_2}{1 + s \cdot R_2 \cdot C_2}}{\left(\dfrac{R_2}{1 + s \cdot R_2 \cdot C_2}\right) + R_1 + \dfrac{1}{s \cdot C_1}} = -0.1111 - 0.38447i$$

$$\frac{\dfrac{N \cdot V_s}{R_1 \cdot C_2}}{s^2 + s \cdot \left(\dfrac{R_2 \cdot C_1 + R_1 \cdot C_1 + R_2 \cdot C_2}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right) + \dfrac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}} = -0.1111 - 0.38447i$$

The two equations for the voltage over capacitor 2 are assigned the same variable values and compared. The final answers for the equations are the same implying that they are equivalent.

$N := 4.99 \quad V_s := 12 \quad R_1 := 2.89 \quad R_2 := 7.63 \quad C_1 := 2.38 \quad C_2 := 5.27 \quad s := (8.7)$

$$\frac{N \cdot V_s}{s} \cdot \frac{\dfrac{R_2}{1 + s \cdot R_2 \cdot C_2}}{\left(\dfrac{R_2}{1 + s \cdot R_2 \cdot C_2}\right) + R_1 + \dfrac{1}{s \cdot C_1}} = 0.05057$$

$$\frac{\dfrac{N \cdot V_s}{R_1 \cdot C_2}}{s^2 + s \cdot \left(\dfrac{R_2 \cdot C_1 + R_1 \cdot C_1 + R_2 \cdot C_2}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right) + \dfrac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}} = 0.05057$$

New variable values are fed into the same equations and again their result is the same. Note that this time an entirely real value for "s" is used.

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$$v_c(s) := \frac{\dfrac{N \cdot V_s}{R_1 \cdot C_2}}{s^2 + s \cdot \left(\dfrac{R_2 \cdot C_1 + R_1 \cdot C_1 + R_2 \cdot C_2}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right) + \dfrac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}}$$

$$\left(\frac{e^{-a \cdot t} - e^{-b \cdot t}}{b - a}\right)$$

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$N := 3.25 \quad V_s := 5.42 \qquad R_1 := 3 \qquad R_2 := 4.5 \qquad C_1 := 2.65 \qquad C_2 := 1.38 \qquad s := (1.7 + i \cdot 2.6)$

$$s^2 + s\cdot\left(\frac{R_2\cdot C_1 + R_1\cdot C_1 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right) + \frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2} = 0$$

The roots of the denominator are taken and compared to the given root values. The results are of opposite signs but that is due to a differing solution format

$$\left[\begin{array}{l} \frac{-1}{\left[2\cdot\left(R_1\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_2\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_1\cdot C_1\right)\right]} \ldots \\ + \left[\frac{1}{2}\cdot\frac{\sqrt{R_2^{\,2}\cdot C_1^{\,2} + 2\cdot R_2\cdot C_1^{\,2}\cdot R_1 + 2\cdot R_2^{\,2}\cdot C_1\cdot C_2 + R_1^{\,2}\cdot C_1^{\,2} - 2\cdot R_1\cdot R_2\cdot C_1\cdot C_2 + R_2^{\,2}\cdot C_2^{\,2}}}{\left[R_1\cdot\left[R_2\cdot\left(C_1\cdot C_2\right)\right]\right]}\right] \\ \frac{-1}{\left[2\cdot\left(R_1\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_2\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_1\cdot C_1\right)\right]} \ldots \\ + \frac{-1}{2}\cdot\frac{\sqrt{R_2^{\,2}\cdot C_1^{\,2} + 2\cdot R_2\cdot C_1^{\,2}\cdot R_1 + 2\cdot R_2^{\,2}\cdot C_1\cdot C_2 + R_1^{\,2}\cdot C_1^{\,2} - 2\cdot R_1\cdot R_2\cdot C_1\cdot C_2 + R_2^{\,2}\cdot C_2^{\,2}}}{\left[R_1\cdot\left[R_2\cdot\left(C_1\cdot C_2\right)\right]\right]} \end{array}\right]$$

$$\begin{array}{l} \frac{-1}{\left[2\cdot\left(R_1\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_2\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_1\cdot C_1\right)\right]} \ldots \\ + \left[\frac{1}{2}\cdot\frac{\sqrt{R_2^{\,2}\cdot C_1^{\,2} + 2\cdot R_2\cdot C_1^{\,2}\cdot R_1 + 2\cdot R_2^{\,2}\cdot C_1\cdot C_2 + R_1^{\,2}\cdot C_1^{\,2} - 2\cdot R_1\cdot R_2\cdot C_1\cdot C_2 + R_2^{\,2}\cdot C_2^{\,2}}}{\left[R_1\cdot\left[R_2\cdot\left(C_1\cdot C_2\right)\right]\right]}\right] \end{array} = -0.04161$$

$$\begin{array}{l} \frac{-1}{\left[2\cdot\left(R_1\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_2\cdot C_2\right)\right]} - \frac{1}{\left[2\cdot\left(R_1\cdot C_1\right)\right]} \ldots \\ + \frac{-1}{2}\cdot\frac{\sqrt{R_2^{\,2}\cdot C_1^{\,2} + 2\cdot R_2\cdot C_1^{\,2}\cdot R_1 + 2\cdot R_2^{\,2}\cdot C_1\cdot C_2 + R_1^{\,2}\cdot C_1^{\,2} - 2\cdot R_1\cdot R_2\cdot C_1\cdot C_2 + R_2^{\,2}\cdot C_2^{\,2}}}{\left[R_1\cdot\left[R_2\cdot\left(C_1\cdot C_2\right)\right]\right]} \end{array} = -0.48675$$

$$a := -\left[\frac{-\left(\frac{R_2\cdot C_1 + R_1\cdot C_1 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right) + \sqrt{\left(\frac{R_2\cdot C_1 + R_1\cdot C_1 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right)^2 - \frac{4}{R_1\cdot R_2\cdot C_1\cdot C_2}}}{2}\right]$$

$$b := -\left[\frac{-\left(\frac{R_2\cdot C_1 + R_1\cdot C_1 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right) - \sqrt{\left(\frac{R_2\cdot C_1 + R_1\cdot C_1 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right)^2 - \frac{4}{R_1\cdot R_2\cdot C_1\cdot C_2}}}{2}\right]$$

$a = 0.04161$

$b = 0.48675$

$$v_c(t) := \frac{N \cdot V_s}{R_1 \cdot C_2} \cdot \left(\frac{e^{-a \cdot t} - e^{-b \cdot t}}{b - a} \right) \cdot \Phi(t)$$

The constants "a" & "b" are calculated then used to determine the capcitor voltage at various times. It can be seen that for these values, the voltage looks to be a single oscillitory function,i.e. it rises then ramps off to zero.

$v_c(1) = 3.29404$	$v_c(12) = 5.77344$	$v_c(24) = 3.52077$	$v_c(50) = 1.19333$
$v_c(3) = 6.21741$	$v_c(15) = 5.11392$	$v_c(27) = 3.10762$	$v_c(100) = 0.14898$
$v_c(6) = 6.93127$	$v_c(18) = 4.51793$	$v_c(30) = 2.74291$	$v_c(150) = 0.0186$
$v_c(9) = 6.45296$	$v_c(21) = 3.98867$	$v_c(40) = 1.8092$	$v_c(1000) = 0$

$V := 2.45 \qquad \tau_r := 6.34 \qquad C := 1.4 \qquad Z_0 := 8.23 \qquad s := (3.88 + i \cdot 2.6)$

$$\frac{V}{\tau_r} \cdot \frac{1 - e^{-\tau_r \cdot s}}{s^2} \cdot \left(\frac{-s \cdot C \cdot Z_0}{2 + s \cdot C \cdot Z_0} \right) = -0.00685 + 0.01576i$$

$$\frac{-C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} + \frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} \cdot e^{-\tau_r \cdot s} = -0.00685 + 0.01576i$$

The two equations for output frequency response are assigned variable values and solved for. In this case the results are alike which implies that the equations are in fact equivalent.

$V := 12.45 \qquad \tau_r := 16.34 \qquad C := 11.4 \qquad Z_0 := 18.23 \qquad s := (13.88 + i \cdot 12.6)$

$$\frac{V}{\tau_r} \cdot \frac{1 - e^{-\tau_r \cdot s}}{s^2} \cdot \left(\frac{-s \cdot C \cdot Z_0}{2 + s \cdot C \cdot Z_0} \right) = -0.00021 + 0.00216i$$

$$\frac{-C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} + \frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} \cdot e^{-\tau_r \cdot s} = -0.00021 + 0.00216i$$

The two equations for output frequency response are assigned new, greater, variable values and solved for again. Once again the results show that the equations are equivalent.

$V := 2.45 \qquad \tau_r := 6.34 \qquad C := 1.4 \qquad Z_0 := 8.23 \qquad s := (3.88)$

$$\frac{V}{\tau_r} \cdot \frac{1 - e^{-\tau_r \cdot s}}{s^2} \cdot \left(\frac{-s \cdot C \cdot Z_0}{2 + s \cdot C \cdot Z_0} \right) = -0.02457$$

$$\frac{-C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} + \frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} \cdot e^{-\tau_r \cdot s} = -0.02457$$

$V := 12.45 \qquad \tau_r := 16.34 \qquad C := 11.4 \qquad Z_0 := 18.23 \qquad s := (13.88)$

$$\frac{V}{\tau_r} \cdot \frac{1 - e^{-\tau_r \cdot s}}{s^2} \cdot \left(\frac{-s \cdot C \cdot Z_0}{2 + s \cdot C \cdot Z_0} \right) = -0.00395$$

$$\frac{-C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} + \frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{\frac{2}{C \cdot Z_0}}{s \cdot \left(s + \frac{2}{C \cdot Z_0} \right)} \cdot e^{-\tau_r \cdot s} = -0.00395$$

The two equations for the output frequency response are assigned new variable values but this time "s" is entirely real. In each of the two cases here, the results are alike which almost certifies that the equations are in fact equivalent.

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$V := 12.45 \qquad \tau_r := 16.34 \qquad C := 11.4 \qquad Z_0 := 18.23 \qquad s := (13.88)$

$$\frac{V}{\tau_r}\cdot\frac{1-e^{-\tau_r \cdot s}}{s^2}\cdot\left(\frac{-s\cdot C\cdot Z_0}{2+s\cdot C\cdot Z_0}\right)$$

$$v_r(t) := -C\cdot Z_0\cdot\frac{V}{\tau_r}\cdot\left[\frac{1}{2}-\frac{1}{2}\cdot\exp\left[\frac{-2}{(C\cdot Z_0)}\cdot t\right]-\Phi(t-\tau_r)\cdot\left[\frac{1}{2}-\frac{1}{2}\cdot\exp\left[\frac{-2}{(C\cdot Z_0)}\cdot(t-\tau_r)\right]\right]\right]$$

$v_r(0) = 0 \qquad v_r(6) = -4.44212 \qquad v_r(20) = -11.12176$

$v_r(3) = -2.25312 \qquad v_r(9) = -6.56883 \qquad v_r(1000) = -0.00089$

$$v_{r1}(t) := -\frac{C\cdot Z_0\cdot V}{2\cdot\tau_r}\cdot\left[1-e^{-\frac{t}{\left(\frac{C\cdot Z_0}{2}\right)}}\right]\cdot\Phi(t)+\frac{C\cdot Z_0\cdot V}{2\cdot\tau_r}\cdot\left[1-e^{-\frac{(t-\tau_r)}{\left(\frac{C\cdot Z_0}{2}\right)}}\right]\cdot\Phi(t-\tau_r)$$

The inverse transform of the given function is evaluated against the given result transform to prove that the correct transform is taken.

$v_{r1}(0) = 0 \qquad v_{r1}(6) = -4.44212 \qquad v_{r1}(20) = -11.12176$

$v_{r1}(3) = -2.25312 \qquad v_{r1}(9) = -6.56883 \qquad v_{r1}(1000) = -0.00089$

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$$v_{r1}(t) := \frac{-C\cdot Z_0\cdot V}{2\cdot\tau_r}\cdot\left[1-e^{-\frac{t}{\left(\frac{C\cdot Z_0}{2}\right)}}\right]\cdot\Phi(t)+\frac{C\cdot Z_0\cdot V}{2\cdot\tau_r}\cdot\left[1-e^{-\frac{(t-\tau_r)}{\left(\frac{C\cdot Z_0}{2}\right)}}\right]\cdot\Phi(t-\tau_r)$$

$v_{r1}(0) = 0$	$v_{r1}(\tau_r) = -11.52047$	$v_{r1}(60) = -7.56822$
$v_{r1}(1) = -0.75828$	$v_{r1}(30) = -10.10133$	$v_{r1}(100) = -5.15009$
$v_{r1}(5) = -3.71947$	$v_{r1}(40) = -9.17453$	$v_{r1}(1000) = -0.00089$
$v_{r1}(10) = -7.26419$	$v_{r1}(50) = -8.33276$	$v_{r1}(10000) = 0$

The values for the inverse transform of the frequency response are calculated and an oscillatory response can be seen with a maximum at τ_r.

$$v_{r2}(t) := \left[\frac{-V}{\tau_r}\cdot\exp\left[-2\cdot\frac{t}{(C\cdot Z_0)}\right]\cdot\Phi(t)-\frac{1}{2}\cdot C\cdot Z_0\cdot\frac{V}{\tau_r}\cdot\left[1-\exp\left[-2\cdot\frac{t}{(C\cdot Z_0)}\right]\right]\text{[redacted]}(t)\right]\ \ldots$$
$$+\frac{V}{\tau_r}\cdot\exp\left[-2\cdot\frac{(t-\tau_r)}{(C\cdot Z_0)}\right]\cdot\Phi(t-\tau_r)\ \ldots$$
$$+\frac{1}{2}\cdot C\cdot Z_0\cdot\frac{V}{\tau_r}\cdot\left[1-\exp\left[-2\cdot\frac{(t-\tau_r)}{(C\cdot Z_0)}\right]\right]\cdot\mathrm{Dirac}(t-\tau_r)$$

Attempted to take the derivative of the transform response but the impulse function term constantly throws errors. I can't type the impulse function in or force it. From observation, however, the given equation for the derivative looks correct. The impulse terms drive to zero anyhow.

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$V := 12.45 \qquad \tau_r := 16.34 \qquad C := 11.4 \qquad Z_0 := 18.23 \qquad s := (13.88)$

$$v_{r3}(t) := -\frac{V}{\tau_r} \cdot e^{-\frac{t}{\left(\frac{C \cdot Z_0}{2}\right)}} \cdot \Phi(t) + \frac{V}{\tau_r} \cdot e^{-\frac{(t-\tau_r)}{\left(\frac{C \cdot Z_0}{2}\right)}} \cdot \Phi(t-\tau_r)$$

$v_{r3}(3) = -0.74025$

$$v_r(\tau_{r1}) := \left[-\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \left[1 - e^{\frac{-\tau_{r1}}{\left(\frac{C \cdot Z_0}{2}\right)}} \right] \cdot \Phi(\tau_{r1}) \right] + 0$$

$v_r(0) = 0 \qquad v_r(8) = -5.86674 \qquad v_r(-5) = 0$

$v_r(3) = -2.25312 \qquad v_r(50) = -30.24001 \qquad v_r(-20) = 0$

$$v_r(\tau_{r0}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \left[1 - e^{-\frac{\tau_{r0}}{\left(\frac{C \cdot Z_0}{2}\right)}} \right]$$

$v_r(0) = 0 \qquad v_r(8) = -5.86674 \qquad v_r(-5) = 3.90281$

$v_r(3) = -2.25312 \qquad v_r(50) = -30.24001 \qquad v_r(-20) = 16.80399$

The peak value functions are compared and the result of each is the same. However, note that the function without the heaviside step function can be used to solve at negative times

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$V := 12.45 \quad \tau_r := 1.02 \quad C := 71.4 \quad Z_0 := 18.23$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \left[1 - \left[1 - \frac{\tau_r}{\left(\frac{C \cdot Z_0}{2}\right)}\right]\right]$$

$v_r(0) = -12.45 \quad v_r(6) = -12.45 \quad v_r(20) = -12.45$

$v_r(3) = -12.45 \quad v_r(9) = -12.45 \quad v_r(1000) = -12.45$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{2 \cdot \tau_r}{C \cdot Z_0} \qquad \frac{C \cdot Z_0}{2} = 650.811$$

$-V = -12.45 \qquad \tau_r = 1.02$

The function is checked using the predetermined knowledge that the result should always be the negative value of "V". The function is checked at several times as well as with several different variable values, mainly "C". Also the conditions of $C^*Z_0/2 >> \tau_r$ are verified.

$V := 2.45 \quad \tau_r := 0.34 \quad C := 51.4 \quad Z_0 := 35.23$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \left[1 - \left[1 - \frac{\tau_r}{\left(\frac{C \cdot Z_0}{2}\right)}\right]\right]$$

$v_r(0) = -2.45 \quad v_r(6) = -2.45 \quad v_r(20) = -2.45$

$v_r(3) = -2.45 \quad v_r(9) = -2.45 \quad v_r(1000) = -2.45$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{2 \cdot \tau_r}{C \cdot Z_0} \qquad \frac{C \cdot Z_0}{2} = 905.411$$

$-V = -2.45 \qquad \tau_r = 0.34$

$V := 4.82 \quad \tau_r := 1.37 \quad C := 19.04 \quad Z_0 := 52.24$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \left[1 - \left[1 - \frac{\tau_r}{\left(\frac{C \cdot Z_0}{2}\right)}\right]\right]$$

$v_r(0) = -4.82 \quad v_r(6) = -4.82 \quad v_r(20) = -4.82$

$v_r(3) = -4.82 \quad v_r(9) = -4.82 \quad v_r(1000) = -4.82$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot \frac{2 \cdot \tau_r}{C \cdot Z_0} \qquad \frac{C \cdot Z_0}{2} = 497.3248$$

$-V = -4.82 \qquad \tau_r = 1.37$

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$V := 12.45 \quad \tau_r := 46.34 \quad C := .04 \quad Z_0 := 1.3$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot (1 - 0)$$

$v_r(0) = -0.00699 \quad v_r(6) = -0.00699 \quad v_r(20) = -0.00699$

$v_r(3) = -0.00699 \quad v_r(9) = -0.00699 \quad v_r(1000) = -0.00699$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r}$$

$v_r(0) = -0.00699 \quad v_r(6) = -0.00699 \quad v_r(20) = -0.00699$

$v_r(3) = -0.00699 \quad v_r(9) = -0.00699 \quad v_r(1000) = -0.00699$

$$\frac{C \cdot Z_0}{2} = 0.026$$

$\tau_r = 46.34$

The function is checked using assigned values for variables. The function is checked at several times as well as with different constant values, mainly "C". Also the conditions of $C^*Z_0/2 << \tau_r$ are verified.

$V := 12.45 \quad \tau_r := 58.31 \quad C := .09 \quad Z_0 := .3$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r} \cdot (1 - 0)$$

$v_r(0) = -0.00288 \quad v_r(6) = -0.00288 \quad v_r(20) = -0.00288$

$v_r(3) = -0.00288 \quad v_r(9) = -0.00288 \quad v_r(1000) = -0.00288$

$$v_r(\tau_{r1}) := -\frac{C \cdot Z_0 \cdot V}{2 \cdot \tau_r}$$

$v_r(0) = -0.00288 \quad v_r(6) = -0.00288 \quad v_r(20) = -0.00288$

$v_r(3) = -0.00288 \quad v_r(9) = -0.00288 \quad v_r(1000) = -0.00288$

$$\frac{C \cdot Z_0}{2} = 0.0135$$

$\tau_r = 58.31$

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$R_1 := 3.68 \qquad R_2 := 8.37 \qquad C := 6.73 \qquad L := 4.81 \qquad s := (4 + i \cdot 2.22)$

$$\frac{\dfrac{R_2 \cdot s \cdot L}{R_2 + s \cdot L}}{\dfrac{R_2 \cdot s \cdot L}{R_2 + s \cdot L} + \dfrac{R_1 \cdot \dfrac{1}{s \cdot C}}{R_1 + \dfrac{1}{s \cdot C}}} = 0.996 + 3.068 \cdot 10^{-3} i$$

The three equations of the transfer function are compared using assigned variable values. All three equations return the same response thus the equations are most likely equivalent.

$$\frac{R_2 \cdot s \cdot L \cdot (1 + s \cdot R_1 \cdot C)}{R_2 \cdot s \cdot L \cdot (1 + s \cdot R_1 \cdot C) + R_1 \cdot (R_2 + s \cdot L)} = 0.996 + 3.068 \cdot 10^{-3} i$$

$$\frac{s^2 + \dfrac{1}{R_1 \cdot C} \cdot s}{s^2 + \dfrac{R_1 + R_2}{R_1 \cdot R_2 \cdot C} \cdot s + \dfrac{1}{L \cdot C}} = 0.996 + 3.068 \cdot 10^{-3} i$$

$R_1 := 12.56 \qquad R_2 := 23.37 \qquad C := 5.73 \qquad L := 4.81 \qquad s := (4)$

$$\frac{\dfrac{R_2 \cdot s \cdot L}{R_2 + s \cdot L}}{\dfrac{R_2 \cdot s \cdot L}{R_2 + s \cdot L} + \dfrac{R_1 \cdot \dfrac{1}{s \cdot C}}{R_1 + \dfrac{1}{s \cdot C}}} = 0.996$$

The three equations of the transfer function are compared once again using new assigned variable values. All three equations again return the same response thus the equations can be considered eqivalent. Note that this time the value for "s" is entirely real.

$$\frac{R_2 \cdot s \cdot L \cdot (1 + s \cdot R_1 \cdot C)}{R_2 \cdot s \cdot L \cdot (1 + s \cdot R_1 \cdot C) + R_1 \cdot (R_2 + s \cdot L)} = 0.996$$

$$\frac{s^2 + \dfrac{1}{R_1 \cdot C} \cdot s}{s^2 + \dfrac{R_1 + R_2}{R_1 \cdot R_2 \cdot C} \cdot s + \dfrac{1}{L \cdot C}} = 0.996$$

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$R_1 := 12.56$ $R_2 := 23.37$ $C := 5.73$ $L := 4.81$ $s := (4)$ $c := 0$ $d := 1$ $h := 0$

$k := 3.3$ $D := 3.89$ $\theta := .25 \cdot \pi$

$b := k$ $g := \frac{1}{R_1 \cdot C}$ $a := \frac{R_1 + R_2}{2 \cdot R_1 \cdot R_2 \cdot C}$ $\beta := \sqrt{\frac{1}{L \cdot C} - a^2}$

$$A := \frac{-g \cdot (a - i \cdot \beta) + (a - i \cdot \beta)^2}{\beta \cdot (b - a + i \cdot \beta)^2}$$

$$V_L := \frac{D}{(s+k)^2} \cdot \frac{s^2 + \frac{1}{R_1 \cdot C} \cdot s}{s^2 + \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C} \cdot s + \frac{1}{L \cdot C}}$$

$$v_L(t) := D \cdot \left[A \cdot e^{-a \cdot t} \cdot \sin(\beta \cdot t + \theta) + \frac{-g \cdot b + b^2}{(b-a)^2 + \beta^2} \cdot t \cdot e^{-b \cdot t} \ldots \atop + \left[\frac{g - 2 \cdot b}{(b-a)^2 + \beta^2} - 2 \cdot \frac{\left(-g \cdot b + b^2\right) \cdot (a - b)}{\left[(b-a)^2 + \beta^2\right]^2} \right] \cdot e^{-b \cdot t} \right] \cdot \Phi(t)$$

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$R := 3.68 \qquad r := 9.37 \qquad C := 5.55 \qquad s := (2.41 + i \cdot 7.65)$

$$\frac{\dfrac{R}{1+s\cdot C\cdot R}}{\left(\dfrac{R}{1+s\cdot C\cdot R}+r\right)} = 7.36954\cdot 10^{-4} - 2.27492\cdot 10^{-3}i$$

$$\frac{1}{r\cdot C}\cdot\frac{1}{\left(\dfrac{r+R}{r\cdot R}\right)\cdot\dfrac{1}{C}+s} = 7.36954\cdot 10^{-4} - 2.27492\cdot 10^{-3}i$$

$$\frac{1}{r\cdot C}\cdot\frac{1}{\left[\dfrac{1}{\left(\dfrac{r\cdot R}{r+R}\right)\cdot C}+s\right]} = 7.36954\cdot 10^{-4} - 2.27492\cdot 10^{-3}i$$

Three transient functions for the circuit are compared after being assigned variable values. All of the functions return the same result even when the values of the variables are changed. Note that real and complex values are used to verify that the expressions are equivalent.

$R := .68 \qquad r := .37 \qquad C := .55 \qquad s := (.4)$

$$\frac{\dfrac{R}{1+s\cdot C\cdot R}}{\left(\dfrac{R}{1+s\cdot C\cdot R}+r\right)} = 0.61519$$

$$\frac{1}{r\cdot C}\cdot\frac{1}{\left(\dfrac{r+R}{r\cdot R}\right)\cdot\dfrac{1}{C}+s} = 0.61519$$

$$\frac{1}{r\cdot C}\cdot\frac{1}{\left[\dfrac{1}{\left(\dfrac{r\cdot R}{r+R}\right)\cdot C}+s\right]} = 0.61519$$

$R := 23.68 \qquad r := 29.37 \qquad C := 25.55 \qquad s := (22.4)$

$$\frac{\dfrac{R}{1+s\cdot C\cdot R}}{\left(\dfrac{R}{1+s\cdot C\cdot R}+r\right)} = 5.94839\cdot 10^{-5}$$

$$\frac{1}{r\cdot C}\cdot\frac{1}{\left(\dfrac{r+R}{r\cdot R}\right)\cdot\dfrac{1}{C}+s} = 5.94839\cdot 10^{-5}$$

$$\frac{1}{r\cdot C}\cdot\frac{1}{\left[\dfrac{1}{\left(\dfrac{r\cdot R}{r+R}\right)\cdot C}+s\right]} = 5.94839\cdot 10^{-5}$$

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$A := 5 \qquad T := 4$

The inverse laplace transform of the output voltage is taken. Several values of the inverse are calculated. The result is oscillatory. The results for both the given equations and the transform done on MathCad yeild the same results

$$V_0(s) := A \cdot \frac{\frac{2\cdot\pi}{T}}{s^2 + \left(\frac{2\cdot\pi}{T}\right)^2} \cdot \frac{1}{r\cdot C} \cdot \frac{1}{\left[\frac{1}{\left(\frac{r\cdot R}{r+R}\right)\cdot C} + s\right]}$$

$$\begin{aligned} v_0(t) := &\left[T^4\cdot A\cdot R\cdot r\cdot \frac{\sin\left(2\cdot\frac{\pi}{T}\cdot t\right)}{\left(4\cdot r^2\cdot R^2\cdot C^2\cdot \pi^2\cdot T^2 + T^4\cdot r^2 + 2\cdot T^4\cdot r\cdot R + T^4\cdot R^2\right)}\right] \ldots \\ &+ \left[T^4\cdot A\cdot R^2\cdot \frac{\sin\left(2\cdot\frac{\pi}{T}\cdot t\right)}{\left(4\cdot r^2\cdot R^2\cdot C^2\cdot \pi^2\cdot T^2 + T^4\cdot r^2 + 2\cdot T^4\cdot r\cdot R + T^4\cdot R^2\right)}\right] \ldots \\ &+ -\left[2\cdot T^3\cdot A\cdot \pi\cdot R^2\cdot r\cdot C\cdot \frac{\cos\left(2\cdot\frac{\pi}{T}\cdot t\right)}{\left(4\cdot r^2\cdot R^2\cdot C^2\cdot \pi^2\cdot T^2 + T^4\cdot r^2 + 2\cdot T^4\cdot r\cdot R + T^4\cdot R^2\right)}\right] \ldots \\ &+ \left[2\cdot T\cdot r^2\cdot R^3\cdot C^2\cdot A\cdot \pi\cdot \frac{\exp\left[\frac{-1}{(r\cdot R)}\cdot\frac{(r+R)}{C}\cdot t\right]}{\left(4\cdot r^3\cdot R^3\cdot C^3\cdot \pi^2 + T^2\cdot r^3\cdot R\cdot C + 2\cdot T^2\cdot r^2\cdot R^2\cdot C + T^2\cdot R^3\cdot r\cdot C\right)}\right] \end{aligned}$$

$v_0(0) = 0$ $\qquad v_0(3) = 4.19595\cdot 10^{-3}$ $\qquad v_0(10) = 8.3589\cdot 10^{-3}$

$v_0(1) = 4.23725\cdot 10^{-3}$ $\qquad v_0(4) = -5.03538\cdot 10^{-5}$ $\qquad v_0(50) = 7.89547\cdot 10^{-3}$

$v_0(2) = 8.45842\cdot 10^{-3}$ $\qquad v_0(5) = 4.18705\cdot 10^{-3}$ $\qquad v_0(100) = -1.09483\cdot 10^{-3}$

$$v_0(t) := \frac{2\cdot\pi\cdot A}{T\cdot r\cdot C}\cdot\left[\frac{e^{-\frac{t}{\left(\frac{r\cdot R}{r+R}\right)\cdot C}} + \frac{T}{2\cdot\pi\cdot\left(\frac{r\cdot R}{r+R}\right)\cdot C}\cdot\sin\left(\frac{2\cdot\pi}{T}\cdot t\right) - \cos\left(\frac{2\cdot\pi}{T}\cdot t\right)}{\left[\frac{1}{\left(\frac{r\cdot R}{r+R}\right)\cdot C}\right]^2 + \left(\frac{2\cdot\pi}{T}\right)^2}\right]$$

$v_0(0) = 0$ $\qquad v_0(3) = 4.19595\cdot 10^{-3}$ $\qquad v_0(10) = 8.3589\cdot 10^{-3}$

$v_0(1) = 4.23725\cdot 10^{-3}$ $\qquad v_0(4) = -5.03538\cdot 10^{-5}$ $\qquad v_0(50) = 7.89547\cdot 10^{-3}$

$v_0(2) = 8.45842\cdot 10^{-3}$ $\qquad v_0(5) = 4.18705\cdot 10^{-3}$ $\qquad v_0(100) = -1.09483\cdot 10^{-3}$

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$$v_0\left(\frac{T}{2}\right) = 8.45842 \cdot 10^{-3}$$

$$\frac{2\cdot\pi\cdot A}{T\cdot r\cdot C}\cdot\left[\frac{e^{-\frac{T}{2\cdot\left(\frac{r\cdot R}{r+R}\right)\cdot C}}+1}{\left[\frac{1}{\left(\frac{r\cdot R}{r+R}\right)\cdot C}\right]^2+\left(2\cdot\frac{\pi}{T}\right)^2}\right] = 8.45842 \cdot 10^{-3}$$

The value of $v_0(T/2)$ does in fact equal the given function.

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$$\frac{v_0\left(\frac{T}{2}\right)}{s}\cdot\frac{R}{R+\frac{1}{s\cdot C}} = 3.7758 \cdot 10^{-4}$$

$$\frac{v_0\left(\frac{T}{2}\right)}{s}\cdot\frac{s\cdot R\cdot C}{1+(s\cdot R\cdot C)} = 3.7758 \cdot 10^{-4}$$

All three functions for the output voltage in the frequency-domain are equal.

$$v_0\left(\frac{T}{2}\right)\cdot\frac{1}{s+\frac{1}{R\cdot C}} = 3.7758 \cdot 10^{-4}$$

$$v(s) := v_0\left(\frac{T}{2}\right)\cdot\frac{1}{s+\frac{1}{R\cdot C}}$$

$$v(t) := v_0\left(\frac{1}{2}\cdot T\right)\cdot\exp\left[\frac{-1}{(R\cdot C)}\cdot t\right]$$

The inverse laplace transform of the output voltage function is taken and the results are compared to the given form. The values check out but only for the values T/2<t<T. Also note that the time shift has been romoved for evaluation.

$$v(3) = 8.41658 \cdot 10^{-3}$$

$$v(t) := v_0\left(\frac{T}{2}\right)\cdot e^{\frac{-t}{R\cdot C}}$$

$$v(3) = 8.41658 \cdot 10^{-3}$$

$$\frac{T}{2} = 2$$

$$T = 4$$