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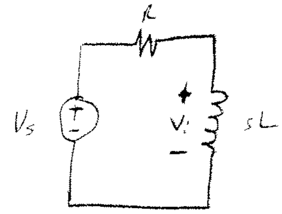
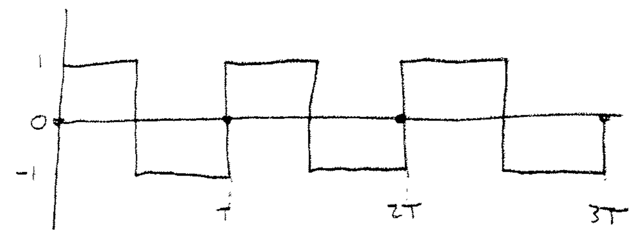




$$V_s = u(t) - 2u\left(t - \tfrac{T}{2}\right) + 2u(t - T) - 2u\left(t - \tfrac{3T}{2}\right) + 2u(t - 2T) - 2u\left(t - \tfrac{5T}{2}\right) + 2u(t - 3T)$$

$$V_i = V_s \frac{sL}{sL + R} = V_s \frac{s}{s + \frac{R}{L}}$$

$0 \le t \le \frac{T}{2}$

USING #6

$$\left(\frac{s}{s + \frac{R}{L}}\right) u(t) = \left(\frac{s}{s + \frac{R}{L}}\right)\left(\frac{1}{s}\right) u(t) = \frac{1}{s + \frac{R}{L}} u(t)$$

USING # 41 $\quad e^{-at} u(t) \Leftrightarrow \frac{1}{s + a}$

SUBSTITUTE FOR a $\quad \boxed{e^{-\frac{R}{L}t} u(t) \Leftrightarrow \frac{1}{s + \frac{R}{L}}}$

$\frac{T}{2} \le t \le T$

$$-2u\left(t - \tfrac{T}{2}\right) \frac{s}{s + \frac{R}{L}}$$

USING PAIR #7 $\quad u(t - a) \Leftrightarrow \frac{e^{-as}}{s}$

AND SUBSTITUTE FOR a $\quad u\left(t - \frac{T}{2}\right) \Leftrightarrow \frac{e^{-\frac{T}{2}s}}{s}$

AND USING PROPERTY #3 TO FACTOR IN SCALAR

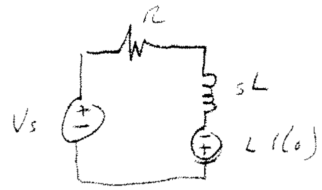
$$\frac{2e^{-\frac{T}{2}s}}{s} \cdot \frac{s}{s + \frac{R}{L}} = \frac{-2e^{-\frac{T}{2}s}}{s + \frac{R}{L}}$$

USING PAIR # 42 $\quad e^{-a(t-b)} u(t-b) \Leftrightarrow \frac{e^{-bs}}{s + a}$

SUBSTITUTE FOR a, AND USE PROPERTY #3 FOR SCALAR MULTIPLICATION WE GET:

$$-2e^{-\frac{R}{L}\left(t-\frac{T}{2}\right)} \Leftrightarrow \frac{-2e^{-\frac{T}{2}s}}{s}$$

BUT WE MUST ALSO TAKE INTO ACCOUNT THE INITIAL VALUE GIVEN FROM THE PREVIOUS TIME INTERVAL. TO CONSIDER THIS WE MUST FOLLOW THE INDUCTOR TRANSFORM MODEL.



THIS MEANS THAT WE ADD THE VOLTAGE ACROSS THE INDUCTOR FOR OUR SECOND TIME PERIOD ($\frac{T}{2} \rightarrow T$) AND ADD IT WITH ITS INITIAL VALUE, BUT TO FIND ITS INITIAL CONDITION WE MUST FIND THE FINAL VALUE FROM THE PREVIOUS TIME PERIOD ($0 \rightarrow \frac{T}{2}$).

$e^{-\frac{R}{L}\left(\frac{T}{2}\right)}u(t)$ = FINAL VALUE OF PREVIOUS TIME PERIOD ($0 \rightarrow \frac{T}{2}$).

$$\boxed{\left[-2e^{-\frac{R}{L}\left(t-\frac{T}{2}\right)} + e^{-\frac{R}{L}\left(\frac{T}{2}\right)}\right]u(t) = V_i(t)} \quad \left(\frac{T}{2} \leq t \leq T\right)$$

$T \le t \le \frac{3T}{2}$

$$V_i(t) = 2u(t-T) \quad \frac{S}{S+\frac{R}{L}}$$

USING PAIR #7 $\quad u(t-a) \Leftrightarrow \frac{e^{-as}}{S}$

SUBSTITUTE FOR a, AND USE PROPERTY #3 TO FACTOR IN SCALAR

$$2u(t-T) \Leftrightarrow \frac{e^{-Ts}}{S}$$

$$V_i(t) = \frac{2e^{-Ts}}{\cancel{S}} \quad \frac{\cancel{S}}{S+\frac{R}{L}}$$

$$= \frac{2e^{-Ts}}{S+\frac{R}{L}}$$

USING PROPERTY # 42 $\quad e^{-a(t-b)}u(t-b) \Leftrightarrow \frac{e^{-bs}}{S+a}$

SUBSTITUTE FOR a, AND USE PROPERTY # TO FACTOR IN SCALAR

$$V_i(t) = 2e^{-\frac{R}{L}(t-T)}\,u(t)$$

BUT AGAIN WE MUST FOLLOW THE SAME MODEL MENTIONED ON PAGE TWO. SO FOLLOWING THE SAME ANALYSIS WE GET

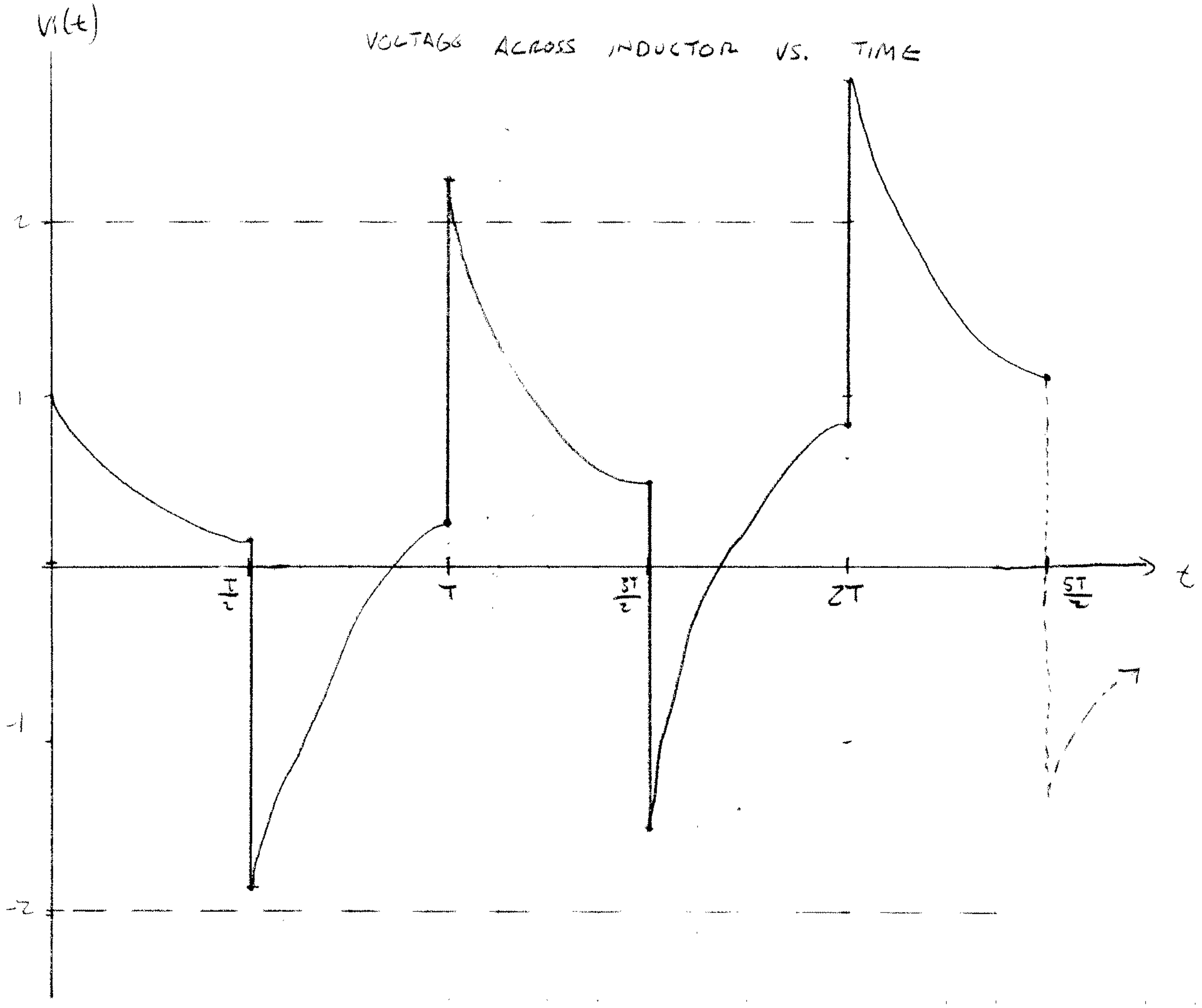
$$V_i(t) = \left[-2e^{-\frac{R}{L}(T-\frac{T}{2})} + e^{-\frac{R}{L}(\frac{T}{2})}\right]u(t)$$

IS THE FINAL VALUE OF $t = T$

SO $$\boxed{V_i(t) = \left[-2e^{-\frac{R}{L}(\frac{T}{2})} + e^{-\frac{R}{L}(\frac{T}{2})} + 2e^{-\frac{R}{L}(t-T)}\right]u(t)}$$

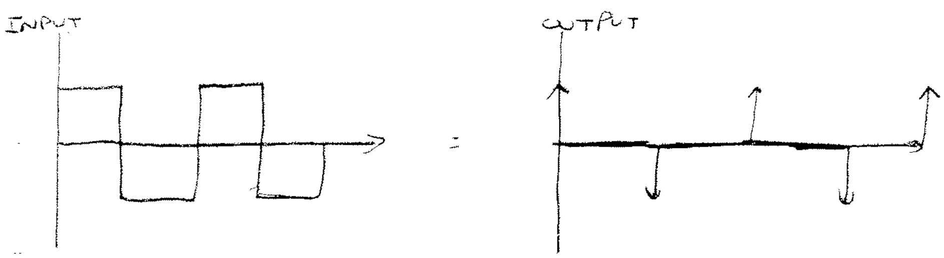
WHERE $T \le t \le \frac{3T}{2}$

VOLTAGE ACROSS INDUCTOR VS. TIME

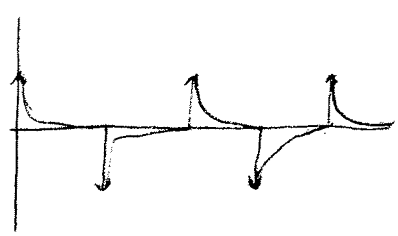


AS YOU CAN SEE FROM THE GRAPH THE VOLTAGE ACROSS THE INDUCTOR INCREASES AS TIME INCREASES, THIS IS BECAUSE OF THE INITIAL VOLTAGE ACROSS THE INDUCTOR FROM THE PREVIOUS TIME PERIOD.

PART B A DIFFERENTIATING CIRCUIT WOULD TRANSFORM OUR INPUT VOLTAGE AS FOLLOWS



OUR CIRCUIT DOES THIS AS WELL. BUT IT LOOKS MORE LIKE THE FOLLOWING



THEREFORE WE CAN CLASSIFY OUR CIRCUIT AS A DIFFERENTIATING CIRCUIT.

PART C SINCE WE ARE GIVEN $R \gg L$, THEN WE CAN EVALUATE $\tau = \frac{L}{R}$ AS RELATIVELY SMALL. USING THE TABLE ON PAGE 318 IN THE "TRANSIENT BEHAVIOR IN THE TIME DOMAIN" SECTION, WE CAN VERIFY THAT WITH A SERIES RL CIRCUIT WITH A SMALL τ, THE INDUCTOR VOLTAGE IS PROPORTIONAL TO THE DERIVATIVE OF THE INPUT VOLTAGE.

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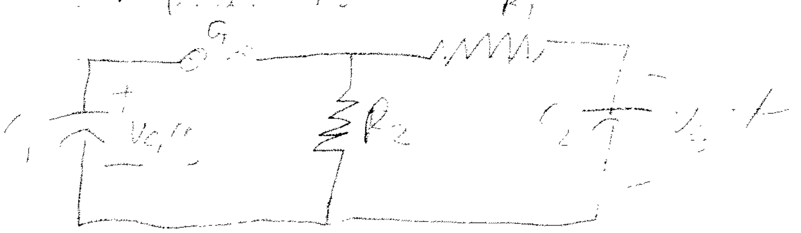
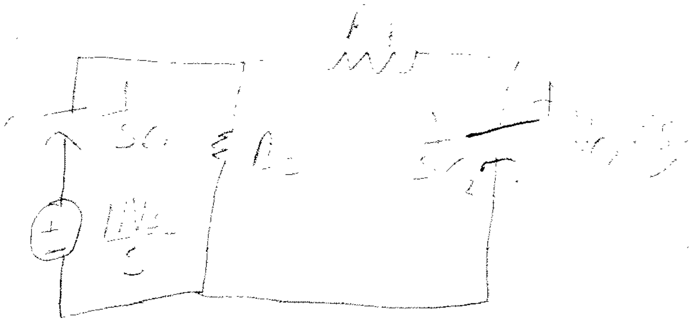
Figure 13.0





$V_0 = \frac{11 V_s}{s} \cdot \frac{(R_1 + \frac{1}{sC_2}) \parallel R_2}{(R_1 + \frac{1}{sC_2}) \parallel R_2 + \frac{1}{sC_1}} \left(\frac{\frac{1}{sC_2}}{R_1 + \frac{1}{sC_2}} \right)$ ✓

[illegible]

$= \frac{11 V_s}{s} \cdot \frac{\frac{R_1 R_2 sC_2 + R_2}{R_1 sC_2 + 1 + R_2 sC_2}}{\frac{R_1 R_2 sC_2 + R_2}{R_1 sC_2 + R_2 sC_2} + \frac{1}{sC_2}} \left(\frac{\frac{1}{sC_2}}{R_1 + \frac{1}{sC_2}} \right)$

$$= \frac{11 V_s C_1 R_2}{(R_2 C_1 R_1 C_2) s^2 + s(R_2 C_1 + R_1 C_2 + R_2 C_2) + 1}$$

$$= \frac{\frac{11 V_s C_1 R_2}{R_2 C_1 R_1 C_2}}{s^2 + s\frac{(R_2 C_1 + R_1 C_2 + R_2 C_2)}{(R_2 C_1 R_1 C_2)} + \frac{1}{R_2 C_1 R_1 C_2}} = \frac{\frac{11 V_s}{R_1 C_2}}{s^2 + s\frac{(R_2 C_1 + R_1 C_2 + R_2 C_2)}{[illegible]} + \frac{1}{[illegible]}}$$

[illegible]

$$\frac{e^{-at}-e^{-bt}}{b-a}$$ [illegible]

[illegible]

$$= \frac{[illegible]}{k_1 k_2}\left(\frac{e^{-at}-e^{-bt}}{b-a}\right) [illegible]$$

$$\frac{N \cdot V_s}{s} \cdot \left[\left[\frac{\dfrac{R_1 \cdot R_2 \cdot s \cdot C_2 + R_2}{R_1 \cdot s \cdot C_2 + 1 + R_2 \cdot s \cdot C_2}}{\dfrac{R_1 \cdot R_2 \cdot s \cdot C_2 + R_2}{R_1 \cdot s \cdot C_2 + 1 + R_2 \cdot s \cdot C_2} + \dfrac{1}{s \cdot C_1}} \right] \cdot \left[\frac{\dfrac{1}{s \cdot C_2}}{R_1 + \dfrac{1}{s \cdot C_2}} \right] \right]$$

$$C_1 \cdot R_2 \cdot N \cdot \frac{V_s}{\left(R_2 \cdot s^2 \cdot C_1 \cdot R_1 \cdot C_2 + R_2 \cdot s \cdot C_1 + s \cdot R_1 \cdot C_2 + 1 + s \cdot R_2 \cdot C_2 \right)}$$

$R_1 := 1\cdot 10^2 \qquad R_2 := 1\cdot 10^3 \qquad C_1 := 1\cdot 10^{-6} \qquad C_2 := 10\cdot 10^{-6} \qquad NV_s := 30\cdot 10^3$

$$a := -\frac{-\left(\frac{C_1\cdot R_2 + R_1\cdot C_2 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right) - \sqrt{\left(\frac{C_1\cdot R_2 + R_1\cdot C_2 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right)^2 - \frac{4}{R_1\cdot R_2\cdot C_1\cdot C_2}}}{2}$$

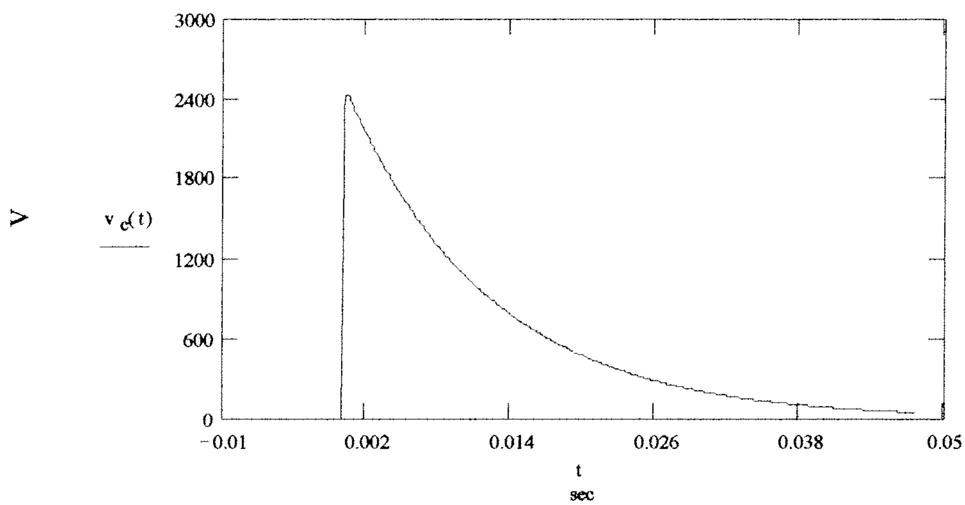
$$\frac{1}{a} = 8.392\cdot 10^{-5}$$

$$b := -\frac{-\left(\frac{C_1\cdot R_2 + R_1\cdot C_2 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right) + \sqrt{\left(\frac{C_1\cdot R_2 + R_1\cdot C_2 + R_2\cdot C_2}{R_1\cdot R_2\cdot C_1\cdot C_2}\right)^2 - \frac{4}{R_1\cdot R_2\cdot C_1\cdot C_2}}}{2}$$

$$\frac{1}{b} = 0.012$$

$$t := 0, \frac{1}{100}\cdot \max\left[\left[\left|\begin{pmatrix}\frac{1}{a}\\ \frac{1}{b}\end{pmatrix}\right|\right]\right] .. 4\cdot \max\left[\left[\left|\begin{pmatrix}\frac{1}{a}\\ \frac{1}{b}\end{pmatrix}\right|\right]\right]$$

$$v_c(t) := \frac{NV_s}{R_1\cdot C_2}\cdot\left(\frac{e^{-a\cdot t} - e^{-b\cdot t}}{b-a}\right)\cdot\Phi(t)$$



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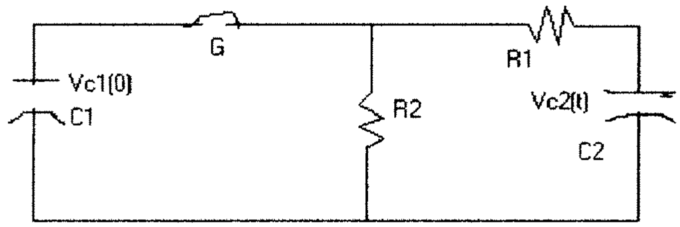
Problem 13-20



The solution for the voltage across the capacitor Cs is obtained using voltage division twice. The first division gets me voltage across R2 (which is the same as the voltage across R1+C2. The second voltage division divides the voltage across R2 between C2 and R1. The first voltage division is the first bracketed part of the equation. For notation, I used absolute value bars to indicate parallelism. So in the following equation, R2 is in parallel with (R1+ 1/S*C2).

$$V_{c2}(s) = \left[\frac{R_2 \cdot \left| R_1 + \frac{1}{S \cdot C_2} \right|}{R_2 \cdot \left| R_1 + \frac{1}{S \cdot C_2} \right| + \frac{1}{S \cdot C_1}}\right] \cdot \left[\frac{\frac{1}{S \cdot C_2}}{\frac{1}{S \cdot C_2} + R_1}\right] \cdot \frac{NV_s}{S}$$

Expand out the parallel resistor equations:

$$V_{c2}(s) = \frac{\dfrac{R_2 \cdot R_1 + \frac{R_2}{S \cdot C_2}}{R_2 + R_1 + \frac{1}{S \cdot C_2}}}{\dfrac{R_2 \cdot R_1 + \frac{R_2}{S \cdot C_2}}{R_2 + R_1 + \frac{1}{S \cdot C_2}} + \frac{1}{S \cdot C_1}} \cdot \frac{\frac{1}{S \cdot C_2}}{\frac{1}{S \cdot C_2} + R_1} \cdot \frac{N \cdot V_s}{S}$$

Simplifying the above equation results in: (MathCAD simplification)

$$V_{c2}(s)=R_2 \cdot \frac{C_1}{\left(R_2 \cdot S^2 \cdot C_1 \cdot R_1 \cdot C_2 + R_2 \cdot S \cdot C_1 + R_2 \cdot S \cdot C_2 + R_1 \cdot S \cdot C_2 + 1\right)} \cdot N \cdot V_s$$

The function corresponding to this Laplace transform is number 125.

$$\left(\frac{e^{-a \cdot t} - e^{-b \cdot t}}{b - a}\right) \cdot \Phi(t) \qquad a \neq b \qquad \Longleftrightarrow \qquad \frac{1}{(s+a)\cdot(s+b)}$$

We are interested in the denominator so we can find a and b (1/a and 1/b are time constants):

$$R_2 \cdot S^2 \cdot C_1 \cdot R_1 \cdot C_2 + R_2 \cdot S \cdot C_1 + R_2 \cdot S \cdot C_2 + R_1 \cdot S \cdot C_2 + 1$$

$$\text{Denominator} = S^2 + \frac{R_2 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2}{R_2 \cdot C_1 \cdot R_1 \cdot C_2} \cdot S + \frac{1}{R_2 \cdot C_1 \cdot R_1 \cdot C_2}$$

The next page shows the equations for a and b, and the graph of the output from this voltage zapper.

$R_1 := 100 \quad R_2 := 1000 \quad C_1 := 0.000010 \quad C_2 := 0.000001 \quad NV_s := 30000$

$$a := -\frac{\left(\frac{C_1 \cdot R_2 + C_2 \cdot R_2 + C_2 \cdot R_1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right) - \sqrt{\left(\frac{C_1 \cdot R_2 + C_2 \cdot R_2 + C_2 \cdot R_1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right)^2 - \frac{4}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}}}{2}$$

$$b := -\frac{\left(\frac{C_1 \cdot R_2 + C_2 \cdot R_2 + C_2 \cdot R_1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right) + \sqrt{\left(\frac{C_1 \cdot R_2 + C_2 \cdot R_2 + C_2 \cdot R_1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}\right)^2 - \frac{4}{R_1 \cdot R_2 \cdot C_1 \cdot C_2}}}{2}$$

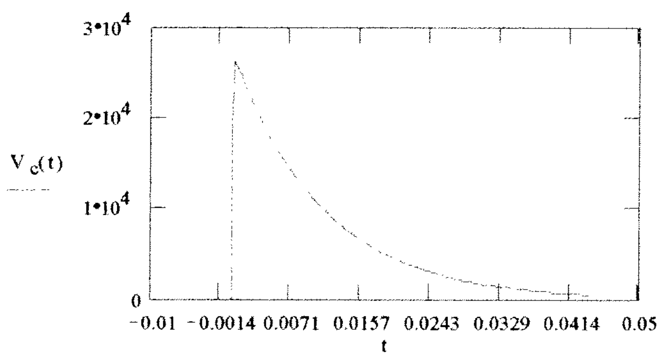
$$\frac{1}{a} = 9.083 \cdot 10^{-5}$$

$$\frac{1}{b} = 0.011$$

$$t := 0, \frac{1}{100} \cdot \max\left[\left[\begin{vmatrix} \frac{1}{a} \\ \frac{1}{b} \end{vmatrix}\right]\right] .. 4 \cdot \max\left[\left[\begin{vmatrix} \frac{1}{a} \\ \frac{1}{b} \end{vmatrix}\right]\right]$$

$$V_c(t) := \left[\frac{NV_s}{R_1 \cdot C_2} \cdot \left(\frac{e^{-a \cdot t} - e^{-b \cdot t}}{b - a}\right) \cdot \Phi(t)\right]$$

This equation is the time domain solution.



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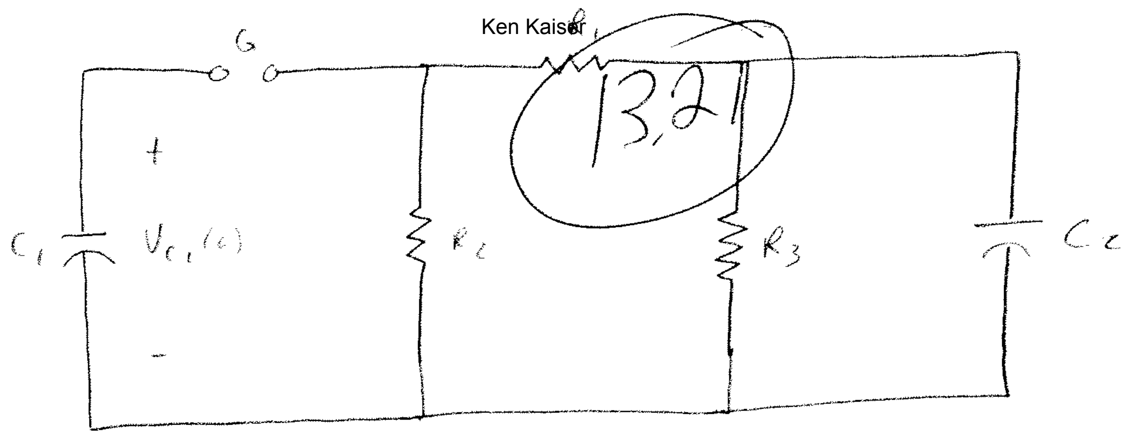
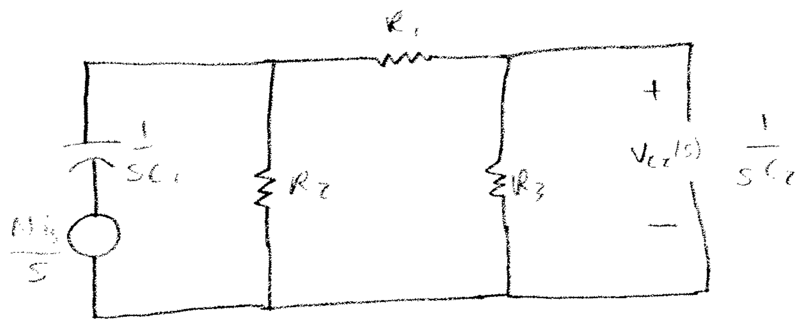


$R_1 = 100\,\Omega$ $R_2 = 1k\Omega$ $C_1 = 10\mu F$ $C_2 = 1\mu F$ $V_{C1} = 50k\,V$



ALSO

$V_{C1} = NV_s$

SIMPLIFY THE EQUATIONS IN MATHCAD

SEE NEXT PAGE

$$V_{C2} := \frac{N \cdot V_s}{s} \cdot \left[\frac{\left[\frac{1}{R_2} + \frac{1}{R_1 + \left(\frac{1}{R_3} + s \cdot C_2 \right)^{-1}} \right]^{-1}}{\left[\frac{1}{R_2} + \frac{1}{R_1 + \left(\frac{1}{R_3} + s \cdot C_2 \right)^{-1}} \right]^{-1} + \frac{1}{s \cdot C_2}} \right] \cdot \left[\frac{\left(\frac{1}{R_3} + s \cdot C_2 \right)^{-1}}{\left(\frac{1}{R_3} + s \cdot C_2 \right)^{-1} + R_1} \right]$$

$$V_{C2} := V_s \cdot N \cdot R_2 \cdot C_2 \cdot \frac{R_3}{\left(R_2 \cdot s \cdot C_2 \cdot R_1 + R_2 \cdot s^2 \cdot C_2{}^2 \cdot R_1 \cdot R_3 + 2 \cdot R_2 \cdot s \cdot C_2 \cdot R_3 + R_1 + R_1 \cdot s \cdot C_2 \cdot R_3 + R_3 + R_2 \right)}$$

TABLE #125 IS USED

Ken Kaiser

$$\left[\frac{e^{-at}-e^{-bt}}{b-a}\right]u(t) \quad a \neq b \quad \Leftrightarrow \quad \frac{1}{(s+a)(s+b)}$$

$$V_{C2} = \frac{V_S N R_2 R_3 C_2}{s^2(R_1R_2R_3C_2^2) + s(2R_2R_3C_2 + R_1R_3C_2 + R_1R_2C_2) + (R_1+R_2+R_3)} \Leftrightarrow \frac{1}{(s+a)(s+b)}$$

$$s+a=0 \qquad s+b=0$$

$$s=-a \qquad s=-b$$

THE QUADRATIC FORMULA IS USED TO OBTAIN a & b.

$$a = -\frac{-(2R_2R_3C_2+R_1R_3C_2+R_1R_2C_2)+\sqrt{(2R_2R_3C_2+R_1R_3C_2+R_1R_2C_2)^2-4(R_1R_2R_3C_2^2)(R_1+R_2+R_3)}}{2(R_1R_2R_3C_2^2)}$$

$$b = -\frac{-(2R_2R_3C_2+R_1R_3C_2+R_1R_2C_2)-\sqrt{(2R_2R_3C_2+R_1R_3C_2+R_1R_2C_2)^2-4(R_1R_2R_3C_2^2)(R_1+R_2+R_3)}}{2(R_1R_2R_3C_2^2)}$$

THE FUNCTION FOR $V_c(t)$ IS.

$$V_c(t) = NV_SR_2R_3C_2\left(\frac{e^{-at}-e^{-bt}}{b-a}\right)u(t) \qquad a \neq b$$

USING THE GIVEN VALUES THE FOLLOWING ANALYSIS IS OBTAINED.

$R_1 := 100$

$R_2 := 1000$

$C_1 := 10 \cdot 10^{-6}$

$C_2 := 1 \cdot 10^{-6}$

$NV_s := 30000$

$R_3 := 100$

$$a := -\frac{-\left(2 \cdot R_2 \cdot R_3 \cdot C_2 + R_1 \cdot R_3 \cdot C_2 + R_1 \cdot R_2 \cdot C_2\right) + \sqrt{\left(2 \cdot R_2 \cdot R_3 \cdot C_2 + R_1 \cdot R_3 \cdot C_2 + R_1 \cdot R_2 \cdot C_2\right)^2 - 4 \cdot \left(R_1 \cdot R_2 \cdot R_3 \cdot C_2^{\,2}\right) \cdot \left(R_1 + R_2 + R_3\right.}}{2 \cdot \left(R_1 \cdot R_2 \cdot R_3 \cdot C_2^{\,2}\right)}$$

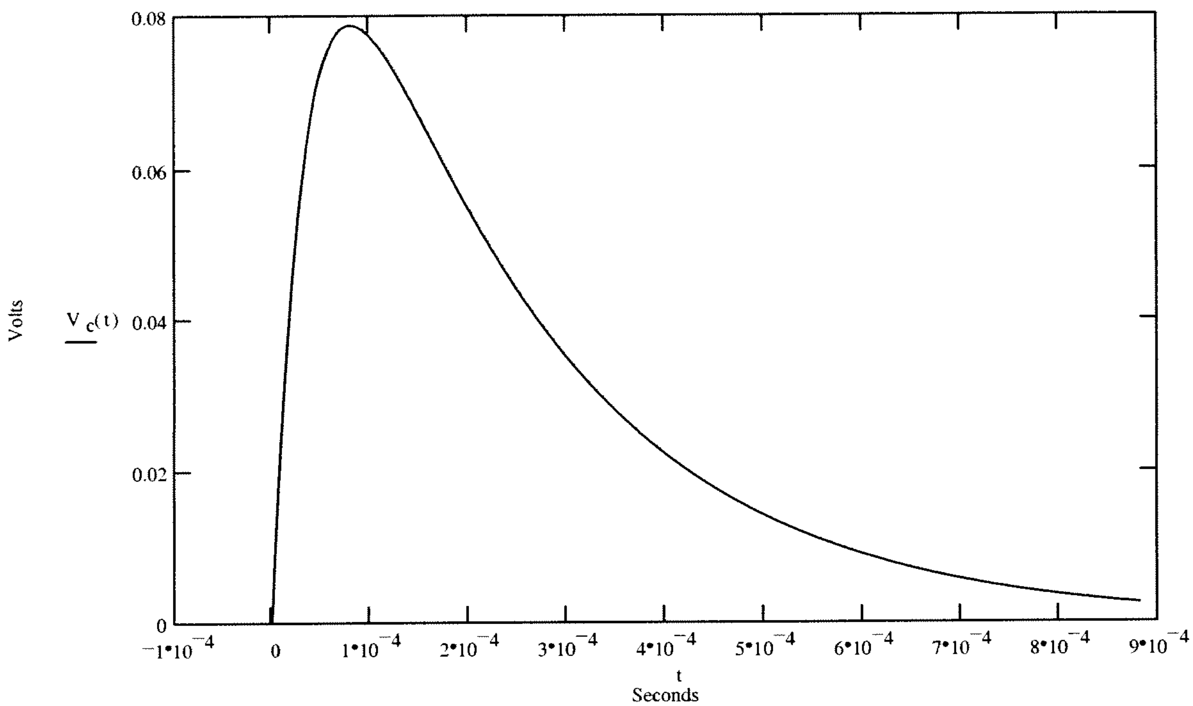
$$\frac{1}{a} = 2.205 \cdot 10^{-4}$$

$$b := -\frac{-\left(2 \cdot R_2 \cdot R_3 \cdot C_2 + R_1 \cdot R_3 \cdot C_2 + R_1 \cdot R_2 \cdot C_2\right) - \sqrt{\left(2 \cdot R_2 \cdot R_3 \cdot C_2 + R_1 \cdot R_3 \cdot C_2 + R_1 \cdot R_2 \cdot C_2\right)^2 - 4 \cdot \left(R_1 \cdot R_2 \cdot R_3 \cdot C_2^{\,2}\right) \cdot \left(R_1 + R_2 + R_3\right.}}{2 \cdot \left(R_1 \cdot R_2 \cdot R_3 \cdot C_2^{\,2}\right)}$$

$$\frac{1}{b} = 3.778 \cdot 10^{-5}$$

$$t := 0, \frac{1}{100} \cdot \max\left[\begin{bmatrix} \frac{1}{a} \\ \frac{1}{b} \end{bmatrix}\right] .. 4 \cdot \max\left[\begin{bmatrix} \frac{1}{a} \\ \frac{1}{b} \end{bmatrix}\right]$$

$$V_c(t) := \left(NV_s \cdot R_2 \cdot R_3 \cdot C_2\right) \cdot \left(\frac{e^{-a \cdot t} - e^{-b \cdot t}}{b - a}\right) \cdot \Phi(t)$$


0.08
0.06
0.04
0.02
0
Volts
Vc(t)
−1•10−4
0
1•10−4
2•10−4
3•10−4
4•10−4
5•10−4
6•10−4
7•10−4
8•10−4
9•10−4
t
Seconds

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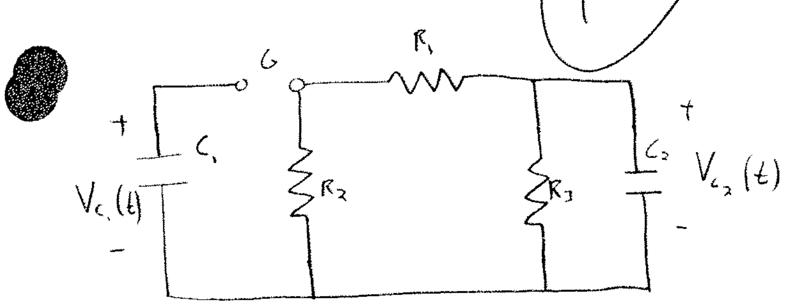
Problem 13.21



$C_1 = 10\,\mu F$

$C_2 = 1\,\mu F$

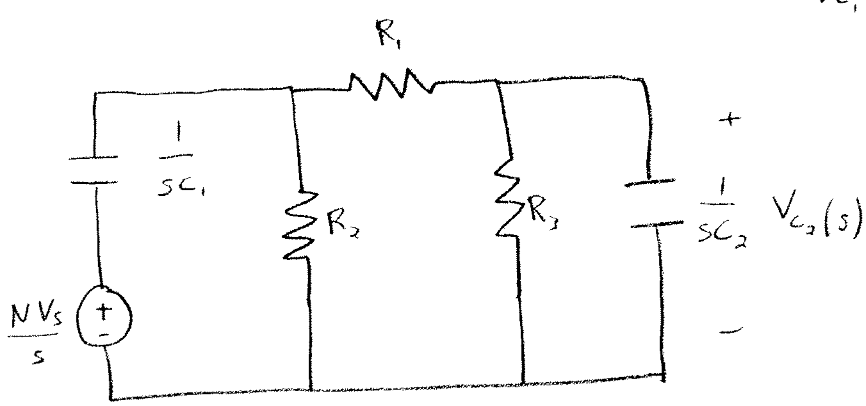
$R_1 = 100\,\Omega$

$R_2 = 1\,k\Omega$

$R_3 = 100\,\Omega$

Number change

$V_{C_1}(0) = 30\,kV$



$$V_{C_2}(s) = \frac{NV_s}{s} \times \left[\frac{R_2 // \left(R_1 + R_3 // \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + R_2 // \left(R_1 + R_3 // \frac{1}{sC_2}\right)}\right]\left[\frac{R_3 // \frac{1}{sC_2}}{R_1 + R_3 // \frac{1}{sC_2}}\right]$$

$$V_{C_2}(s) = \frac{NV_s}{s}\left[\frac{R_2 // \left(R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}\right)}{\frac{1}{sC_1} + R_2 // \left(R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}\right)}\right]\left[\frac{\frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}}{R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}}\right]$$

$$V_{C_2}(s) = \frac{NV_s}{s}\left[\frac{\dfrac{R_2\left(R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}\right)}{R_2 + R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}}}{\dfrac{1}{sC_2} + \left[\dfrac{R_2\left(R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}\right)}{\left(R_2 + R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}\right)}\right]}\right]\left[\frac{\frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}}{R_1 + \frac{R_3 \frac{1}{sC_2}}{R_3 + \frac{1}{sC_2}}}\right]$$

$$V_{C_2}(s) = \frac{V_s N R_2 R_3 C_2}{(R_1 R_2 R_3 C_2^2)s^2 + (R_1 C_2 R_3 + 2R_2 R_3 C_2 + R_1 R_2 C_2)s + (R_1 + R_2 + R_3)}$$

$$V_{C_2}(s) = \frac{V_s N R_2 R_3 C_2}{s^2 + \left(\frac{R_1 C_2 R_3 + 2R_2 R_3 C_2 + R_1 R_2 C_2}{R_1 R_2 R_3 C_2^2}\right)s + \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3 C_2^2}}$$

$$V_{C_2}(s) = \frac{V_s N R_2 R_3 C_2}{s^2 + 2\left(\frac{R_1 R_3 + 2R_2 R_3 + R_1 R_2}{2R_1 R_2 R_3 C_2}\right)s + \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3 C_2^2}}$$

This is in the form of Eq. # 154.

$$\left[\frac{1}{\sqrt{\omega_0^2 - \alpha^2}} e^{-\alpha t} \sin\left(\sqrt{\omega_0^2 - \alpha^2}\right)t\right]u(t) \Longleftrightarrow \frac{1}{s^2 + 2\alpha s + \omega_0^2}$$

where: $\alpha = \left(\frac{R_1 R_3 + 2R_2 R_3 + R_1 R_2}{2R_1 R_2 R_3 C_2}\right)$ and $\omega_0^2 = \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3 C_2^2}$

$\sqrt{\omega_0^2 - \alpha^2}$ will give an imaginary solution so we need to rewrite in exponential form.

We can use:

$$\left(\frac{e^{-at} - e^{-bt}}{b - a}\right)u(t) \Longleftrightarrow \frac{1}{(s+a)(s+b)}$$

Solving for a and b in MathCAD

$$a = -\left(\frac{-R_1R_3C_2 - 2R_2R_3C_2 - R_2C_2R_1 + C_2\sqrt{R_1^2R_3^2 - 2R_1^2R_2R_3 + 4R_2^2R_3^2 + R_2^2R_1^2}}{2R_2C_2^2R_1R_3}\right)$$

$$b = -\left(\frac{-R_1R_3C_2 - 2R_2R_3C_2 - R_2C_2R_1 - C_2\sqrt{R_1^2R_3^2 - 2R_1^2R_2R_3 + 4R_2^2R_3^2 + R_2^2R_1^2}}{2R_2R_1R_3C_2^2}\right)$$

$$V_{C_2}(t) = V_sNR_2R_3C_2\left(\frac{e^{-at} - e^{-bt}}{b-a}\right)u(t) \qquad a \neq b$$

Plotting this time function will give the response of the Voltage across C_2.

$= 1 \quad C_1 := 10\cdot 10^{-6} \quad C_2 := 1\cdot 10^{-6} \quad R_1 := 100 \quad R_2 := 1000 \quad R_3 := 100 \quad V_s := 30\cdot 10^{3}$

$$V_{C2}(s) := \frac{N\cdot V_s}{s}\cdot\left[\frac{\dfrac{R_2\cdot\left[R_1+\dfrac{R_3\cdot\frac{1}{s\cdot C_2}}{R_3+\frac{1}{s\cdot C_2}}\right]}{R_2+R_1+\dfrac{R_3\cdot\frac{1}{s\cdot C_2}}{R_3+\frac{1}{s\cdot C_2}}}}{\left(\dfrac{1}{s\cdot C_2}\right)+\left[\dfrac{R_2\cdot\left[R_1+\dfrac{R_3\cdot\frac{1}{s\cdot C_2}}{R_3+\frac{1}{s\cdot C_2}}\right]}{R_2+R_1+\dfrac{R_3\cdot\frac{1}{s\cdot C_2}}{R_3+\frac{1}{s\cdot C_2}}}\right]}\right]\cdot\left[\frac{\dfrac{R_3\cdot\frac{1}{s\cdot C_2}}{R_3+\frac{1}{s\cdot C_2}}}{R_1+\dfrac{R_3\cdot\frac{1}{s\cdot C_2}}{R_3+\frac{1}{s\cdot C_2}}}\right]$$

$$V_{C2}(s) := V_s\cdot N\cdot R_2\cdot C_2\cdot\frac{R_3}{\left(2\cdot R_2\cdot R_3\cdot s\cdot C_2+R_2+R_1\cdot R_3\cdot s\cdot C_2+R_1+R_3+R_2\cdot s^2\cdot C_2^{\,2}\cdot R_1\cdot R_3+R_2\cdot s\cdot C_2\cdot R_1\right)}$$

$$V_{C2}(s) := V_s\cdot N\cdot R_2\cdot C_2\cdot\frac{R_3}{\left[R_2\cdot s^2\cdot C_2^{\,2}\cdot R_1\cdot R_3+\left(R_1\cdot R_3\cdot C_2+2\cdot R_2\cdot R_3\cdot C_2+R_2\cdot C_2\cdot R_1\right)\cdot s+R_3+R_2+R_1\right]}$$

$$\left[R_2\cdot s^2\cdot C_2^{\,2}\cdot R_1\cdot R_3+\left(R_1\cdot R_3\cdot C_2+2\cdot R_2\cdot R_3\cdot C_2+R_2\cdot C_2\cdot R_1\right)\cdot s+R_3+R_2+R_1\right]=0$$

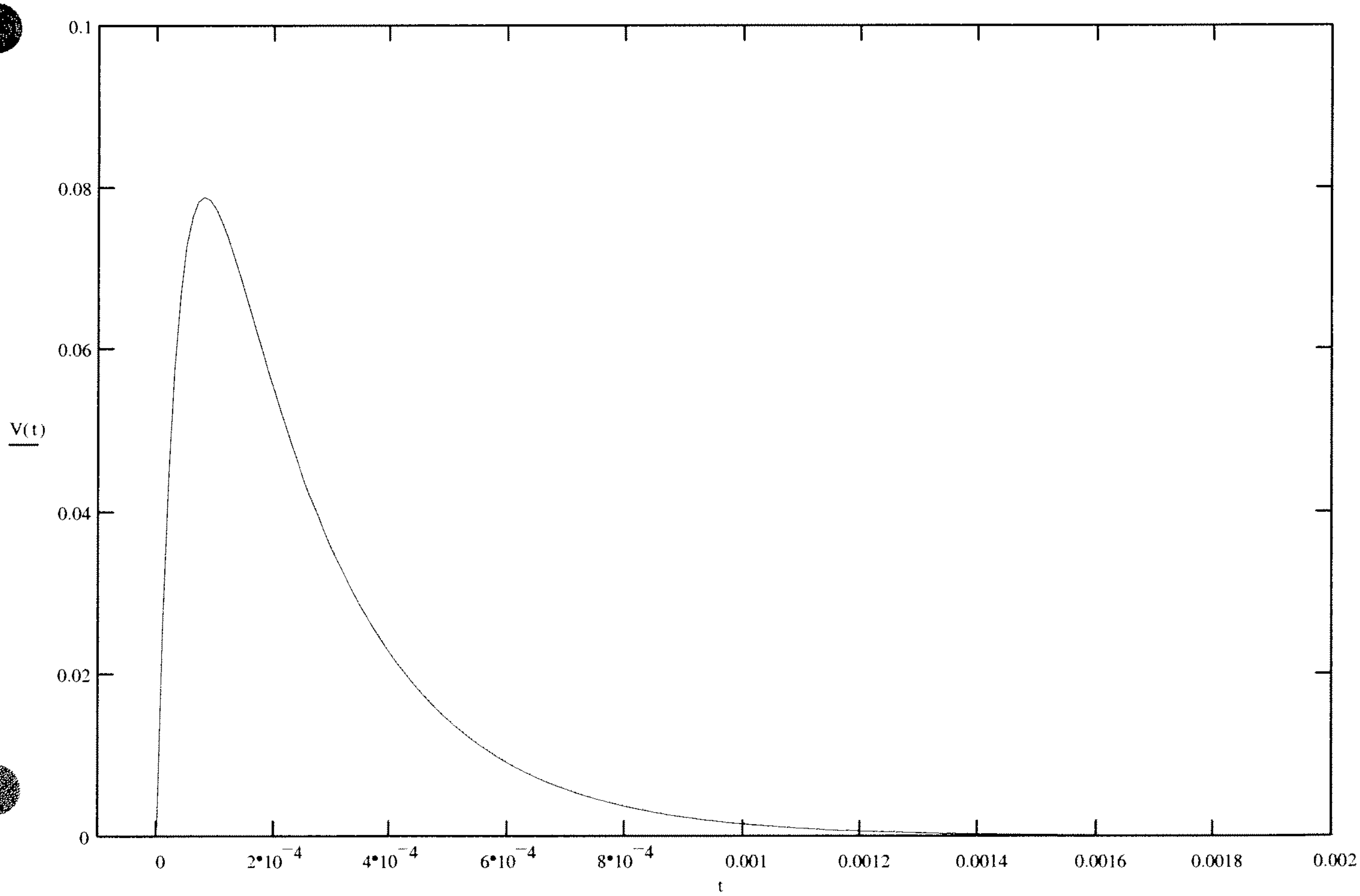
$$\begin{bmatrix}\dfrac{1}{\left[2\cdot\left[R_2\cdot\left[C_2^{\,2}\cdot\left(R_1\cdot R_3\right)\right]\right]\right]}\cdot\left(-R_1\cdot R_3\cdot C_2-2\cdot R_2\cdot R_3\cdot C_2-R_2\cdot C_2\cdot R_1+C_2\cdot\sqrt{R_1^{\,2}\cdot R_3^{\,2}-2\cdot R_1^{\,2}\cdot R_3\cdot R_2+4\cdot R_2^{\,2}\cdot R_3^{\,2}+R_2^{\,2}\cdot R_1^{\,2}}\right.\\ \dfrac{1}{\left[2\cdot\left[R_2\cdot\left[C_2^{\,2}\cdot\left(R_1\cdot R_3\right)\right]\right]\right]}\cdot\left(-R_1\cdot R_3\cdot C_2-2\cdot R_2\cdot R_3\cdot C_2-R_2\cdot C_2\cdot R_1-C_2\cdot\sqrt{R_1^{\,2}\cdot R_3^{\,2}-2\cdot R_1^{\,2}\cdot R_3\cdot R_2+4\cdot R_2^{\,2}\cdot R_3^{\,2}+R_2^{\,2}\cdot R_1^{\,2}}\right.\end{bmatrix}$$

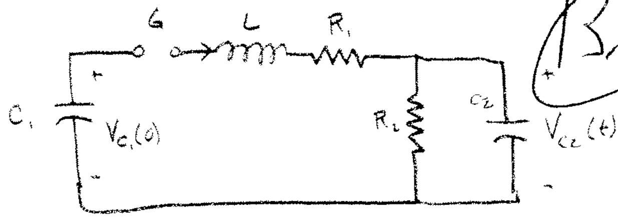
$$a := -\left[\frac{1}{\left[2\cdot\left[R_2\cdot\left[C_2^{\,2}\cdot\left(R_1\cdot R_3\right)\right]\right]\right]}\cdot\left(-R_1\cdot R_3\cdot C_2 - 2\cdot R_2\cdot R_3\cdot C_2 - R_2\cdot C_2\cdot R_1 + C_2\cdot\sqrt{R_1^{\,2}\cdot R_3^{\,2} - 2\cdot R_1^{\,2}\cdot R_3\cdot R_2 + 4\cdot R_2^{\,2}\cdot R_3^{\,2} + R_2^{\,2}\cdot R_1^{\,2}}\right.\right.$$

$$b := -\left[\frac{1}{\left[2\cdot\left[R_2\cdot\left[C_2^{\,2}\cdot\left(R_1\cdot R_3\right)\right]\right]\right]}\cdot\left(-R_1\cdot R_3\cdot C_2 - 2\cdot R_2\cdot R_3\cdot C_2 - R_2\cdot C_2\cdot R_1 - C_2\cdot\sqrt{R_1^{\,2}\cdot R_3^{\,2} - 2\cdot R_1^{\,2}\cdot R_3\cdot R_2 + 4\cdot R_2^{\,2}\cdot R_3^{\,2} + R_2^{\,2}\cdot R_1^{\,2}}\right.\right.$$

$$t := 0, 0.00001 .. 0.1$$

$$V(t) := V_s\cdot N\cdot R_3\cdot R_2\cdot C_2\cdot\frac{e^{-a\cdot t} - e^{-b\cdot t}}{b - a}\cdot\Phi(t)$$





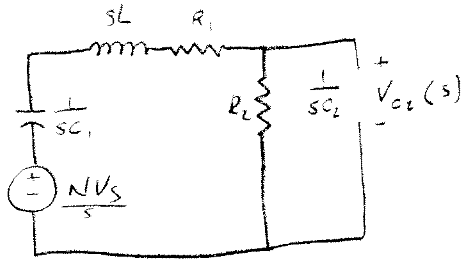
Ken Kaiser

$R_1 = 100\,\Omega$ $\quad C_2 = 1\,\mu F$

$R_2 = 1\,k\Omega$ $\quad V_{c_1}(0) = 30\,eV$

$L = 1\,\mu H$

$C_1 = 10\,\mu F$



$$V_{C_2}(s) = \frac{NV_s}{s} \cdot \left(\frac{R_2 \| \frac{1}{sC_2}}{R_2 \| \frac{1}{sC_2} + \frac{1}{sC_1} + sL + R_1} \right)$$

$$V_{C_2}(s) = \frac{NV_s}{s} \cdot \frac{\frac{R_2}{1 + sR_2C_2}}{\frac{R_2}{1+sR_2C_2} + R_1 + \frac{1}{sC_1} + sL}$$

$$V_{C_2}(s) = \frac{C_1 R_2 N V_s}{(R_2 sC_1 + R_1 sC_1 + R_1 s^2 C_1 C_2 R_2 + 1 + sR_2C_2 + s^2 LC_1 + s^3 LC_1C_2R_2)}$$

Ken Kaiser

$$\therefore V_{C_2}(s) = \frac{NV_s}{s} \cdot \frac{\dfrac{R_2}{1+sR_2C_2}}{\dfrac{R_2}{1+sR_2C_2} + R_1 + \dfrac{1}{sC_1} + sL}$$

$$= \frac{NV_s}{s} \cdot \frac{R_2}{R_2 + R_1(1+sR_2C_2) + \frac{1}{sC_1}(1+sR_2C_2) + sL(1+sR_2C_2)}$$

$$= \frac{NV_sR_2}{sR_2 + sR_1(1+sR_2C_2) + \frac{1}{C_1}(1+sR_2C_2) + s^2L(1+sR_2C_2)}$$

$$= \frac{C_1NV_sR_2}{C_1sR_2 + C_1sR_1(1+sR_2C_2) + 1 + sR_2C_2 + C_1s^2L(1+sR_2C_2)}$$

$$= \frac{C_1NV_sR_2}{sC_1R_2 + sC_1R_1 + s^2C_1C_2R_1R_2 + 1 + sR_2C_2 + s^2C_1L + s^3C_1C_2LR_2}$$

$$= \frac{C_1NV_sR_2}{s^3C_1C_2LR_2 + s^2(C_1C_2R_1R_2 + C_1L) + s(C_1R_2 + C_1R_1 + C_2R_2) + 1}$$

THIS IS THE EQUATION I USED

$$= \frac{\dfrac{NV_s}{C_2L}}{s^3 + s^2\left(\dfrac{C_1C_2R_1R_2 + C_1L}{C_1C_2LR_2}\right) + s\left(\dfrac{C_1R_2 + C_1R_1 + C_2R_2}{C_1C_2LR_2}\right) + \dfrac{1}{C_1C_2LR_2}}$$

=

FROM THE TABLE

$$\left[-\frac{(c-b)e^{-at} + (a-c)e^{-bt} + (b-a)e^{-ct}}{(b-a)(c-b)(a-c)}\right]u(t) \Longleftrightarrow \frac{1}{(s+a)(s+b)(s+c)}$$

[redacted]
Circuit II
Problem 13-22

Solving for s using simple equation

$A \cdot s^3 + B \cdot s^2 + C \cdot s + D = 0$

Go to next page. (long solutions)

$R_1 := 100 \quad C_1 := 10 \cdot 10^{-6}$

$R_2 := 1 \cdot 10^3 \quad C_2 := 1 \cdot 10^{-6}$

$L := 1 \cdot 10^{-6}$

Since Mathcad will not solve the following equation directly,
use the symbolic solutions for s based on the simple equation, and plug in A,B,C,D.

$$s^3 + s^2 \cdot \frac{C_1 \cdot C_2 \cdot R_1 \cdot R_2 + C_1 \cdot L}{C_1 \cdot C_2 \cdot L \cdot R_2} + s \cdot \left(\frac{C_1 \cdot R_2 + C_1 \cdot R_1 + C_2 \cdot R_2}{C_1 \cdot C_2 \cdot L \cdot R_2} \right) + \frac{1}{C_1 \cdot C_2 \cdot L \cdot R_2} = 0$$

$A := 1 \qquad A = 1$

$$B := \frac{C_1 \cdot C_2 \cdot R_1 \cdot R_2 + C_1 \cdot L}{C_1 \cdot C_2 \cdot L \cdot R_2} \qquad B = \frac{C_1 \cdot C_2 \cdot R_1 \cdot R_2 + C_1 \cdot L}{C_1 \cdot C_2 \cdot L \cdot R_2} = 1 \cdot 10^8$$

$$C := \frac{C_1 \cdot R_2 + C_1 \cdot R_1 + C_2 \cdot R_2}{C_1 \cdot C_2 \cdot L \cdot R_2} \qquad C = \frac{C_1 \cdot R_2 + C_1 \cdot R_1 + C_2 \cdot R_2}{C_1 \cdot C_2 \cdot L \cdot R_2} = 1.2 \cdot 10^{12}$$

$$D := \frac{1}{C_1 \cdot C_2 \cdot L \cdot R_2} \qquad D = \frac{1}{C_1 \cdot C_2 \cdot L \cdot R_2} = 1 \cdot 10^{14}$$

Now, plug A,B,C,D into equation and solve for s

$$s^3 + 1 \cdot 10^8 \cdot s^2 + 1.2 \cdot 10^{12} \cdot s + 1 \cdot 10^{14} = 0$$

$$\begin{pmatrix} -83.920216400883 \\ -99987998.569657897723 \\ -11917.510125701393 \end{pmatrix}$$

$a := 83.920216400883$

$b := 99987998.569657897723$

$c := 11917.510125701393$

$$A\cdot s^3 + B\cdot s^2 + C\cdot s + D = 0$$

$$\left(\frac{1}{6}\cdot\frac{C}{A^2}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^3}{A^3} + \frac{1}{18}\cdot\sqrt{4\cdot C^3\cdot \ldots}\right.$$

$$\frac{-1}{2}\cdot\left(\frac{1}{6}\cdot\frac{C}{A^2}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^3}{A^3} + \frac{1}{18}\cdot\sqrt{4\cdot C^3\cdot A - C^2\cdot B^2 - 18\cdot C\cdot B\cdot A\cdot D + 27\cdot D^2\cdot A^2 + 4\cdot D\cdot B^3}\cdot\frac{\sqrt{3}}{A^2}\right)^{\left(\frac{1}{3}\right)} + \frac{1}{2}\cdot\frac{\left(\ldots\right.}{\left(\frac{1}{6}\cdot\frac{C}{A^2}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^3}{A^3} + \frac{1}{18}\cdot\sqrt{4\cdot C^3\cdot A \ldots}\right.}$$

$$\frac{-1}{2}\cdot\left(\frac{1}{6}\cdot\frac{C}{A^2}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^3}{A^3} + \frac{1}{18}\cdot\sqrt{4\cdot C^3\cdot A - C^2\cdot B^2 - 18\cdot C\cdot B\cdot A\cdot D + 27\cdot D^2\cdot A^2 + 4\cdot D\cdot B^3}\cdot\frac{\sqrt{3}}{A^2}\right)^{\left(\frac{1}{3}\right)} + \frac{1}{2}\cdot\frac{\left(\ldots\right.}{\left(\frac{1}{6}\cdot\frac{C}{A^2}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^3}{A^3} + \frac{1}{18}\cdot\sqrt{4\cdot C^3\cdot A \ldots}\right.}$$

$$\ldots A - C^2 \cdot B^2 - 18 \cdot C \cdot B \cdot A \cdot D + 27 \cdot D^2 \cdot A^2 + 4 \cdot D \cdot B^3 \cdot \frac{\sqrt{3}}{A^2}\Bigg)^{\left(\frac{1}{3}\right)} - \frac{\left(\frac{1}{3} \cdot \frac{C}{A} - \frac{1}{9} \cdot \frac{B^2}{A^2}\right)}{\left(\frac{1}{6} \cdot \frac{C}{A^2} \cdot B - \frac{1}{2} \cdot \frac{D}{A} - \frac{1}{27} \cdot \frac{B^3}{A^3} + \frac{1}{18} \cdot \sqrt{4 \cdot C^3 \cdot A - C^2 \cdot B^2 - 18 \cdot C \cdot B \cdot A \cdot D + 27 \cdot D^2 \cdot A^2 + 4 \cdot D \cdot B^3} \cdot \frac{\sqrt{3}}{A^2} \ldots}$$

$$\frac{\ldots \frac{C}{A} - \frac{1}{9} \cdot \frac{B^2}{A^2}\Big)}{\ldots C^2 \cdot B^2 - 18 \cdot C \cdot B \cdot A \cdot D + 27 \cdot D^2 \cdot A^2 + 4 \cdot D \cdot B^3 \cdot \frac{\sqrt{3}}{A^2}\Big)^{\left(\frac{1}{3}\right)}} - \frac{1}{3} \cdot \frac{B}{A} + \frac{1}{2} \cdot i \cdot \sqrt{3} \cdot \left[\left(\frac{1}{6} \cdot \frac{C}{A^2} \cdot B - \frac{1}{2} \cdot \frac{D}{A} - \frac{1}{27} \cdot \frac{B^3}{A^3} + \frac{1}{18} \cdot \sqrt{4 \cdot C^3 \cdot A - C^2 \cdot B^2 - 18 \cdot C \cdot B \cdot A \cdot D + 27 \cdot D^2 \cdot A \ldots}\right.\right.$$

$$\frac{\ldots \frac{C}{A} - \frac{1}{9} \cdot \frac{B^2}{A^2}\Big)}{\ldots C^2 \cdot B^2 - 18 \cdot C \cdot B \cdot A \cdot D + 27 \cdot D^2 \cdot A^2 + 4 \cdot D \cdot B^3 \cdot \frac{\sqrt{3}}{A^2}\Big)^{\left(\frac{1}{3}\right)}} - \frac{1}{3} \cdot \frac{B}{A} - \frac{1}{2} \cdot i \cdot \sqrt{3} \cdot \left[\left(\frac{1}{6} \cdot \frac{C}{A^2} \cdot B - \frac{1}{2} \cdot \frac{D}{A} - \frac{1}{27} \cdot \frac{B^3}{A^3} + \frac{1}{18} \cdot \sqrt{4 \cdot C^3 \cdot A - C^2 \cdot B^2 - 18 \cdot C \cdot B \cdot A \cdot D + 27 \cdot D^2 \cdot A \ldots}\right.\right.$$

$$\cdots^{\left(\frac{1}{3}\right)} - \frac{1}{3}\cdot\frac{B}{A}$$

$$\cdots^{2} + 4\cdot D\cdot B^{3}\cdot\frac{\sqrt{3}}{A^{2}}\Bigg)^{\left(\frac{1}{3}\right)} + \frac{\left(\frac{1}{3}\cdot\frac{C}{A} - \frac{1}{9}\cdot\frac{B^{2}}{A^{2}}\right)}{\left(\frac{1}{6}\cdot\frac{C}{A^{2}}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^{3}}{A^{3}} + \frac{1}{18}\cdot\sqrt{4\cdot C^{3}\cdot A - C^{2}\cdot B^{2} - 18\cdot C\cdot B\cdot A\cdot D + 27\cdot D^{2}\cdot A^{2} + 4\cdot D\cdot B^{3}}\cdot\frac{\sqrt{3}}{A^{2}}\right)^{\left(\frac{1}{3}\right)}}$$

$$\cdots^{2} + 4\cdot D\cdot B^{3}\cdot\frac{\sqrt{3}}{A^{2}}\Bigg)^{\left(\frac{1}{3}\right)} + \frac{\left(\frac{1}{3}\cdot\frac{C}{A} - \frac{1}{9}\cdot\frac{B^{2}}{A^{2}}\right)}{\left(\frac{1}{6}\cdot\frac{C}{A^{2}}\cdot B - \frac{1}{2}\cdot\frac{D}{A} - \frac{1}{27}\cdot\frac{B^{3}}{A^{3}} + \frac{1}{18}\cdot\sqrt{4\cdot C^{3}\cdot A - C^{2}\cdot B^{2} - 18\cdot C\cdot B\cdot A\cdot D + 27\cdot D^{2}\cdot A^{2} + 4\cdot D\cdot B^{3}}\cdot\frac{\sqrt{3}}{A^{2}}\right)^{\left(\frac{1}{3}\right)}}$$

Ken Kaiser

$R_1 := 100 \quad R_2 := 1\cdot10^3 \quad C_1 := 10\cdot10^{-6} \quad C_2 := 1\cdot10^{-6} \; L := 1\cdot10^{-6} \quad NV_s := 30\cdot10^3$

Values for a,b,c

[redacted]
Circuits II
Problem 13-22

$$s^3 + 1\cdot10^8\cdot s^2 + 1.2\cdot10^{12}\cdot s + 1\cdot10^{14} = 0$$

$$\begin{pmatrix} -83.920216400883 \\ -99987998.569657897723 \\ -11917.510125701393 \end{pmatrix}$$

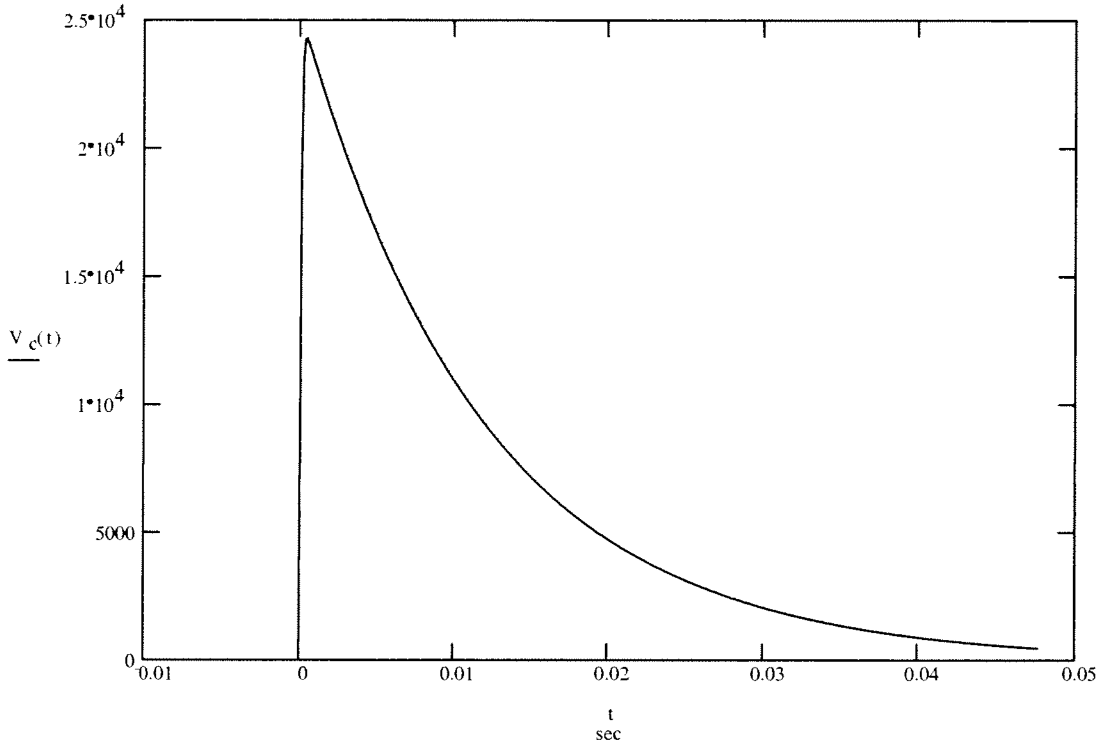
$a := 83.920216400883$ $\qquad \frac{1}{a} = 0.012$

$b := 99987998.569657897723$ $\qquad \frac{1}{b} = 1\cdot10^{-8}$

$c := 11917.510125701393$ $\qquad \frac{1}{c} = 8.391\cdot10^{-5}$

$$t := 0, \frac{1}{100}\cdot\max\left[\left[\begin{bmatrix} \left|\frac{1}{a}\right| \\ \left|\frac{1}{b}\right| \\ \left|\frac{1}{c}\right| \end{bmatrix}\right]\right] .. 4\cdot\max\left[\left[\begin{bmatrix} \left|\frac{1}{a}\right| \\ \left|\frac{1}{b}\right| \\ \left|\frac{1}{c}\right| \end{bmatrix}\right]\right]$$

$$V_c(t) := \frac{-NV_s}{C_2\cdot L}\cdot\left[\frac{(c-b)\cdot e^{-a\cdot t} + (a-c)\cdot e^{-b\cdot t} + (b-a)\cdot e^{-c\cdot t}}{(b-a)\cdot(c-b)\cdot(a-c)}\right]\cdot\Phi(t)$$



2.5•10^4
2•10^4
1.5•10^4
1•10^4
5000
0
V c(t)
-0.01
0
0.01
0.02
0.03
0.04
0.05
t
sec