Fourier Transform

$$e^{-at^2} \rightarrow \sqrt{\frac{\pi}{a}}\, e^{-\frac{\omega^2}{4a}} \qquad a>0$$

? $$e^{jat^2} \rightarrow \sqrt{\frac{\pi}{+ja}}\, e^{-j\frac{\omega^2}{4a}}$$

$$= \sqrt{\frac{\pi}{a}}\, e^{j\frac{\pi}{4}}\, e^{-j\frac{\omega^2}{4a}}$$

$$= \sqrt{\frac{\pi}{a}}\, e^{-j\left(\frac{\omega^2}{4a}+\frac{\pi}{4}\right)}$$

Sketch

$$\frac{A}{\tau_1} e^{+\pi \frac{|t-\tau_d|}{\tau_1^2}}$$

$$A = \int h(t) w \, dt$$

~~$y \Leftrightarrow Y(\omega)$~~

Since $\frac{dx}{dt} \Leftrightarrow j\omega X(\omega)$

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$$\Rightarrow \int_{-\infty}^{\infty} \left[\frac{dx}{dt}\right]^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |j\omega X(\omega)|^2 \, d\omega$$

also

$$\int_{-\infty}^{\infty} [t\,h(t)]^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left| j \frac{d H(\omega)}{d\omega} \right|^2 d\omega$$

Let $x(t) = 2e^{-3t}u(t)$, $\frac{dx}{dt} = -6e^{-3t}u(t) + 2\delta(t)$

$$X(\omega) = \frac{2}{3+j\omega}$$

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} 4e^{-6t}u(t)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{2}{3+j\omega}\right|^2 d\omega$$

$$\Rightarrow ? \int_{-\infty}^{\infty} \left[-6e^{-3t}u(t) + 2\delta(t)\right]^2 dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|j\omega \times \frac{2}{3+j\omega}\right|^2 d\omega$$ ✓

$$-6e^{-3t}u(t) + 2\delta(t) \Leftrightarrow \frac{-6}{3+j\omega} + 2 = \frac{-6}{3+j\omega} + \frac{6+j2\omega}{3+j\omega}$$

$$= \frac{j2\omega}{3+j\omega}$$

Ken Kaiser

Let $x(t) = 2e^{-3t}u(t)$

$$tx(t) = 2te^{-3t}u(t)$$

$$X(\omega) = \frac{2}{3+j\omega}$$

$$F\{tx(t)\} = F\{2te^{-3t}u(t)\} = \frac{2}{(3+j\omega)^2}$$

$$j\frac{dH(\omega)}{d\omega} = j\left(\frac{(3+j\omega)-2j}{(3+j\omega)^2}\right)$$

$$= j\left(\frac{-2j}{(3+j\omega)^2}\right) = \frac{2}{(3+j\omega)^2}$$

Table Item Number 3

$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$ $\quad \dfrac{1}{(5 + i \cdot \omega)^2}$

let a be 2

$2 \cdot \left(t \cdot e^{-5 \cdot t} \cdot \Phi(t)\right)$ $\quad \dfrac{2}{(5 + i \cdot \omega)^2}$

$$\frac{2 \cdot \left[\dfrac{1}{(5 + i \cdot \omega)^2}\right]}{\dfrac{2}{(5 + i \cdot \omega)^2}}$$

1

pg. 448 Fourier Transform Table #3

Ken Kaiser

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$\frac{1}{(5 + i \cdot \omega)^2} \cdot 3$$

$$\frac{3}{(5 + i \cdot \omega)^2}$$

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot 3$$

$$3 \cdot t \cdot \exp(-5 \cdot t) \cdot \Phi(t)$$

$$\frac{3}{(5 + i \cdot \omega)^2}$$

$$\frac{\frac{3}{(5 + i \cdot \omega)^2}}{\frac{3}{(5 + i \cdot \omega)^2}}$$

$$1$$

Ken Kaiser

table item #4, p9. 448

$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot 3$ $\quad$ $t \cdot e^{-7 \cdot t} \cdot \Phi(t) \cdot 5$

$$\frac{3}{(5 + i \cdot \omega)^2} \qquad \frac{5}{(7 + i \cdot \omega)^2}$$

$$\frac{1}{(5 + i \cdot \omega)^2} \cdot 3 \qquad \frac{1}{(7 + i \cdot \omega)^2} \cdot 5$$

$$\frac{\frac{3}{(5 + i \cdot \omega)^2} + \frac{5}{(7 + i \cdot \omega)^2}}{\frac{1}{(5 + i \cdot \omega)^2} \cdot 3 + \frac{1}{(7 + i \cdot \omega)^2} \cdot 5}$$

1

pg. 448 Fourier Transform Properties

Ken Kaiser

7

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$\frac{1}{(5 + i \cdot t)^2}$$

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$(-\omega) \cdot e^{-5 \cdot (-\omega)} \cdot \Phi(-\omega) \cdot 2 \cdot \pi$$

$$\frac{1}{(-5 - i \cdot t)^2}$$

Inverse Laplace

$$\frac{\frac{1}{(-5 - i \cdot t)^2}}{\frac{1}{(5 + i \cdot t)^2}}$$

$$1$$

Table Item Number 8

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$(-t) \cdot e^{-5 \cdot (-t)} \cdot \Phi(-t)$$

$$\frac{1}{(-5 + i \cdot \omega)^2}$$

$$\frac{\frac{1}{(5 + i \cdot \omega)^2}}{\frac{1}{(-5 + i \cdot \omega)^2}}$$

1

Table Item Number 9

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \qquad \frac{1}{(5 + i \cdot \omega)^2}$$

a is equal to 2

$$(2 \cdot t) \cdot e^{-5 \cdot (2 \cdot t)} \cdot \Phi(2 \cdot t) \qquad \frac{2}{(10 + i \cdot \omega)^2}$$

$$\frac{\frac{1}{|2|} \cdot \frac{1}{\left(5 + i \cdot \frac{\omega}{2}\right)^2}}{\frac{2}{(10 + i \cdot \omega)^2}}$$

1

$$-2 \cdot \frac{\left(-25 - 10 \cdot i \cdot \omega + \omega^2\right)}{(10 + i \cdot \omega)^2}$$

Table Item Number 10

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$(t - a) \cdot e^{-5 \cdot (t - a)} \cdot \Phi(t - a)$$

$$i \cdot \exp(5 \cdot a) \cdot \left[\frac{-i}{(5 + i \cdot \omega)^2} \cdot \exp(-i \cdot \omega \cdot a - 5 \cdot a) - \frac{i}{(5 + i \cdot \omega)} \cdot a \cdot \exp(-i \cdot \omega \cdot a - 5 \cdot a) \right] - a \cdot \frac{\exp(5 \cdot a)}{(5 + i \cdot \omega)} \cdot \exp(-i \cdot \omega \cdot a - 5 \cdot a)$$

$$\frac{\frac{1}{(5 + i \cdot \omega)^2} \cdot e^{-i \cdot \omega \cdot a}}{i \cdot \exp(5 \cdot a) \cdot \left[\frac{-i}{(5 + i \cdot \omega)^2} \cdot \exp(-i \cdot \omega \cdot a - 5 \cdot a) - \frac{i}{(5 + i \cdot \omega)} \cdot a \cdot \exp(-i \cdot \omega \cdot a - 5 \cdot a) \right] - a \cdot \frac{\exp(5 \cdot a)}{(5 + i \cdot \omega)} \cdot \exp(-i \cdot \omega \cdot a - 5 \cdot a)}$$

1

let a be equal to 2

$$(t - 2) \cdot e^{-5 \cdot (t - 2)} \cdot \Phi(t - 2)$$

$$i \cdot \exp(10) \cdot \left[\frac{-i}{(5 + i \cdot \omega)^2} \cdot \exp(-2 \cdot i \cdot \omega - 10) - 2 \cdot \frac{i}{(5 + i \cdot \omega)} \cdot \exp(-2 \cdot i \cdot \omega - 10) \right] - 2 \cdot \frac{\exp(10)}{(5 + i \cdot \omega)} \cdot \exp(-2 \cdot i \cdot \omega - 10)$$

$$\frac{\frac{1}{(5 + i \cdot \omega)^2} \cdot e^{-i \cdot \omega \cdot 2}}{i \cdot \exp(10) \cdot \left[\frac{-i}{(5 + i \cdot \omega)^2} \cdot \exp(-2 \cdot i \cdot \omega - 10) - 2 \cdot \frac{i}{(5 + i \cdot \omega)} \cdot \exp(-2 \cdot i \cdot \omega - 10) \right] - 2 \cdot \frac{\exp(10)}{(5 + i \cdot \omega)} \cdot \exp(-2 \cdot i \cdot \omega - 10)}$$

1

Table Item Number 12

$e^{-j\alpha t} x(t) \Rightarrow X(\omega + \alpha)$ a real

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$ ✓

let a be equal to 2

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot e^{-i \cdot 2 \cdot t}$$ ✓

$$\frac{1}{(5 + i \cdot (\omega + 2))^2}$$

$$\frac{\frac{1}{(5 + i \cdot (\omega + 2))^2}}{\frac{1}{(5 + i \cdot (\omega + 2))^2}}$$ ✓

1

p. 449

Fourier Transform Property Table

Ken Kaiser

#13

$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$

$i := \sqrt{-1}$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot t$

$t^2 \cdot \exp(-5 \cdot t) \cdot \Phi(t)$

$$\left[\frac{d}{d\omega} \frac{1}{(5 + i \cdot \omega)^2}\right] \cdot i$$

$$\frac{2}{(5 + i \cdot \omega)^3}$$ ✓

$$-2 \cdot \frac{i}{(5 + i \cdot \omega)^3} \cdot i$$ ✓

$$\frac{\frac{2}{(5 + i \cdot \omega)^3}}{-2 \cdot \frac{i}{(5 + i \cdot \omega)^3} \cdot i}$$ ✓

$$\frac{i}{i}$$

$$\frac{i}{i} = 1$$

Table Item Number 14

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

let n be equal to 3

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot t^3$$

$$\frac{24}{(5 + i \cdot \omega)^5}$$

$$\frac{\left[\frac{d^3}{d\,\omega^3} \frac{1}{(5 + i \cdot \omega)^2}\right] \cdot i^3}{\frac{24}{(5 + i \cdot \omega)^5}}$$

1

pg 449

Fourier Transform Property Table

Ken Kaisen

#15

$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$

$i := \sqrt{-1}$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$\frac{d}{dt} t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2} \cdot i \cdot \omega$$

$$\frac{1}{(5 + i \cdot \omega)} - \frac{5}{(5 + i \cdot \omega)^2}$$

$$\frac{\frac{1}{(5 + i \cdot \omega)^2} \cdot i \cdot \omega}{\frac{1}{(5 + i \cdot \omega)} - \frac{5}{(5 + i \cdot \omega)^2}}$$

$-i \cdot i$

$-i \cdot \sqrt{-1} = 1$

Table Item Number 16

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

let n be equal to 3

$$\frac{d^3}{dt^3} t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$-10 + i \cdot \omega + \frac{75}{(5 + i \cdot \omega)} - \frac{125}{(5 + i \cdot \omega)^2}$$

$$\frac{\frac{1}{(5 + i \cdot \omega)^2} \cdot (i \cdot \omega)^3}{-10 + i \cdot \omega + \frac{75}{(5 + i \cdot \omega)} - \frac{125}{(5 + i \cdot \omega)^2}}$$

1

Table Item Number 18

$$\left[\left(t\cdot e^{-5\cdot t}\right)+\left(t\cdot e^{-3\cdot t}\right)\right]\cdot\Phi(t)$$

$$\frac{1}{(5+i\cdot\omega)^2}+\frac{1}{(3+i\cdot\omega)^2}$$

even odd

$$\left[\int_0^{10}\left(t\cdot e^{-5\cdot t}\right)dt\right]+\left[\int_{-\infty}^{10}\left(t\cdot e^{-3\cdot t}\right)dt\right]$$

$$2\cdot\pi\cdot\left(\frac{-51}{25}\cdot\exp(-50)+\frac{1}{25}-\infty\right)\cdot\mathrm{Dirac}(\omega)$$

$$\frac{\dfrac{1}{(5+i\cdot\omega)^2}+\dfrac{1}{(3+i\cdot\omega)^2}}{i\cdot\omega}$$

$$2\cdot\pi\cdot\left(\frac{-51}{25}\cdot\exp(-50)+\frac{1}{25}-\infty\right)\cdot\mathrm{Dirac}(\omega)$$

0

Table Item Number 20

$$e^{-5\cdot t}\cdot\Phi(t)$$

$$\frac{1}{(5+i\cdot\omega)}$$

$$e^{-5\cdot 0}\cdot\Phi(t)$$

$$\pi\cdot\mathrm{Dirac}(\omega)-\frac{i}{\omega}$$

$$\frac{\int_{-10}^{10}\frac{1}{(5+i\cdot\omega)}\,d\omega\cdot\frac{1}{2\cdot\pi}}{\pi\cdot\mathrm{Dirac}(\omega)-\frac{i}{\omega}}$$

$$\mathrm{atan}(2)\cdot\frac{\omega}{(\pi\cdot(\pi\cdot\Delta(\omega)\cdot\omega-i))}$$

$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{j\omega t}\,d\omega$

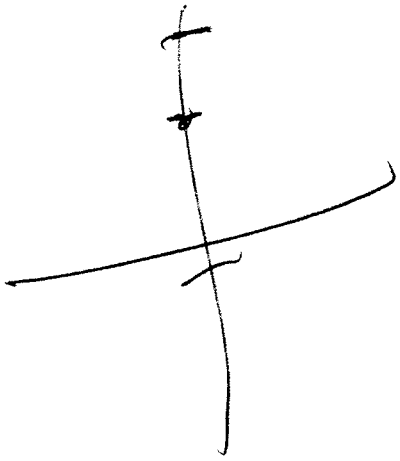
$x(0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)\,d\omega$

$x(0)=e^{0}u(0)=1$

$x(t)=e^{-5t}u(t)$

$X(\omega)=\frac{1}{5+j\omega}$

$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{5+j\omega}\,d\omega$



pg. 449 Fourier Transform Property Table

Ken Kaiser

#21

$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$ $\quad i := \sqrt{-1}$ $\quad \dfrac{1}{(5 + i \cdot \omega)^2}$

$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot \sin(3 \cdot t)$

$$\left[\frac{1}{\left[(5 + i \cdot (\omega - 3))^2\right]} + \frac{-1}{\left[(5 + i \cdot (\omega + 3))^2\right]}\right] \cdot \frac{1}{2 \cdot i}$$

$$\frac{-1}{2} \cdot \frac{i}{(5 + i \cdot \omega - 3i)^2} + \frac{1}{2} \cdot \frac{i}{(5 + i \cdot \omega + 3i)^2}$$

$$\frac{\left[\dfrac{1}{\left[(5 + i \cdot (\omega - 3))^2\right]} + \dfrac{1}{\left[(5 + i \cdot (\omega + 3))^2\right]} \cdot -1\right] \cdot \dfrac{1}{2 \cdot i}}{\dfrac{-1}{2} \cdot \dfrac{i}{(5 + i \cdot \omega - 3i)^2} + \dfrac{1}{2} \cdot \dfrac{i}{(5 + i \cdot \omega + 3i)^2}}$$

$$\frac{i}{i} = 1$$

Table Item Number 22

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

let a be equal to 3

$$\left(t \cdot e^{-5 \cdot t} \cdot \Phi(t)\right) \cdot \cos(3 \cdot t)$$

$$\frac{1}{\left[2 \cdot (5 + i \cdot \omega - 3i)^2\right]} + \frac{1}{\left[2 \cdot (5 + i \cdot \omega + 3i)^2\right]}$$

$$\frac{\frac{1}{2} \cdot \left[\frac{1}{(5 + i \cdot (\omega - 3))^2} + \frac{1}{(5 + i \cdot (\omega + 3))^2}\right]}{\frac{1}{\left[2 \cdot (5 + i \cdot \omega - 3i)^2\right]} + \frac{1}{\left[2 \cdot (5 + i \cdot \omega + 3i)^2\right]}}$$

1

pg. 449 Fourier Transform Property Table #23

Ken Kaiser

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \qquad i := \sqrt{-1}$$

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot \cos(3 \cdot t) \cdot \cos(3 \cdot t)$$

$$\frac{1}{\left[4 \cdot (5 + i \cdot \omega - 6i)^2\right]} + \frac{1}{\left[4 \cdot (5 + i \cdot \omega + 6i)^2\right]} + \frac{1}{\left[2 \cdot (5 + i \cdot \omega)^2\right]}$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$\frac{1}{2} \cdot \frac{1}{(5 + i \cdot \omega)^2} + \frac{1}{4} \cdot \left[\frac{1}{(5 + i \cdot (\omega - 2 \cdot 3))^2} + \frac{1}{(5 + i \cdot (\omega + 2 \cdot 3))^2}\right]$$

$$\frac{\left(\omega^4 - 168 \cdot \omega^2 + 680i \cdot \omega + 1723 - 20i \cdot \omega^3\right)}{\left[(5 + i \cdot \omega)^2 \cdot \left[(5 + i \cdot \omega - 6i)^2 \cdot (5 + i \cdot \omega + 6i)^2\right]\right]}$$

$$\frac{\dfrac{\left(\omega^4 - 168 \cdot \omega^2 + 680i \cdot \omega + 1723 - 20i \cdot \omega^3\right)}{\left[(5 + i \cdot \omega)^2 \cdot \left[(5 + i \cdot \omega - 6i)^2 \cdot (5 + i \cdot \omega + 6i)^2\right]\right]}}{\dfrac{1}{\left[4 \cdot (5 + i \cdot \omega - 6i)^2\right]} + \dfrac{1}{\left[4 \cdot (5 + i \cdot \omega + 6i)^2\right]} + \dfrac{1}{\left[2 \cdot (5 + i \cdot \omega)^2\right]}}$$

1

I personally double-checked these results

Table Item Number 24

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$t^2 \cdot e^{-3 \cdot t} \cdot \Phi(t)$$

$$\frac{2}{(3 + i \cdot \omega)^3}$$

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t) \cdot \left(t^2 \cdot e^{-3 \cdot t} \cdot \Phi(t)\right)$$

$$\text{fourier}\left(t^3 \cdot \exp(-5 \cdot t) \cdot \Phi(t)^2 \cdot \exp(-3 \cdot t), t, \omega\right)$$

$$\frac{\frac{1}{(5 + i \cdot \omega)^2} \cdot \frac{2}{(3 + i \cdot \omega)^3}}{\text{fourier}\left(t^3 \cdot \exp(-5 \cdot t) \cdot \Phi(t)^2 \cdot \exp(-3 \cdot t), t, \omega\right)}$$

$$\frac{1}{\left[3 \cdot \left[(5 + i \cdot \omega)^2 \cdot (3 + i \cdot \omega)^3\right]\right]} \cdot (8 + i \cdot \omega)^4$$

Table Item Number 26

$$t \cdot e^{-5 \cdot t} \cdot \Phi(t)$$

$$\frac{1}{(5 + i \cdot \omega)^2}$$

$$\int_{-10}^{10} \left(\left| t \cdot e^{-5 \cdot t} \cdot \Phi(t) \right| \right)^2 dt$$

$$\frac{\frac{1}{2 \cdot \pi} \cdot \int_{-10}^{10} \left[\left| \frac{1}{(5 + i \cdot \omega)^2} \right| \right]^2 d\omega}{\blacksquare}$$

Table 2
#27

$$e^{-5\cdot t}\cdot\Phi(t)$$

$$\frac{1}{(5+i\cdot\omega)}$$

$$e^{-3\cdot t}\cdot\Phi(t)\cdot i \qquad -i\cdot e^{-3\cdot t}\cdot\Phi(t)$$

$$\frac{i}{(3+i\cdot\omega)} \qquad \frac{-i}{(3+i\cdot\omega)}$$

$$\int_{-\infty}^{\infty} e^{-5\cdot t}\cdot\Phi(t)\cdot\left(-i\cdot e^{-3\cdot t}\cdot\Phi(t)\right)dt$$

$$\frac{-1}{8}\cdot i$$

$$\frac{1}{2\cdot\pi}\cdot\int_{-\infty}^{\infty}\frac{1}{(5+i\cdot\omega)}\cdot\frac{-i}{(3+i\cdot\omega)}\,d\omega$$

$$0$$

Table Item Number 30

$\sin(t)\cdot e^{-5\cdot t}\cdot\Phi(t)$ — not an odd function

$$\frac{-1}{2}\cdot\frac{i}{(5+i\cdot\omega-i)}+\frac{1}{2}\cdot\frac{i}{(5+i\cdot\omega+i)}$$

$\sin(-t)\cdot e^{-5\cdot -t}\cdot\Phi(-t)$

$$\frac{-1}{2}\cdot\frac{i}{(-5+i\cdot\omega-i)}+\frac{1}{2}\cdot\frac{i}{(-5+i\cdot\omega+i)}$$

$-\left(\sin(t)\cdot e^{-5\cdot t}\cdot\Phi(t)\right)$

$$\frac{1}{2}\cdot\frac{i}{(5+i\cdot\omega-i)}-\frac{1}{2}\cdot\frac{i}{(5+i\cdot\omega+i)}$$

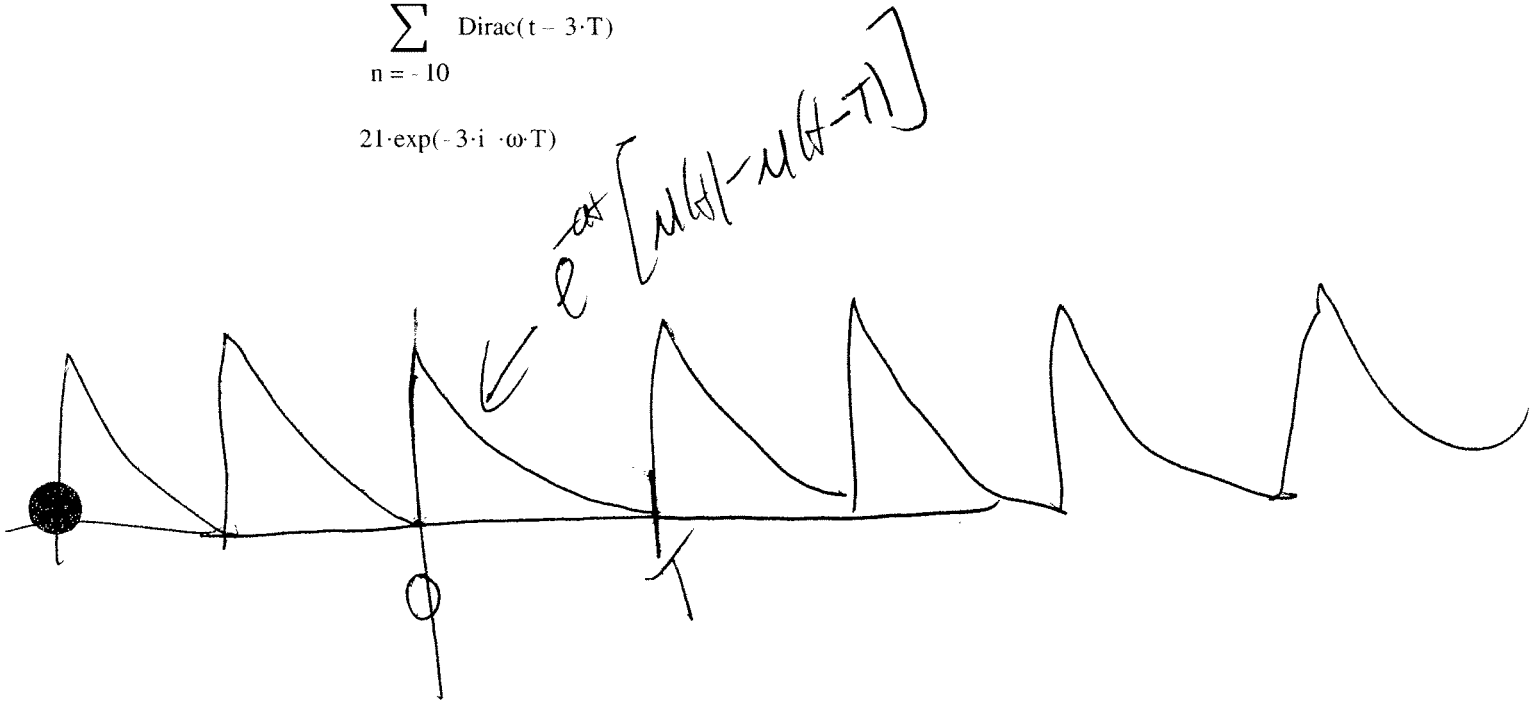
Table Item Number 32

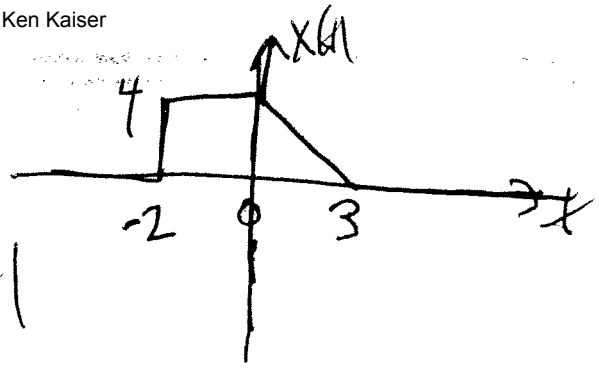
Dirac(t)

1

$$\sum_{n=-10}^{10} \text{Dirac}(t - 3\cdot T)$$

$21\cdot\exp(-3\cdot i\cdot\omega\cdot T)$





$x(t) = x_e(t) + x_o(t)$

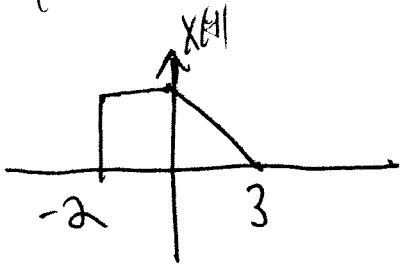
$x(t) = 4[u(t+2) - u(t)] + \left(\frac{-4}{3}t + 4\right)[u(t) - u(t-3)]$

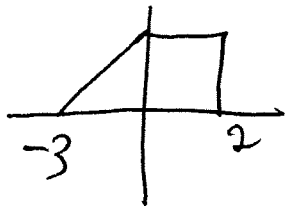
$X(\omega)$

$$x_e \equiv \frac{x(t) + x(-t)}{2}$$

$$x_o = \frac{x(t) - x(-t)}{2}$$

$x_e + x_o = x(t)$





$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

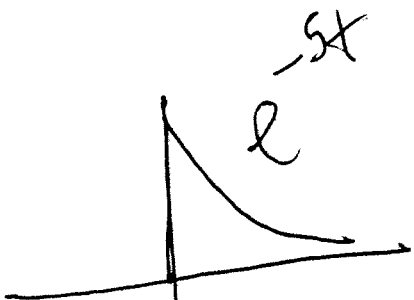
$$x_e(-t) = \frac{x(-t) + x(t)}{2} \checkmark$$

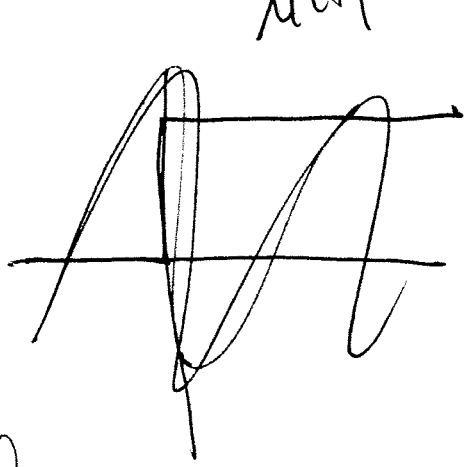
$$x_o(t) = \frac{x(-t) - x(t)}{2}$$

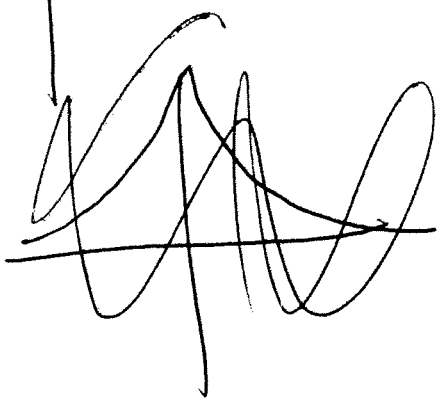
$$f(t) = e^{-5t}u(t)$$

$$g(t) = u(t)$$

$$\frac{1 - e^{-5t}}{5}$$







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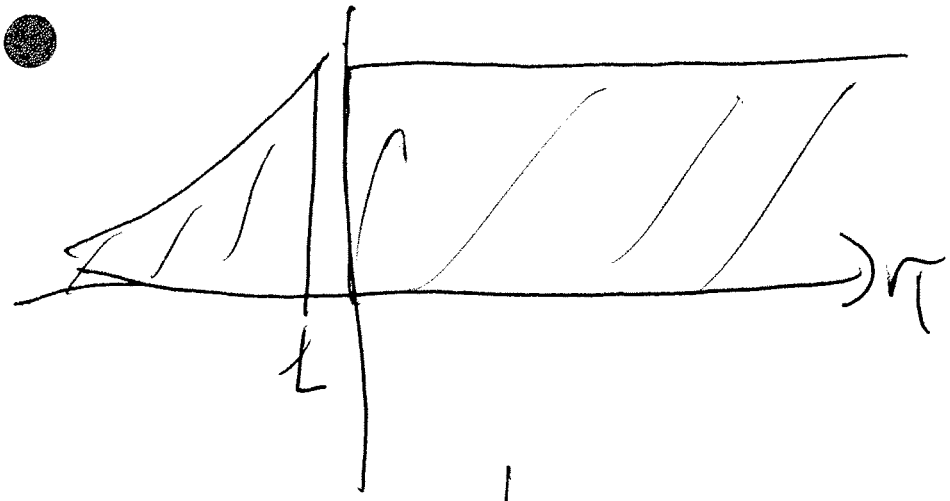
$$e^{-5(t-\tau)} = e^{5t} e^{-5\tau}$$



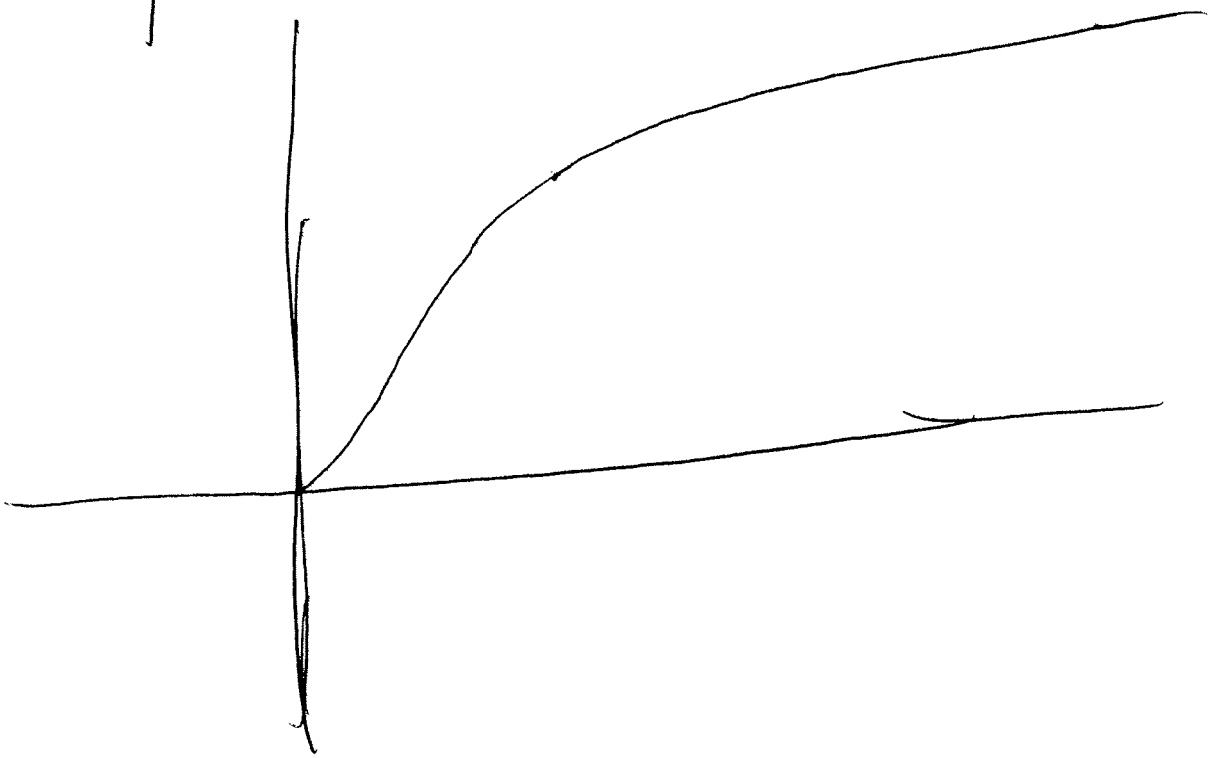


Table 2 #6

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{3+jw} e^{jwt}\,dw$$

$$x(t) = jt$$

$$= j e^{-5t} u(t)$$

$$x^*(t) = -j e^{-5t} u(t)$$

$$X(w) = \frac{1}{5+jw}$$

$$X^*(w) = \frac{1}{5-jw}$$

$$X^*(-w) = \frac{1}{5+jw}$$

Table 1
Off cd

$$\frac{d}{dx}\text{Dirac}(x)$$

$$i \cdot \omega$$

$$\frac{d^3}{dx^3}\text{Dirac}(x)$$

$$-i \cdot \omega^3$$

$$\frac{d^4}{dx^4}\text{Dirac}(x)$$

$$\omega^4$$

Dirac(x)

1

$$\frac{d^2}{dx^2}\text{Dirac}(x-2)$$

$$-\omega^2 \cdot \exp(-2 \cdot i \cdot \omega)$$

$$\frac{d^3}{dx^3}\text{Dirac}(x-2)$$

$$-i \cdot \omega^3 \cdot \exp(-2 \cdot i \cdot \omega)$$

$$\frac{d^2}{dx^2}\text{Dirac}(x-4)$$

$$-\omega^2 \cdot \exp(-4 \cdot i \cdot \omega)$$

$$\frac{d^3}{dx^3}\text{Dirac}(x-4)$$

$$-i \cdot \omega^3 \cdot \exp(-4 \cdot i \cdot \omega)$$

$\Phi(x)$

$\pi \cdot \mathrm{Dirac}(\omega) - \frac{i}{\omega}$

$$\Phi(x-2)$$

$$\exp(-2\cdot i\cdot\omega)\cdot\left(\pi\cdot\mathrm{Dirac}(\omega)-\frac{i}{\omega}\right)$$

$$\Phi(x+2)$$

$$\exp(2\cdot i\cdot\omega)\cdot\left(\pi\cdot\mathrm{Dirac}(\omega)-\frac{i}{\omega}\right)$$

$$\Phi(x-8)$$

$$\exp(-8\cdot i\cdot\omega)\cdot\left(\pi\cdot\mathrm{Dirac}(\omega)-\frac{i}{\omega}\right)$$

$$\frac{2}{\omega \cdot \sqrt{-1}}$$

$$\Phi(t) - \Phi(-t)$$

4

$8 \cdot \pi \cdot \mathrm{Dirac}(\omega)$

12

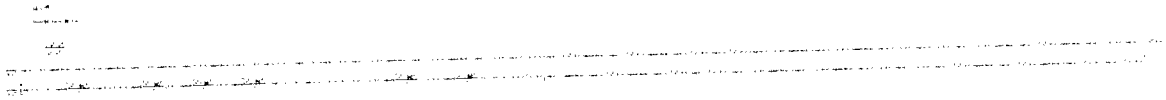
$24 \cdot \pi \cdot \mathrm{Dirac}(\omega)$

$$x \cdot \exp(-2 \cdot |x|)$$

$$-8 \cdot \frac{i}{\left(4 + \omega^2\right)^2} \cdot \omega$$

$$x \cdot \exp(-6 \cdot |x|)$$

$$-24 \cdot \frac{i}{\left(36 + \omega^2\right)^2} \cdot \omega$$



$$\frac{1}{(\alpha + \sqrt{-1}\ \omega)\ (B + \sqrt{-1}\ \omega)}$$

$$\frac{1}{2}\ \frac{\left(-\exp(-B\ t)\ \Phi(t) - \exp(-B\ t)\ \Phi(t)\ \text{signum}(\text{Re}(B)) + \exp(-B\ t)\ \Phi(-t) - \exp(-B\ t)\ \Phi(-t)\ \text{signum}(\text{Re}(B)) + \exp(-\alpha\ t)\ \Phi(t) + \exp(-\alpha\ t)\ \Phi(t)\ \text{signum}(\text{Re}(\alpha)) - \exp(-\alpha\ t)\ \Phi(-t) + \exp(-\alpha\ t)\ \Phi(-t)\ \text{signum}(\text{Re}(\alpha))\right)}{(-\alpha + B)}$$

$$\frac{1}{2}\ \left(\left(-\exp(-B\ t)\ \Phi(t) - \exp(-B\ t)\ \Phi(t)\ \text{signum}\left(\frac{\left(B^2 + (|B|)^2\right)^{\frac{1}{2}}}{B}\right) + \exp(-B\ t)\ \Phi(-t) - \exp(-B\ t)\ \Phi(-t)\ \text{signum}\left(\frac{\left(B^2 + (|B|)^2\right)^{\frac{1}{2}}}{B}\right) + \exp(-\alpha\ t)\ \Phi(t) + \exp(-\alpha\ t)\ \Phi(t)\ \text{signum}\left(\frac{\left(\alpha^2 + (|\alpha|)^2\right)^{\frac{1}{2}}}{\alpha}\right) - \exp(-\alpha\ t)\ \Phi(-t) + \exp(-\alpha\ t)\ \Phi(-t)\ \text{signum}\left(\frac{\left(\alpha^2 + (|\alpha|)^2\right)^{\frac{1}{2}}}{\alpha}\right)\right)\right)$$

$$\frac{e^{\alpha\ t} - e^{B\ t}}{B - \alpha}\ \Phi(t)$$

$$\frac{1}{(-\alpha + B)}\left(\frac{1}{(2\ \pi)}\left(2\ \pi^2\ \text{Dirac}(\omega - i\ \alpha) - 2\ i\ \frac{\pi}{(\omega - i\ \alpha)}\right) - \frac{1}{(2\ \pi)}\left(2\ \pi^2\ \text{Dirac}(\omega - i\ B) - 2\ i\ \frac{\pi}{(\omega - i\ B)}\right)\right)_{\omega}$$

$$\cos(4 \cdot x)$$

$$\pi \cdot \text{Dirac}(\omega - 4) + \pi \cdot \text{Dirac}(\omega + 4)$$

$$\cos(7 \cdot x)$$

$$\pi \cdot \text{Dirac}(\omega - 7) + \pi \cdot \text{Dirac}(\omega + 7)$$

$$\sin(3 \cdot x)$$

$$-i \cdot \pi \cdot \mathrm{Dirac}(\omega - 3) + i \cdot \pi \cdot \mathrm{Dirac}(\omega + 3)$$

$$\sin(6 \cdot x)$$

$$-i \cdot \pi \cdot \mathrm{Dirac}(\omega - 6) + i \cdot \pi \cdot \mathrm{Dirac}(\omega + 6)$$

$$\sum_{n=-\infty}^{\infty} \text{Dirac}(t - n \cdot T)$$

$$\sum_{n=-\infty}^{\infty} \exp(-i \cdot n \cdot T \cdot \omega)$$

$$\frac{2 \cdot \pi}{T} \cdot \sum_{n=-\infty}^{\infty} \text{Dirac}\left(\omega - \frac{2 \cdot \pi \cdot n}{T}\right)$$

$$\frac{1}{T} \cdot \sum_{n=-\infty}^{\infty} \exp\left(2 \cdot i \cdot \pi \cdot \frac{n}{T} \cdot t\right)$$

$$\exp(i \cdot 3 \cdot x)$$

$$2 \cdot \pi \cdot \mathrm{Dirac}(\omega + 3 \cdot i \cdot i)$$

$$\exp(i \cdot -4 \cdot x)$$

$$2 \cdot \pi \cdot \mathrm{Dirac}(\omega - 4 \cdot i \cdot i)$$

$$\frac{2 \cdot \sin\left(\frac{\omega \cdot T}{2}\right)}{\omega}$$

$$\frac{1}{2} \cdot \Phi\left(t + \frac{1}{2} \cdot T\right) - \frac{1}{2} \cdot \Phi\left(-t - \frac{1}{2} \cdot T\right) - \frac{1}{2} \cdot \Phi\left(t - \frac{1}{2} \cdot T\right) + \frac{1}{2} \cdot \Phi\left(-t + \frac{1}{2} \cdot T\right)$$

$|x|$

$$\frac{-2}{\omega^2}$$

$$x \cdot \Phi(x)$$

$$i \cdot \left(\pi \cdot \mathrm{Dirac}(1, \omega) + \frac{i}{\omega^2} \right)$$

$$i \cdot \left[\frac{\left(\pi \cdot \Delta(1, \omega) \cdot \omega^2 + i \right)}{\omega^2} \right]$$

$$x^3 \cdot \Phi(x)$$

$$-i \cdot \left(\pi \cdot \mathrm{Dirac}(3,\omega) + 6 \cdot \frac{i}{\omega^4} \right)$$

$$-i \cdot \left[\frac{\left(\pi \cdot \Delta(3,\omega) \cdot \omega^4 + 6 \cdot i \right)}{\omega^4} \right]$$

$$x^4 \cdot \Phi(x)$$

$$\pi \cdot \mathrm{Dirac}(4,\omega) - 24 \cdot \frac{i}{\omega^5}$$

$$\pi \cdot \Delta(4,\omega) - 24 \cdot \frac{i}{\omega^5}$$

x^2

$-2 \cdot \pi \cdot \mathrm{Dirac}(2, \omega)$

$-2 \cdot \pi \cdot \Delta(2, \omega)$

x^5

$2 \cdot i \cdot \pi \cdot \mathrm{Dirac}(5, \omega)$

$2 \cdot i \cdot \pi \cdot \Delta(5, \omega)$

$$T \cdot \left[\frac{\sin\left(\frac{\omega \cdot T}{2}\right)}{\left(\frac{\omega \cdot T}{2}\right)} \right]^2$$

$$\frac{-1}{2} \cdot \frac{\left(\left(-t \cdot \Phi(t+T) + t \cdot \Phi(-t-T) - T \cdot \Phi(t+T) + T \cdot \Phi(-t-T) + 2 \cdot t \cdot \Phi(t) - 2 \cdot t \cdot \Phi(-t) - t \cdot \Phi(t-T) + t \cdot \Phi(-t+T) + T \cdot \Phi(t-T) - T \cdot \Phi(-t+T)\right)\right)}{T}$$

$$\begin{bmatrix} 0 \\ \frac{-1}{(2 \cdot T)} \end{bmatrix}$$

$$1 - \frac{|t|}{T}$$

$$2 \cdot \pi \cdot \mathrm{Dirac}(\omega) + \frac{2}{\left(T \cdot \omega^2\right)}$$

$$\exp(-2 \cdot x) \cdot \Phi(x)$$

$$\frac{1}{(2 + i \cdot \omega)}$$

$$\exp(-7 \cdot x) \cdot \Phi(x)$$

$$\frac{1}{(7 + i \cdot \omega)}$$

$$\exp(2 \cdot x) \cdot \Phi(-x)$$

$$\frac{1}{(2 - i \cdot \omega)}$$

$$\exp(7 \cdot x) \cdot \Phi(-x)$$

$$\frac{1}{(7 - i \cdot \omega)}$$

$$x \cdot \exp(-3 \cdot x) \cdot \Phi(x)$$

$$\frac{1}{(3 + i \cdot \omega)^2}$$

$$x \cdot \exp(-8 \cdot x) \cdot \Phi(x)$$

$$\frac{1}{(8 + i \cdot \omega)^2}$$

$$\exp(-2 \cdot x) \cdot \cos(10 \cdot x) \cdot \Phi(x)$$

$$\frac{1}{\left[2 \cdot (2 - 10 \cdot i + i \cdot \omega)\right]} + \frac{1}{\left[2 \cdot (2 + 10 \cdot i + i \cdot \omega)\right]}$$

$$\frac{(2 + i \cdot \omega)}{\left[(2 - 10 \cdot i + i \cdot \omega) \cdot (2 + 10 \cdot i + i \cdot \omega)\right]}$$

$$\frac{(2 + i \cdot \omega)}{\left(104 + 4 \cdot i \cdot \omega - \omega^2\right)}$$

$$\exp(-3 \cdot x) \cdot \sin(100 \cdot x) \cdot \Phi(x)$$

$$\frac{-1}{2} \cdot \frac{i}{(3 - 100 \cdot i + i \cdot \omega)} + \frac{1}{2} \cdot \frac{i}{(3 + 100 \cdot i + i \cdot \omega)}$$

$$\cos(z \cdot t) \cdot \Phi(t)$$

$$\frac{1}{(4 \cdot \pi)} \cdot \left[2 \cdot \pi^2 \cdot \text{Dirac}(\omega - z) - 2 \cdot i \cdot \frac{\pi}{(\omega - z)} \right] + \frac{1}{(4 \cdot \pi)} \cdot \left[2 \cdot \pi^2 \cdot \text{Dirac}(\omega + z) - 2 \cdot i \cdot \frac{\pi}{(\omega + z)} \right]$$

$$\sin(z \cdot t) \cdot \Phi(t)$$

$$\frac{-1}{4} \cdot \frac{i}{\pi} \cdot \left[2 \cdot \pi^2 \cdot \text{Dirac}(\omega - z) - 2 \cdot i \cdot \frac{\pi}{(\omega - z)} \right] + \frac{1}{4} \cdot \frac{i}{\pi} \cdot \left[2 \cdot \pi^2 \cdot \text{Dirac}(\omega + z) - 2 \cdot i \cdot \frac{\pi}{(\omega + z)} \right]$$

$$\cos\left(a \cdot t^2\right)$$

$$\left(\frac{\pi}{|a|}\right)^{\left(\frac{1}{2}\right)} \cdot \exp\left(\frac{-1}{4} \cdot \frac{\omega^2}{|a|}\right)$$

$$\sqrt{\frac{\pi}{a}} \cdot \cos\left(\frac{\omega^2}{4 \cdot a} + \frac{\pi}{4}\right)$$

$$\frac{1}{4} \cdot \left(\frac{\pi}{a}\right)^{\left(\frac{1}{2}\right)} \cdot \sqrt{2} \cdot \sqrt{4} \cdot (\pi \cdot |a|)^{\left(\frac{1}{2}\right)} \cdot \frac{\exp\left(-t^2 \cdot |a|\right)}{\pi}$$

$$\sin\left(a \cdot t^2\right)$$

$$0$$

$$\sqrt{\frac{\pi}{a}} \cdot \cos\left(\frac{\omega^2}{4 \cdot a} + \frac{\pi}{4}\right)$$

$$\frac{1}{4} \cdot \left(\frac{\pi}{a}\right)^{\left(\frac{1}{2}\right)} \cdot \sqrt{2} \cdot \sqrt{4} \cdot \left(\pi \cdot |a|\right)^{\left(\frac{1}{2}\right)} \cdot \frac{\exp\left(-t^2 \cdot |a|\right)}{\pi}$$

$$\frac{1}{2^2 + x^2}$$

$$\frac{1}{2} \cdot \exp(2 \cdot \omega) \cdot \pi \cdot \Phi(-\omega) + \frac{1}{2} \cdot \exp(-2 \cdot \omega) \cdot \pi \cdot \Phi(\omega)$$

$$\frac{1}{5^2 + x^2}$$

$$\frac{1}{5} \cdot \exp(5 \cdot \omega) \cdot \pi \cdot \Phi(-\omega) + \frac{1}{5} \cdot \exp(-5 \cdot \omega) \cdot \pi \cdot \Phi(\omega)$$

$$\frac{\sin(6 \cdot x)}{6 \cdot x}$$

$$\frac{1}{12} \cdot \pi \cdot \left(\Phi(-\omega + 6) - \Phi(\omega - 6)\right) - \frac{1}{12} \cdot \pi \cdot \left(\Phi(-\omega - 6) - \Phi(\omega + 6)\right)$$

$$\frac{-1}{6} \cdot \pi \cdot \Phi(\omega - 6) + \frac{1}{6} \cdot \pi \cdot \Phi(\omega + 6)$$

$$\exp\left(-3^2 \cdot x^2\right)$$

$$\frac{1}{9} \cdot \sqrt{9} \cdot \pi^{\left(\frac{1}{2}\right)} \cdot \exp\left(\frac{-1}{36} \cdot \omega^2\right)$$

$$\exp\left(-6^2 \cdot x^2\right)$$

$$\frac{1}{36} \cdot \sqrt{36} \cdot \pi^{\left(\frac{1}{2}\right)} \cdot \exp\left(\frac{-1}{144} \cdot \omega^2\right)$$

$$e^{-a^2 \cdot t^2}$$

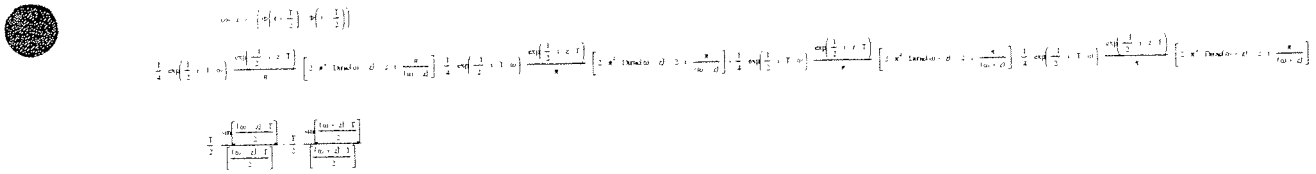
$$\left[\frac{\pi}{(|a|)^2}\right]^{\left(\frac{1}{2}\right)} \cdot \exp\left[\frac{-1}{4} \cdot \frac{\omega^2}{(|a|)^2}\right]$$

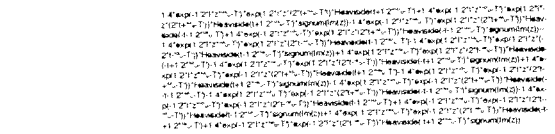
$$\exp(-3 \cdot |x|)$$

$$\frac{6}{\left(9 + \omega^2\right)}$$

$$\exp(-10 \cdot |x|)$$

$$\frac{20}{\left(100 + \omega^2\right)}$$





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$$\frac{2}{(j\omega + a)^3}$$

$$\frac{2}{(\sqrt{-1} \cdot \omega + a)^3}$$

$$\frac{1}{2} \cdot \exp(-a \cdot t) \cdot t^2 \cdot \left(\Phi(t) + \Phi(t) \cdot \text{signum}(\text{Re}(a)) - \Phi(-t) + \Phi(-t) \cdot \text{signum}(\text{Re}(a))\right)$$

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Figure 12

$a := 1$

$\omega := -4 \cdot a, -4 \cdot a + \frac{a}{1000} .. 4 \cdot a$

$M(\omega) := \frac{1}{\sqrt{a^2 + \omega^2}}$

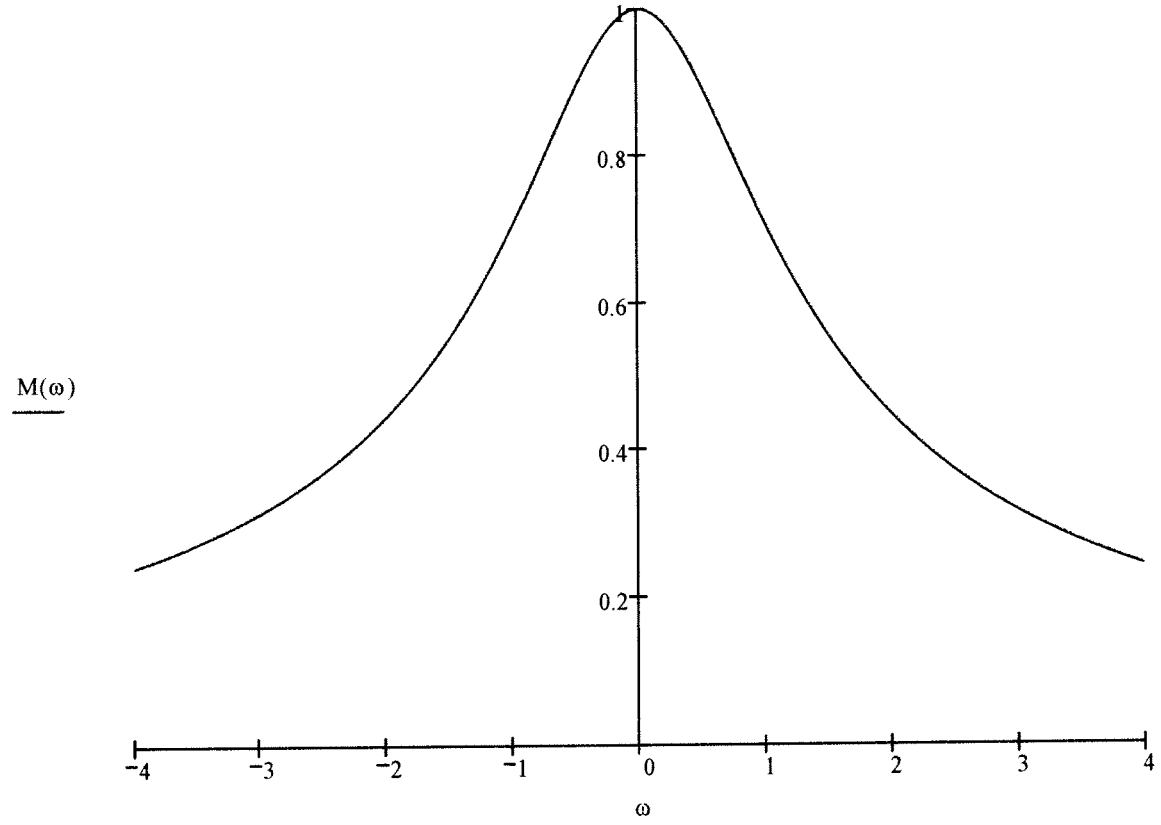

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Figure 12

$a := 1$

$w := -4 \cdot a, -4 \cdot a + \frac{a}{1000} .. 4 \cdot a$

$M(\omega) := -\text{atan}\left(\frac{\omega}{a}\right) \cdot \frac{180}{\pi}$