P 514
Figure 12 f

Ken Kaiser

$$h(t) = e^{-8t} \sinh(6t)\, u(t)$$

$$a = 8$$
$$b = 6$$

$$\tau_d = \frac{2(8)}{|8^2 - 6^2|}$$

$$= .57$$

$$\tau_r = \frac{2\sqrt{\pi}}{|8^2-6^2|}\sqrt{8^2-6^2}$$

$$= 1.26 \approx 1.3$$

4187

$$\frac{\dfrac{c^2\cdot(b-a)^2}{\left(a^2+w^2\right)\cdot\left(b^2+w^2\right)}}{\left[\dfrac{c\cdot(b-a)}{(a+w)\cdot(b+w)}\right]^2}$$

$$\frac{1}{\left[\left(a^2+w^2\right)\cdot\left(b^2+w^2\right)\right]}\cdot(a+w)^2\cdot(b+w)^2$$

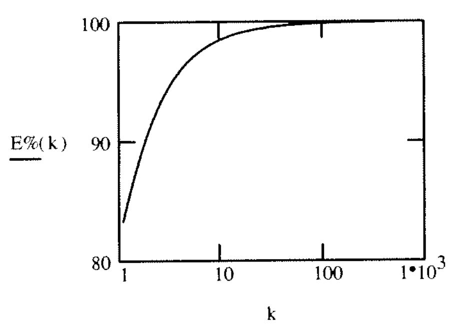
Math cad 4
Proof
p. 489

Ken Kaiser

$C := 1 \qquad \alpha := 1 \qquad \infty := 10^7$

$k := 1.1, 1.15 .. 400$

$$E\%(k) := \frac{\frac{(k-1)\cdot C^2}{\pi\cdot k\cdot\alpha\cdot(k+1)}\cdot(k\cdot atan(k) - atan(1))}{\left[\frac{(k-1)\cdot C^2}{\pi\cdot k\cdot\alpha\cdot(k+1)}\right]\cdot(k\cdot atan(\infty) - atan(\infty))}\cdot 100$$



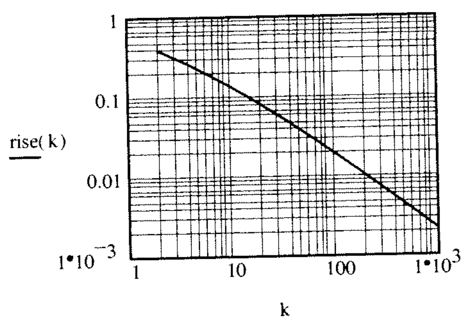
p 492
Top

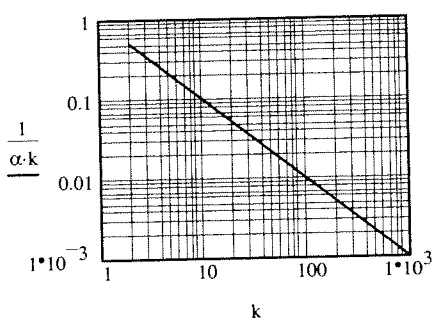
$\alpha := 1 \qquad h := 1 \cdot 10^{-8}$

$k := 2, 4 .. 1000$

$$A(k) := \frac{-\ln(k)}{k-1} \qquad tm(k) := \frac{-A(k)}{\alpha} \qquad d(t,k) := \frac{t}{tm(k)} \qquad xr(t,k) := \frac{e^{A(k)\cdot d(t,k)} - e^{k\cdot A(k)\cdot d(t,k)}}{e^{A(k)} - e^{k\cdot A(k)}}$$

$$rise(k) := root(xr(h,k) - 0.9, h) - root(xr(h,k) - 0.1, h)$$





p 4 92
Bottom

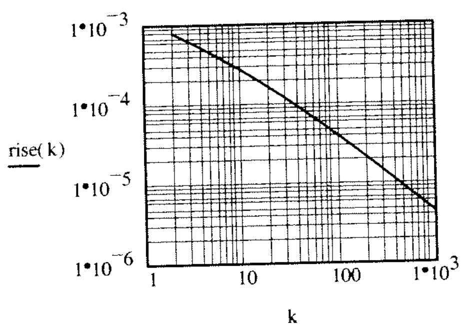
Ken Kaiser

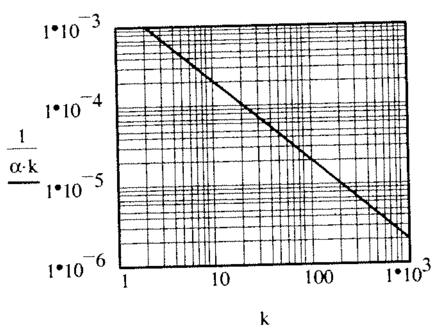
$\alpha := 500 \qquad h := 1\cdot 10^{-10}$

$k := 2, 4 .. 1000$

$A(k) := \dfrac{-\ln(k)}{k-1} \qquad tm(k) := \dfrac{-A(k)}{\alpha} \qquad d(t,k) := \dfrac{t}{tm(k)} \qquad xr(t,k) := \dfrac{e^{A(k)\cdot d(t,k)} - e^{k\cdot A(k)\cdot d(t,k)}}{e^{A(k)} - e^{k\cdot A(k)}}$

$rise(k) := root(xr(h,k) - 0.9, h) - root(xr(h,k) - 0.1, h)$





p 497

$$\frac{\int_0^{\infty} \left(t^2\right)\cdot e^{-3\cdot t}\,dt}{\int_0^{\infty} (t)\cdot e^{-3\cdot t}\,dt} = \frac{2}{3}$$

.66666666666666666667

$$\frac{\int_0^{\infty} \left(t^2\right)\cdot e^{-5\cdot t}\,dt}{\int_0^{\infty} (t)\cdot e^{-5\cdot t}\,dt} = \frac{2}{5}$$

.4

Ken Kaiser

p498

efunda
engineering fundamentals

Series Exp

Series Expansion of Exponential and Logarithmic Functions

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \ldots\right]$$

$$= 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{x-1}{x+1}\right)^{2n-1} \quad (x > 0)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \ldots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{x-1}{x}\right)^n \quad (x > \frac{1}{2})$$

$$\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}(x-1)^n \quad (0 < x \le 2)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}x^n \quad (|x| < 1)$$

$x = -st$

$e^x = e^{-st}$

$= 1 + \frac{(-st)}{1!} + \frac{(-st)^2}{2!} + \frac{(-st)^3}{3!}$

$= 1 - st + \frac{(st)^2}{2} - \frac{(st)^3}{3!}$

498

Return

The binomial series for $(1+x)^{-4}$

Just as before, we use the pattern to write down the coefficients, this time replacing n by -4 everywhere. Here's the formula and what it gives us:

$$1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$(1+x)^{-4} = 1 - 4x + \frac{(-4)\times(-5)}{2!}x^2 + \frac{(-4)\times(-5)\times(-6)}{3!}x^3 + \ldots$$

Again we can simplify those coefficients, to get:

$(1+x)^{-4}=1-4x+10x^2-20x^3+\ldots$

Notice that this time the terms are alternatively positive and negative.

Return

$x = \frac{s}{a}$

$n = -2$

$(1+x)^n$

$= (1+\frac{s}{a})^{-2}$

$= 1 + (-2)\frac{s}{a} + \frac{-2(-2-1)}{2}\left(\frac{s}{a}\right)^2$

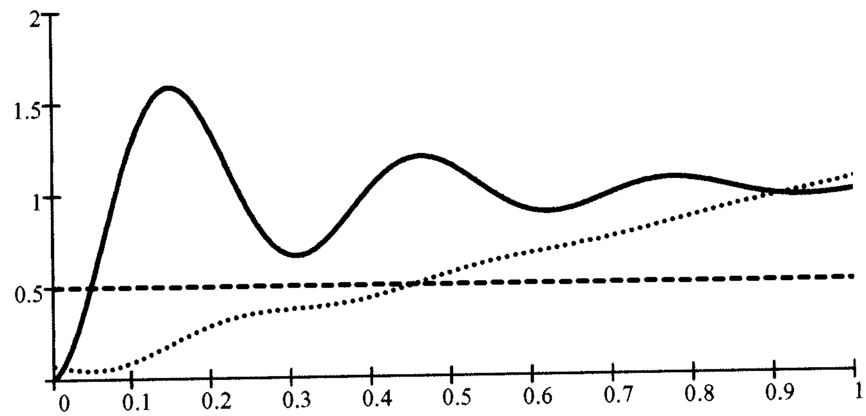
Figure 6
p 501

$a := 3.5$

$t := 0, \frac{1}{400} .. 2$

$y(t) := 1 - e^{-a \cdot t} \cdot \cos(20 \cdot t) \quad \text{diff}(t,\tau) := y(t) - \Phi(t - \tau)$

$\varepsilon(\tau) := \int_0^2 \text{diff}(t,\tau)^2 \, dt$



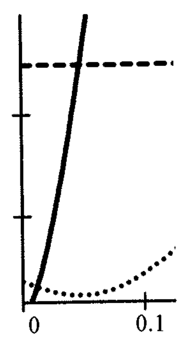


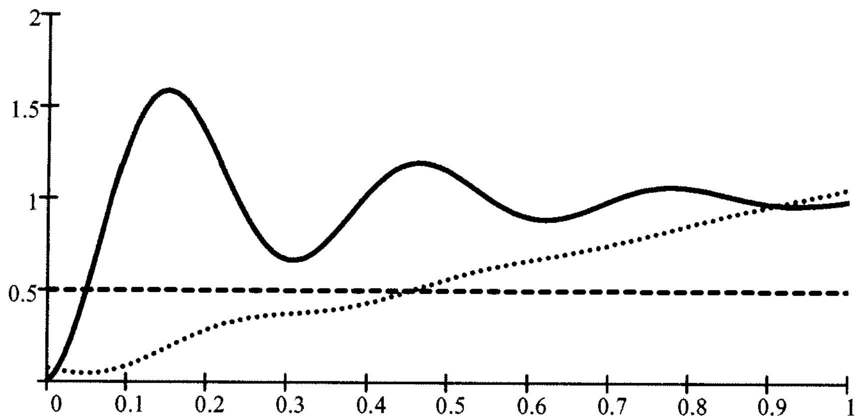
Figure 6
p 501

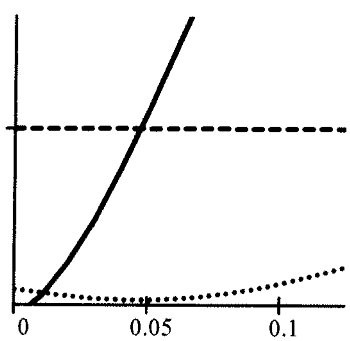
$a := 3.5$

$t := 0, \frac{1}{100} .. 2$

$y(t) := 1 - e^{-a \cdot t} \cdot \cos(20 \cdot t) \quad diff(t, \tau) := y(t) - \Phi(t - \tau)$

$\varepsilon(\tau) := \int_0^2 diff(t, \tau)^2 \, dt$





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Taylor test

$\cos(w \cdot t)$

$1 + \left(\frac{-1}{2} \cdot w^2\right) \cdot t^2 + \left(\frac{1}{24} \cdot w^4\right) \cdot t^4 + O\left(t^6\right)$

$\sin(w \cdot t)$

$w \cdot t + \left(\frac{-1}{6} \cdot w^3\right) \cdot t^3 + \left(\frac{1}{120} \cdot w^5\right) \cdot t^5 + O\left(t^6\right)$

$5 \cdot (.1) = -5 \cdot \left[e^{-a \cdot t} + \left(a \cdot t \cdot e^{-a \cdot t}\right) - 1\right]$ $a = w_0$

$\begin{bmatrix} \frac{-.39165871526656812942}{a} \\ \frac{.53181160838961202 01}{a} \end{bmatrix} \rightarrow t_{10\%}$

$5 \cdot (.9) = -5 \cdot \left[e^{-a \cdot t} + \left(a \cdot t \cdot e^{-a \cdot t}\right) - 1\right]$

$\begin{bmatrix} \frac{-.96177875825320056806}{a} \\ \frac{3.8897201698674290579}{a} \end{bmatrix} - t_{90\%}$

$\frac{3.8897201698674290579}{a} - \frac{.53181160838961202 01}{a}$

$\frac{3.3579085614778170378}{a}$

$\approx \frac{3.4}{a} = \frac{3.4}{w_0}$

Ken Kaiser

$$\frac{2.5}{5} = 0.5$$

$$5\cdot\left[1-\left(e^{-3\cdot.12}\cdot\cos(10\cdot.12)\right)-\frac{3}{10}\cdot e^{-3\cdot.12}\cdot\sin(10\cdot.12)\right] = 2.761$$

$$5\cdot\left[1-\left(e^{-3\cdot.114}\cdot\cos(10\cdot.114)\right)-\frac{3}{10}\cdot e^{-3\cdot.114}\cdot\sin(10\cdot.114)\right] = 2.549$$

td approx equal to .114 not .13

509
Top proof

Ken Kaiser

$$\left[\left[\left[(1) - e^{-3\cdot t}\cdot\cos(10\cdot t)\right] - \left(\frac{3}{10}\cdot e^{-3\cdot t}\cdot\sin(10\cdot t)\right)\right]\cdot 5\right]$$

t = .11

$$\frac{109}{2}\cdot\exp(-3\cdot.11)\cdot\sin(10\cdot.11)$$

← cosine terms drop out

34.918701679377100196

$$\frac{5}{34.918701679377100196}$$

.14318974530925895708

Ken Kaiser

513 Top ~~of~~

$$\left[1-\left(\frac{s+a}{b}\right)^2\right]^{-1}$$

$$\left[1+-\left(\frac{s+a}{b}\right)^2\right]^{-1};\quad (1+x)^n \quad \text{where } x=-\left(\frac{s+a}{b}\right)^2,\ n=-1$$

$$1+nx+\frac{n(n-1)}{2}x^2$$

$$1+-1\cdot-\left(\frac{s+a}{b}\right)^2+\frac{-1\cdot(-1-1)}{2}\cdot\left[-\left(\frac{s+a}{b}\right)^2\right]^2$$

$$1+\left(\frac{s+a}{b}\right)^2+\frac{2}{2}\cdot\left[-\left(\frac{s+a}{b}\right)^2\right]^2$$

$$1+\left(\frac{s+a}{b}\right)^2+\left(\frac{s+a}{b}\right)^4$$

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Q

$$\frac{\frac{5}{(7+3)^2-5^2}}{\left[\frac{-5^{-1}}{1-\left(\frac{7+3}{5}\right)^2}\right]}=1$$

1

$$\frac{-5^{-1}\cdot\left[1+\left(\frac{7+3}{5}\right)^2+\left(\frac{7+3}{5}\right)^4\right]}{\frac{5}{3^2-5^2}-\left[\frac{2\cdot3\cdot5}{\left(3^2-5^2\right)^2}\cdot7\right]+\left[\frac{5}{\left(3^2-5^2\right)^2}+\frac{2\cdot3^2\cdot5}{\left(3^2-5^2\right)^3}\right]\cdot7^2}=3.353$$

should = 1

3.353

My Work Mathcad expand to series

$$\frac{-\frac{1}{b}}{1-\left(\frac{s+a}{b}\right)^2} = \frac{-1}{b\left(1-\frac{a^2}{b^2}\right)} + \frac{2}{b\left(-b^2+a^2\right)}\frac{a}{\left(1-\frac{a^2}{b^2}\right)}$$

$$- \left[\frac{\frac{-1}{b\left(-b^2+a^2\right)} + \frac{4a^2}{b\left(-b^2+a^2\right)^2}}{1-\frac{a^2}{b^2}}\right] s^2 + \ldots$$

$$= \frac{-b}{b^2-a^2} + \frac{2ab}{\left(-b^2+a^2\right)\left(b^2-a^2\right)} s$$

$$+ - \left[\frac{\frac{-b}{-b^2+a^2} + \frac{4a^2b}{\left(-b^2+a^2\right)^2}}{b^2-a^2}\right] s^2$$

$$= \frac{b}{a^2-b^2} - \frac{2ab}{\left(a^2-b^2\right)\left(a^2-b^2\right)} s \mp \left[\frac{b}{\left(a^2-b^2\right)^2} + \frac{-4a^2b}{\left(a^2-b^2\right)^3}\right]$$

P514
Figure 12

Ken Kaiser

$a := 8 \qquad b := 6$

$$\frac{\int_0^{10} t\cdot\left(e^{-a\cdot t}\cdot\sinh(b\cdot t)\right)dt}{\int_0^{10} e^{-a\cdot t}\cdot\sinh(b\cdot t)\,dt} = 0.571$$

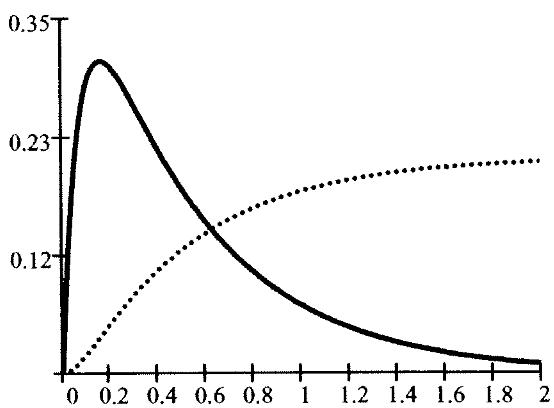
$$\frac{\int_0^{4} e^{-a\cdot t}\cdot\sinh(b\cdot t)\,dt}{2} = 0.107$$

$$\int_0^{x} e^{-a\cdot t}\cdot\sinh(b\cdot t)\,dt$$

$t := 0, \frac{1}{100} .. 3$

$h(t) := e^{-a\cdot t}\cdot\sinh(b\cdot t)$

$g(x) := \frac{1}{2}\cdot\frac{[-b\cdot\exp[-x\cdot(b+a)] - a\cdot e\ldots}{}$



$c := 0.423$

$g(12) = 0.214$

$$\int_0^{c} e^{-a\cdot t}\cdot\sinh(b\cdot t)\,dt = 0.107$$

$h(0.517) = 0.177$

$$\int_c^{10} e^{-a\cdot t}\cdot\sinh(b\cdot t)\,dt = 0.107$$

$h(4) = 1.677 \times 10^{-4}$

$$\int_0^{4} h(t)\,dt = 0.214$$

$$2\cdot\frac{a}{a^2 - b^2} = 0.571$$

$$\frac{0.214}{2} = 0.107$$

$$\frac{2\cdot\sqrt{\pi}}{}\cdot\sqrt{a^2 + b^2} = 1.266$$

$\ldots \sqrt{1 \ldots \left(\frac{b}{\ldots}\right)^2} \ldots$ $\int^{0.571}$

$$\frac{\text{xp}[-x\cdot(-b+a)] + a\cdot\exp[-x\cdot(b+a)] - b\cdot\exp[-x\cdot(-b+a)] + 2\cdot b]}{[(b+a)\cdot(-b+a)]}$$

$a^2 - b^2$ ·

$0.57 \cdot \sqrt{\pi} \cdot \sqrt{1 + \left(\frac{}{a}\right)} = 1.263$

$\int_0 h(t)\,dt = 0.135$

$$\frac{\int_0^m t \cdot \left(e^{-a \cdot t}\right) dt}{\int_0^m \left(e^{-a \cdot t}\right) dt}$$

$$\frac{(\exp(-a \cdot m) \cdot a \cdot m + \exp(-a \cdot m) - 1)}{[a \cdot (\exp(-a \cdot m) - 1)]}$$ this is equal to 1/a when m is large

$$\frac{\int_0^{\frac{1}{a}} t \cdot \left(e^{-a \cdot t}\right) dt}{\int_{\frac{1}{a}}^{\frac{4}{a}} \left(e^{-a \cdot t}\right) dt} = 0.094$$

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(38)

Ken Kaiser

$$\sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^\infty t^2\cdot c^2\cdot \exp\left(-2\cdot\frac{t}{3}\right)dt}{\int_0^\infty c^2\cdot \exp\left(-2\frac{t}{3}\right)dt}-\left(\frac{\int_0^\infty t^2\cdot c^2\cdot \exp\left(-2\cdot\frac{t}{3}\right)dt}{\int_0^\infty c^2\cdot \exp\left(-2\frac{t}{3}\right)dt}\right)^2}$$

$$\sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^\infty t^2\cdot c^2\cdot \exp\left(-2\cdot\frac{t}{3}\right)dt}{\int_0^\infty c^2\cdot \exp\left(-2\frac{t}{3}\right)dt}-\left(\frac{\int_0^\infty t^2\cdot c^2\cdot \exp\left(-2\cdot\frac{t}{3}\right)dt}{\int_0^\infty c^2\cdot \exp\left(-2\frac{t}{3}\right)dt}\right)^2}$$

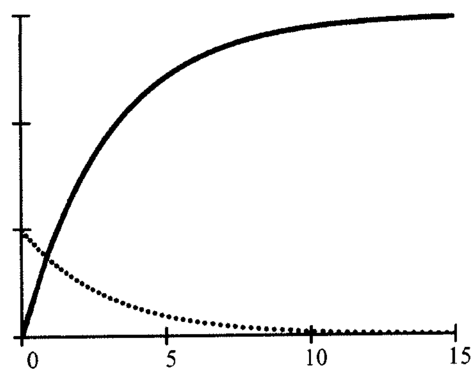
$$\frac{3}{2}\cdot i\cdot\sqrt{2}\cdot\pi^{\left(\frac{1}{2}\right)}\cdot\sqrt{7}$$

Pg
516
Figure 13

$t := 0, 0.001 .. 15$

$h(t) := 1 \cdot e^{-\frac{1}{3} \cdot t}$

$$\frac{\int_0^{10} t \cdot h(t)\,dt}{\int_0^{10} h(t)\,dt} = 2.63$$

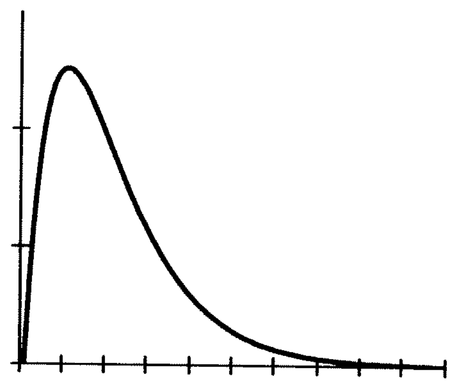


P 517
Figure 14

$a := 10 \quad b := 2$

$t := 0, \frac{1}{100} .. 3$

$h(t) := e^{-a\cdot(t-2)}\cdot 1\cdot \sinh(b\cdot t) + e^{-a\cdot 1.3\cdot(t-1)}\cdot \Phi(t)\cdot -20$



$\int_0^x e^{-a\cdot t}\cdot \sinh(b\cdot t)\,dt$

$g(x) := \frac{1}{2}\cdot \frac{[-b\cdot \exp[-x\cdot(b+a)] - a\cdot e\text{x}}{}$

$g(12) = 0.021$

$$\frac{\text{xp}[-x\cdot(-b + a)] + a\cdot\exp[-x\cdot(b + a)] - b\cdot\exp[-x\cdot(-b + a)] + 2\cdot b]}{[(b + a)\cdot(-b + a)]}$$

Figure 19
p529

$j := \sqrt{-1}$

$$\frac{1}{(s+a)^{10}}$$

$$\frac{1}{362880}\cdot t^{9}\cdot \exp(-a\cdot t)$$

$a := 10$

$t := -0.2, \frac{1}{100} - 0.2 .. 3$

$y_1(t) := \exp(-a\cdot t)\cdot \Phi(t)$ $y_2(t) := t\cdot \exp(-a\cdot t)\cdot \Phi(t)$ $y_3(t) := \frac{1}{2}\cdot t^{2}\cdot \exp(-a\cdot t)\cdot \Phi(t)$

$y_4(t) := \frac{1}{6}\cdot t^{3}\cdot \exp(-a\cdot t)\cdot \Phi(t)$ $y_{10}(t) := \frac{1}{362880}\cdot t^{9}\cdot \exp(-a\cdot t)\cdot \Phi(t)$

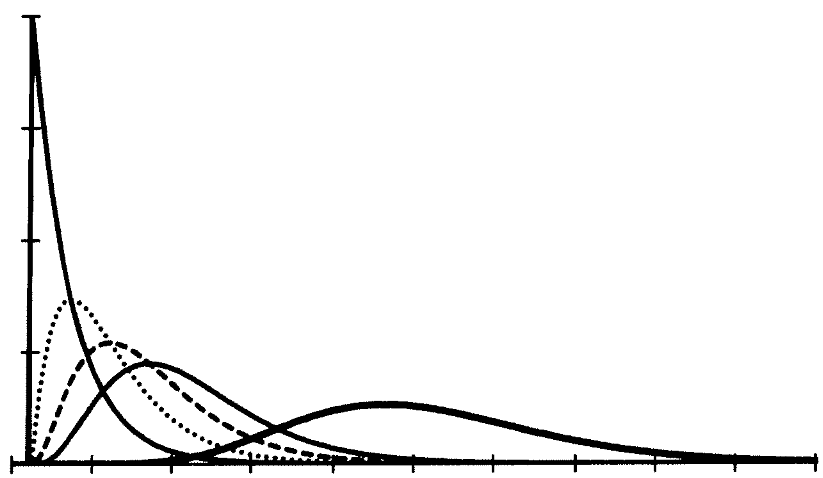


Figure 20

p 530

$j := \sqrt{-1}$

$$\frac{1}{(s+a)^{10}}$$

$$\frac{1}{362880} \cdot t^9 \cdot \exp(-a \cdot t)$$

$n := 10$

$t := -0.2, \frac{1}{100} - 0.2 .. 6$

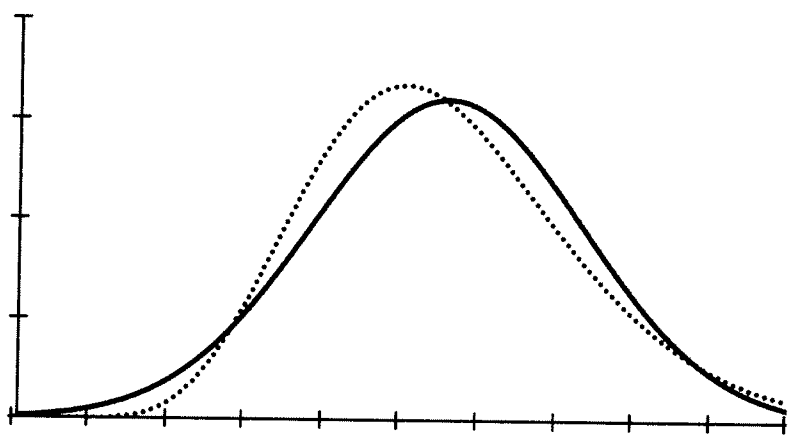
$a := 3$

$\tau_r := \frac{\sqrt{2 \cdot \pi \cdot n}}{a}$ $\tau_d := \frac{n}{a}$ $A := \frac{1}{a^n}$

$$y_{Gn10}(t) := \frac{A}{\tau_r} \cdot e^{\frac{(t-\tau_d)^2 \cdot - \pi}{\tau_r^2}} \cdot \Phi(t)$$

$$y_{G10}(t) := \frac{A}{\tau_r} \cdot e^{\frac{-\pi \cdot (t-\tau_d)^2}{\tau_r^2}} \cdot \Phi(t)$$

$$y_{10}(t) := \frac{1}{362880} \cdot t^9 \cdot \exp(-a \cdot t) \cdot \Phi(t)$$



, 533
Proof

$$\left(\left|\frac{1}{2+i\ \cdot w}\right|\right)^2$$

$$\left(\frac{1}{\sqrt{4+w^2}}\right)^2$$

$$\frac{1}{\left(4+w^2\right)}$$

$$\int_0^{\infty} \frac{1}{\left(4+w^2\right)}\, dw\cdot 4$$

π

3.14

p 534
Figure 22

$\omega := 0, 0.01 .. 10$

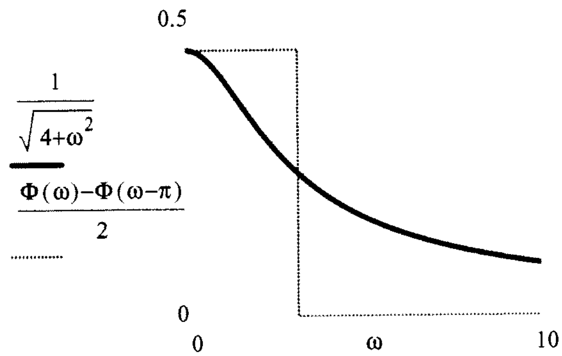


Table 3
Standard deviation
of rise time

$$\sqrt{\frac{\displaystyle\int_0^{\infty}\left(t-a\cdot\sqrt{\frac{2}{\pi}}\right)^2\cdot e^{-\frac{t^2}{2\cdot a^2}}\,dt}{\displaystyle\int_0^{\infty} e^{-\frac{t^2}{2\cdot a^2}}\,dt}\cdot\sqrt{2\cdot\pi}}$$

$$\sqrt{\operatorname{limit}\left[\frac{1}{2}\cdot a^2\cdot\frac{\left(-2\cdot t\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)\cdot\sqrt{\pi}+a\cdot\pi\cdot\sqrt{2}\cdot\operatorname{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)+4\cdot a\cdot\sqrt{2}\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)+2\cdot a\cdot\sqrt{2}\cdot\operatorname{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)-4\cdot a\cdot\sqrt{2}\right)}{\sqrt{\pi}},t=\infty,\text{left}\right]\cdot\frac{2^{\left(\frac{1}{4}\right)}}{\left[\sqrt{a\cdot\pi}^{\left(\frac{1}{4}\right)}\right]}\cdot\sqrt{2\cdot\pi}}$$

$$\sqrt{\frac{1}{2}\cdot a^2\cdot\frac{\left(a\cdot\pi\cdot\sqrt{2}\cdot 1+2\cdot a\cdot\sqrt{2}\cdot 1-4\cdot a\cdot\sqrt{2}\right)}{\sqrt{\pi}}\cdot\frac{2^{\left(\frac{1}{4}\right)}}{\left[\sqrt{a\cdot\pi}^{\left(\frac{1}{4}\right)}\right]}\cdot\sqrt{2\cdot\pi}}$$

$$\sqrt{2}\cdot a\cdot\sqrt{\pi-2}$$

My work

table 3 again

Causal Gaussian Pulse:

$$h(t) = e^{-\frac{t^2}{2\cdot a^2}} \cdot u(t)$$

$$w(t) = \int_0^t h(t)\,dt$$

$$\int_0^t e^{-\frac{t^2}{2\cdot a^2}}\,dt$$

$$\frac{1}{2}\cdot\sqrt{\pi}\cdot\sqrt{2}\cdot a\cdot \operatorname{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right) = a\cdot\sqrt{\frac{\pi}{2}}\cdot \operatorname{erf}\left(\frac{t}{a\cdot\sqrt{2}}\right)$$

Therefore, w(t) is equal to

$$a\cdot\sqrt{\frac{\pi}{2}}\cdot \operatorname{erf}\left(\frac{t}{a\cdot\sqrt{2}}\right)\cdot u(t)$$

$u(t) := 1$

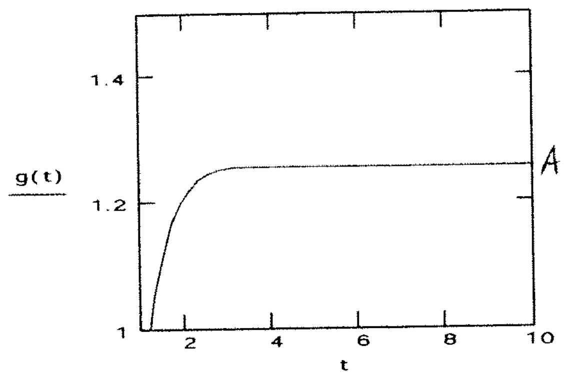
$a := 1$

$T := 20$

$t := -T, -T + \frac{T}{100} .. T$

$$g(t) := a\cdot\sqrt{\frac{\pi}{2}}\cdot \operatorname{erf}\left(\frac{t}{a\cdot\sqrt{2}}\right)\cdot u(t)$$

Ken Kaiser



$a = 1$

$A = a\sqrt{\frac{\pi}{2}} = \sqrt{\frac{\pi}{2}} \approx 1.2533$

* also, when $t \to \infty$

$erf\left(\frac{t}{a\cdot\sqrt{2}}\right) = 1$

$\therefore A = a\sqrt{\frac{\pi}{2}} \cdot erf\left(\frac{t}{a\cdot\sqrt{2}}\right)$ → 1

$= a\sqrt{\pi/2}$

[redacted]

50 Percent Delay Time

$$a \cdot \sqrt{\frac{\pi}{2}} \cdot \mathrm{erf}\left(\frac{t}{a \cdot \sqrt{2}}\right)$$

when t = τd = 0.68*a

$$\mathrm{erf}\left(\frac{0.68 \cdot a}{a \cdot \sqrt{2}}\right) = .50349553909285894221$$

this is approximately 1/2

for 50% delay time, we would expect to see w(τd) to equal 0.5*a*(pi/2)^0.5

[redacted]

Centroid Delay Time

$$\tau_d = \frac{\int_0^\infty t \cdot h(t)\,dt}{\int_0^\infty h(t)\,dt}$$

therefore,

$$\tau_d = \frac{\int_0^\infty t \cdot e^{\frac{-t^2}{2\cdot a^2}}\,dt}{\int_0^\infty e^{\frac{-t^2}{2\cdot a^2}}\,dt}$$

[redacted]

$$\frac{\lim_{t \to \infty^-} -\exp\left(\frac{-1}{2} \cdot \frac{t^2}{a^2}\right) \cdot a^2 + a^2}{\sqrt{\pi}} \cdot \frac{\sqrt{2}}{\left(a \cdot \lim_{t \to \infty^-} \operatorname{erf}\left(\frac{1}{2} \cdot \frac{\sqrt{2}}{a} \cdot t\right)\right)}$$

$$\frac{-0 \cdot a^2 + a^2}{\sqrt{\pi}} \cdot \frac{\sqrt{2}}{a \cdot 1}$$

$$\frac{a}{\sqrt{\pi}} \cdot \sqrt{2} = 0.8 \cdot a$$

Average Delay Time

$w(\tau_d) = \frac{1}{2}$ From derivation on pages 499-501, average delay time is equal to 50% delay time

[redacted]

10-90 Percent Rise Time

t(90%) and t(10%) were solved through trial and error

$$\frac{2}{\sqrt{\pi}} \cdot \int_0^{\frac{1.645 \cdot a}{a \cdot \sqrt{2}}} e^{-\lambda^2} \, d\lambda \qquad t_{90\,\%} = 1.645 \cdot a$$

.90003018892175726944

$$\frac{2}{\sqrt{\pi}} \cdot \int_0^{\frac{0.1257 \cdot a}{a \cdot \sqrt{2}}} e^{-\lambda^2} \, d\lambda \qquad t_{10\,\%} = 0.1257 \cdot a$$

.10003059813234277297

$\tau r = t(90\%) - t(10\%) = 1.5193\ a$

[redacted]

<u>Slope-based Rise Time</u>
<u>@ 50% Delay Time</u>

$\tau d = 0.68*a$

$A = a*(\pi/2)$^0.5

$$\tau_r = \frac{A}{\frac{dy(t)}{dt}} \quad ,t = t_d$$

$$\frac{dy(t)}{dt} = h(t)$$

$$\tau_r = \frac{A}{h(t)} \quad ,t = t_d$$

$$\tau_r = \frac{a \cdot \sqrt{\frac{\pi}{2}}}{e^{\frac{-(0.68 \cdot a)^2}{2 \cdot a^2}}} = 1.5793152272905349559 \cdot a$$

Standard Deviation Rise Time

$$\tau_r \quad = \quad \sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^{\infty}\left(t - a\cdot\sqrt{\frac{2}{\pi}}\right)^2\cdot\left(e^{-\frac{t^2}{2\cdot a^2}}\right)dt}{\int_0^{\infty}\left(e^{-\frac{t^2}{2\cdot a^2}}\right)dt}}$$

$$2^{\left(\frac{3}{4}\right)}\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\frac{\sqrt{\text{limit}\left[\frac{1}{2}\cdot a^2\cdot\frac{\left(-2\cdot t\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)\cdot\sqrt{\pi}+a\cdot\pi\cdot\sqrt{2}\cdot\text{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)+4\cdot a\cdot\sqrt{2}\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)+2\cdot a\cdot\sqrt{2}\cdot\text{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)-4\cdot a\cdot\sqrt{2}\right)}{\sqrt{\pi}}, t=\infty, \text{left}\right]}}{\sqrt{a}}$$

As t --> ∞

$$2^{\left(\frac{3}{4}\right)}\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\sqrt{\frac{\text{limit}\left[\frac{1}{2}\cdot a^2\cdot\frac{\left(-2\cdot t\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)\cdot\sqrt{\pi}+a\cdot\pi\cdot\sqrt{2}\cdot\text{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)+4\cdot a\cdot\sqrt{2}\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)+2\cdot a\cdot\sqrt{2}\cdot\text{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)-4\cdot a\cdot\sqrt{2}\right)}{\sqrt{\pi}},t=\infty,\text{left}\right]}{\sqrt{a}}}$$

$$2^{\left(\frac{3}{4}\right)}\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\sqrt{\frac{\frac{1}{2}\cdot a^2\cdot\frac{\left(-2\cdot t\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)\cdot\sqrt{\pi}+a\cdot\pi\cdot\sqrt{2}\cdot\text{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)+4\cdot a\cdot\sqrt{2}\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)+2\cdot a\cdot\sqrt{2}\cdot\text{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{a}\cdot t\right)-4\cdot a\cdot\sqrt{2}\right)}{\sqrt{\pi}}}{a}}$$

$$2^{\left(\frac{3}{4}\right)}\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\sqrt{\frac{\frac{1}{2}\cdot a^2\cdot\frac{\left(-2\cdot t\cdot 0\cdot\sqrt{\pi}+a\cdot\pi\cdot\sqrt{2}\cdot 1+4\cdot a\cdot\sqrt{2}\cdot 0+2\cdot a\cdot\sqrt{2}\cdot 1-4\cdot a\cdot\sqrt{2}\right)}{\sqrt{\pi}}}{a}} = \sqrt{2}\cdot a\cdot\sqrt{\pi-2}$$

$$= a\cdot\sqrt{2\cdot\pi-4}$$

Standard Deviation Rise Time for h^2(t)

$$\tau_d = \frac{\int_0^{\infty} t \cdot h(t)^2 \, dt}{\int_0^{\infty} h(t)^2 \, dt}$$

$$\tau_d = \frac{\int_0^{\infty} t \cdot \left(e^{-\frac{t^2}{2 \cdot a^2}}\right)^2 dt}{\int_0^{\infty} \left(e^{-\frac{t^2}{2 \cdot a^2}}\right)^2 dt} = \frac{a}{\sqrt{\pi}}$$

$$\tau_r = \sqrt{2\cdot\pi}\cdot\sqrt{\frac{\displaystyle\int_0^{\infty}\left(t-\frac{a}{\sqrt{\pi}}\right)^2\cdot\left(e^{-\frac{t^2}{2\cdot a^2}}\right)^2 dt}{\displaystyle\int_0^{\infty}\left(e^{-\frac{t^2}{2\cdot a^2}}\right)^2 dt}}$$

$$2\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\frac{\sqrt{\text{limit}\left[\frac{-1}{2}\cdot a^2\cdot t\cdot\exp\left(\frac{-t^2}{a^2}\right)+\frac{1}{4}\cdot a^3\cdot\sqrt{\pi}\cdot\text{erf}\left(\frac{1}{a}\cdot t\right)+\frac{1}{\exp\left(\frac{t^2}{a^2}\right)}\cdot\frac{a^3}{\sqrt{\pi}}+\frac{1}{\left(2\cdot\sqrt{\pi}\right)}\cdot a^3\cdot\text{erf}\left(\frac{1}{a}\cdot t\right)-\frac{a^3}{\sqrt{\pi}},t=\infty,\text{left}\right]}}{\sqrt{a}}$$

As t --> ∞

$$2\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\sqrt{\frac{\frac{-1}{2}\cdot a^2\cdot t\cdot \exp\left(\frac{-t^2}{a^2}\right)+\frac{1}{4}\cdot a^3\cdot\sqrt{\pi}\cdot \operatorname{erf}\left(\frac{1}{a}\cdot t\right)+\frac{1}{\exp\left(\frac{t^2}{a^2}\right)}\cdot\frac{a^3}{\sqrt{\pi}}+\frac{1}{\left(2\cdot\sqrt{\pi}\right)}\cdot a^3\cdot \operatorname{erf}\left(\frac{1}{a}\cdot t\right)-\frac{a^3}{\sqrt{\pi}}}{a}}$$

$$2\cdot\pi^{\left(\frac{1}{4}\right)}\cdot\sqrt{\frac{\frac{-1}{2}\cdot a^2\cdot t\cdot 0+\frac{1}{4}\cdot a^3\cdot\sqrt{\pi}\cdot 1+0\cdot\frac{a^3}{\sqrt{\pi}}+\frac{1}{\left(2\cdot\sqrt{\pi}\right)}\cdot a^3\cdot 1-\frac{a^3}{\sqrt{\pi}}}{a}}$$

$$a\cdot\sqrt{\pi-2}$$

Table 7

2) Centroid Delay Time

$$\tau_d := \frac{\int_0^\infty t\cdot h(t)\,dt}{\int_0^\infty h(t)\,dt} \qquad h(t) := e^{-\left(\frac{t^2}{2\cdot a^2}\right)}$$

$$\frac{\int_0^\infty t\cdot e^{-\left(\frac{t^2}{2\cdot a^2}\right)}\,dt}{\int_0^\infty e^{-\left(\frac{t^2}{2\cdot a^2}\right)}\,dt} \qquad \frac{\lim_{t\to\infty} -a^2\cdot\exp\left(\frac{-1}{2}\cdot\frac{t^2}{a^2}\right)+a^2}{\lim_{t\to\infty} \frac{1}{2}\cdot a\cdot\sqrt{2}\cdot\pi^{\frac{1}{2}}\cdot\operatorname{erf}\left(\frac{1}{2}\cdot\frac{t}{a}\cdot\sqrt{2}\right)}$$

$$\frac{\lim_{t\to\infty} -a^2\cdot 0+a^2}{\lim_{t\to\infty} \frac{1}{2}\cdot a\cdot\sqrt{2}\cdot\pi^{\frac{1}{2}}\cdot 1} \qquad \frac{-a^2\cdot 0+a^2}{\frac{1}{2}\cdot a\cdot\sqrt{2}\cdot\pi^{\frac{1}{2}}\cdot 1} \qquad \frac{1}{\pi^{\frac{1}{2}}}\cdot a\cdot\sqrt{2} \qquad = 0.79788*a$$

✓

3) Average Delay Time

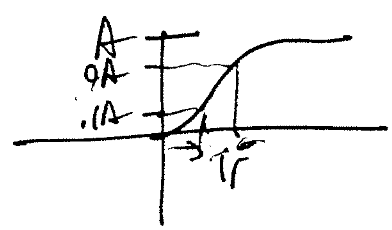
From page 501 in the text, the average delay time is equal to the 50% delay time, or for this example, approximately 0.68*a

✓

4) 10-90% Delay Time

$$\left[1-e^{-\left(\frac{t^2}{2\cdot a^2}\right)}\right] = 0.1 \qquad \text{solve for } t = \begin{bmatrix} \left(-2\cdot\ln\left(\frac{9}{10}\right)\right)^{\frac{1}{2}}\cdot a \\ -\left(-2\cdot\ln\left(\frac{9}{10}\right)\right)^{\frac{1}{2}}\cdot a \end{bmatrix} \qquad = 0.459*a = t\ 10\%$$

? should be w(t) not h(t)?



$\operatorname{erf}\left(\frac{1.65a}{\sqrt{2}a}\right) = .901$

$\operatorname{erf}\left(\frac{.126a}{\sqrt{2}a}\right) = .1$ ✓

$(1.65 - .126)a = 1.5a$

$$\left[1 - e^{-\left(\frac{t^2}{2\cdot a^2}\right)}\right] = 0.9 \quad \text{solve for } t = \begin{pmatrix} \sqrt{2}\cdot\ln(10)^{\frac{1}{2}}\cdot a \\ -\sqrt{2}\cdot\ln(10)^{\frac{1}{2}}\cdot a \end{pmatrix} = 2.14*a = t\,90\%$$

t 90% - t 10% = (2.14-.459)a = 1.68*a

5) Slope based rise time @ 50% Delay Time

$$\tau_r := \frac{A}{h(t)}$$

A = final value = a(π/2)^(1/2)   t = 0.68*a

$$\frac{a\cdot\sqrt{\frac{\pi}{2}}}{e^{-\left[\frac{(.68\cdot a)^2}{2\cdot a^2}\right]}} \quad 1.5793152272905349559\cdot a$$

please if you get a chance see me about this result

6) Standard Deviation Rise Time

$$\sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^{\infty}\left[t-\left(a\cdot\sqrt{\frac{2}{\pi}}\right)\right]^2\cdot e^{-\left[\frac{(t\cdot a)^2}{2\cdot a^2}\right]}\,dt}{\int_0^{\infty} e^{-\left[\frac{(t\cdot a)^2}{2\cdot a^2}\right]}\,dt}}$$

$$\sqrt{2}\cdot\pi^{\frac{1}{2}}\cdot\left[\left[\frac{1}{2}\cdot\sqrt{2}\cdot\frac{(2\cdot a^2+\pi)}{\pi^{\frac{1}{2}}} - \frac{2}{\pi^{\frac{1}{2}}}\cdot a\cdot\sqrt{2}\right]\cdot\frac{\sqrt{2}}{\pi^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

$$\sqrt{2}\cdot(2\cdot a^2+\pi-4\cdot a)^{\frac{1}{2}}$$

$$(4\cdot a^2+2\pi-8\cdot a)^{\frac{1}{2}}$$

No limit expressions? I didn't get this?

My Mathcad again

a = 3

$3\sqrt{2}(\pi-2)^{1/2}$

$= 3(2\pi-4)^{1/2}$

a = 5

$5\sqrt{2}(-2+\pi)^{1/2}$

This was done using MathCAD 2001, and the program simplified what is shown above automatically, except for the very last simplification. This does not agree with the book.

erf(∞) = 1

$te^{-at}\big|_{t=\infty} = 0$

I obtain

$$\frac{2^{3/4}\pi^{1/4}}{\sqrt{a}}\sqrt{\frac{\frac{1}{2}a^2 a\pi\sqrt{2} + 2a\sqrt{2} - 4a\sqrt{2}}{\sqrt{\pi}}} = a\sqrt{2}\sqrt{\pi-2} = a\sqrt{2\pi-4}$$

7) Standard Deviation Rise Time for h^2(t)

$$\sqrt{2\cdot\pi}\cdot\sqrt{\frac{\int_0^{\infty}\left[t-\left(a\cdot\sqrt{\frac{2}{\pi}}\right)\right]^2\cdot\left[e^{-\left[\frac{(t\cdot a)^2}{2\cdot a^2}\right]}\right]^2 dt}{\int_0^{\infty}\left[e^{-\left[\frac{(t\cdot a)^2}{2\cdot a^2}\right]}\right]^2 dt}}$$

$$2\cdot\pi^{\frac{1}{2}}\cdot\left[\frac{\left[\frac{1}{4}\cdot\frac{\left(4\cdot a^2+\pi\right)}{\pi^{\frac{1}{2}}}-\frac{1}{\pi^{\frac{1}{2}}}\cdot a\cdot\sqrt{2}\right]}{\pi^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

$$\left(4\cdot a^2+\pi-4\cdot a\cdot\sqrt{2}\right)^{\frac{1}{2}}$$

$$\sqrt{4a^2+\pi-4a\sqrt{2}}$$

p51a
Table 2
w(t) proof

$$\int_0^t t\cdot e^{-a\cdot t}\,dt$$

$$\int t\cdot e^{-a\cdot t}\,dt$$

$$-\exp(-a\cdot t)\cdot\frac{(a\cdot t+1)}{a^2}$$

$$-e^{-a\cdot t}\cdot\frac{a\cdot t+1}{a^2} \quad \text{for t from 0 to t}$$

$$-e^{-a\cdot t}\cdot\frac{a\cdot t+1}{a^2} - -e^{-a\cdot 0}\cdot\frac{a\cdot 0+1}{a^2}$$

$$\frac{-(a\cdot t\cdot\exp(-a\cdot t)+\exp(-a\cdot t)-1)}{a^2}$$

$$= \frac{-ate^{-at} - e^{-at} + 1}{a^2}$$

$$= \frac{1 - e^{-at} - ate^{-at}}{a^2}$$

10 - 90% Rise time

Table 2

Ken Kaiser

$$T_r = t_{90\%} - t_{10\%}$$

$$.1\left(\frac{1}{a^2}\right) = \left(\frac{1 - e^{-at_{10\%}} - at_{10\%}e^{-at_{10\%}}}{a^2}\right)$$

$$.1 = 1 - e^{-at_{10\%}} - at_{10\%}e^{-at_{10\%}}$$

$$.1 - 1 = -e^{-at_{10\%}} - at_{10\%}e^{-at_{10\%}}$$

$$-.9 = -(e^{-at_{10\%}} + at_{10\%}e^{-at_{10\%}})$$

$$.9 = e^{-at_{10\%}} + at_{10\%}e^{-at_{10\%}}$$

$$.9 = e^{-at_{10\%}}(1 + at_{10\%})$$

$$\frac{.9}{1 + at_{10\%}} = e^{-at_{10\%}} \rightarrow \text{mathcad}$$

$$.9\left(\frac{1}{a^2}\right) = \frac{1 - e^{-at_{10\%}} - at_{10\%}e^{-at_{10\%}}}{a^2}$$

$$.9 = 1 - e^{-at_{10\%}} - at_{10\%}e^{-at_{10\%}}$$

$$.1 - 1 = -e^{-at_{10\%}} - at_{10\%}e^{-at_{10\%}}$$

$$-.1 = -e^{-at_{10}}(1 + at_{10\%})$$

$$\frac{.1}{1 + at_{10\%}} = e^{-at_{10\%}} \rightarrow \text{mathcad}$$

: 10-90% rise time cont

Ken Kaiser

Table 2

$$\frac{.9}{1+a\cdot t}=e^{-a\cdot t}$$

$$\begin{bmatrix} \frac{-.39165871526656812942}{a} \\ \frac{.53181160838961 20201}{a} \end{bmatrix}$$

$$\frac{.1}{1+a\cdot t}=e^{-a\cdot t}$$

$$\begin{bmatrix} \frac{-.96177875825320056806}{a} \\ \frac{3.8897201698674290579}{a} \end{bmatrix}$$

$$\frac{3.8897201698674290579}{a}-\frac{.53181160838961 20201}{a}=\frac{3.358}{a}\approx\frac{3.4}{a}$$

Ken Kaiser

Table 4

$$\omega(t) = \int_{-\infty}^{t} h(t)\,dt$$

$$\omega(t) = \int_{-\infty}^{t} e^{-\frac{t^2}{2a^2}}\,dt$$

⇓

$$a\sqrt{\frac{\pi}{2}}\left[\operatorname{erf}\left(\frac{t}{a\sqrt{2}}\right)+1\right] = \int_{-\infty}^{t} e^{-\frac{t^2}{2a^2}}\,dt$$

From the approximation table, if $t \to \infty$ then the expression for the error function can be approximated to 1

⇓

$$a\sqrt{\frac{\pi}{2}}\,[1+1] = \int_{-\infty}^{t} e^{-\frac{t^2}{2a^2}}\,dt$$

$$\frac{a\sqrt{\pi}\,2}{\sqrt{2}} = a\sqrt{2\pi}$$

$$a\sqrt{2\pi} = a\sqrt{2\pi}$$

$$\int_{-\infty}^{\infty} e^{-\frac{t^2}{2 \cdot a^2}}\,dt$$

$$\sqrt{2} \cdot a \cdot \sqrt{\pi}$$

* Since these expressions yield the same result, the expression for $\omega(t)$ must be correct

50% Delay Time

The value of the step response is equal to one half of the final value of the step response.

$$\Rightarrow \omega(t) = a\sqrt{\frac{\pi}{2}}\left[\mathrm{erf}\left(\frac{t}{a\sqrt{2}}\right)+1\right] = \frac{a\sqrt{2\pi}}{2}$$

final value divided by 2

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\lambda^2}\,d\lambda \Rightarrow \frac{2}{\sqrt{\pi}}\int_0^{\frac{t}{a\sqrt{2}}} e^{-\lambda^2}\,d\lambda = \frac{a\sqrt{2\pi}}{2}$$

Given

$$a\cdot\frac{\sqrt{2\cdot\pi}}{2} = a\cdot\sqrt{\frac{\pi}{2}}\cdot\left(\frac{2}{\sqrt{\pi}}\cdot\int_0^{\frac{\tau_d}{a\cdot\sqrt{2}}} e^{\lambda^2}\,d\lambda + 1\right)$$

Find $\tau_d \rightarrow 0$

$\mathrm{erf}(x) = 1$ as $x \rightarrow \infty$

so if $t \rightarrow \infty$

then $\omega(\infty) = a\sqrt{\frac{\pi}{2}}\,[1+1]$

$$= \frac{a\sqrt{\pi}\,2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = a\sqrt{2\pi}$$

final value

Centroid Delay Time

$$\tau_d = \frac{\int_0^\infty t\,h(t)\,dt}{\int_0^\infty h(t)\,dt} \qquad h(t) = e^{-\frac{t^2}{2a^2}}$$

$$\tau_d = \frac{\int_{-\infty}^{\infty} t\cdot e^{\frac{t^2}{2\cdot a^2}}\,dt}{\int_{-\infty}^{\infty} e^{\frac{t^2}{2\cdot a^2}}\,dt}$$

$\tau_d = 0$

Average Delay Time

* same as the centroid Delay time

$$\frac{\text{Final Value}}{2} = \int_0^{\tau_d} h(t)\,dt$$

$$\frac{}{2} = w(t)\Big|_{\tau_d = 0}$$

$$a\cdot\sqrt{\frac{\pi}{2}}\cdot\left[\frac{2}{\sqrt{\pi}}\cdot\int_0^{\frac{0}{a\cdot\sqrt{2}}} e^{-\lambda^2}\,d\lambda + 1\right]$$

$$\frac{\text{Final Value}}{2} = \frac{1}{2}\cdot a\cdot\sqrt{2}\cdot\sqrt{\pi}$$

$$\frac{a\sqrt{2\pi}}{2} = \frac{a\sqrt{2\pi}}{2}$$

10-90% Rise Time

$$\tau_r = t_{90\%} - t_{10\%}$$

Final Value

$$(0.1)(a\sqrt{2\pi}) = \omega(t_{10\%})$$

$$0.1 = \frac{\omega(t_{10\%})}{a\sqrt{2\pi}} \Rightarrow \frac{a\cdot\sqrt{\frac{\pi}{2}}\cdot\left[\frac{2}{\sqrt{\pi}}\cdot\int_0^{\frac{-1.2816\cdot a}{a\cdot\sqrt{2}}} e^{-\lambda^2}\,d\lambda + 1\right]}{a\cdot\sqrt{2\cdot\pi}}$$

$t_{10\%} = -1.2816a$

.10000904996608381492

$$(0.9)(a\sqrt{2\pi}) = \omega(t_{90\%})$$

$$0.9 = \frac{\omega(t_{90\%})}{a\sqrt{2\pi}} \Rightarrow \frac{a\cdot\sqrt{\frac{\pi}{2}}\cdot\left[\frac{2}{\sqrt{\pi}}\cdot\int_0^{\frac{1.2816\cdot a}{a\cdot\sqrt{2}}} e^{-\lambda^2}\,d\lambda + 1\right]}{a\cdot\sqrt{2\cdot\pi}}$$

$t_{90\%} = 1.2816a$

.90000849990232483384

$$\tau_r = 1.2816a - {}^{-}1.2816a$$

$$\boxed{\tau_r = 2.5632\,a}$$

Slope-Based Rise Time

$$\tau_r = \frac{\text{final value of step response}}{h(t)\big|_{\tau_d = 0}}$$

$$\tau_r = \lim_{t \to 0} \frac{a\cdot\sqrt{2\cdot\pi}}{e^{\frac{t^2}{2\cdot a^2}}}$$

$$\tau_r = a\cdot\sqrt{2\cdot\pi} \cong 2.507\,a$$

Standard Deviation Rise Time

$$\tau_r = \sqrt{2\pi}\,\sqrt{\frac{\int_{-\infty}^{\infty}(t-\tau_d)^2\,h(t)\,dt}{\int_{-\infty}^{\infty}h(t)\,dt}} \qquad \tau_d = 0 \qquad h(t) = e^{-\frac{t^2}{2a^2}}$$

$$\tau_r = \sqrt{2\cdot\pi\cdot\frac{\int_{-\infty}^{\infty} t^2\cdot e^{-\frac{t^2}{2\cdot a^2}}\,dt}{\int_{-\infty}^{\infty} e^{-\frac{t^2}{2\cdot a^2}}\,dt}}$$

$$\tau_r = a\cdot\sqrt{2\cdot\pi} \cong 2.507\,a$$

Standard Deviation Rise Time for h^2(t)

$$\tau_r = \sqrt{2\pi}\sqrt{\frac{\int_{-\infty}^{\infty}(t-\tau_d)^2 h^2(t)\,dt}{\int_{-\infty}^{\infty} h^2(t)\,dt}} \quad \text{where} \quad \tau_d = \frac{\int_{-\infty}^{\infty} t\,h^2(t)\,dt}{\int_{-\infty}^{\infty} h^2(t)\,dt}$$

$$\tau_d = \frac{\int_{-\infty}^{\infty} t \cdot \left(e^{\frac{t^2}{2\cdot a^2}}\right)^2 dt}{\int_{-\infty}^{\infty} \left(e^{\frac{t^2}{2\cdot a^2}}\right)^2 dt}$$

$$\tau_d = 0$$

$$\tau_r = \sqrt{2\cdot\pi\cdot\frac{\int_{-\infty}^{\infty} t^2 \cdot \left(e^{\frac{t^2}{2\cdot a^2}}\right)^2 dt}{\int_{-\infty}^{\infty} \left(e^{\frac{t^2}{2\cdot a^2}}\right)^2 dt}}$$

$$\tau_r = a\cdot\sqrt{\pi} \cong 1.77\,a$$

P 525
5th line
proof

Ken Kaiser

$$+ \frac{-2 \left. \frac{dH_1(\omega)}{d\omega} \right|_{\omega=0} \left. \frac{dH_2(\omega)}{d\omega} \right|_{\omega=0}}{H_1(0)\, H_2(0)}$$

$$j \frac{dH}{d\omega} = \int_{-\infty}^{\infty} t h(t) dt$$

$$\frac{dH}{d\omega} = \frac{\int_{-\infty}^{\infty} t h(t) dt}{j}$$

$$\frac{1}{j^2} = \frac{1}{-1} = -1$$

$$= -2 \frac{\int_{-\infty}^{\infty} t h_1(t) dt}{j H_1(0)} \cdot \frac{\int_{-\infty}^{\infty} t h_2(t) dt}{j H_2(0)}$$

$$= +2 \frac{\int_{-\infty}^{\infty} t h_1(t) dt}{H_1(0)} \cdot \frac{\int_{-\infty}^{\infty} t h_2(t) dt}{H_2(0)}$$

p 535
Figure 23 & 24.

$$\frac{\sin\left(\frac{\omega T}{2}\right)}{T}$$

Ken Kaiser

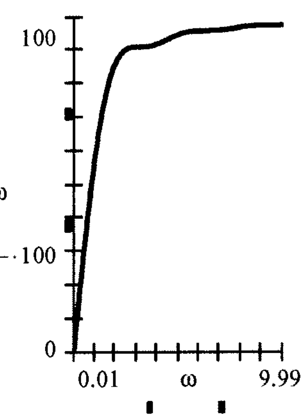
$T := 2$

$$\int_0^{10} \left(\left| \frac{2 \cdot \sin\left(\omega \cdot \frac{T}{2}\right)}{\omega} \right| \right)^2 d\omega = 6.075$$

$$\int_0^{3.14} \left(\left| \frac{2 \cdot \sin\left(\omega \cdot \frac{T}{2}\right)}{\omega} \right| \right)^2 d\omega = 5.673$$

$\omega := 0.01, 0.03 .. 10$

$$\frac{\int_0^{\omega} \left(\left| \frac{2 \cdot \sin\left(\omega \cdot \frac{T}{2}\right)}{\omega} \right| \right)^2 d\omega}{6.23} \cdot 100$$

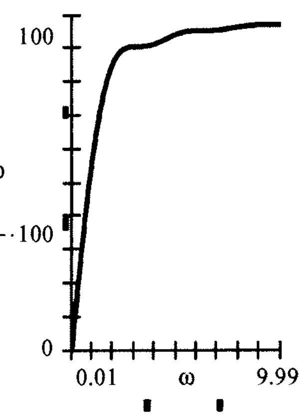


$$\int_{0}^{80}\left(\left|\frac{2\cdot\sin(\omega)}{\omega}\right|\right)^{2}d\omega = 6.258$$

$$\int_{0}^{3.14}\left(\left|\frac{2\cdot\sin(\omega)}{\omega}\right|\right)^{2}d\omega = 5.673$$

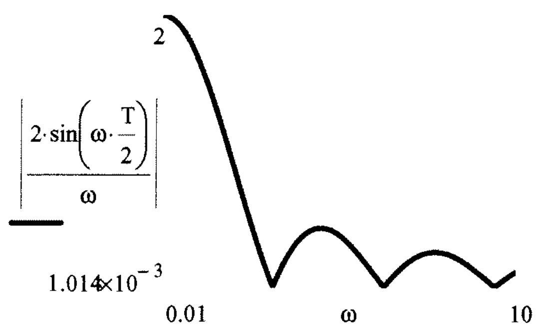
$$\omega := 0.01, 0.03 .. 10$$

$$\frac{\int_{0}^{\omega}\left(\left|\frac{2\cdot\sin(\omega)}{\omega}\right|\right)^{2}d\omega}{6.26}\cdot 100$$



$\omega := 0.01, 0.02 .. 10$

$T := 2$



Verification of Percent Energy vs. $BW_{z\%}$ plot on page ~~533~~ 536

$$BW := 0, .01 .. 10$$

$$\omega := 0, 6.34 .. 2 \cdot \pi$$

$$z\%(\omega) := 100 \cdot \frac{\frac{1}{\pi} \cdot \int_{0}^{BW} \left(\left| \frac{2 \cdot \sin(\omega)}{\omega} \right| \right)^{2} d\omega}{\frac{1}{\pi} \cdot \int_{.01}^{1000} \left(\left| \frac{2 \cdot \sin(\omega)}{\omega} \right| \right)^{2} d\omega}$$